Zeroing in on Exclusively Exclusive Content*

Christopher Kennedy
University of Chicago

DRAFT: January 11, 2022

Abstract

Bylinina & Nouwen (2018) use the NPI-licensing properties of *zero* to argue that plural count nouns must be analyzed as having domains with the structure of a complete lattice, containing minimal objects whose count is zero. In this paper, I consider a wider array of data which indicates that the traditional analysis of plural count noun domains as join semi-lattices, lacking minimal objects, is correct. It follows from this view that sentences in which *zero* combines with a plural count noun are contradictions, but they come to have contingent truth conditions, I claim, because exhaustification can sometimes return exclusively exclusive content. Building on recent proposals by Bassi, Del Pinal & Sauerland (2021), I argue that the content of exhaustification just is exclusion of alternatives, and whether the prejacent is entailed depends on whether the exclusive proposition is not-at-issue vs. at-issue.

1 Introduction

> *"If the force of the exclusive proposition is to exclude everything other than what is named in or by the subject-term from 'sharing in the predicate,' that is no reason for reading in an implication that something named by the subject-term does 'share in the predicate;' and we certainly cannot exclude from our logic predicables that are not true of anything."* — Geach 1962, p. 208

Bylinina & Nouwen (2018) (BN) discuss two puzzles involving the numeral *zero* and plural count nouns in sentences like (1), one empirical and one theoretical.

(1) Zero signals have been detected by our instruments.

On the empirical side, *zero* is similar to the negative indefinite quantifier *no* in creating downward entailing contexts in both its restriction and its scope:

(2) a. Zero signals have been detected by our instruments. →
Zero (weak) signals have been detected (this week) by our (most sensitive) instruments.

b. No signals have been detected by our instruments. →
No (weak) signals have been detected (this week) by our (most sensitive) instruments.

*Acknowledgements.

But as pointed out by Zeijlstra (2007), *zero* is unlike *no* in failing to license negative polarity items (NPIs):[1]

(3) a. # Zero signals have been detected by any of our our instruments.
    b. No signals have been detected by any of our our instruments.

(4) a. # Zero signals have ever been detected by our instruments.
    b. No signals have ever been detected by any of our our instruments.

(5) a. # Zero signals at all have been detected by our instruments.
    b. No signals at all have been detected by our instruments.

On the theoretical side, *zero* presents a challenge for contemporary views of the semantics of numerals and plurals. The standard view of plurals since at least Link 1983 is that their domains have the structure of a join semi-lattice, containing the set of all objects that can be constructed by the join operation out of the atomic objects in the domain of the corresponding singular noun. Such denotations lack minimal elements, i.e. they bottom out in atoms, and not in an empty object "$\perp$" that is a proper part of all other objects.

As for numerals, a large body of work has converged on the idea that numerals saturate a predicate in the nominal projection whose meaning can be characterized as an extensive measure function over the objects in the domain of a plural noun. In some analyses, this predicate is the denotation of the numeral itself (Landman 2003, 2004; Rothstein 2011; Ionin & Matushansky 2006); in others, it is introduced by some other expression in the nominal projection and takes a numeral as its argument (Cresswell 1976; Krifka 1989; Hackl 2000).

Putting these two ideas together, the semantic content of a sentence like (6a) with arbitrary numeral N is as in (6b), where n is a number, # is the measure function in the nominal projection and x ranges over objects in the domain of the plural noun.

(6) a. N signals have been detected.
    b. $\exists x[\#(x) = n \wedge \textbf{signals}(x) \wedge \textbf{detected}]$

Non-*zero* numerals such as *three* can then be analyzed in one of two ways. (See Buccola & Spector 2016 and Bylinina & Nouwen 2020 for good discussion of the various implementations of these general ideas, and comparisons between approaches.) The first is as expressions that directly fill in the value for n, deriving "lower-bounded" truth conditions like (7b); such meanings can then be enriched either pragmatically or through compositional exhaustification to derive upper bounds and "two-sided" truth conditions.

(7) $\exists x[\#(x) = 3 \wedge \textbf{signals}(x) \wedge \textbf{detected}(x)]$

The second approach treats numerals as generalized quantifiers over numbers (which are a special case of the semantic type of degrees), which take scope and compose with a property of numbers derived by abstracting over the number position, as shown in (8a). This approach allows for the possibility of analyzing numerals as directly introduce upper bounds, as in (8b), which gives back the two-sided truth conditions in (8c) after it composes with its scope.

[1]Contra Chen (2018), who — incorrectly in my view, based on examples like (5a) — claims that *zero* licenses NPIs in its restriction when it composes with a plural count noun. (5a) contrasts crucially with examples involving other kinds of nouns, as we will see in Section 3.

(8) a. $\lambda n \exists x[\#(x) = n \wedge \textbf{signals}(x) \wedge \textbf{detected}(x)]$

b. $[\![three]\!] = \lambda P.max\{n \mid P(n)\} = 3$

c. $max\{n \mid \exists x[\#(x) = n \wedge \textbf{signals}(x) \wedge \textbf{detected}(x)]\} = 3$

The trouble with *zero* is that neither of these approaches seems to work. First, since the domain of a plural noun bottoms out in atoms, there is no object x in the domain of *signals* such that $\#(x) = 0$, which means that (9a) is a contradiction. And second, assuming that *max* is defined only for non-empty sets, (9b) is undefined.

(9) a. $\exists x[\#(x) = 0 \wedge \textbf{signals}(x) \wedge \textbf{detected}]$

b. $max\{n \mid \exists x[\#(x) = n \wedge \textbf{signals}(x) \wedge \textbf{detected}(x)]\} = 0$

This is not to say that it is impossible to define a meaning for *zero*, consistent with the rest of the above assumptions about plurals and numeral semantics, which returns the correct truth-conditions. We could, for example, stipulate that $max(\varnothing) = 0$ (i.e., that the metalanguage *max* isn't really a maximality function). Or even easier, we could give quantificational *zero* the denotation in (10a), which would derive (10b) as the truth conditions for (1) (cf. Chen 2018).

(10) a. $[\![zero]\!] = \lambda P.\forall n > 0 : \neg P(n)$

b. $\forall n > 0 : \neg\exists x[\#(x) = n \wedge \textbf{signals}(x) \wedge \textbf{detected}(x)]$

However, as pointed out by BN, this kind of move — and indeed any analysis that resolves the problem of contradiction/undefinedness by assigning *zero* a quantificational denotation such that its semantic contribution is to exclude propositions involving non-zero values for n — would leave it a complete mystery why *zero* fails to license NPIs. In any such analysis, the scope of *zero*, on any theory of NPI-licensing, should be one in which such expressions flourish. Based on this fact, BN conclude that *zero* cannot have such a meaning.

Instead, they argue, *zero* sentences must be analyzed as in (9a), and they propose to resolve the problem of contradiction by discarding the standard treatment of plural nouns. Instead of analyzing plural noun denotations as semi-lattices, BN propose, they should be analyzed as complete lattices, with a bottom element $\bot$ which is such that $\#(\bot) = 0$. If this is correct, then (9a) is not a contradiction, but a tautology. $\bot$ is a part of any object x in the domain of a plural noun, so (9a) is true not only when no signals have been detected, but also when one signal has been detected, or two, or three, and so on.

The basic semantic content of a *zero* sentence, on this view, is not particularly useful. However, as BN point out, it can be made useful by exhaustification. For present purposes, let us assume with Fox & Spector (2018) that exhaustification involves insertion of an operator with the meaning in (11): $[\![exh]\!](p)$ entails p and excludes all of alternatives to p that p does not entail.

(11) $[\![exh]\!] = \lambda p \lambda w.p(w) \wedge \forall p' \in ALT(p) : p \nvDash p' \rightarrow \neg p'(w)$

Let us further assume, for the sake of simplicity, that the relevant alternatives in sentences involving numerals are all and only the sentences with other numerals.[2] With these assumptions in hand, exhaustification of (9a) delivers the truth conditions in (12).

[2]For the case of *zero*, this assumption will do no harm. It will eventually be important, however, to assess the proposal I will advance in Section 4 in the context of a more sophisticated theory of alternatives.

(12) $\exists x[\#(x) = 0 \wedge \mathbf{signals}(x) \wedge \mathbf{detected}] \wedge$
$\forall n > 0 : \neg\exists x[\#(x) = n \wedge \mathbf{signals}(x) \wedge \mathbf{detected}]$

It should be clear that (12) is both contingent and provides correct truth conditions for (1): the prejacent is a tautology, so it idles, but the exclusive proposition is equivalent to (10b).

And crucially, unlike an analysis in which the truth conditions are derived from a meaning for *zero* like (10a), this analysis also provides a basis for explaining the difference between *zero* and *no* with respect to NPI licensing. Specifically, BN follow Gajewski 2011 in taking the licensing conditions for NPIs to be such that at least the condition in (13-i) must be satisfied, where an environment is non-trivially downward-entailing just in case it is downward-entailing and not also upward-entailing.[3] (Strong NPIs must further satisfy the condition in (13-ii).)

(13) Given a structure $[_\alpha$ *exh* $[_\beta$ ... $[_\gamma$ NPI] ...]]:
(i) the environment γ is non-trivially downward-entailing in β
(ii) the environment γ is non-trivially downward-entailing in α

In the case of *no*, both of these conditions are met: *no* creates a non-trivially downward-entailing context, and exhaustification does not change things. In the case of *zero*, however, things are different. Although condition (13-ii) is met in virtue of exhaustification, condition (13-i) is not: since the prejacent of exhaustification is a tautology, it is both downward *and* upward entailing.

If Bylinina and Nouwen's analysis is correct, *zero* sentences provide a compelling argument for two different, theoretically significant conclusions. First, plurals must be analyzed as having denotations with the structure of a complete lattice, rather than a semi-lattice. And second, *zero* — and by extension, all numerals — must be assigned a semantic analysis that derives lower-bounded truth conditions, since, as we saw above, a quantificational analysis involving upper-bounded truth conditions incorrectly predicts *zero* to license NPIs, regardless of which approach to plurals we adopt.

The purpose of this paper is to argue that although Bylinina and Nouwen's basic idea about why *zero* fails to license NPIs is fundamentally correct, their proposals about plurals and the semantics of *zero* are not. Instead, I will provide empirical evidence which indicates first that plural nouns have denotations with the structure of a join semi-lattice, as is standardly assumed, and second that *zero* should be assigned a quantificational denotation that introduces an upper bound. The evidence comes from two sources, which I discuss in Sections 2 and 3, respectively: the interaction of *zero* with exclusives like *only* (Elliott 2019), and the fact that *zero* actually does license NPIs, but only when it composes with "abstract" nouns which, I will suggest, we have independent reasons to analyze as having domains that include $\perp$. In other words, I will argue that when nouns have the kinds of domains that BN propose for plural count nouns, *zero* is an NPI-licenser. This is exactly what we would expect on an upper-bounding quantificational analysis, as we have seen, but not on a lower-bounded, number-denoting analysis like BN's.

[3] I use Gajewski's licensing conditions in this paper because this is what BN use, but as far as I can tell, other theories could work just as well, such as one based on non-veridicality (Giannakidou 1998) or one based on scope-marking (Barker 2018), provided they include a means of deriving licensing domains and are made sensitive to non-trivial — or what I will refer to below as *contingent* — entailment.

Taken together, these observations will lead to a semantic analysis of *zero* in which sentences like (1) do, in fact, turn out to be contradictions in virtue of their unenriched semantic content. In Section 4, I will resolve this apparent paradox by proposing, in line with the quote from Peter Geach above, that natural language includes a mechanism for mapping sentences to "partial" exhaustifications, consisting exclusively of the exclusive proposition, not the prejacent. Geach himself intended his remarks as a proposal about the semantic contribution of *only*, and a similar analysis of *only* is proposed by McCawley (1981, pp. 226-227). Horn (1996) argues that such an analysis is untenable as an analysis of *only*, but I will argue that it is not untenable — it is in fact necessary — to detach entailment of the prejacent from sentence-level exhaustification, leaving only the exclusive proposition behind, in order to account for the behavior of *zero*. In effect, I will argue that exhaustification must be broken down into two separate semantic components — entailment of the prejacent, and exclusion of alternatives — which sometimes come apart. That these components come apart has recently been claimed by Del Pinal (2021) and Bassi et al. (2021), who argue for an analysis of *exh* in which the exclusive proposition is removed from the at-issue content and treated as a presupposition. In this paper, I will argue that *zero* sentences like (1) provide evidence for another option: treating the exclusive proposition as the exclusive at-issue content, and eliminating entailment of the prejacent entirely. This analysis will support an account of *zero*'s failure to license NPIs (when it does so fail) that has essentially the same structure as the one proposed by Bylinina & Nouwen (2018).

2 *Only zero*

Elliott (2019) observes that modification of *zero* with *only* in sentences like (1) results in unacceptability; the same is true for exclusives like *just* and *solely*:

(14) # Only/just/solely zero signals have been detected by our instruments.

As Elliott points out, this fact is a complete mystery on the BN analysis of *zero* and plurals. On a standard semantics for exclusives, an utterance of *only* p presupposes p and asserts that no stronger alternative to p is true. On BN's analysis, the prejacent in (14) is a tautology, so the presupposition is always satisfied, and the exclusive component is equivalent to exhaustification. (14) should therefore be fine, and in fact equivalent to (1), given that its presuppositional component cannot fail to be satisfied. One could potentially appeal to the fact that *zero* triggers obligatory exhaustification, plus some sort of principle of redundancy avoidance to explain the infelicity of (14). However, given that *only* and *just* can also give rise to a scalar inference, which in this case would serve to emphasize the smallness of the (zero) number of signals detected (see e.g. Beaver & Clark 2008; Coppock & Beaver 2011), this explanation does not seem particularly promising. In contrast, the unacceptability of (14) follows straightforwardly on a standard analysis of plurals and a semantic analysis of *zero* in which the prejacent in (14) is contradictory: the presupposition of the exclusive can never be satisfied.[4]

[4] The unacceptability of (14) also follows, for similar reasons, if *only* does not presuppose the prejacent but rather entails it, a view with a history dating back to the Thirteenth Century, as documented by Horn (1996). It does not follow, however, on an analysis of *only* like the one suggested by Geach (1962) and McCawley (1981), in which *only* neither presupposes nor entails the prejacent.

Elliott also observes that it is not the case that there is a blanket ban on modification of *zero* by *only*, and uses the contrast between (15a) and (15b) to suggest that *only zero* is infelicitous just in the special case that *zero* picks out a scalar endpoint.

(15) a. The water here has only ever risen to zero_F degrees.
b. # The water here has only ever risen by zero_F degrees.

The # in (15b) is Elliott's judgment; my own judgment is that although this example is a bit odd — it is a cheeky, Manner-violating way of saying that the water temperature has stayed the same — it is more acceptable than (14) and other examples in which *only zero* is associated with a plural count noun. And in fact, it is possible to find quite acceptable, naturally occurring examples of *only zero* in a range of different contexts, all of which have in common the fact that *zero* names a scalar endpoint.

The first context is one that is semantically parallel to Elliott's (15b), involving the differential measure phrase in a comparative. Here *zero* names the minimal element of a scale that begins with the degree to which the standard of comparison has the compared property, as shown by the examples in (16):

(16) a. Guess what the difference in labor cost is to install crappy heat cable versus quality heat cable? Zero. It costs zero dollars more to install long-lasting, efficient ice dam heat tape than it does the cheap stuff.
b. [The Southern Border Wall] is now ZERO inches longer than the Border Wall was that existed before Trump's Inauguration.
c. In June 2024, Donald Trump will be 78 years old, or zero years older than Joe Biden is now.

And sure enough, modification by *only* is possible here:

(17) a. Infinitely better functionality and if you wait for a sale only 3/4 the price, or worst case scenario only $0 more.
b. For longer lasting polish, you may upgrade to gel polish for only $0 more.

Only is also acceptable in examples in which *zero* is clearly picking out the endpoint of a scale, as in the following examples involving scales of cost, percentage, chance and probability:

(18) a. Universidad Teologica del Caribe students pay only $0 to live on campus.
b. Most of the scoring for the Golden Eagles comes from their starting lineup, as only zero percent of Oral Roberts' points come from bench players
c. ... due to novel drugs invented to combat leukemic cells and rescue the small patients which 10-20 years ago had only zero chance to survive.
d. The above [model] would be fully ranked for any $K \geqslant 3$, except at a few particular parametric values that could occur with only zero probability, due to the noises in the data model

Taken together, these examples indicate that *only zero* is, in fact, acceptable when it composes with an expression that uses a scale that has an endpoint: zero degree of comparative difference, zero chance/probability/cost/percent, and so forth. This should actually come as no surprise, since such scales easily allow for the possibility of (contingent) satisfaction of both the presupposition and the entailments of the exclusive. But these examples only serve to reinforce the problem that examples like (14) present for BN's analysis: if their proposals about the semantics of plural count nouns and *zero* are correct, there should be no difference in acceptability between (14) and the sentences in (17) and (18).

A final challenge for their analysis comes from the fact that *only zero* and *just zero* are acceptable when they compose with "abstract" mass nouns like *effort, interest* and *talent*, but not when they compose with "concrete" mass nouns like *salt, alcohol* and *oxygen*, as shown by the contrast between the examples in (19) and (20).[5]

(19) a. If the compensation scheme is a fixed salary, the employee puts in only zero effort and the solution is not efficient.

b. I've read the site a lot, but to have just ZERO interest when a woman is basically saying, "Look, I want to hook up. Your place or mine?"

c. Although I am mostly enjoying classics, I honestly have only zero talent in music.

(20) a. # The most important characteristic of Tuscan bread is that it contains only ZERO salt.

b. # It's safest to drive when you have only zero alcohol in your body

c. # When the water reaches full saturation temperature (boiling point), theoretically there is only zero oxygen left in the water.

Abstract and concrete mass nouns differ in several systematic ways, including acceptability with negative quantifiers in Romance (Tovena 2001), interaction with degree modifiers and comparative morphemes in Wolof (Baglini 2015), and acceptability with modifiers that pick out qualitative gradability in noun denotations (Morzycki 2012; Francez & Koontz-Garboden 2017). Several accounts of these differences have been proposed in the literature (including in the works just cited), which agree broadly on the idea that these nouns are associated with scales that rank objects in the extension of the noun according to an intensive measure function, in contrast to the extensive measure associated with concrete mass nouns and plurals. The contrast between the examples in (19) and (20) further indicate that the denotations of abstract mass nouns are such that they include objects which their associated intensive measures relate to a minimum value on the corresponding intensive measure scale.

In other words, abstract mass nouns behave exactly as expected if they have denotations like the ones BN propose for plural count nouns, but concrete mass nouns — and plural count nouns — do not. The conclusion to draw from these observations is that traditional analysis of plurals and concrete mass nouns is correct: they have the structure of a join semi-lattice, and lack minimal elements.

[5] *Zero* is otherwise acceptable with concrete mass nouns: van der Bent (2016) shows that *zero* occurs in corpora with all classes of nouns, and (18a-c) were all constructed by adding *only* to naturally occurring sentences. I have not found naturally occurring examples of *only zero N* for *N* a concrete mass noun.

3 *Zero doubt at all*

The previous section raised a challenge for BN's analysis of plural count nouns; in this section, I will consider data which also calls into question their analysis of *zero* as semantically picking out a lower bound. Recall that the motivation for this analysis was the fact that *zero* contrasts with *no* in failing to license NPIs:

(21) a. # Zero signals at all have ever been detected by any of our our instruments.
b. No signals at all have ever been detected by any of our our instruments.

In BN's analysis, (21a) introduces tautological truth conditions because 1) the lower-bounded semantics for *zero* returns truth conditions which require there to be a plurality of detected signals whose size is at least zero, and 2) since $\perp$ is a part of every object, these truth conditions are satisfied no matter how big the plurality of signals detected actually is. (21a) comes to have contingent (and downward-entailing) truth conditions through exhaustification, but this is not enough to satisfy the condition in (13-i), which applies to all NPIs.

In the previous section, I argued that the acceptability of *only zero* with nouns that introduce scalar concepts like *probability* and *chance*, and with "abstract" mass nouns like *talent, interest* and *effort*, indicates that such nouns have denotations that include objects that are minimal relative to their associated intensive measures. I further concluded that the fact that plural count nouns and concrete mass nouns are not acceptable with *only zero* shows that they do not have denotations with minimal elements, and so BN's proposal about the meanings of such nouns is incorrect. This means that their account of *zero*'s failure to license NPIs, which I just sketched, cannot be correct either.

But the logic of BN's analysis surely *is* correct. If *zero* merely introduces a lower bound in virtue of its semantics, and if the licensing conditions for NPIs require satisfaction of (13-i), then *zero* should give rise to tautological truth conditions anytime it composes with a noun whose denotation contains minimal elements, and so should fail to license NPIs. And so if *zero* introduces a lower bound in virtue of its semantics, it should fail to license NPIs when it composes with nouns like *probability, chance, talent, interest, effort* and so forth, if our conclusions about these nouns in Section 2 are correct. But this is not the case, as shown by the following examples (see also Chen 2018):

(22) *any*
a. I have zero sympathy for any voter of the red wall that voted for this lying charlatan and suffers because of his shite policies.
b. I have zero patience for any person who believes that they are "making a difference" by generalizing and spreading hate about a specific group of people.
c. I had zero familiarity with any characters in the main cast.
d. I have zero respect for any parent fighting against masks in schools.

(23) *ever*
a. The bones, which seem to have zero possibility of ever knowing life again, reveal how Israel feels about their discipline and exile.
b. Many widows and widowers have zero interest in ever engaging in a romantic relationship with another person.

c. I'm thinking of your highness
And crying long upon the loss
I've found
And on the plus and minus
It's a zero chance of ever
Turning this around. (Soundgarden 'Zero Chance', from the *Down on the Upside* album)

(24) *at all*

a. Today I'll show you how to make your Kraft Mac and Cheese Better and DELICIOUS with almost ZERO effort at all!

b. I suppose it can sound a bit shocking to hear someone say with absolute certainty — with zero doubt at all — that there is nothing scary in the world and limitations don't exist.

c. I'm the biggest Rangers fan so trust me when I say they have zero chance at all as long as Quinn is still there coach.

As I said above, the logic of BN's analysis is correct. So if it is the case that the nouns in the examples above include minimal elements in their domains, which the *only zero* facts suggest, then it cannot be the case that *zero* merely introduces a lower bound in virtue of its semantics: if it did, these sentences would be tautologous in virtue of their semantic content, and would fail to license NPIs, for the reasons BN say. Instead, it must have a denotation that imposes an upper bound, and so creates a non-trivially downward-entailing context all on its own in these examples.

Based on observations about similar data, Chen (2018) proposes a denotation for *zero* like the one in (10a), repeated below as (25).

(25) $[\![zero]\!] = \lambda P.\forall n > 0 : \neg P(n)$

But this can't be right. Such a denotation would be perfectly compatible with plural count nouns, even on a standard semantics based on a join semi-lattice, and so would incorrectly predict NPIs to be acceptable across the board.[6]

Likewise, an analysis based on Kennedy's (2015) treatment of non-*zero* numerals in terms of a maximality function is doomed:

(26) $[\![zero]\!] = \lambda P.max\{n \mid P(n)\} = 0$

This kind of analysis faces two problems. If we adopt a standard semantics for plurals, then whenever a *zero* sentence like (1) is intuitively true, the scope of *zero* denotes the empty set: if zero messages were detected, then there is no number n such that there is a plurality of messages of size n that were detected. In that case, however, there is nothing to maximize over, and so composition of *zero* with a plural count noun should be undefined. If we adopt BN's analysis of nouns in terms of complete lattices, we avoid this problem, and in fact derive the right truth conditions, but we also then satisfy the licensing condition in (13-i) and so make the wrong predictions about NPI licensing.

There is, however, a quantificational denotation for *zero* that does the right thing, namely the one in (27), which says that *zero* is true of a property of numbers (degrees)

[6] On the other hand, (25) is a perfectly good denotation for *no*, if Alrenga & Kennedy (2014) are correct that it should be analyzed as a degree quantifier.

P just in case the supremum, or least upper bound, of the set of numbers that satisfy P is 0.

(27) $[\![zero]\!] = \lambda P.sup\{n \mid P(n)\} = 0$

When P picks out a non-empty set of (positive) numbers, (27) is equivalent to (26): $[\![zero]\!](P)$ is true just in case the maximum number that satisfies P is zero, which is just to say that *zero* will (contingently) require that the only number that satisfies its scope is 0. When *zero* composes with a predicate that includes minimal elements in its domain, as in the examples above, this will lead to contingently downward-entailing truth conditions, and NPIs are correctly predicted to be acceptable.

But crucially, unlike (26), (27) is defined when there are no numbers that satisfy P. Any real number is an upper bound of $\varnothing$, and the least upper bound is $-\infty$.[7] But since $-\infty$ is not equal to 0, $[\![zero]\!](P)$ is false whenever there are no numbers that satisfy P. What this means is that, on the analysis I am proposing here, use of *zero* with a plural or concrete mass noun leads to *contradiction*, not to tautology, as in BN's analysis.

For illustration, consider (1), which is assigned the truth conditions in (28).

(28) $sup\{n \mid \exists x[\#(x) = n \wedge \textbf{signals}(x) \wedge \textbf{detected}(x)]\} = 0$

If, say, three signals were detected, then the set of numbers true of the scope of *zero* is $\{1, 2, 3\}$, and (28) is false. The same holds for any other non-zero number of signals detected. If no signals were detected, then — assuming there is no minimal element in the domain of *signals* — the set of numbers true of the scope of *zero* is empty, and (28) is again false, because $sup(\varnothing) = -\infty$ does not equal 0.[8]

This, I claim, is why *zero* fails to license NPIs with plural count nouns and concrete mass nouns. My explanation is essentially the same as the one given by Bylinina & Nouwen (2018), with one modification. On BN's analysis, *zero* is trivially downward entailing over its scope: for scope terms ϕ, ψ involving plural count nouns, *zero* ϕ is

[7] Of course $-\infty$ is not a number, but it is standardly treated as such by mathematicians to describe the algebra on infinites and in the theory of measure. The union of the set of real numbers with the infinity elements $\{-\infty, +\infty\}$ is called the *affinely extended real number system* (see Cantrell, David W. "Affinely Extended Real Numbers." From *MathWorld – A Wolfram Web Resource*, created by Eric W. Weisstein. https://mathworld.wolfram.com/AffinelyExtendedRealNumbers.html). Thanks to Julian Kennedy for helpful discussion.

[8] Whether all numerals should be analyzed in terms of *sup* is a question that I will leave for another time. Bylinina and Nouwen (2018, footnote 14) suggest that this will lead to problems in examples involving modals, but as I noted above, as long as the set picked out by P is finite, *sup* will work just like *max*.

Let me also mention here that using *sup* in the semantics of modified numerals like *fewer than ten* takes away one of the independent arguments that BN bring to bear in favor of putting $\perp$ in the denotation of plural count nouns. (And See Alrenga, Kennedy & Merchant 2012 for independent arguments that the meaning of comparatives should be stated in terms of *sup*.) Citing Buccola & Spector 2016, BN note that a semantics for *fewer than ten* based on maximality, together with a standard semantics for plurals, fails to capture the downward monotonicity of such expressions, since a sentence like *"fewer than ten students passed the test"* fails to come out true (and is in fact undefined, for the reasons noted above) when no students passed. This problem goes away if the denotation of *students* contains $\perp$. But this problem also goes away, on a standard semantics for plurals, if the meaning of *fewer than ten* is defined in terms of *sup*, since $-\infty$ is, in fact, less than 10.

Finally, let me point out that a denotation based on *sup* supports the same type-shifting-based account of the relation between quantificational denotations for numerals and singular term denotations (as numbers) that Kennedy (2015) describes for a maximality-based approach. Application of Partee's (1987) **BE** and **iota** type-shifting operators to the denotation for *zero* in (27) returns the number 0.

guaranteed to entail *zero* ψ, no matter the logical relation between ϕ and ψ, because the propositions denoted by each expression are both guaranteed to be true. On the analysis presented here, *zero* is *vacuously* downward-entailing over its scope: for scope terms ϕ, ψ involving plural count nouns, *zero* ϕ is guaranteed to vacuously entail *zero* ψ, no matter the logical relation between ϕ and ψ, because the propositions denoted by each expression are both guaranteed to be *false*.

Given these observations, I propose to minimally revise the licensing conditions on NPIs as in (29), where an environment is contingently downward-entailing just in case it is both non-trivially downward-entailing and non-vacuously downward-entailing.

(29) Given a structure $[_\alpha$ *exh* $[_\beta$ ... $[_\gamma$ NPI $]$ ... $]]$:
  (i) the environment γ is contingently downward-entailing in β
  (ii) the environment γ is contingently downward-entailing in α

(29) handles all the cases that (13) was designed to handle (involving Strawson-downward-entailment, definite descriptions, and so forth), and also accounts for *zero*'s pattern of NPI licensing, given its denotation in (27). When *zero* composes withan expression whose domain contains minimal elements, such as an abstract mass noun, it creates a contingently downward-entailing context and licenses NPIs. When it composes with an expression whose domain does not contain minimal elements, such as a plural count noun or concrete mass noun, it leads to contradiction and so creates a non-contingently (vacuously) downward-entailing context, and fails to license NPIs.[9]

We are now left with one very large open question: why aren't sentences in which *zero* composes with a plural count noun or concrete mass noun heard as contradictions? The next section provides an answer.[10]

[9] Let me quickly note that this analysis predicts no difference between distributive and collective predicates with respect to NPI licensing, since the semantics for *zero* in (27) will render sentences with the latter just as contradictory as those with the former. Chen (2018) suggests that NPIs are in fact fine in examples involving collective predicates, pointing to (i).

(i) Zero soldiers surrounded any castle.

I, however, do not find (i) particularly acceptable, and I hear a very clear contrast between the *no* and *zero* variants of the following:

(ii) a. Before Jones played Smith in the championship, no/#zero friends had ever met in a final before.
  b. No/#Zero nonbelievers have ever gathered in these hallowed halls.
  c. No/#Zero trees had yet surrounded any of the houses.

[10] I should note that that the argument for a quantificational analysis of *zero* that I presented in this section relies on the assumption that the nominal projections in these examples should be analyzed in the same way that as those involving plurals and concrete mass nouns, i.e. as introducing relations between objects/stuff and degrees, with existential quantification over the former. On this view, an example like (ia) has the truth conditions in (ib) on a non-quantificational analysis of *zero*, and the truth conditions in (ic) on a quantificational analysis, where μ is the intensive ordering associated with the noun *sympathy*; (ib) is a tautology on the assumption that any portion of sympathy with intensive measure n includes a subpart with intensive measure $m \leqslant n$, but (ic) is contingent (cf. Francez & Koontz-Garboden 2017).

(i) a. Kim has zero sympathy.
  b. $\exists x[\mathbf{have}(\mathbf{kim}, x) \wedge \mathbf{sympathy}(x) \wedge \mu(x) = 0]$

4 Exclusively exclusive content

I have argued that the distribution of exclusives with *zero* and the differential acceptability of NPIs with *zero* in examples involving abstract nouns vs. plural count nouns and concrete mass nouns are best explained by adopting a semantic analysis of *zero* as a degree quantifier, and a semantic analysis of plural count nouns (and concrete mass nouns) as having domains with the structure of a join semi-lattice. If this is correct, the logical form of (1) is as in (30a) (ignoring the *by*-phrase for simplicity), which has the truth conditions in (30b).

(30) a. zero [n signals have been detected]

b. $sup\{n \mid \exists x[\#(x) = n \wedge \mathbf{signals}(x) \wedge \mathbf{detected}(x)]\} = 0$

As we saw above, (30b) is a contradiction. It is false when some non-zero number n of signals are detected, because then the set of numbers that satisfies the scope term is the set $\{1, ...n\}$, and the supremum of such a set is not 0. But it is also false when there are no signals detected, because then the set of numbers that satisfy the scope term is empty, and the supremum of the empty set is $-\infty$, which is likewise not equal to 0. I used this fact to explain *zero*'s inability to license NPIs, by adopting the revised licensing conditions in (29) stated in terms of contingent downward-entailingess, but now I need to say why examples like (1), in actual use, have contingent truth conditions: true when no signals are detected, false otherwise.

Recall that BN showed that the tautologous denotations assigned to *zero* sentences on their analysis — which are no more useful than contradictory ones — can be saved by exhaustification: exhaustification of e.g. *"zero signals have been detected"* derives a proposition that is true just in case it is false that n signals have been detected, for all $n > 0$. I will make the same claim here: *zero* sentences come to have contingent truth conditions, of exactly this sort, through exhaustification. But in order to derive this result, I need to make two adjustments to the theory of exhaustification, one rather minor, the other less so.

The minor adjustment concerns the characterization of the exclusive proposition. Intuitively, we want exhaustification of a *zero* sentence to exclude all alternatives involving other numerals, just as in BN's analysis. The version of *exh* I adopted earlier in (11) follows Fox & Spector (2018) in excluding all propositions that are not entailed by the prejacent. But if *zero* sentences are contradictions, as I have claimed, then there are no alternatives that they do not *vacuously* entail, and so exhaustification won't exclude anything.

There is a (relatively) simple solution to this problem, however, which is to suppose that the entailment relation that exhaustification is sensitive to — just like the one needed

c. $sup\{n \mid \exists x[\mathbf{have}(\mathbf{kim}, x) \wedge \mathbf{sympathy}(x) \wedge \mu(x) = n]\} = 0$

Another possibility, however, is that the difference between abstract nouns like *chance* and *sympathy* on the one hand, and concrete nouns like *signals* and *salt* on the other, is that the former are just predicates of degrees and impose upper bounds in virtue of their semantics, so that e.g. *zero sympathy* picks out only degrees of sympathy that are equal to 0. If this is the case, even a non-quantificational, number/degree-denoting semantics for *zero* would create a non-trivially downward-entailing context when it composes with these nouns. This would eliminate my argument for the quantificational analysis of *zero* in (27), but it would leave my account of *zero*'s failure to license NPIs with plural count nouns untouched, since such sentences will be assigned contradictory truth conditions — and so will be vacuously downward-entailing — regardless of whether *zero* denotes the quantifier in (27) or the number 0.

for NPI-licensing, or the one that is relevant for considerations of Scope Economy, according to Fleisher (2013) — is a *contingent* one, as defined in (31) (Smiley 1959, p. 240).

(31) p contingently entails q, $p \vDash q$, iff $p \rightarrow q$ is a tautology, and either:
  (i) neither $\neg p$ nor q are tautologies, or
  (ii) $p \rightarrow q$ is a substitution instance of some tautology $p' \rightarrow q'$, where neither $\neg p'$ nor q' are tautologies.

(31i) is the basic condition which ensures that, in general, entailment fails when either the premises are a contradiction or the conclusion is a tautology; (31ii) allows for the special case that a conclusion or tautology still counts as entailing itself.[11] Redefining entailment in this way makes intuitive sence since exhaustification is ultimately about excluding informationally stronger alternatives, and contradictions (and tautologies) are informationally useless on their own. It will, moreover, have no effect on standard examples of exhaustification, in which the prejacent is contingent. But when applied to a *zero* sentence like (30a) as in (32a), the result will be exclusion of all non-*zero* alternatives, as in (32b).

(32) a. *exh* [zero [n signals have been detected]]
  b. $sup\{n \mid \exists x[\#(x) = n \wedge \mathbf{signals}(x) \wedge \mathbf{detected}(x)]\} = 0 \wedge$
  $\forall m > 0 : \neg sup\{n \mid \exists x[\#(x) = n \wedge \mathbf{signals}(x) \wedge \mathbf{detected}(x)]\} = m$

This brings us to the second, more substantive adjustment to the theory of exhaustification. The standard view, exemplified by (32b), is that exhaustification returns the conjunction of the prejacent and the exclusive proposition. But in a *zero* sentence, the prejacent is a contradiction, and the conjunction of a contradiction with another proposition is still a contradiction. What we need to do is eliminate the content of the prejacent entirely, leaving behind only the content of the exclusive proposition:

(33) $\forall m > 0 : \neg sup\{n \mid \exists x[\#(x) = n \wedge \mathbf{signals}(x) \wedge \mathbf{detected}(x)]\} = m$

(33) is false whenever some positive number n of signals have been detected, because in such a case the supremum is n itself, and thus it is not the case that there is no $m > 0$ such that $n = m$. But if no signals are detected, the supremum is $-\infty$, and since it is is the case that there is no $m > 0$ such that $-\infty = m$, (33) comes out as true.

To explain how a sentence with the semantic content in (30b) can come to have the meaning in (33), I propose that natural language includes a mechanism for mapping sentences to their corresponding exclusive content, exclusively, as argued (incorrectly, according to Horn (1996)) by Geach (1962) and McCawley (1981) for *only*. The intuition is that natural languages include sentences that are useless in virtue of their basic semantic

[11] The reasons for adopting this kind of definition are nicely stated by Clark (1967) (who further elaborates Smiley's definition to account for other cases), who observes that "...if A entails B, then A & –B inconsistent. But the converse does not hold: (A & -A) & –B is inconsistent, but we do not want to allow that (i) A & –A entails B. For we see that the inconsistency belongs merely to A & –A; A & -A is not inconsistent *with* –B in the same way as A is inconsistent *with* –A. The extreme solution, which disallows entailments with non-contingent antecedents or consequents, would certainly debar (i), but it would also debar (ii) A & –A entails A & –A, because of the intrinsic inconsistency of the antecedent and of the consequent and in spite of the additional inconsistency *between* A & –A and –(A & –A)."

content, but are useful in virtue of their exclusionary content, relative to alternatives, and that the grammar provides the means for deriving such content. This idea is, in fact, precisely the same as the one that BN rely on to explain why *zero* sentences are not heard as tautologies, though for them it is enough to rely on the standard theory of exhaustification for this purpose. My claim is that *zero* sentences reveal something more interesting: that language includes a mechanism for making use of "predicables that are not true of anything," to use Geach's phrasing, by mapping them to exclusively exclusive content.

My implementation of this proposal builds on recent work by Bassi et al. (2021), who argue for a reanalysis of *exh*, which they dub "*pex*," in which the exclusive component is not part of the operator's at-issue content, but is rather a presupposition. This analysis resolves a number of empirical challenges for the standard analysis of exhaustification, such as the disappearance of exclusive inferences in downward-entailing contexts and some complex patterns of projection, and they explain the fact that exclusive inferences often "feel" like at-issue content through appeal to systematic global accommodation.[12]

Here I would like to suggest a variation on the Bassi et al. analysis: *the substantive content of the exhaustification operator just is the exclusive proposition.* But this content can be either presuppositional (or otherwise not-at-issue), as in (34a), or at-issue, as in (34b).

(34) $[\![exh]\!] =$

a. $\lambda p \lambda w : \forall p' \in ALT(p) : p \not\models p' \rightarrow \neg p'(w).p(w)$ DEFAULT

b. $\lambda p \lambda w . \forall p' \in ALT(p) : p \not\models p' \rightarrow \neg p'(w)$ SPECIAL CASE

(34a) is the default, and is equivalent to Bassi et al.'s *pex*; here entailment of the prejacent just reflects the absence of any contribution to, or modification of, the at-issue content of the target of exhaustification. (34b) is the special case, and returns exclusively exclusive at-issue content. This option is equivalent to the Geach/McCawley *only*, and is exactly what we need to account for the truth conditions and NPI-licensing properties of *zero* sentences.

The final piece of the puzzle is an explanation of why (34a) is the default interpretation of *exh*, and why (34b) is available only in special cases. For example, $exh(p \vee q)$ would be compatible with both p and q being false if *exh* could be interpreted as in (34b), which is obviously not an option we want to allow for sentences involving disjunction.[13] And Chierchia's (2013) account of the unacceptability of NPIs in upward-entailing contexts would be unavailable if (34b) were freely available. Briefly, this account says that an NPI like *any* is an existential quantifier with a maximally wide domain, and is obligatorily

[12]In fact, Bassi et al. (2021) allow for the possibility that the exclusive proposition is not a standard presupposition, but rather some other kind of not-at-issue content. For the purposes of this paper, this distinction does not matter, nor does the difference between the rather simplistic characterization of exclusive content that I have been using here, and the more fine-grained one used in that paper, which also introduces relevance-based context sensitivity.

[13]But see Bar-Lev and Fox (2020, footnote 46), for examples in which free choice inferences — which in their approach come from the exclusive proposition — can be true, while the both conjuncts of the disjunction from which they are derived can be false. (Thanks to Danny Fox for bringing this example to my attention.) Such cases may provide additional evidence for the possibility of interpreting exhaustification as in (34b), but showing that this is the case will require taking into account a sophisticated algorithm for calculating alternatives than the one I have considered here.

associated with exhaustification over domain alternatives. (35a) is fine because the exclusive proposition — that there is no alternative domain such that there aren't signals in that domain that Kim detected — is entailed by the prejacent.

(35) a. *exh* [Kim didn't detect any signals]
b. # *exh* [Kim detected any signals]

In contrast, (35b) is bad in Chierchia's analysis because the exclusive proposition — that there is no alternative domain such that there are signals in that domain that Kim detected — contradicts the prejacent. But this is the case only if *exh* is interpreted as in (34a); if it were interpreted as in (34b), (35b) would be contingent, and would in fact mean that Kim detected no signals, which would clearly be the wrong result.

What is different from both of these examples and the case of *zero* sentences is that in the former, the constituent with which *exh* composes is contingent, while in the latter, it is a contradiction. Given that the interpretation of *exh* in (34a) returns the content of its complement as its exclusive at-issue contribution, it is natural to make the additional assumption that such a use not only presupposes the exclusive proposition, but also the contingency of its complement. At the same time, it equally natural to assume that the use in (34b) cannot possibly come with such a contingency presupposition. If this is correct, then the principle of Maximize Presupposition (Heim 1991; Sauerland 2008) dictates that exhaustification must be understood as in (34a) whenever the prejacent is contingent. We then derive the desired result that (34b) may be used only in the special case of "predicables that are not true of anything."[14]

5 Conclusion

I have argued that the variable NPI licensing properties of *zero*, and the variable acceptability of exclusive modification of *zero* with different classes of nouns, are best explained by a semantic analysis of *zero* as an upper-bounding degree quantifier and a traditional semantic analysis of plural count nouns (and concrete mass nouns) in which their denotations have the structure of join semi-lattices, which lack minimal elements. If these claims are correct, then simple sentences like *"zero signals were detected"* are contradictions in virtue of their basic semantic content, but they come to have contingent truth conditions, I claim, because the grammar of exhaustification allows for the possibility of returning exclusively exclusive at-issue content as a special case.

It must be admitted that this is a rather big conclusion to draw from the behavior of a rather small word, and so it is appropriate to ask: is this conclusion justified? Morzycki (2017) suggests that *zero* is a "semantic virus" — a kind of add-on to the basic vocabulary and meanings of English, with unusual properties that sets it outside the core vocabulary and meanings of numerals, plurals, measure terms and so forth. But as we have learned all too well during the first part of the third decade of the Twenty-First Century, viruses require a certain kind of compatibility with their hosts, and are able to thrive by taking advantage of existing structures and operations. If my interpretation of the facts involving *zero* are correct, then we have reason to think that Geach and McCawley were on the

[14] Or, of course, predicables that are true of everything, as BN originally proposed for *zero*. But sinice such cases are compatible with entailment of the prejacent, they provide less clear evidence for exclusively exclusive content.

right track in proposing that natural language includes mechanisms that allow for the expression of exclusively exclusive content.

References

Alrenga, Peter & Christopher Kennedy. 2014. *No More* shall we part: Quantifiers in English comparatives. *Natural Language Semantics* 22. 1–53. http://dx.doi.org/10.1007/s11050-013-9099-4.

Alrenga, Peter, Christopher Kennedy & Jason Merchant. 2012. A new standard of comparison. In Nathan Arnett & Ryan Bennett (eds.), *Proceedings of 30th west coast conference on linguistics*, 32–42. Somerville, MA: Cascadilla Press.

Baglini, Rebekah. 2015. *Stative predication and semantic ontology: A cross-linguistic study*: University of Chicago dissertation.

Bar-Lev, Moshe & Danny Fox. 2020. Free choice, simplification, and innocent inclusion. *Natural Language Semantics* 28. 175–223.

Barker, Chris. 2018. Negative polarity as scope marking. *Linguistics and Philosophy* 41. 483–510.

Bassi, Itai, Guillermo Del Pinal & Uli Sauerland. 2021. Presuppositional exhaustification. *Semantics and Pragmatics* 14(11). http://dx.doi.org/https://doi.org/10.3765/sp.14.11.

Beaver, David & Brady Clark. 2008. *Sense and sensitivity: How focus determines meaning*. Chichester: Wiley Blackwell.

van der Bent, Renate. 2016. The meaning and use of zero: An exploration of the semantic interpretation of the number word *zero*. BA thesis.

Buccola, Brian & Benjamin Spector. 2016. Modified numerals and maximality. *Linguistics and Philosophy* 39(3). 151–199.

Bylinina, Lisa & Rick Nouwen. 2018. On "zero" and semantic plurality. *Glossa* 3(1). 1–23.

Bylinina, Lisa & Rick Nouwen. 2020. Numeral semantics. *Language and Linguistics Compass* http://dx.doi.org/https://doi.org/10.1111/lnc3.12390.

Chen, Sherry Yong. 2018. *Zero* degrees: numerosity, intensification, and negative polarity. In *Proceedings of CLS 54*, 53–67. Chicago, IL: Chicago Linguistics Society.

Chierchia, Gennaro. 2013. *Logic in grammar*. Oxford, UK: Oxford University Press.

Clark, Michael. 1967. The general notion of entailment. *The Philosophical Quarterly* 17(68). 231–245.

Coppock, Elizabeth & David Beaver. 2011. Sole sisters. In Neil Ashton, Anca Chereches & David Lutz (eds.), *Proceedings of Semantics and Linguistic Theory 21*, 197–211.

Cresswell, M. J. 1976. The semantics of degree. In Barbara Partee (ed.), *Montague grammar*, 261–292. New York: Academic Press.

Del Pinal, Guillermo. 2021. Oddness, modularity and exhaustification. *Natural Language Semantics* 29. 115–158.

Elliott, Patrick. 2019. #only zero. *Snippets* 35. 1–2.

Fleisher, Nicholas. 2013. Comparative quantifiers and negation: Implications for scope economy. *Journal of Semantics* 32. 139–171.

Fox, Danny & Benjamin Spector. 2018. Economy and embedded exhaustification. *Natural Language Semantics* 26. 1–50.

Francez, Itamar & Andrew Koontz-Garboden. 2017. *Semantics and morphosyntactic variation: Qualities and the grammar of property concepts*. Oxford, UK: Oxford University Press.

Gajewski, Jon. 2011. Licensing strong NPIs. *Natural Language Semantics* 19(2). 109–148.

Geach, Peter T. 1962. *Reference and generality*. Ithaca, NY: Cornell University Press.

Giannakidou, Anastasia. 1998. *Polarity sensitivity as (non)veridical dependency*. Amsterdam and Philadelphia: John Benjamins.

Hackl, Martin. 2000. *Comparative quantifiers*: Massachusetts Institute of Technology dissertation.

Heim, Irene. 1991. Articles and definiteness. In Armin von Stechow & Dieter Wunderlich (eds.), *Semantics. an international handbook of contemporary research*, 487–535. Berlin: Walter de Gruyter & Co.

Horn, Laurence. 1996. Exclusive company: *Only* and the dynamics of vertical inference. *Journal of Semantics* 13. 1–40.

Ionin, Tania & Ora Matushansky. 2006. The composition of complex cardinals. *Journal of Semantics* 23. 315–360.

Kennedy, Christopher. 2015. A "de-Fregean" semantics (and neo-Gricean pragmatics) for modified and unmodified numerals. *Semantics and Pragmatics* 8(10). 1–44.

Krifka, Manfred. 1989. Nominal reference, temporal constitution and quantification in event semantics. In Renate Bartsch, Johann van Benthem & Peter van Emde Boas (eds.), *Semantics and contextual expression*, 75–115. Stanford, CA: CSLI Publications.

Landman, Fred. 2003. Predicate argument mismatches and the adjectival theory of indefinites. In Martine Coene & Yves d'Hulst (eds.), *From NP to DP*, vol. 1, 211–238. John Benjamins. http://dx.doi.org/10.1075/la.55.10lan.

Landman, Fred. 2004. *Indefinites and the type of sets*. Oxford University Press. http://dx.doi.org/10.1002/9780470759318.

Link, Godehard. 1983. The logical analysis of plurals and mass terms: A lattice theoretical approach. In R. Bäuerle, C. Schwarze & A. von Stechow (eds.), *Meaning, use, and the interpretation of language*, 302–323. Berlin: de Gruyter.

McCawley, James D. 1981. *Everything that linguists have always wanted to know about logic* *but were ashamed to ask*. Chicago: University of Chicago Press.

Morzycki, Marcin. 2012. The several faces of adnominal degree modification. In Jaehoon Choi, E. Alan Hogue, Jeffrey Punske, Deniz Tat, Jessamyn Schertz & Alex Trueman (eds.), *Proceedings of WCCFL 29*, 187–195. Somerville, MA: Cascadilla Press.

Morzycki, Marcin. 2017. Some viruses in the semantics. In *A schrift to fest Kyle Johnson*, 281–291. Linguistics Open Access Publications.

Partee, Barbara H. 1987. Noun phrase interpretation and type-shifting principles. In Jeroen Groenendijk et al. (eds.), *Studies in discourse representation theory and the theory of generalized quantifiers*, 115–143. Dordrecht: Foris. http://dx.doi.org/10.1002/9780470758335.ch15.

Rothstein, Susan. 2011. Counting, measuring and the semantics of classifiers. *The Baltic International Yearbook of Cognition, Logic and Communication* 6(0).

Sauerland, Uli. 2008. Implicated presuppositions. In Anita Steube (ed.), *The discourse potential of underspecified structures*, 581–600. De Gruyter. http://dx.doi.org/doi:10.1515/9783110209303.4.581. https://doi.org/10.1515/9783110209303.4.581.

Smiley, T. J. 1959. Entailment and deducibility. *Proceedings of the Aristotelian Society* 233–254.

Tovena, Lucia. 2001. Between mass and count. In K. Megerdoomian & L.A. Bar-el (eds.), *Proceedings of WCCFL 20*, 565–578. Somerville, MA: Cascadilla Press.

Zeijlstra, Hedde. 2007. Zero licensers. *Snippets* 16. 21–22.

ck@uchicago.edu

Department of Linguistics
University of Chicago
1115 E. 58th Street
Chicago, IL 60637