On the Monotonicity of Polar Adjectives

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1 The monotonicity properties of polar adjectives

Negative polarity item licensing (1)-(4) and entailment patterns (5)-(8) indicate that certain adjectives generate monotone decreasing contexts, while others generate monotone increasing contexts (Seuren 1978, Ladusaw 1979, Linebarger 1980, Sánchez-Valencia 1994).

(1) It is difficult/*easy for Tim to admit that he has *ever* been wrong.
(2) It was foolish/*clever of her to *even bother to lift a finger* to help.
(3) It is strange/*typical that *any* of those papers were accepted.
(4) It’s sad/*great that you *even* have to talk to *any* of these people *at all*.

(5) It's dangerous to drive in Rome. ==> <=/=
It's dangerous to drive fast in Rome.
(6) It's safe to drive in Des Moines. =/=> <==
It's safe to drive fast in Des Moines.
(7) It’s strange to see Frances playing electric guitar. ==> <=/=
It’s strange to see Frances playing electric guitar poorly.
(8) It’s common to see Frances playing electric guitar. =/=> <==
It’s common to see Frances playing electric guitar poorly.

Monotonicity properties are one of several factors that have traditionally been used to classify gradable adjectives according to their "logical polarity" (in the sense of H. Klein 1996). Adjectives which license negative polarity items and downward entailments in clausal complements, such as *difficult* and *strange*, are classified as "negative", while adjectives which do not license negative polarity items but do permit upward inferences, such as *easy* and *common*, are classified as "positive" (see Seuren 1978 for early discussion of this issue). This paper addresses the question of how this distinction should be formally represented within a larger theory of the meaning of gradable adjectives. I demonstrate that the monotonicity properties of polar adjectives follows from an independently motivated representation of logical polarity as a sortal distinction between positive and negative adjectives, in a model in which gradable adjectives denote relations between individuals and intervals on a scale (“extents”), rather than relations between individuals and points on a scale (“degrees”), as traditionally assumed.

The paper is organized as follows. First, I the basics of traditional degree-based semantic analyses of gradable adjectives, and I show that although this type of analysis supports an explanation of monotonicity properties of gradable adjectives, it is empirically unsatisfactory, because it fails to account for the anomaly of comparatives constructed out of adjectives of opposite polarity. I then introduce an alternative analysis of gradable adjectives and adjectival polarity, based on an algebra of extents, that provides the basis for an account of the anomalous comparatives mentioned above. Finally, I demonstrate that this representation of adjectival polarity has the additional positive consequence of deriving the monotonicity properties of polar adjectives.

2 Degrees

2.1 The algebra of degrees

A fundamental problem introduced by gradable adjectives like *difficult*, *tall*, and *interesting* is that they are inherently vague: the criteria for deciding whether an utterance of e.g. *x is difficult* or *it is difficult to p* is true may vary according to factors external to the adjective, such as the meaning of *x* or *p* and the context of utterance. A basic requirement of any theory of the semantics of gradable adjectives is that it must provide a means of capturing this variability, while at the same time ensuring that for any particular context of utterance, sentences of this type have a definite interpretation.

One approach to this problem maintains that gradable adjectives have a more complex meaning than might initially be assumed: they are not simple predicates, but rather relations between individuals and some kind of abstract representation of measurement (see e.g., Seuren 1973, Cresswell 1976; see E. Klein 1991 for general discussion).[1] On this view, the logical representation of a sentence like (9) is (10), where m_s represents the "measure argument" of the adjective *tall*.

(9) Carmen is tall.
(10) $tall(Carmen,m_s)$

The role of the "measure argument" is to provide a means of fixing the extension of the adjective by introducing a "standard" value against which the target of predication is compared. For example, the truth conditions of the expression in (10) can be stated as follows: *Carmen is tall* is true just in case Carmen is at least as tall as m_s; i.e., just in case Carmen's measure of tallness is at least as great as the standard, where the standard value is determined contextually.[2]

A question that immediately arises in this type of analysis concerns the formal representation of measures: just what kinds of objects are they, and does their structure determine (or affect) aspects of the interpretation of gradable adjectives? One answer to this question, developed in Cresswell 1976, builds on the intuition that "[w]hen we make comparisons we have in mind points on a scale" (Cresswell 1976:266). Cresswell formalizes the notion of measure in terms of an ontology of such points, starting from the general observation that every gradable adjective supports a partial ordering on its domain along some *dimension*, a general property that permits grading. Objects in the domain of the adjective *tall* (entities that can have some kind of vertical extent, such as dogs, stars, amoebae, etc., but not charity, disgust, or syntax), for example, can be ordered along a dimension of *height.* Noting that a partial ordering on a set A with respect to a dimension δ partitions A into equivalence classes of objects under $\leq_\delta$, Cresswell defines a *scale* S_δ as the set of all equivalence classes under $\leq_\delta$, which is itself a totally ordered set. A *degree* of φ*-ness*, where φ is a gradable adjective associated with S_δ is a member of S_δ. That is, a degree on S_δ corresponds to an equivalence class of objects under $\leq_\delta$. It is not necessary to use equivalence classes directly in the semantics; degrees may instead be construed as abstract objects that are associated with the equivalence classes determined by $\leq_\delta$ by a one-to-one mapping.

Given this general characterization of scales and degrees, gradable adjectives can be analyzed as expressions that define relations between individuals and degrees (see e.g. Cresswell 1976, Hellan 1981, Hoeksema 1983, von Stechow 1984a, Heim 1985, Lerner & Pinkal 1992, Moltmann 1992, Rullmann 1995, Gawron 1995 and others; see Atlas 1984 for a related analysis). The basic account of the semantics of positive adjectives in this system parallels the one sketched above. The logical representation of an example like (11) is (12), where $d_{s(long)}$ represents the degree argument of *long* and denotes a contextually determined standard of "longness".[3]

(11) *The Brothers Karamazov* is long.

(12) $long(BK,d_{s(long)})$

(11) is true just in case *The Brothers Karamazov* is at least as long as $d_{s(long)}$; i.e., if and only if $d_{s(long)} \leq d_{BK}$ holds, where d_{BK} is the degree of *The Brothers K.*'s length.

Negative adjectives are analyzed in roughly the same way, with one important modification: the relation between the standard and the degree of the subject's *adjectiveness* must be reversed (cf. Gawron 1995). (13) illustrates this point.

(13) *The Dream of a Ridiculous Man* is short.

(14) $short(DRM,d_{s(short)})$

Given the analysis of (11) outlined above, (13) should be true just in case the length of *The Dream of a Ridiculous Man* is at least as great a standard of shortness (for Russian novels; this may or may not be the same degree as $d_{s(long)}$ for a given context; see E. Klein 1980 for discussion). The partial ordering relation involved in the truth conditions for this example must be the reverse of the corresponding relation in (11), however. For example, if the standard of shortness for Russian novels in some context is 45 pages, then (13) is true just in case *The Dream of a Ridiculous Man* is 45 pages long or *less*. Taking (14) to be the logical representation of (13), then, we can say that (13) is true just in case $d_{s(short)} \leq d_{DRM}$.

Sentences like (15)-(16) show that the relational difference between positive and negative adjectives extents to the comparative forms as well.

(15) *The Brothers Karamazov* is longer than *The Dream of a Ridiculous Man*.

(16) *The Dream of a Ridiculous Man* is shorter than *The Brothers Karamazov*.

For perspicuity, I will adopt two fairly common assumptions about the semantic analysis of comparatives (though I should point out that the arguments for the characterization of adjectival polarity that I will develop in this paper are independent of the specific semantic analysis of comparatives that I will use in the discussion). First, I assume that the comparative clause (the complement of *than* or *as*) denotes a definite description of a (maximal) degree (see e.g., Russell 1905, von Stechow 1984a, Rullmann 1995, Kennedy 1997c). Second, I will adopt an analysis of comparatives in terms of restricted existential quantification over degrees, in which the semantic function of a comparative is to restrict the possible value of the degree argument of a gradable adjective to only those degrees which stand in some relation–the relation introduced by the comparative morpheme–to the degree denoted by the comparative clause (Heim 1985). The logical representation of an example like (15), given these two assumptions, can be can be formulated as in (17).[4]

(17) $\exists d[d > \iota d'.long(Dream,d')][long(BK,d)]$

According to (17), (15) is true just in case there is a degree *d* which exceeds the (maximal) degree of *The Dream of a Ridiculous Man*'s length, and *The Brothers K* is *d* long.

As was the case with the absolute, the ordering relation imposed by the comparative morpheme appears to be dependent on the ordering relation associated with the adjective it modifies. In order to get the correct truth conditions for (16), the ordering relation must be reversed: (16) should be true just in case the length of *The Dream of a Ridiculous Man* is less than the length of *The Brothers K.* If the ordering associated with the comparative morpheme is reversed when it modifies a negative adjective, as in (18), then we get the right results.

(18) $\exists d[d < \iota d'.short(BK,d')][short(Dream,d)]$

According to (18), (16) is true just in case there is a degree which is ordered below the degree of *The Brothers K*'s shortness, and *The Dream of a Ridiculous Man* is that short.

2.2 Degrees, polar opposition, and monotonicity

The relational difference between positive and negative adjectives illustrated by the examples in the previous section suggests a way of formally representing the logical polarity of gradable adjectives within a degree algebra. Given the definition of a scale as a linearly ordered set of degrees along some dimension, and the analysis of gradable adjectives as relations between individuals and degrees, we can characterize adjectival polarity in the following way: assume that antonymous pairs of adjectives define mappings between objects and the same scale (for example, *tall* and *short* both define relations between objects and degrees on a scale of *height* ; cf. Rullmann 1995), but they differ in that the ordering relations on the scale are reversed. That is, for any antonymous pair of adjectives φ_{pos} and φ_{neg} associated with a scale S, φ_{pos} denotes a relation between objects and $\langle S,<_{\delta}\rangle$, while φ_{pos} denotes a relation between objects and $\langle S,>_{\delta}\rangle$. On this view, the set of positive degrees D_{pos} and the set of negative degrees D_{neg} on a scale S stand in the dual relation. Moreover, since there is a one-to-one mapping between the two sets (the identity function), D_{pos} and D_{neg} are isomorphic.

This characterization of adjectival polarity has three important results. First, it explains why the ordering is reversed when computing the meaning of positive and negative absolute and comparative adjectives. Second, it correctly predicts that statements like (19) are valid.

(19) *The Dream of a Ridiculous Man* is shorter than *The Brothers Karamazov* if and only if *The Brothers Karamazov* is longer than *The Dream of a Ridiculous Man*.

Degrees of longness and shortness are the same objects, therefore, given the difference in the ordering relations associated with positive and negative adjectives, the two conjuncts in (19) are true in exactly the same situations. Third, and most importantly for the current discussion, the formalization of adjectival polarity outlined here has the additional positive consequence that negative adjectives are monotone decreasing.

To see why, consider an arbitrary case of ordering along a dimension, for example, *safety*. If *b* is safer than *a*, then the following relation holds:

(20) $a <_{safety} b$

Assume that every gradable adjective is associated with a function which maps individuals to degrees.[5] If the ordering relation associated with a negative adjectives is the dual of the relation associated with its positive counterpart, then whenever (20) holds, (21)-(22) also hold (where SAFE and DANGEROUS denote functions from individuals to degrees).

(21) SAFE(a) < SAFE(b)
(22) DANGEROUS(b) < DANGEROUS(a)

Given the standard definitions in (23), it follows that positive adjectives are monotone increasing and negative adjectives are monotone decreasing.

(23) a. A function f is *monotone increasing* iff: $a < b \rightarrow f(a) < f(b)$
b. A function f is *monotone decreasing* iff: $a < b \rightarrow f(b) < f(a)$

Although this is a positive result, it must be acknowledged that the monotonicity of negative adjectives does not follow from an independent aspect of the algebra of degrees. Rather, it is a definitional property of negative adjectives. The facts discussed so far indicate that within a degree algebra, the assumption that negative adjectives are scale reversing is necessary in order to construct a semantics that has the correct truth conditions for both positive and negative adjectives in the absolute and comparative forms. Nothing about the system itself requires negative adjectives to be monotone decreasing; rather, it is the data which force this assumption. An alternative situation would be one in which independently motivated aspects of the formalism have the additional consequence that negative adjectives are scale reversing. In section 4, I will introduce an algebra of measures which does have this property; before I do this, however, I will turn to a set of empirical data which calls into question the descriptive adequacy of the degree-based characterization of adjectival polarity presented here.

3 Cross-polar anomaly

(24)-(27) show that comparative constructions formed out of positive and negative pairs of adjectives are anomalous; in the discussion that follows, I will refer to this phenomenon as *cross-polar anomaly* (see Kennedy 1997b for more detailed discussion of this phenomenon and the problems it poses for a degree algebra).

(24) #*The Brothers Karamazov* is longer than *The Idiot* is short.
(25) #Mike is shorter than Carmen is tall.
(26) #The surf at Maverick's is twice as high as the surf at Seabright is low.
(27) #Maureen is even less exciting than Maurice is dull.

These examples should be compared to acceptable sentences like (28)-(29), in which the polarity of the adjectives are the same.

(28) The desk is longer than the table is wide.
(29) Luckily, the ficus was shorter than the doorway was low.

The puzzle of sentences like these is that the very same assumption which leads to the right semantics for positive and negative adjectives, an explanation of the validity of (19), and an account of the monotonicity of negative adjectives–the assumption that positive and negative degrees on a scale are the same objects–makes the wrong prediction in cases of comparison between adjectives of opposite polarity. If positive and negative degrees are the same objects, then, given the analysis of typical comparatives like (15), (24) should be true when the point on a scale of length corresponding to the longness of *The Brothers Karamazov* exceeds the point on a scale of length corresponding to the shortness of *The Idiot*. Consider the logical representation of (24), shown in (30).

(30) $\exists d[d > \iota d'.short(Idiot,d')][long(BK,d)]$

Given the analysis of comparatives sketched in section 2.1, (24) should not only be interpretable, *it should be logically equivalent to (15)*. The same argument can be made for examples in which the adjectives are reversed. This result points out a fundamental flaw in the degree approach: because positive and negative degrees denote the same objects, there is no way to rule out comparisons like (24)-(27).

It should be noted that within a degree algebra the anomaly of (24)-(27) cannot be reduced to the better-known anomaly of (31)-(32), which are examples of *incommensurability*, or incompatibility of scales (see E. Klein 1980, 1991 for discussion).

(31) #Mike is taller than Carmen is clever.
(32) #*The Idiot* is more tragic than my copy of *The Brothers Karamazov* is heavy.

The oddness of these examples follows from general properties of ordering relations and the fairly natural assumption that the adjectives *tall* and *clever* are associated with distinct scales (i.e., scales along different dimensions). The comparative in (31) restricts the possible values of the degree variable be a degree which exceeds the maximum degree of Carmen's cleverness. Since ordering relations are defined only for objects in the same ordered set, only degrees of cleverness satisfy this restriction. The result is that the comparative restricts the degree argument of *tall* to be a degree of cleverness. Degrees of cleverness are not of the appropriate sort for the adjective *tall*, however, triggering a sortal mismatch (this type of explanation can be applied to (32), as well).[6]

An incommensurability explanation of cross-polar anomaly would start from the same claim: antonymous pairs of adjectives like *long* and *short* are associated with distinct scales, and so their degrees cannot be compared. Such an account is unavailable in the degree approach, however, because it presupposes that there is no mapping of the degrees on the scales associated with the incomparable adjectives onto some common scale appropriate to both adjectives. When such a mapping can be defined, comparison is possible, as shown by typical examples of comparative subdeletion like (33).

(33) Mike's hands are wider than Carmen's are long.

(33) is interpretable despite the fact that the compared adjectives *wide* and *long* use different scales because degrees of width and length can be mapped onto a common scale–call it *linear measurement*–which can be used to calculate the denotation of the comparative.

In order for an incommensurability explanation of (24)-(27) to go through, then, it must be demonstrated that for any for positive-negative pair of adjectives, there is no possible mapping between D_{pos} and D_{neg}. If such a mapping can be defined, then these constructions should be amenable to the same sort of analysis as (33). The problem is that given the assumption that D_{pos} and D_{neg} are related by the dual relation–an assumption that is necessary to explain the validity of (19)–there is a mapping from positive to negative degrees: the identity function (see Rullmann 1995 for additional discussion of this point). The incommensurability explanation fails because any algebra of degrees which explains the validity of (19), a minimal requirement of descriptive adequacy, also permits a mapping between positive and negative degrees, with the consequence that (24) should be fully interpretable.

Although it fails to explain the anomaly of (24)-(27), the proposal that positive and negative adjectives are incomparable contains an interesting hypothesis: that degrees of tallness and degrees of shortness are basically different sorts of objects, and cross-polar anomaly can be explained in terms of this difference. Without an ad hoc stipulation to this effect, however, there is no way to build such a sortal difference into an algebra of degrees. What is required to make this type of explanation work is an elaboration of the algebra of "measures" in such a way that the measures associated with positive and negative adjectives are structurally distinct. In the following section, I will describe such an elaboration, and I will show that this distinction not only provides an explanation of cross-polar anomaly, it also entails that positive adjectives are monotone increasing and negative adjectives are monotone decreasing.

4 Extents

4.1 Extents and polar opposition

Seuren (1978, 1984) proposes that a gradable adjective does not denote a relation between an individual and a degree (a point on a scale), but rather a relation between an individual and an *extent*, where an extent is an interval on a scale (see also von Stechow 1984b and Löbner 1990; see Bierwisch 1989 for a similar proposal). As in the degree approach, every gradable adjective is associated with a scale, and scales are differentiated from one another through association with a dimension (e.g., *height, reading length, complexity, abstractness,* etc). The difference is that the scalar values that provide the "measure argument" of a gradable adjective are not points, but rather intervals. More formally, let a *scale* $\langle S,<_\delta\rangle$ be a dense, linearly ordered set of points along a dimension δ. An *extent E* on $\langle S,<_\delta\rangle$ is a nonempty, convex subset of S, i.e., a subset of S with the following property: $\forall p_1,p_2 \in E \forall p_3 \in S[p_1<p_3<p_2 \rightarrow p_3 \in E]$ (cf. Landman 1991:110; this is simply the definition of *interval* for a linearly ordered set of points). Finally, let a *proper extent* on $\langle S,<_\delta\rangle$ be a nonempty, convex proper subset of S.

In general terms, an extent, like a degree, is an abstract representation of the measure of an object's "adjectiveness". The advantage of formalizing these measures as intervals, rather than as points, is that a basic structural distinction can be made between different sorts of extents, and this distinction can be used as the basis for a theory of adjectival polarity. Following Seuren (1978, 1984) and von Stechow (1984b), I will distinguish two sorts of extents: *positive extents* and *negative extents*, defined in (34)-(35).

(34) A *positive extent* on $\langle S,<_\delta\rangle$ is a proper extent on $\langle S,<_\delta\rangle$ that ranges from the lower end of the scale to some positive point.

(35) A *negative extent* on $\langle S,<_\delta\rangle$ is a proper extent on $\langle S,<_\delta\rangle$ that ranges from some positive point to the upper end of the scale.

The primary distinction between positive and negative extents is *perspective*. The definitions in (34)-(35) entail that the complement of any positive extent on a scale is a negative extent, and vice versa. The intuition which this is meant to capture is that although positive and negative pairs of adjectives are fundamentally similar in meaning (they map objects onto the same scale), they are used in a very different way. Specifically, they are used to express *complementary* perspectives on a scale. For example, the sentences *Carmen is beautiful* and *Mike is ugly* both provide information about the attractiveness of the individuals denoted by the subjects, but the information is qualitatively different in each case. The positive adjective *beautiful* conveys information about the attractiveness an object has, while the negative adjective *ugly* conveys information about the attractiveness an object does not have (cf. von Stechow 1984b:196).[7]

To make this idea more precise, assume that for any object a which can be ordered along some dimension δ, there is a function d from a to a unique point on the scale $\langle S,<_\delta\rangle$. Let the positive extent of a with respect to $\langle S,<_\delta\rangle$, $\textsc{pos}_\delta(a)$, be the interval $\{p \in \langle S,<_\delta\rangle \mid p \leq d(a)\}$, and let the negative extent of a with respect to $\langle S,<_\delta\rangle$, $\textsc{neg}_\delta(a)$, be the interval $\{p \in \langle S,<_\delta\rangle \mid d(a) \leq p\}$. Thus for any a, $\textsc{pos}_\delta(a)$ and $\textsc{neg}_\delta(a)$ represent (join) complementary perspectives on a with respect $\langle S,<_\delta\rangle$.

This discussion highlights the primary difference between an extent ontology and a degree ontology. Because degrees are defined as points on a scale, there is no way to differentiate positive and negative degrees: for any object a, $deg_{pos}(a)$ and $deg_{neg}(a)$ denote the same object. In contrast, because extents have additional structure (the intervals ranging to one or the other end of the scale), positive and negative extents are distinct objects: for any a ordered along a dimension δ, $\textsc{pos}_\delta(a) \neq \textsc{neg}_\delta(a)$.[8] More generally, for any scale $\langle S,<_\delta\rangle$, the set of positive extents on $\langle S,<_\delta\rangle$, E_{pos}, and the set of negative extents on $\langle S,<_\delta\rangle$, E_{neg}, are disjoint subsets of the total set of extents on $\langle S,<_\delta\rangle$.

This result provides the basis for a theory of polar opposition. Assume that the set of relations between individuals and extents can be sorted according to their range, with the following result: positive adjectives denote gradable properties whose range is E_{pos}; negative adjectives denote gradable properties whose range is E_{neg}. In other words, logical polarity is represented as a sortal distinction between positive and negative adjectives: positive adjectives denote relations between individuals and positive extents; negative adjectives denote relations between individuals and negative extents.

An immediate advantage of this approach is that propositions constructed out of positive and negative adjectives can be analyzed in exactly the same way–it is not necessary to stipulate that the ordering relation used when computing the interpretations of absolute and comparative adjectives must be reversed in the negative form. The interpretation of the absolute is straightforward: a sentence of the form *a is* φ, φ a positive or negative gradable adjective, is true just in case the extent to which a is φ includes the value of the extent argument of φ. For example, consider the interpretation of (36) in the situation represented by (38), where e_{B^+} denotes the positive extent of *The Brothers K*'s length, e_{B^-} its negative extent, $e_{s(long)}$ a contextually determined standard of longness (of book-reading), and $e_{s(short)}$ a standard of shortness.

(36) *The Brothers Karamazov* is long.

(37) $long(BK,e_{s(long)})$

(38) LENGTH: 0 ————————————————————→ ∞

0 ———————— e_{B^+} ————————•———— e_{B^-} —————→ ∞

0 —————— $e_{s(long)}$ ——————•

•——————— $e_{s(short)}$ ———————→ ∞

Since *long* is a positive adjective, its extent argument must be a positive extent. (36) is true iff e_{B^+}, the extent of *The Brothers K*'s longness, includes $e_{s(long)}$. This relation holds in (38), so in this context (36) is true.

Negative adjectives are analyzed in the same way. The logical form of (39) is (40), where the extent argument of the negative adjective *short* is a contextually determined standard of shortness.

(39) *The Brothers Karamazov* is short.

(40) $short(BK, e_{s(short)})$

(39) is false in the context of (38), because e_{B^-}, the negative extent of *The Brothers K*'s length, does not include $e_{s(short)}$.

Note that although the positive and negative projections of an individual on a scale are complementary, by definition (this is illustrated by e_{B^+} and e_{B^-} in (38)), the standard values associated with antonymous adjectives need not be. It must be the case, however, that in any particular context, the maximal element of the positive standard must at least *exceed* the minimal element of the negative standard, as in (38), in order to capture the fact that antonymous adjectives are *contraries*, not *contradictories*: although *x is* φ_{pos} entails *x is not* φ_{neg}, it is not the case that *x is not* φ_{pos} entails *x is* φ_{neg} (and vice-versa; see Bierwisch 1989). For example, in a context in which the standards for longness and shortness (of book-reading) are as in (38) and the projection of *Notes from the Underground* is as shown in (41), it is true that *Notes from the Underground* is nether long nor short.[9]

(41) LENGTH: 0 ————————————————————→ ∞

0 ————— e_{Notes^+} ——————•——————— e_{Notes^-} ————————→ ∞

0 —————— $e_{s(long)}$ ——————•

•——————— $e_{s(short)}$ ———————→ ∞

The analysis of comparatives is somewhat more complex, but again, positives and negatives can be analyzed in the same way. The analysis of comparatives as restricted quantification structures can be straightforwardly recast in an algebra of extents by formulating the truth conditions of comparatives with *more* in terms of proper inclusion. (Similarly, comparatives of equality (comparatives with *as*) can be analyzed in terms of (non-proper) set inclusion, or, alternatively, in terms of set equality.) For illustration, consider the analysis of (42) in the context represented by (43), where e_{B^+} and e_{B^-} are as defined above, and e_{D^+} and e_{D^-} represent the positive and negative extents of *The Dream of a Ridiculous Man*'s length, respectively.

(42) *The Brothers Karamazov* is longer than *The Dream of a Ridiculous Man*.

(43) LENGTH: $0 \longrightarrow \infty$

0 —— e_{B^+} ——•—— e_{B^-} —— $\longrightarrow \infty$

0 —— e_{D^+} ——•—— e_{D^-} —— $\longrightarrow \infty$

Assuming a mapping from syntactic structure to logical form analogous to the one adopted in section 2, the logical form of (42) is (44).

(44) $\exists e[e \supset \iota e'.long(Dream,e')][long(BK,e)]$

According to (44), (42) is true just in case there is an extent which properly includes the extent of *The Dream of a Ridiculous Man*'s longness, and *The Brothers K.* is that long. These conditions are met in the context represented by (43), because e_{B^+} properly includes e_{D^+}. Crucially, because the adjective in the restriction is positive, the possible values of the extent argument of the adjective which the comparative modifies are restricted to be positive extents, since only a positive extent properly includes another positive extent.[10]

Negative comparatives are treated in the same way. The logical form of an example like (45) is (46).

(45) *The Dream of a Ridiculous Man* is shorter than *The Brothers Karamazov*.

(46) $\exists e[e \supset \iota e'.short(BK,e')][short(Dream,e)]$

(45) is true just in case there is an extent which properly includes the maximal extent of *The Brothers K.*'s shortness, and *The Dream of a Ridiculous Man* is that short. These conditions are met in (43), because e_{D^-} properly includes e_{B^-}. Again, note the consequence of restricted quantification: since the adjective in the restriction is negative, the possible values of the extent argument of the main predicate are restricted to be negative extents, since only a negative extent properly includes another negative extent.

(42) and (45) illustrate another important aspect of the analysis, namely that it explains the validity of (19), repeated below.

(19) *The Dream of a Ridiculous Man* is shorter than *The Brothers Karamazov* iff *The Brothers Karamazov* is longer than *The Dream of a Ridiculous Man*.

A statement like (19) can be paraphrased as: "the extent to which *The Dream of a Ridiculous Man* is short properly includes the extent to which *The Brothers K.* is short if and only if the extent to which *The Brothers K.* is long properly includes the extent to which *The Dream of a Ridiculous Man* is long". More generally, statements like (19) can be viewed as substitution instances of (47), where POS and NEG are defined as functions from individuals to extents.

(47) $\textsc{pos}(a) \supset \textsc{pos}(b)$ iff $\textsc{neg}(b) \supset \textsc{neg}(a)$

Since positive and negative extents are join complementary, (47) is clearly valid.

The upshot of this discussion is that with respect to the interpretation of positive and negative absolute and comparative adjectives, the extent algebra has at least the same range of empirical coverage as the degree algebra. In the next section, I will demonstrate that the empirical coverage of the extent approach actually exceeds that of the degree analysis, because it supports an explanation of cross-polar anomaly.

4.2 Extents and cross-polar anomaly

In section 3, I considered and rejected the possibility of an explanation of the anomaly of sentences like (48) within the degree approach in terms of incommensurability.

(48) #*The Brothers K* is longer than *The Idiot* is short.

The basic idea behind this type of explanation is that degrees of e.g. *tallness* and *shortness* are of different sorts of objects, and so are not comparable. I argued that this explanation is unavailable in the degree approach because the algebra of degrees does not distinguish between positive and negative degrees on the same scale. In contrast, a fundamental property of the algebra of extents is that positive and negative extents *are* different sorts of objects. Whereas the degree approach sorts adjectives only according to scale (as in the case of *clever* and *tall*, see the discussion of (31-32) in section 3), the extent approach sorts adjectives both according to the scale with which they are associated *and* according to the sort of extent argument they require. The latter distinction provides the basis for an explanation of cross-polar anomaly.

According to the analysis of the comparative construction outlined above, the logical form of (48) is (49).

(49) $\exists e[e \supset \iota e'.short(Idiot,e')][long(BK,e)]$

According to (49), (48) is true just in case there is an extent which properly includes the extent of *The Idiot's* shortness, and *The Brothers Karamazov* is long to that extent. Since the only sorts of extents which satisfy the conditions imposed by the restriction are negative extents, the comparative requires the argument of the positive adjective *long* to be a negative extent. The restriction of the possible values of the extent variable introduced by *long* to a negative extent triggers a sortal mismatch, with the result that this sentence is semantically anomalous.

The same explanation applies to sentences in which the polar adjectives are reversed, such as (50).

(50) #*The Idiot* is shorter than *The Brothers K* is long.

(51) $\exists e[e \supset \iota e'.long(BK,e')][short(Idiot,e)]$

(50) is true if and only if there is an extent which properly includes the extent of *The Brothers K*'s longness, and *The Idiot* is that short. Only positive extents include a positive extent, therefore the comparative construction forces the argument of the negative adjective *short* to be a positive extent. This violates the sortal requirements of *short*, with the result that (50) is anomalous.[11]

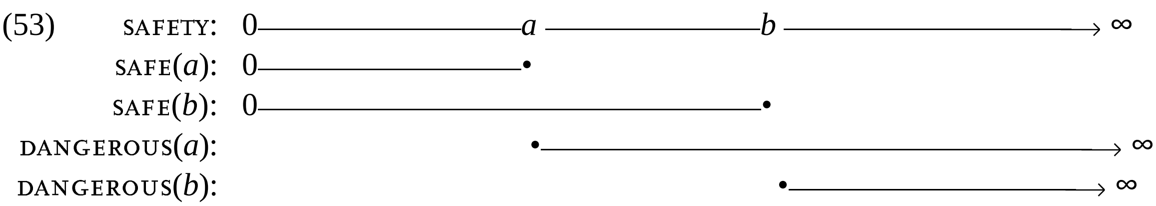
The explanation of cross-polar anomaly outlined here is available precisely because the algebra of extents permits a structural distinction between positive and negative extents, which in turn provides the basis for the formalization of adjectival polarity presented in the previous section. Indeed, the fact that this anomaly is observed in the first place actually provides support for the extent approach. If positive and negative adjectives are distinguished by the sort of extent argument they take, and if comparatives restrict the possible value of the extent argument, then sortal mismatches are predicted to arise exactly in the context of comparatives involving polar opposites.

5 Extents and monotonicity

The sortal distinction between positive and negative extents made within an extent ontology has the consequence that positive adjectives are monotone increasing and negative adjectives are monotone decreasing. Consider again the case of an ordering along a dimension of *safety*, as discussed in section 2:

(52) $a <_{safety} b$

In line with the earlier discussion, I will assume that gradable adjectives are associated with functions from individuals to extents. When the relation between *a* and *b* shown in (52) holds, then the values of SAFE(a) and SAFE(b), and DANGEROUS(a) and DANGEROUS(b) (i.e., the positive and negative extents of a and b on the scale of safety) are as in (53).

(53)


Assuming a standard Boolean ordering on extents as in (54), whenever (52) holds, the relations in (55)-(56) also obtain.

(54) $[e < e']$ iff $[e \cap e' = e$ and $e \neq e']$
(55) SAFE(a) < SAFE(b)
(56) DANGEROUS(b) < DANGEROUS(a)

The positive adjective *safe* preserves the ordering on a and b, but the negative adjective *dangerous* reverses it. Therefore *safe* is monotone increasing, and *dangerous* is monotone decreasing. This result does not follow from some property of *safe* and *dangerous*; rather it is a general consequence of the hypothesis that logical polarity is represented as a sortal distinction between gradable adjectives, as defined in section 4.1. What distinguishes this result from the one obtained in the degree approach is that it follows directly from the algebra of extents, not from prior assumptions about implicit ordering relations associated with the adjectives *safe* and *dangerous*.[12]

6 Conclusion

Taking differences in monotonicity properties of gradable adjectives as a starting point, this paper considered two alternative representations of logical polarity. The degree analysis makes the assumption that negative adjectives are scale reversing at a basic level; rather than explaining why negative adjectives are monotone decreasing, it defines them as such. The extent approach, on the other hand, derives the monotonicity properties of polar adjectives from an independently motivated structural distinction between positive and negative extents, a distinction that provides the basis for an explanation of cross-polar anomaly. It is important to note that the proposals I have made here do not *predict* whether an adjective is going to be positive or negative, nor are they intended to. The goal is rather to develop and motivate a formal representation of adjectival polarity that accounts for the logical properties of gradable predicates in as robust and explanatorily adequate a way as possible.

The facts I have considered in this paper show that a formal system in which polarity is represented as a sortal distinction between gradable adjectives achieves this goal.

Notes

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1. See McConnell-Ginet 1973, Kamp 1975, E. Klein 1980, Van Benthem 1983, and Sánchez-Valencia 1994 for a different type of approach, in which gradable adjectives are analyzed as vague (possibly partial) functions from individuals to truth values (but see Kennedy 1997a for empirical arguments against this sort of approach).

2. For the purposes of this paper, we can assume that the standard value is determined contextually, although there are problems for this simplistic view (see e.g. Klein 1980). See Cresswell 1976, E. Klein 1980, 1991, von Stechow 1984a, Ludlow 1989, Kennedy 1997a for general discussion of this issue.

3. This example illustrates the fact that a particular adjective may be associated with different scales in different contexts. The scale of length involved in (11) could take several possible values, such as "length as a function of reading time", or "length as a function of number of pages", which is the scale I assume to be relevant here (see McConnell-Ginet 1973, Kamp 1975, E. Klein 1980, Pustejovsky & Boguraev 1993, and Kennedy 1997a for relevant discussion of this issue).

4. I ignore here questions regarding the derivation of (17) from the syntactic representation of (15), in particular, the question of whether some kind of ellipsis operation is involved (as in e.g. Gawron 1995), or whether some kind of “direct” interpretation is possible (as in e.g. Klein 1980, Hoeksema 1983, Heim 1985, and Kennedy 1997a). These questions, while of fundamental importance to our understanding of the syntax-semantics interface, do not bear directly on the issues addressed in this paper.

5. This function is implicit in the statement of the truth conditions for the absolute form outlined in section 2.1. It some analyses, it is stated as an explicit component of the meaning of a gradable adjective (see e.g. Bierwisch 1989); in others, gradable adjectives are actually analyzed as functions from objects to degrees (Bartsch and Vennemann 1973, Kennedy 1997a).

6. According to the discussion in section 2.1, an expression of the form $\varphi(a,d)$ (such as

the nuclear scope of a comparative, on the analysis adopted here) is true just in case the degree to which a is φ is at least as great as d. If d is a degree on a scale distinct from the scale associated with φ, the partial ordering relation encoded in the truth conditions will not be defined for these two degrees.

7. For dimensional adjectives like *tall*, *short*, *long* etc., at least, a second distinguishing characteristic is boundedness (assuming that dimensional gradable adjectives are distinguished from non-dimensional gradable adjectives in being associated with a scale which has a lower bound (cf. Bierwisch 1989)). Positive extents have both a greatest lower bound (0) and a least upper bound (the requirement that a positive extent be a proper extent forces it to have an upper bound), therefore they are bounded. Negative extents, on the other hand, are by definition unbounded: although they must have a greatest lower bound in order to be proper, they have no upper bound.

8. Though for any a, $\textsc{neg}(a) = -\textsc{pos}(a) \cup \{d(a)\}$, and vice-versa. This follows from the fact that $\textsc{pos}(a)$and $\textsc{neg}(a)$ are join complementary: $\textsc{pos}(a) \cup \textsc{neg}(a) = S$, but $\textsc{pos}(a) \cap \textsc{neg}(a) = d(a)$.

9. It should be noted that a degree-based analysis must stipulate the same sort of relation between positive and negative standards.

10. For two extents X and Y, X $\supset$ Y iff X $\cup$ Y = X. If X $\cup$ Y = X, then if Y has no lower bound (i.e., if Y is positive), X must also have no lower bound, so X must be positive as well. (This argument holds *mutatis mutandis* for scales in which positive extents are lower closed by 0.) Similarly, if Y has no upper bound (if Y is negative), then X also must not have an upper bound. Therefore X must be negative.

11. An interesting result of this analysis is that cross-polar anomaly receives the same type of explanation as the well-known anomaly of examples like (i) (cf. (ii)).

(i) #The *Dream of a Ridiculous Man* is 21 pages short.
(ii) The *Brothers Karamazov* is 762 pages long.

These examples show that positive adjectives license overt measure phrases while negative adjectives do not. Assuming that measure phrases such as *762 pages*, *six feet three inches*, *twenty-nine years*, etc. may refer to positive extents, which, in the case of dimensional adjectives at least, are bounded and therefore measurable (cf. note 7), but not to negative extents, which are unbounded and so not measurable, the contrast between (i) and (ii) can be explained in terms of sortal mismatch (see von Stechow 1984b for a similar analysis). (i) is interpretable because the measure phrase *762 pages* provides an appropriate argument for the positive adjective *long*. (ii), on the other hand, is anomalous because *21 pages* refers to a positive extent, which is not the right sort of argument for the negative adjective *short*. As pointed out by Rullmann (1995), this contrast is unexplained in a degree approach, in which positive and negative degrees are the same objects.

12. Note that if the nominal determiners *many* and *few* are analyzed as positive and negative gradable predicates, respectively (cf. E. Klein 1980), then it should be possible to derive their monotonicity properties in the same way.

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