The Number of Meanings of English Number Words

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The Issue

Sentences with number words appear to give rise to three different kinds of scalar inferences in different contexts of use:

(1) a. Kim has three children. *two-sided*

b. Kim needs to get three As. *lower bounded*

c. Kim may enroll in three courses. *upper bounded*

Most researchers have treated scalar variability as a contextual phenomenon, involving either implicature or enrichment.

Today I want to consider the possibility that these facts are are best explained by a fully semantic, scope-based analysis.

The Plan

1. The "classic" neo-Gricean account of number word meaning, and challenges to it from:
   - Semantic/pragmatic data
   - Experimental data
2. Alternatives to the Classic Analysis and their drawbacks
3. The Scopal Analaysis
   - Number words as scope-taking degree quantifiers
   - Accounting for the observed patterns of data
   - Interactions with numeral modifiers
4. Discussion

The Classic Analysis

"Numbers, then, or rather sentences containing them, assert lower-boundedness — at least n — *and given tokens of utterances containing cardinal numbers may, depending on the context, implicate upper-boundedness* — at most n — *so that the number may be interpreted as denoting an exact quantity."* (Horn, 1972, p. 33)

(2)
a. John read three of the articles, if not more/#fewer.
b. John read many of the articles, if not most/#few of them.
c. John read most of the articles, if not all/#many of them.

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Analytical options: *Five olives fell*

(3) *Quantificational determiner*

a. $[\![\text{five}]\!] = \lambda P \lambda Q.|P \cap Q| \geq / = 5$

b. $|olives' \cap fell'| \geq / = 5$ 1/2-sided

(4) *Cardinality predicate*

a. $[\![\text{five}]\!] = \lambda x.\#(x) = / \geq 5$

b. $\exists x[\#(x) = / \geq 5 \wedge olives'(x) \wedge fell'(x)]$ 1-sided

Analytical options: *Five olives fell*

(5) *Singular term*
$[\![\text{five}]\!] = 5$

(6) a. *Parameterized quantificational determiner*

i. $[\![\text{many}]\!] = \lambda n \lambda P \lambda Q.|P \cap Q| \geq / {=} n$

ii. $[\![\text{five } \cancel{\text{many}}]\!] = \lambda P \lambda Q.|P \cap Q| \geq / {=} 5$ 1/2-sided

b. *Parameterized cardinality predicate*

i. $[\![\text{many}]\!] = \lambda n \lambda x.\#(x) = / \geq n$

ii. $[\![\text{five } \cancel{\text{many}}]\!] = \lambda x.\#(x) = / \geq 5$ 1-sided

c. *Nominal measure function*

i. $[\![\text{olives}]\!] = \lambda n \lambda x.\#(x) = / \geq n \wedge olives'(x)$

ii. $[\![\text{five olives}]\!] = \lambda x.\#(x) = / \geq 5 \wedge olives'(x)$ 1-sided

Problems for the Classic Analysis

- Semantic and pragmatic evidence for two-sided content
- Typology of numeral modifiers
- Experimental evidence against a Quantity-based account of two-sided inferences
- Experimental data bearing on the status of bounding inferences

Affirmation and denial

According to Horn (1972), the response in (9b) is metalinguistic:

(7) Do you have three children?

(8) a. No, I have two.
b. No, I have four.
c. Yes, in fact I have four.

Likewise, (10a-b) have a different status: the former is a real negation/denial, and the latter is metalinguistic.

(9) How many pupils are there in your class?
a. 31. No wait, 33.
b. 31. No wait, 29.

This doesn't obviously accord with intuition.

Affirmation and denial

And similar examples show clear asymmetries between numerals and other scalar terms:

(10) Neither of us have three kids: she has two and I have four.

(11)
a. ?? Neither of us started the book: she was too busy to read it, and I finished it.
b. ?? Neither of us tried to climb the mountain: she had a broken leg, and I reached the summit.
c. ?? Neither of us used to smoke: she never started, and I still do.

Modals

In some examples, modals interact with lower-bounded content:

(12) a. In Britain, you have to be 18 to drive a car.
b. Mary needs to receive 3 As on her final grade report in order to get into Oxford.

But in others, they appear to require two-sided content:

(13) a. In "Go Fish", each player must start with seven cards.
b. Abstracts are required to be two pages long.

Modals

In still others, we seem to have upper-bounded content:

(14) a. She can have 2000 calories without putting on weight.
b. The council houses are big enough for families with three kids.
c. You may attend six courses.

However, these cases are not problematic for the Classic Analysis, and indeed do not distinguish it from recently proposed alternatives.

Collective vs. distributive predicates

Koenig (1991) observes that 1-sided readings are possible only with distributive predicates; not with collective ones:

(15)	a. Three men carried umbrellas up the stairs.	$\models$
	b. Two men carried umbrellas up the stairs.	
(16)	a. Three men carried a grand piano up the stairs.	$\not\models$
	b. Two men carried a grand piano up the stairs.	
(17)	a. Four cards of the same suit didn't fall on the table.	$\models$
	b. Five cards of the same suit didn't fall on the table.	
(18)	a. Four cards of the same suit don't make a flush.	$\not\models$
	b. Five cards of the same suit don't make a flush.	

Numeral modifiers

Koenig (1991) also points out that the Classic Analysis gives rise to a somewhat odd semantic classification of numeral modifers:

(19) ⟦three⟧: $0 - - -_{1} - - -_{2} - - \bullet_{3} \longrightarrow \infty$

- *at most*, *exactly*, and comparatives modify the content of the numeral
- *at least*, on the other hand, is an "implicature suspender"
- *approximately* is a "slack regulator" (cf. Lasersohn, 1999)

Numeral modifiers

In fact, the classification of modifiers depends a lot on our initial assumptions about number word meaning, so this may not be a fair criticism of the Classic Analysis. That said, recent work has indicated that the following classification is the one that our semantic/pragmatic theory should derive (Musolino, 2004; Geurts and Nouwen, 2007; Geurts et al., 2009; Nouwen, 2010):

- *more than, less/fewer than*
- *at least, at most*
- *exactly, approximately*

Experimental evidence for two-sided content

Over the past decade, a large set of experimental evidence based on different methodologies and studies of both child and adult behavior has emerged which indicates that number words give rise to two-sided interpretations in contexts in which quantity implicatures for other scalar terms are reduced or disappear (Noveck, 2001; Papafragou and Musolino, 2003; Musolino, 2004; Huang and Snedeker, 2009; Geurts et al., 2009).

- Truth Value Judgment Tasks
- "Act-out" Tasks
- Eye-tracking studies

The Covered Box Task

Huang et al. (2009): Since implicatures are not part of truth conditional content, they should be canceled (or not calculated) in contexts in which their addition would lead to incompatibility with the compositionally determined inferences of an utterance.

- Three boxes, two with visible contents, one covered.
- Inference that a unique box contains a quantity of something:

  (20) The box that contains *Q* NPs

- Do subjects select the "mystery box" when neither of the others satisfies a 2-sided interpretation of *Q*?

The Covered Box Task: Scalar Determiners

"Give me the box where Cookie Monster has some of the cookies."

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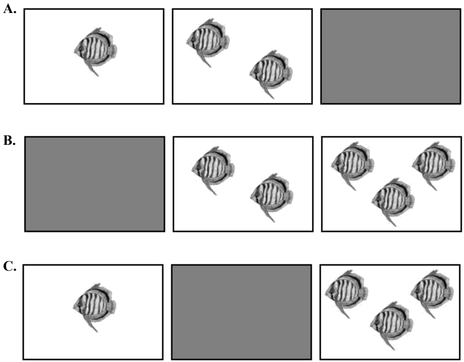
Trial	Children	Adults
A.	some	some
B.	some≈all	some
C.	all	all

The Covered Box Task: Number Words

"Give the the box with two fish."



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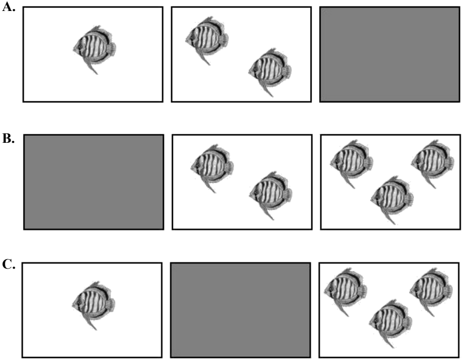

Trial	Children	Adults
A.	two	two
B.	two	two
C.	covered	covered

Implicit vs. explicit bounding

Musolino (2004): Can children correctly assign one-sided interpretations to sentences with number words in contexts that promote such readings in adult language?

Implicit vs. explicit bounding

(21) *At most*
Goofy said the Troll could miss two hoops and still win the coin. Does the Troll win the coin?

(22) *At least*
Goofy said that the Troll had to put two hoops on the pole in order to win the coin. Does the Troll win the coin?

Implicit vs. explicit bounding

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Goofy said that the Troll had to put two hoops on the pole in order to win the coin. Does the Troll win the coin?

	Children	Adults
at least	35%	95%
at most	82.5%	97%
control	98%	100%

Implicit vs. explicit bounding

(21) *At most*
Goofy said the Troll could miss two hoops and still win the coin. Does the Troll win the coin?

(22) *At least*
Let's see if Goofy can help the Troll. The Troll needs two cookies. Does Goofy have two cookies?

	Children	Adults
at least	80%	95%
at most	82.5%	97%
control	98%	100%

Implicit vs. explicit bounding

At the same time, children have a difficult time with sentences in which an upper/bound is explicitly provided by *at most/at least*:

(23) Cookie monster only likes to keep cards with exactly/at least/at most/more than two stars on them. Would he like to keep this card?

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	Children	Adults
exactly	100%	100%
at least	50%	100%
at most	54.1%	95.5%
more than	88%	

Summary

- Semantic evidence for two-sided content.
- Experimental evidence that two-sided readings are not derived via implicature, and that one-sided readings are not accessed.
- Experimental evidence that children can correctly map sentences onto one-sided interpretations in contexts in which adults do so.
- Experimental evidence suggesting that whatever is responsible for one-sided readings does not involve content equivalent to *at least/at most*.

The current market

Three types of alternatives to the Classic Analysis:

- 2-sided content (Breheny, 2008; Koenig, 1991): scalar readings involve particularized CIs.
- 1/2-sided polysemy via type-shifting (Geurts, 2006; Nouwen, 2010): upper-bounded readings involve particularized CIs.
- Contextual enrichment of underspecified representations (Carston, 1998): all readings part of propositional content.

Each has problems with the data we've observed so far.

Assessment

	Q/Neg	Modals	Col/Dist	Box	LB/UB	ClM/ExM
1S-cont	-	-	-/+	-	+/$\sim$	-/
2S-cont	+	-	+/?	+	-/$\sim$	+/
1/2-poly	+	+	?/+	?	+/$\sim$	+/
enrich	+	+	?/+	?	+/$\sim$	+/-

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1/2-poly	+	+	?/+	?	+/$\sim$	+/
enrich	+	+	?/+	?	+/$\sim$	+/-

NB: All analyses derive UB readings with weak modals via scalar implicature; this reading is never a matter of semantic content:

(25) Troll can miss three shots (and still win).

a. $\exists w \in Acc_{w0}[\#(misses)(w) = / \geq 3]$

b. $\forall p \in \{\exists w \in Acc_{w0}[\#(misses)(w) = / \geq n] \mid n > 3\} : \neg p$

This is a potentially serious problem, given the experimental results showing that children resist calculating scalar implicatures.

“Range of permission” sentences

Examples like the following are extremely problematic for an analysis that posits only 2-sided content, since it predicts contradictory truth conditions.

(26) Students are required to enroll in two classes and allowed to enroll in four.

a. $\forall w \in Acc_{w0}[\#(cl)(w) = 2] \wedge \exists w \in Acc_{w0}[\#(cl)(w) = 4]$

b. $\forall w \in Acc_{w0}[\#(cl)(w) \geq 2] \wedge \exists w \in Acc_{w0}[\#(cl)(w) \geq 4]$

Modals and numerals: Another look

(27) $\forall$ *modals*

a. In Britain, you have to be 17 to drive a motorbike.

b. Mary needs three As to get into Oxford.

c. Goofy said that the Troll needs to put two hoops on the pole in order to win the coin.

d. You must take three cards.

e. You are required to enroll in two classes per quarter.

(28) $\exists$ *modals*

a. She can have 2000 calories a day without putting on weight.

b. You may have half the cake.

c. Pink panther said the horse could knock down two obstacles and still win the blue ribbon.

d. You are permitted to take three cards.

e. You are allowed to enroll in four classes per quarter.

The same patterns of interpretation are also observed in sentences containing modals and comparatives, and have been traditionally analyzed in terms of the relative scope of the comparative with respect to the modal (Heim, 2000).

(29) Students are required to enroll in fewer than three classes.

a. $\forall w \in Acc[max\{n|\textit{students enroll in n classes in } w\} < 3]$

b. $max\{n|\forall w \in Acc[\textit{students enroll in n classes in } w]\} < 3$

(30) Students are permitted to enroll in fewer than three classes.

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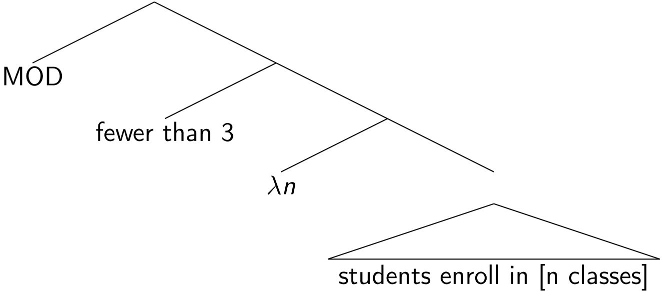
Modals and comparatives

$[\![\text{fewer than 3}]\!] = \lambda D_{\langle d,t \rangle}.max\{n|D(n)\} < 3$

(31) a.



b. $\forall/\exists w \in Acc[max\{n|students\ enroll\ in\ n\ classes\ in\ w\} < 3]$

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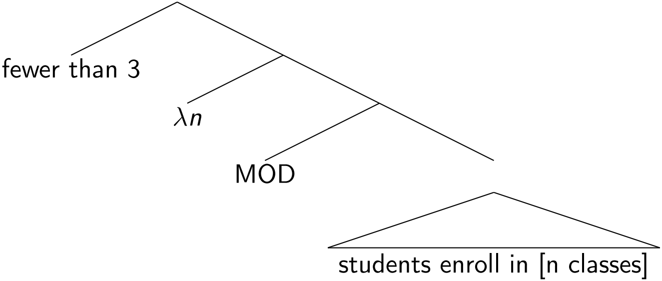


b. $max\{n|\forall/\exists w \in Acc[\textit{students enroll in n classes in w}]\} < 3$

Average sentences

Kennedy and Stanley (2009): In order to provide a compositional analysis of *average* sentences, unmodified number words must be able to scope out of the NPs in which they appear.

(33) a. The average American family has 2.3 children.

b. #The typical American family has 2.3 children.

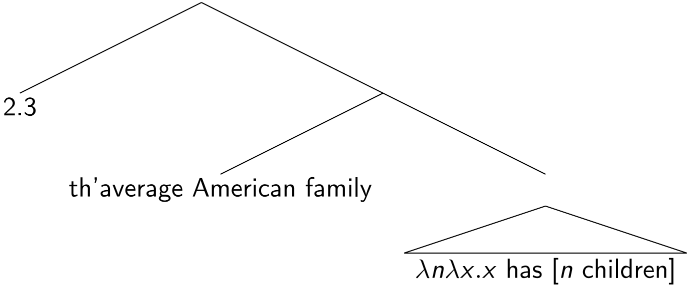
(34) a. On average, American families have 2.3 children.

b. #Usually, American families have 2.3 children.

Average sentences

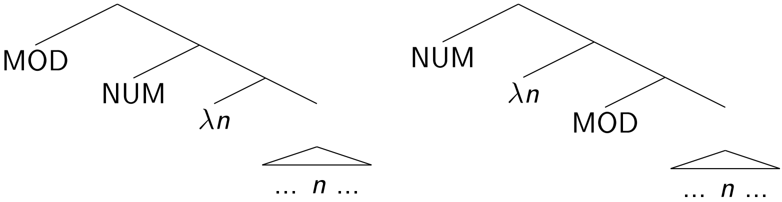
(35) $[\![\text{th'average NP}]\!] = \lambda f_{\langle d,et\rangle} \lambda n. \dfrac{\sum_{x \in NP'} max(f)(x)}{|NP'|} = n$

(36)



The number of meanings of English number words...

...is one, it's 2-sided, and 1-sided *sentence* readings arise through scopal interactions with modals (and presumably other Ops):



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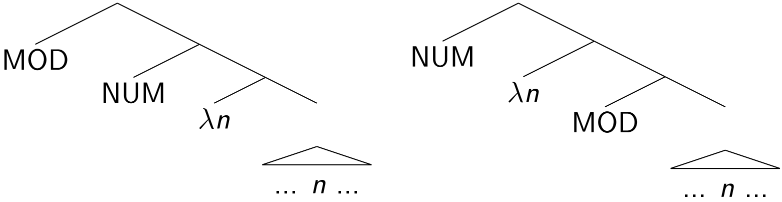

But what *is* the meaning of English number words? K&S conclude that number words are singular terms, but that won't help here.

Number word meaning

1. Relative scope of singular term and modal not truth-conditionally significant, even with type shift:

(37) $9 \Rightarrow \lambda P_{\langle d,t \rangle}.P(9)$

2. We get lower-bounded truth conditions for simple sentences, assuming nominal measure fn or parameterized cardinality predicate:

(38) There are seven planets in the solar system.

a. $[\![\text{seven}]\!] = 7$

b. $[\![\text{planets}]\!] = \lambda n \lambda x.\#(x) = n \wedge \mathit{planets}'(x)$

c. $\exists x[\#(x) = 7 \wedge \mathit{planets}'(x) \wedge \mathit{in\text{-}the\text{-}solar\text{-}system}'(x)]$

Number word meaning

What we want is something that will give us similar truth conditions to comparatives, namely the “Fregean” semantics in (39):

(39) $[\![\text{seven}]\!] = \lambda P_{\langle d,t \rangle}.max\{m \mid P(m)\} = 7$

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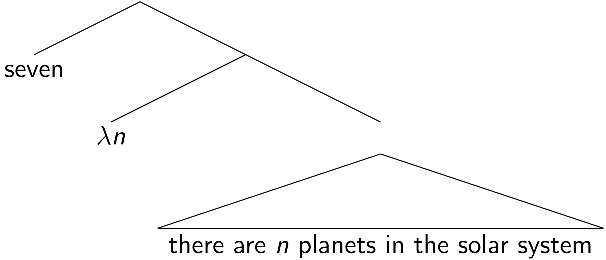
(40) a.

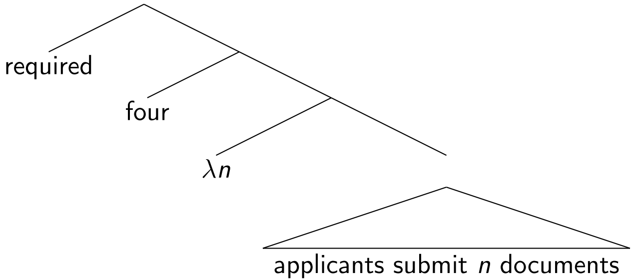


b. $max\{n|\exists x[\#(x) = n \wedge planets'(x) \wedge in\text{-}the\text{-}ss'(x)]\} = 7$

$\forall$ modals

(41) Applicants are required to submit four documents.

a.

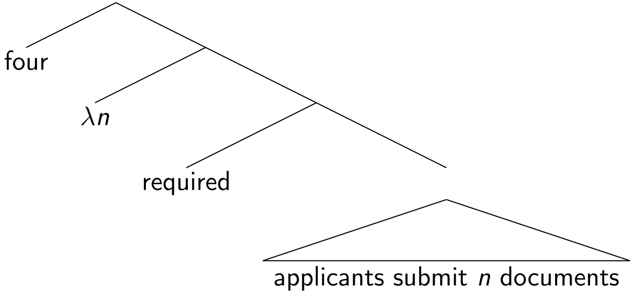


b. $\forall w \in Acc_{w0}[max\{n \mid \exists x[docs_w(x) \wedge \#_w(x) = n \wedge submit_w(x)(a)\} = 4]$

∀ modals

(42) Applicants are required to submit four documents.

a.



b. $max\{n \mid \forall w \in Acc_{w0}[\exists x[docs_w(x) \wedge \#_w(x) = n \wedge submit_w(x)(a)]\} = 4$

$\exists$ modals

(43) Applicants are allowed to submit four documents.

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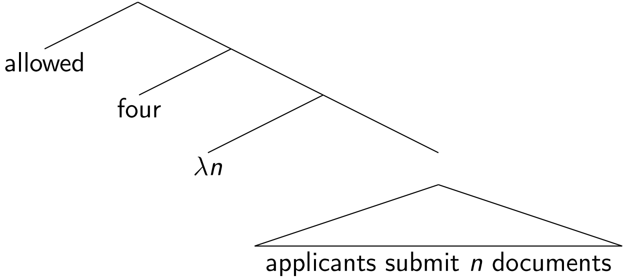

b. $\exists w \in Acc_{w0}[max\{n \mid \exists x[docs_w(x) \wedge \#_w(x) = n \wedge submit_w(x)(a)\} = 4]$

∃ modals

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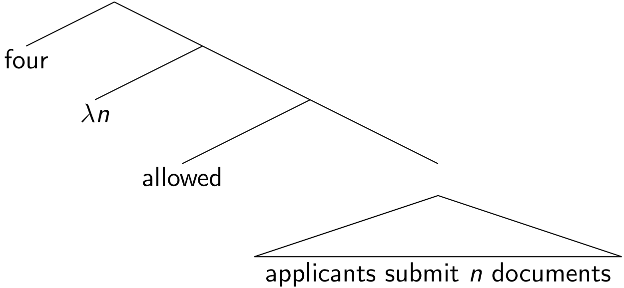

b. $max\{n \mid \exists w \in Acc_{w0}[\exists x[docs_w(x) \wedge \#_w(x) = n \wedge submit_w(x)(a)]\} = 4$

∃ modals

Unlike all other analyses, the “strong” readings of sentences with weak modals are derived as a matter of semantics, rather than as a scalar implicature.

(45) Troll can miss three shots (and still win).

We need to find out whether children systematically say that Troll loses when he misses four or more shots. Stay tuned....

Range of permission sentences

No problem:

(46) a. Students are required to take two classes and allowed to take four.

b. $max\{n \mid \forall w \exists x[\#_w(x) = n \wedge classes_w(x)]\} = 2 \wedge$
$max\{n \mid \exists w \exists x[\#_w(x) = n \wedge classes_w(x)]\} = 4$

Reassessment

	Q/Neg	Modals	Col/Dist	Box	LB/UB	CIM/ExM
1S-cont	-	-	-/+	-	+/∼	-/
2S-cont	+	-	+/?	+	-/∼	+/
1/2-poly	+	+	?/+	?	+/∼	+/
enrich	+	+	?/+	?	+/∼	+/-
scope	+	+	+/+	+	+/+	+/+

Two puzzles about modified numerals

Nouwen (2010) discusses two puzzles about numeral modifiers like *minimally/at least* and *maximally/at most*. First, they are "incompatible with definite amounts:"

(47) I know exactly how much memory my laptop has:

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(47) I know exactly how much memory my laptop has:

a. # It has at most/maximally 8GB of memory.

b. # It has at least/minimally 2GB of memory.

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(47) I know exactly how much memory my laptop has:

a. # It has at most/maximally 8GB of memory.

b. # It has at least/minimally 2GB of memory.

c. It has less than 8GB of memory.

d. It has more than 2GB of memory.

Two puzzles about modified numerals

(48) a. A hexagon has four sides.

b. A hexagon has ten sides.

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b. # A hexagon has maximally/at most ten sides.

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b. # A hexagon has maximally/at most ten sides.

(50) a. A hexagon has more than four sides.
b. A hexagon has fewer than ten sides.

Second, what look to be the most natural analyses derive incorrect truth conditions in sentences with modals:

(51) You are required to register for minimally/at least three classes.

a. $\otimes$ $min\{n \mid \forall w \in Acc \exists x[\#_w(x) = n \wedge classes_w(x)]\} = 3$

b. $\otimes$ $\forall w \in Acc[min\{n \mid \exists x[\#_w(x) = n \wedge classes_w(x)]\} = 3]$

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(52) You are allowed to register for maximally/at most three classes.

a. $max\{n \mid \exists w \in Acc \exists x[\#_w(x) = n \wedge classes_w(x)]\} = 3$

b. $\otimes$ $\exists w \in Acc[max\{n \mid \exists x[\#_w(x) = n \wedge classes_w(x)]\} = 3]$

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a. $max\{n \mid \exists w \in Acc \exists x[\#_w(x) = n \wedge classes_w(x)]\} = 3$

b. $\otimes$ $\exists w \in Acc[max\{n \mid \exists x[\#_w(x) = n \wedge classes_w(x)]\} = 3]$

Nouwen proposes that (51) involves reanalysis of $\Box$ as $\Diamond$, and that (52b) is eliminated by blocking. But there is an alternative....

Extending the current proposal

Hypothesis: Both unmodified and modified numerals have a "Fregean" semantics, but differ in the orderings they introduce:

i. $[\![\textit{three}]\!] = \lambda P_{\langle d,t\rangle}.max\{n \mid P(n)\} = 3$

ii. $[\![\textit{more than three}]\!] = \lambda P_{\langle d,t\rangle}.max\{n \mid P(n)\} > 3$

iii. $[\![\textit{fewer than three}]\!] = \lambda P_{\langle d,t\rangle}.max\{n \mid P(n)\} < 3$

iv. $[\![\textit{at least three}]\!] = \lambda P_{\langle d,t\rangle}.max\{n \mid P(n)\} \geq 3$

v. $[\![\textit{at most three}]\!] = \lambda P_{\langle d,t\rangle}.max\{n \mid P(n)\} \leq 3$

Assume with Cummins and Katsos (2010) that $\geq$ and $\leq$ break down to "$>$ or $=$" and "$<$ or $=$", and so are (i) more complex, and (ii) generate epistemic uncertainty implicatures.

Epistemic uncertainty

(53) a. A hexagon has four sides.

b. $max\{n|a\ hexagon\ has\ n\ sides\} = 4$

Epistemic uncertainty

(53) a. A hexagon has four sides.

b. $max\{n|a\ hexagon\ has\ n\ sides\} = 4$

(54) a. A hexagon has more than four sides.

b. $max\{n|a\ hexagon\ has\ n\ sides\} > 6$

(55) a. A hexagon has fewer than ten sides.

b. $max\{n|a\ hexagon\ has\ n\ sides\} < 10$

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(55) a. A hexagon has fewer than ten sides.

b. $max\{n|a\ hexagon\ has\ n\ sides\} < 10$

(56) a. # A hexagon has at least four sides.

b. $max\{n|a\ hexagon\ has\ n\ sides\} \geq 6$

(57) a. # A hexagon has at most ten sides.

b. $max\{n|a\ hexagon\ has\ n\ sides\} \leq 10$

These are bad because the definitional statement conflicts with the epistemic uncertainty inferences generated by $\geq$ and $\leq$.

Minimum and maximum requirements

Both LFs in (58) forbid registration in fewer than three classes:

(58) You are required to register for minimally/at least three classes.

a. $max\{n \mid \forall w \in Acc \exists x[\#_w(x) = n \wedge classes_w(x)]\} \geq 3$

b. $\forall w \in Acc[max\{n \mid \exists x[\#_w(x) = n \wedge classes_w(x)]\} \geq 3]$

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(58) You are required to register for minimally/at least three classes.

a. $max\{n \mid \forall w \in Acc \exists x[\#_w(x) = n \wedge classes_w(x)]\} \geq 3$

b. $\forall w \in Acc[max\{n \mid \exists x[\#_w(x) = n \wedge classes_w(x)]\} \geq 3]$

The LF (59b) is unusuable, because it tells us nothing about the number of classes that are actually allowed, leaving (59a) as the only understanding of the sentence.

(59) You are allowed to register for maximally/at most three classes.

a. $max\{n \mid \exists w \in Acc \exists x[\#_w(x) = n \wedge classes_w(x)]\} \leq 3$

b. $\# \exists w \in Acc[max\{n \mid \exists x[\#_w(x) = n \wedge classes_w(x)]\} \leq 3]$

Conclusion and questions

The semantic analysis in the following table provides a superior account of the semantic/pragmatic and experimental data, and it has a lot of intuitive appeal, but two big questions remain open.

$$
\boxed{\begin{array}{ll}
[\![\mathit{three}]\!] = & \lambda P_{\langle d,t\rangle}.max\{n \mid P(n)\} = 3 \\
[\![\mathit{more\ than\ three}]\!] = & \lambda P_{\langle d,t\rangle}.max\{n \mid P(n)\} > 3 \\
[\![\mathit{fewer\ than\ three}]\!] = & \lambda P_{\langle d,t\rangle}.max\{n \mid P(n)\} < 3 \\
[\![\mathit{at\ least\ three}]\!] = & \lambda P_{\langle d,t\rangle}.max\{n \mid P(n)\} \geq 3 \\
[\![\mathit{at\ most\ three}]\!] = & \lambda P_{\langle d,t\rangle}.max\{n \mid P(n)\} \leq 3
\end{array}}
$$

1. Is the quantificational meaning of bare numerals basic or derived?
2. Where does maximality come from?

Collective vs. distributive predicates

Stative predicates (without modals) should in general have only 2-sided truth conditions. But with eventive predicates, we expect an interaction with existential quantification over the event variable, but only for distributive predicates:

(60) a. $max\{n \mid \exists e \exists x[\#(x) = n \wedge P(x)(e)]\} = 7$ exactly

b. $\exists e[max\{n \mid \exists x[\#(x) = n \wedge P(x)(e)]\} = 7]$ at least

More than one kind of Q-principle?

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1. There is a general preference to go with the strongest meaning. This preference is blind to the semantics/pragmatics distinction.
2. Sometimes the candidates we are choosing from are based on implicature calculation (scalar or otherwise), which involves lots of reasoning, keeping track of context, and keeping track of the epistemic status of the discourse participants. This is "hard", and that difficulty is seen (in a gradient and adjustable way, depending on what else is going on) in the experiments with children.

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1. There is a general preference to go with the strongest meaning. This preference is blind to the semantics/pragmatics distinction.
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3. Sometimes, however, the candidates we are choosing from are simply two different parses — two LFs. This is at least potentially "easier", since it's just a matter of looking at what the grammar gives you in the first place, though there may be other factors at play, such as "isomorphism". But these factors ought to be of a different sort than the ones involved in (2).

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