Split Scope, Summative Existentials, and the Semantics of “Bare Quantifiers”

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Introduction: Two semantic analysis of “bare quantifiers”
Numerals (and their friends) as “bare quantifiers”

\[ Q_{at,t}(S_{at}) \]  

(cf. Szabó, 2011)
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(1) How many cars did Kim decide to buy?
   a. What is the number of cars such that Kim decided to buy them?
   b. What is the number such that Kim decided to buy that many cars?
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(2) a. Kim has more cars than she has ___ children.
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   b. # American families generally have 2.3 children.
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(3) a. American families on average have 2.3 children.
   b. # American families generally have 2.3 children.

(4) They sought no friends amongst the neighbors, despising them all.
   a. \textit{It is not the case that they tried to find friends amongst the neighbors.}
   b. * \textit{There are no friends amongst the neighbors such that they tried to find them.}
Two bare quantifier semantics for numerals

A “Fregean” Analysis: second-order properties of *individuals*.

(5) \[ \text{three} = \lambda P_{\langle e, t \rangle} \cdot \#\{x \mid P(x)\} = 3 \]

(6) a. Kim read three books
    b. \( \lambda x.\text{read}(x)(k) \land \text{books}(x) \)
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A “De-Fregean” analysis: second-order properties of *degrees*.

(7) \[
\text{three} = \lambda P_{\langle d,t \rangle} \cdot \max\{n \mid P(n)\} = 3
\]

(8)  
   a. Kim read three books
   b. \( \lambda n. \exists x[\text{read}(x)(k) \land \text{books}(x) \land \#(x) = n] \)
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Two bare quantifier semantics for numerals

It’s not clear how to implement the Fregean analysis compositionally. The obvious option is not kosher:

(9) 

\[
\text{three}_{et, t} \rightarrow \lambda x. \text{Kim}_e \rightarrow \text{read}_{e, et} \rightarrow t! \rightarrow x_e \rightarrow \text{books}_{e, t}
\]
It's not clear how to implement the Fregean analysis compositionally. The obvious option is not kosher, or at least not compositional:

\[(9)\]

\[
\begin{aligned}
&\text{three}_{\langle et, t \rangle} \\
&\text{Kim}_{\langle e, t \rangle} \\
&\text{read}_{\langle e, et \rangle} & \text{books}_{\langle e, t \rangle}
\end{aligned}
\]
And type-shifting doesn’t work:

(10)  a. \([\text{no}] = \lambda P.\{x \mid P(x)\} = \emptyset\)

b. \([\text{no}^*] = \lambda P \lambda Q. [\text{no}](\lambda z. P(z) \land Q(z))\)

c. 

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Implementing the de-Fregean semantics

On the other hand, it is clear how to implement the de-Fregean analysis:

1. Add degrees into the model theory, impose the right kind of mereology (total ordering)
2. Sort the domain of degrees into various subdomains (natural numbers/cardinalities, weights, temperatures, ...)
3. Introduce appropriate distinctions between degree-denoting terms and individual-denoting terms in the syntax
Implementing the de-Fregean semantics

(11)  a. Kim is exactly two meters tall.
    b. * Kim is exactly Lee tall.
    c. Kim is exactly as tall as Lee.
(11) a. Kim is exactly two meters tall.
b. * Kim is exactly Lee tall.
c. Kim is exactly as tall as Lee.

(12) a. Kim weighs 40 kilos.
b. ≠ Kim weighs Lee.
c. Kim weighs as much as Lee.
Implementing the de-Fregean semantics

(11) a. Kim is exactly two meters tall.
b. * Kim is exactly Lee tall.
c. Kim is exactly as tall as Lee.

(12) a. Kim weighs 40 kilos.
b. $\neq$ Kim weighs Lee.
c. Kim weighs as much as Lee.

(13) a. Kim read two more books than Lee.
b. * Kim read *Vagueness in Context* and *Thinking about Mathematics* more books than Lee.
c. Kim read as many more books than Lee as Stewart has written since 2000.
In examples in which a numeral looks like a determiner, the degree position comes from an implicit relation between (plural) individuals and degrees: $\text{[MANY]} = \lambda n \lambda x. \#(x) = n$

(14)
The plan for today is to show:

1. That there are good reasons to like the de-Fregrean analysis.

2. That despite those reasons, there is evidence that we need a Fregean analysis *in addition*, and there may even be a grammatical explanation for why it shows up when it does.

3. That it may be that the two meanings are actually variants of a more basic “proto-Fregean” one.
Motivating the de-Fregean semantics
Two-sided meanings

In the de-Fregean analysis, “two-sided” meanings are truth-conditional.

(15)  a. three_{dt,t} [Kim read \( n \) MANY books]
      b. \( \max\{ n \mid \exists x [\text{read}(x)(k) \land \text{books}(x) \land \#(x) = n]\} = 3 \)

In e.g. an alternative in which numerals denote numbers, the semantics derives lower-bounded truth conditions, and two-sided interpretations are pragmatic.

(16)  a. [Kim read three_{d} MANY books]
      b. \( \exists x [\text{read}(x)(k) \land \text{books}(x) \land \#(x) = 3]\)
Two-sided meanings

But there is by now a huge amount of “armchair” and experimental evidence showing that two-sided readings of numerals are preserved when implicatures of other scalar terms disappear.

▶ Interactions with negation
(König, 1991; Horn, 1992, ...)

▶ Interactions with modals
(Geurts, 2006; Breheny, 2008, ...)

▶ Acquisition studies
(Papafragou and Musolino, 2003; Musolino, 2004, ...)

▶ Adult behavioral studies
(Huang et al., 2013; Marty et al., 2013, ...)

▶ ...
(17) a. ?? Neither of us started the book. She was too busy to read it, and I finished it.
b. ?? Neither of us tried to climb the mountain. She had a broken leg, and I easily made it to the summit.
c. ?? Neither of us used to smoke. She never smoked, and I still smoke.
Negation

(17) a. ?? Neither of us started the book. She was too busy to read it, and I finished it.
b. ?? Neither of us tried to climb the mountain. She had a broken leg, and I easily made it to the summit.
c. ?? Neither of us used to smoke. She never smoked, and I still smoke.
d. Neither of us have three kids. She has two and I have four.
In some examples, modals appear to require lower-bounded content; in others, they require two-sided content:

(18)  
   a. Dustin has to get three hits on the last day of the season in order to win the batting title.
   b. Dustin has to get three hits on the last day of the season in order to finish with a batting average of .345.

Adding in the implicature doesn’t get us what we want here:

(19)  
   a. $\otimes \Box_{Baseball}(\geq 3) \land \neg \Box_{Baseball}(> 3)$
   b. $\sqrt{\Box_{Baseball}(\geq 3) \land \Box_{Baseball}(\neg(> 3))}$
In the analysis below, our dependent measure is the proportion of 'No' responses to the puppet's statements, i.e. the subjects' tendency to judge these statements as 'bad' descriptions of the stories they witnessed. The proportions of 'No' responses were entered into an analysis of variance (ANOVA) with two factors: age (5-year-olds vs. adults) and scale type (all, some, three, two, finish, start). The analysis revealed a significant main effect of age ($F(1; 54) = 135.34, P < 0.0001$), a significant main effect of scale type ($F(2; 54) = 13.03, P < 0.0001$) and a reliable interaction between age and scale type ($F(2; 54) = 7.43, P = 0.001$) (see Fig. 3).

In test trials, we found that adult subjects overwhelmingly rejected the puppet's statements in each of the three conditions, i.e. 92.5% of the time in the all, some condition, 100% of the time in the three, two condition and 92.5% of the time in the finish, start condition. Statistical analysis revealed no reliable difference between these rejection rates ($F(2; 27) = 1.92, P = 0.16$). By contrast, we found that while 5-year-olds rejected the puppet's statements in the case of three, two 65% of the time, they almost never did so in the case of all, some and finish, start (12.5% and 10% of the time, respectively). This difference was confirmed statistically ($F(2; 27) = 11.17, P < 0.001$). Pairwise comparisons (Tukey–Kramer) further revealed a reliable difference between three, two–all, some and three, two–finish, start ($P = 0.002$ and $P = 0.001$, respectively) but no reliable difference between all, some and finish, start ($P = 0.77$).

On the control items, adults gave correct responses 100% of the time in the all, some condition, 80% of the time in the three, two condition and 95% of the time in the finish, start condition. No reliable difference was found between these means ($F(2; 27) = 2.43, P = 0.1$). On the same items, children gave correct responses 90% of the time in the all, some condition, 95% of the time in the three, two condition and 85% of the time in the finish, start condition. Specifically, five children rejected the puppet's statements on all four of the test trials, one child on three of the test trials, one child on two of the test trials, one child on one of the test trials and two children rejected none of the test trials. In sum, six of the ten children almost always rejected the puppet's statements (i.e. on three or four of the test trials), three children almost never rejected the puppet's statements (i.e. on either none or one of the test trials) and one child rejected half of the test trials and accepted the other half.
### Upper bounding inferences under memory load

Marty et al. (2013)

<table>
<thead>
<tr>
<th>Picture</th>
<th>Null</th>
<th>Partial</th>
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<tbody>
<tr>
<td>Some</td>
<td>False (4)</td>
<td>True (4)</td>
<td>Target (8)</td>
</tr>
<tr>
<td>Some dots are red.</td>
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<tr>
<td>All</td>
<td>—</td>
<td>False (4)</td>
<td>True (4)</td>
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<tr>
<td>All dots are red.</td>
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<tr>
<th>Picture</th>
<th>Inferior</th>
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<th>Superior</th>
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<tr>
<td>Some</td>
<td>False (8)</td>
<td>True (8)</td>
<td>Target (8)</td>
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<tr>
<td>Bare Numeral 4 dots are red.</td>
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<tr>
<td>Between</td>
<td>False (8)</td>
<td>True (8)</td>
<td>Target (8)</td>
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<tr>
<td>Between 3 and 5 dots are red.</td>
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<tr>
<td>More than</td>
<td>False (4)</td>
<td>—</td>
<td>True (8)</td>
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<tr>
<td>More than 2 dots are red.</td>
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<tr>
<td>Fewer than</td>
<td>True (4)</td>
<td>—</td>
<td>False (8)</td>
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<tr>
<td>Fewer than 6 dots are red.</td>
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Upper bounding inferences under memory load
But we do sometimes get lower bounded readings, especially with universal root modals (Scharten, 1997):

(20)  

a. In Britain, you have to be 17 to drive a motorbike and 18 to drive a car.

b. Mary needs three As to get into Oxford.

c. Goofy said that the Troll needs to put two hoops on the pole in order to win the coin.

d. You must provide three letters of recommendation.

e. You are required to take three classes per quarter.
And conversely, we get upper bounded readings with existential modals:

(21)  a. She can have 2000 calories a day without putting on weight.
    b. You may have half the cake.
    c. Pink panther said the horse could knock down two obstacles and still win the blue ribbon.
    d. You are permitted to take three cards.
    e. You are allowed to enroll in three classes per quarter.
Scope ambiguities with universal modals

Two (nominal *de dicto*) readings for examples with universal (root) modals, a two-sided one and a lower-bounded one:

(22) Dustin has to get three hits.
    a. *has to* > *three*; 2-sided
        □[max{n | ∃x[♯(x) = n ∧ hits(x) ∧ get(x)(d)]} = 3]
    b. *three* > *has to*; lower-bounded
        max{n | □∃x[♯(x) = n ∧ hits(x) ∧ get(x)(d)]} = 3

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<th>c₁</th>
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The analysis also derives two readings for examples with existential (root) modals, a two-sided one and an upper bounded one:

(23) Kim is allowed to enroll in three classes.
    a. \textit{allowed} > \textit{three}; 2-sided (but weak)
        \[\Diamond [\max \{ n \mid \exists x[\#(x) = n \wedge \text{cls}(x) \wedge \text{enrl}(x)(k)] \}] = 3\]
    b. \textit{three} > \textit{allowed}; upper-bounded
        \[\max \{ n \mid \Diamond [\exists x[\#(x) = n \wedge \text{cls}(x) \wedge \text{enrl}(x)(k)]] \} = 3\]
What about Horn’s (1972) original argument for one-sided truth conditions?

(24) Kim took four classes offered by Prof. Jones...
   a. ... if not five.
   b. # ... if not three.

(25) Kim took most classes offered by Prof. Jones...
   a. ... if not all.
   b. # ... if not few.

The de-Fregean semantics should entail an upper bound in (24a).
Partee (1987) defines a set of general type-shifting principles that regulate the distribution and interpretation of quantificational, predicative and referential expressions. Here are two of them:

\[
\begin{align*}
\text{a. } BE &= \lambda Q_{\langle \alpha t, t \rangle} \lambda x_{\alpha} \cdot Q(\lambda y_{\alpha}. y = x) \\
\text{b. } \iota_\alpha &= \lambda P_{\langle \alpha, t \rangle}. \iota x_{\alpha}[P(x)]
\end{align*}
\]

\(BE\) maps e.g. quantificational indefinites or lifted proper names to properties; \(\iota_\alpha\) maps properties that hold of unique objects to singular terms.
Numerals and type-shifting principles

(27) \[ \llbracket \text{four} \rrbracket = \lambda P. \operatorname{max}\{n \mid P(n)\} = 4 \]

(28) \[ BE(\llbracket \text{four} \rrbracket) \]
\[ = [\lambda Q \lambda m. Q(\lambda p.p = m)](\lambda P. \operatorname{max}\{n \mid P(n)\} = 4) \]
\[ = [\lambda m.[\lambda P. \operatorname{max}\{n \mid P(n)\} = 4](\lambda p.p = m)] \quad (\lambda\text{-conversion}) \]
\[ = \lambda m. \operatorname{max}\{n \mid [\lambda p.p = m](n)\} = 4 \quad (\lambda\text{-conversion}) \]
\[ = \lambda m. \operatorname{max}\{n \mid n = m\} = 4 \quad (\lambda\text{-conversion}) \]
\[ = \lambda m. m = 4 \quad \text{(substitution of equivalents)} \]

(29) \[ \iota BE(\llbracket \text{four} \rrbracket) = 4 \]
Two routes to one-sided truth conditions

Lowering *four* all the way to a singular term and saturating the degree argument introduced by MANY:

\[(30)\quad a.\quad \langle e, t \rangle \]

\[
\begin{array}{c}
\langle e, t \rangle \\
\downarrow \\
\text{iota}(BE(\text{four}))_d \\
\downarrow \\
\text{MANY}_{d,et}
\end{array}
\]

\[
\begin{array}{c}
\langle e, t \rangle \\
\downarrow \\
\text{classes}_{et}
\end{array}
\]

b. \[\exists x [\text{took}(x)(k) \land \text{classes}(x) \land \#(x) = 4] \]
Lowering *four* to a degree property, *restricting* the degree argument of *MANY*, and existentially closing it:

\[(31)\]

a. \[\langle d, et \rangle \]

\[\langle d, et \rangle \]

\[\langle d, et \rangle \]

\[\text{classes}_{et} \]

\[\text{BE}(\text{four})_{d,t} \]

\[\text{MANY}_{d,et} \]

b. \[\exists x, n[\text{took}(x)(k) \land \text{classes}(x) \land \#(x) = n \land n = 4] \]
Hofweber (2005):

(32)  a. Sam ate four eggs.
     b. The number of eggs Sam ate is four.
     c. * John ate the number of eggs Sam ate apples.
Maybe the singular term variant of the numeral isn’t used in “ordinary” language?

(34)  
   a. Sam ate four eggs.  \langle dt, t \rangle, \langle d, t \rangle, (d) 
   b. The number of eggs Sam ate is four.  \langle d, t \rangle, (d)
Maybe the singular term variant of the numeral isn’t used in “ordinary” language?

\[(34)\]

a. Sam ate four eggs. \(\langle dt, t\rangle, \langle d, t\rangle, \langle d, t\rangle, (d)\)
b. The number of eggs Sam ate is four. \(\langle d, t\rangle, (d)\)
c. The number of eggs ... is fewer than four. \(\langle d, t\rangle, *d\)
d. The number of eggs ... is at most four. \(\langle d, t\rangle, *d\)
e. The number of eggs ... is between four and six. \(\langle d, t\rangle, *d\)

\[(35)\]

a. \(BE(\llbracket \text{fewer than four} \rrbracket) = \lambda n. n < 4\)
b. \(BE(\llbracket \text{at most four} \rrbracket) = \lambda n. n \leq 4\)
c. \(BE(\llbracket \text{between four and six} \rrbracket) = \lambda n. 4 \leq n \leq 6\)

\(iota\) is undefined for these properties!
“Split scope”

(36) “I served with Jack Kennedy. I knew Jack Kennedy. Jack Kennedy was a friend of mine. Senator, you’re no Jack Kennedy.” — Lloyd Bentsen to Dan Quayle, October 5, 1988

(37)

a. * You are a non-Jack Kennedy.
b. * There is not a Jack Kennedy such that you are him.
c. It is not the case that you are a Jack Kennedy.
(38) In my junior year of college, I took a J-term class that taught me basic PHP/MySQL, tools used to serve more dynamic web content. I became no expert, but this introduction was enough that I could make a rudimentary PHP web app when I started working at Minnehaha full time after graduation.

(39) a. *I became a non-expert.

b. *There is not an expert such that I became him.

c. I didn’t become an expert.
“Split scope”

We need no new taxes.

a. *It must be the case that it is false that there are new taxes.*

b. *It is false that it must be the case that there are new taxes.*
“Split scope”

(41)  a. In order to overtake Canada and get into the top six they will need three wins while Canada has to win no games in the next round.

b. *It has to be the case that there are no games that Canada wins.*

(42)  a. Now that the U.S. has two losses, Canada has to win no games in order to advance to the next round.

b. *It is false that it has to be the case that there are some games that Canada wins.*
“Split scope”

(43)  
   a. You may drink no alcohol, take thoughtful notes, and live vicariously through the rest of the drinking crowd.  
   b. It is allowed for you to drink no alcohol.

(44)  
   a. The restriction of zero tolerance when driving (ZTD) imposed on drivers in Switzerland means that they may drink no alcohol at all before driving and basically consume only moderate amounts of alcohol at any other time.  
   b. It is false that it is allowed for driver to drink alcohol.
NPI *need*

(45)  a. There need be no split scope.
     b. * There need be split scope.

(46)  a. *It is false that it must be the case that there is split scope.*
     b. *It must be the case that it is false that there is split scope.*
(47b) is the meaning of (47a); scoping the entire nominal content above the modal doesn’t really make sense.

(47)   a.  There need be no split scope.
       b.  ¬□∃x[split-scope(x)]

The question is how to get derive this kind of meaning. The standard semantics for *no* is not helpful:

(48)  \[ \{\text{no}\} = \lambda P \lambda Q. \neg \exists x[P(x) \land Q(x)] \]

Existing analyses either literally “split” negation into its component parts (decomposition into ¬ and ∃), or introduce special kinds of quantification.
“What I would like to suggest instead is that scope-splitting (at least sometimes) is DegP-movement ... I question that there is a good analysis of scope-splitting that is a genuine alternative to DegP movement.” (Heim, 2001, pp. 225-226)
Heim doesn’t give an implementation of this idea, but here is one, which is a variant of the bare quantifier, de-Fegrean-style semantics I gave for numerals:

\[(49) \quad \llbracket \text{no}_{\text{Deg}} \rrbracket = \lambda Q \langle d, t \rangle. \{ n \mid Q(n) \} = \emptyset \]

\[(50) \quad \text{There are no cats in this house.} \]
\[(51) \]
a. \text{no}_n \quad \text{[there are } t_n \text{ MANY cats in this house]}
b. \quad \{ n \mid \exists x [\text{cats}(x) \land \#(x) = n \land \text{in}(x)(h)] \} = \emptyset 

\[(52) \]
a. \quad \text{[there are no cats in this house]}
b. \quad \neg \exists x [\text{cats}(x) \land \text{in}(x)(h)] \]
Scope of no: Universal modals

(53) Canada has to win no games.

(54) a. have to [no, [Canada win \( t_n \) games]]
    b. \( \Box[\{n \mid \exists x[\#(x) = n \land \text{games}(x) \land \text{win}(x)(c)]\}] = \emptyset \)

(55) a. no, [have to [Canada win \( t_n \) games]]
    b. \( \{n \mid \Box\exists x[\#(x) = n \land \text{games}(x) \land \text{win}(x)(c)]\} = \emptyset \)
Scope of no: Existential modals

(56) You may drink no alcohol.

(57) a. may [no\textsubscript{n} [you drink t\textsubscript{n} alcohol]]
   b. \(\Diamond\{n \mid \exists x[\#(x) = n \land \text{alcohol}(x) \land \text{drink}(x)(u)]\} = \emptyset\)

(58) a. no\textsubscript{n} [may [you drink t\textsubscript{n} alcohol]]
   b. \(\{n \mid \Diamond\exists x[\#(x) = n \land \text{alcohol}(x) \land \text{drink}(x)(u)]\} = \emptyset\)
“...scope splitting arising with fewer/less comparatives should be considered distinct from scope splitting with NIs. [...] NIs ... do not exhibit morphology related to degree semantics. It does thus not seem justified to extend the analysis of scope splitting based on DegP movement to NIs (although it would be technically feasible).”

(Penka, 2011, p. 162)

It’s true that no is does not properly include degree morphology. But that’s because it can itself be a degree morpheme!
no can saturate degree positions in the syntax:

(59)  
   a. Tsunamis are often no taller than wind waves.  
   b. The tsunami was much/10 meters/a lot taller than the wind wave.

(60)  
   a. Very tall they were, and the Lady no less tall than the Lord;  
   and they were grave and beautiful. (JRRT; *Fellowship*)  
   b. The Lady was somewhat/2cm/slightly less tall than the Lord.

(61)  
   a. In this respect the Royal Academy is no different from any  
   other major museum.  
   b. The Royal Academy is a bit/a lot/quite different from other  
   museums.
And here too we see “split” readings:

(62) a. Your keyword density has to be no higher than a certain percentage. Search engines can tell if a keyword is appearing too many times....

b. Honestly, my attendance has to be no higher than 15-20%. I don’t go because I don’t think lecture is the way I learn best....

(63) a. Shoes with wheels are strictly prohibited. Shoes must have closed toes and heels. Heels or soles may be no higher than 2 inches....

b. With wholesale pricing, you can pay no more than $6.00 [per] bottle.
“Split-scope” phenomena generally show that expressions like numerals and *no* denote “bare” quantifiers (Szabó, 2011).

The patterns of interpretation we’ve seen here and others (e.g. semantics and pragmatics of modified numerals; Kennedy (2015)) show that the right semantics for these expressions is the de-Fregean one.
Summative existentials: Evidence for a (Proto-)Fregean semantics of bare quantifiers
Summative existentials

Szabó (2011); Francez (2015):

(64) At this point in the race, there could be three winners:
   a. Kim, Pat or Lee.
   b. Kim, Pat and Lee.

(65) a. There are three \( x \) such that \( x \) could be the winner.
    b. It could be that there are three \( x \) such that \( x \) is a winner.
There could be three decisions from this meeting: (i) Case Closed (ii) A further review in 6 weeks’ time. (iii) Legal intervention.

Behan said there might be three outcomes of the charette regarding what to do with the Fox site:
1. Do nothing (not acceptable)
2. Hold on to the site for future opportunities
3. Market the site for development

If they are thrown into the air, there can be three results: Head-Head; Head-Tail; Tail-Tail.
(69) Because the claims against the defendants were not tried in 16 separate cases, there should just be one judgment, Weaver, of Armstrong Teasdale, argued. He also said there could be three judgments in the case, with the bond for each capped at $50 million. The plaintiffs’ lawyers filed the claims in three separate lawsuits and originally asked Schaumann to enter three judgments, Weaver said.
Deriving the summative reading

The de-Fregean denotation can’t derive the summative reading, no matter where the numeral takes scope, or whether we lower it to a singular term:

(70) There could be three winners (in this race).
   a. \( \max\{n \mid \Box \exists x[\text{winners}(x)(r) \land \#(x) = n]\} = 3 \)
   b. \( \Box[\max\{n \mid \exists x[\text{winners}(x)(r) \land \#(x) = n]\} = 3] \)
   c. \( \Box[\exists x[\text{winners}(x)(r) \land \#(x) = 3] \)
However, a Fregean bare quantifier semantics can derive the summative reading. It is the result of letting the numeral take scope over the modal:

(71) There could be three winners (in this race).
   a. $\#\{x \mid \Diamond [\text{winners}(x)(r)]\} = 3$
   b. $\Diamond [\#\{x \mid \text{winners}(x)(r)\} = 3]$

It seems that we need a Fregean bare quantifier semantics after all!
But we already saw that the Fregean semantics is compositionally problematic, because the numeral saturates the argument introduced by the noun.

(72)

So why is it (evidently) ok in summative existentials?
As Francez (2015) shows, the pivot must be a relational noun like *winner*, *result*, *outcome*, etc.

(73) Kim is a spy, who is carrying a single object in her bag that is a disguised camera. We’re not sure which object it is, but we know of the objects in the bag that there could be three cameras.
As Francez (2015) shows, the pivot must be a relational noun like *winner, result, outcome*, etc.

(73) Kim is a spy, who is carrying a single object in her bag that is a disguised camera. We’re not sure which object it is, but we know of the objects in the bag that there could be three cameras.

(74a-b) can be synonymous; (75a-b) cannot:

(74)  
   a. Three could be winners.  
   b. There could be three winners.

(75)  
   a. Three could be cameras.  
   b. There could be three cameras.
Non-relational nouns work as expected if we suppose that *there* has to (somehow) associate with an argument slot in the pivot:

(76)  

a.  * there ... [three$_{et,t}$ cameras$_{e,t}$]$_{t}$  
b.  there ... [[three$_{dt,t}$ MANY$_{d,et}$] cameras$_{e,t}$]$_{e,t}$
Non-relational nouns work as expected if we suppose that \textit{there} has to (somehow) associate with an argument slot in the pivot:

\begin{align*}
(76) & \quad \text{a.} \quad * \text{there ... [three}_{\langle et,t \rangle} \text{ cameras}_{\langle e,t \rangle}]_{t} \\
& \quad \text{b.} \quad \text{there ... } [[\text{three}_{\langle dt,t \rangle} \text{ MANY}_{\langle d,et \rangle}] \text{ cameras}_{\langle e,t \rangle}]_{\langle e,t \rangle}
\end{align*}

\textbf{Hypothesis:} in summative existentials, \textit{there} (somehow) gets to associate with the second argument of the relation:

\begin{align*}
(77) & \quad \text{a.} \quad \text{there ... [three}_{\langle et,t \rangle} \text{ winners}_{\langle e,et \rangle}]_{\langle e,t \rangle} \\
& \quad \text{b.} \quad \text{there ... } [[\text{three}_{\langle dt,t \rangle} \text{ MANY}_{\langle d,et \rangle}] \text{ cameras}_{\langle e,et \rangle}]_{\langle e,et \rangle}
\end{align*}

I think this is consistent with Francez' semantics for conditionals, if the relevant “association” principle is syntactic.
If this s on the right track, then need both the Fregean and de-Fregean bare quantifier denotations for numerals:

(78)  

\[ \text{a. } \llbracket \text{three} \rrbracket = \lambda_{(e,t)} \cdot \# \{ x \mid P(x) \} = 3 \]
\[ \text{b. } \llbracket \text{three} \rrbracket = \lambda_{(d,t)} \cdot \max \{ n \mid P(n) \} = 3 \]

The Fregean meaning is going to be ruled out in most cases on compositional grounds, but becomes available in special cases, where saturating the individual argument of the noun doesn’t cause a problem for the rest of the composition.
Numeral pluralism revisited

If this is on the right track, then need both the Fregean and de-Fregean bare quantifier denotations for numerals:

(78)  
  a. $\llbracket \text{three} \rrbracket = \lambda P_{<e,t>} \cdot \# \{x \mid P(x)\} = 3$
  
  b. $\llbracket \text{three} \rrbracket = \lambda P_{<d,t>} \cdot \max \{n \mid P(n)\} = 3$

The Fregean meaning is going to be ruled out in most cases on compositional grounds, but becomes available in special cases, where saturating the individual argument of the noun doesn’t cause a problem for the rest of the composition.

A new question: Do we really need both denotations, or is there a way to unify them? They are awfully similar to each other.
A “proto-Fregean” semantics?

The following denotation, adapted from The Foundations of Arithmetic, seems to get us what we want for both the individual and degree domains, given their respective mereologies:

\[
\begin{align*}
\text{(79)} & \quad \text{a. } \llbracket \text{one} \rrbracket = \lambda P. \exists v [P(v) \land \neg \exists v' [P(v') \land v' > v]] \\
& \quad \text{b. } \llbracket \text{succ(num)} \rrbracket = \lambda P. \exists v [P(v) \land \llbracket \text{num} \rrbracket (\lambda v'. P(v') \land v' \neq v)]
\end{align*}
\]

It’s not plausible to think that the denotation of every natural lg numeral is defined in terms of the denotation of its successor.

But maybe we can see this as providing a conceptual basis for collapsing the Fregean and de-Fregean denotations into a single, type-neutral one?
The End


Kennedy, Christopher. 2015. A “de-Fregean” semantics (and neo-Gricean pragmatics) for modified and unmodified numerals. *Semantics and Pragmatics* 8:1–44.


