What an *Average* Semantics Needs

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1. Introduction

Our topic in this paper is the proper analysis of sentences such as (1a-c).

(1) a. The average American has 2.3 children.
b. The average Freddie Voter belongs to 3.2 airline programs. (www.freddieawards.com/events/17/trivial.htm)
c. Although we did not measure this in our study, I can say from other work the average German sees his doctor 13 times a year, the average Swiss sees his doctor 7.5 times a year, and the average Briton 3.5 times. (British National Corpus)

The mystery of (1a-b) is that it seems that they can be true, even though there is no person in the world that is the reference of the noun phrases *the average American* and *the average Freddie Voter*, and no person in the world that has 2.3 children. (1c) makes a similar point, though in a somewhat different fashion: while the property of seeing one's doctor 7.5 times per year is not incoherent in the same way as the property of having 2.3 children, the truth of (1c) does not commit us to the existence of any individuals who actually have this property.

Both linguists and philosophers have used *average* sentences to draw dramatic conclusions of various sorts. For example, Norbert Hornstein and Noam Chomsky have used this class of sentences to argue that semantic theory does not exploit a reference relation, a relation between words and things (Hornstein 1984, Chomsky 2000). Similarly, some philosophers have used these sentences to argue that singular reference is not ontologically committing. Such conclusions are far too hasty. Those who based dramatic conclusions about the nature of semantics or ontological commitment on *average* sentences are unfairly exploiting the lack of a compositional semantics for sentences of this sort. In this paper, we provide the first such semantics. Moreover, we show that within the referential approach to semantics one can provide a detailed compositional semantics of these constructions that reveals much of specific linguistic interest. Specifically, *average* sentences provide new evidence that the compositional system of natural language must allow for the possibility that the nuclear scope of of a scope-taking term may serve as the argument of a third expression, rather than the scope-taking term itself, a relation that Barker (2007) refers to as PARASITIC SCOPE.

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Before proceeding, we want to distinguish the use of *average* in (1), which is the focus of this paper, from the use in examples like (2).

(2) a. Cruise lines have also decided to target a more average american.
(*www.cruisemates.com/forum/viewtopic.php?t=540988*)

b. The most average American is Bob Burns, a 53-year-old building maintenance supervisor in Windham, Conn.
(*www.cbsnews.com/stories/2005/12/13/eveningnews/main1124183.shtml*)

c. As an outsider Bush looks like a perfectly average American to me. He loves nascar and knows no history or geography...
(*www.lioncity.net/buddhism/lofiversion/index.php/t25936.html*)

In these examples, *average* is a gradable predicate (as shown by the acceptability of degree modification) with a meaning roughly equivalent to *typical*, which can replace it without a significant shift in meaning. In contrast, the sense of *average* we are interested is not gradable and is not synonymous with *typical*: substituting *typical* for *average* in (1a), for example, results in a sentence that is anomalous precisely because it does entail the existence of Americans with 2.3 children. See Kennedy and Stanley (2008) for an extended argument that these two uses of *average* are semantically distinct.

2. Previous analyses

We have a number of distinct complaints about each previous approach to the problem of *average* sentences, but due to space considerations, we leave the task of a rigorous account of the shortcomings of previous accounts to other work (see Kennedy and Stanley 2008). Here we focus here on two common failings. First, all previous theories fail to be compositional. Second, all previous theories fail to account for all of the different contexts in which *average* can occur.

According to Carlson and Pelletier (2002), an expression of the form *the average American* denotes a set of properties. The set of properties it denotes is determined by subjecting the denotation of its nominal argument to what they call a partition function part and a special kind of averaging function ave, as spelled out in (3) (where f_c is a contextually restricted variant of f, the property contributed by the noun; see Carlson and Pelletier (2002), p. 92) :

(3) $[\![average]\!]^c = \lambda f.\{Q \mid Q \in \mathbf{ave}(\mathbf{part}(f_c))\}$

The job of the partition function is to take a common noun denotation and yield an object over which the averaging function can operate. As Carlson and Pelletier (p. 91) write, **part** "...has the dual jobs of (a) finding the appropriate partitions of the properties indicated by the CN it is operating on, and (b) for each partition thus constructed, building the set of ordered pairs made up of individual CNs and value-on-that-partition. For example, if we are computing a semantic value for *the average American*, then $\mathbf{part}(american'_c)$ will first determine what the appropriate partitions of properties for American are — for instance, it will pick out ‘height’, ‘weight’, ‘number of children’, ‘food preferences’, etc., for all those types of properties we are used to seeing in reports of the features of average Americans.”

Furthermore, according to Carlson and Pelletier, for each dimension that is relevant for paritioning the set corresponding to the Common Noun denotation, the part function produces a set of ordered pairs $\langle x, v\rangle$ where x is an individual in the partition, and v is that individual’s “most specific value” along the relevant dimension. So, if Kim has 2 children, then $\mathbf{part}(american'_c)$ will include only $\langle Kim, 2\rangle$ along the “number of children” partition, and no other Kim-containing ordered pairs in that partition.

The job of the averaging function ave is then to range over all of these sets of pairs and “do some computation ... to figure out the average value corresponding to each partition” (p. 92). Let’s assume for the moment that this is simply a matter of summing up the values of the second members of each pair and dividing by the cardinality of the partition set. Assuming that *the* is semantically vacuous in such constructions, *the average American* would end up denoting a set of properties, as shown in (4).

(4) $[\![average]\!]([\![American]\!]) = \mathbf{ave}(\mathbf{part}(american'_c)) =$
$\{\lambda x.x \textit{ has 2.3 children}, \lambda x.x \textit{ weighs 150.25 lbs}, \lambda x.x \textit{ is 64 in tall}, \lambda x.x \textit{ is concerned about the economy}, \lambda x.x \textit{ eats too much fast food}, ...\}$

Thus, on Carlson and Pelletier’s analysis, *the average American* ends up denoting a set of properties, the same semantic type as generalized quantifiers.

However, trouble looms when one reflects on how the set of properties is supposed to be generated from partitions, which are sets of person-number pairs. How do we get from the set of ordered pairs $\langle Kim, 2\rangle$, $\langle Hannah, 8\rangle$, $\langle Bill, 3\rangle$, etc. to the property of having 2.3 children? Pairs like $\langle Kim, 2\rangle$ and $\langle Hannah, 8\rangle$ might be relevant for determining properties like having 2.3 children, but they also might be relevant for determining properties like having 2.3 cars. There is no compositional way to go from a set of person-number pairs to a property. There are various repairs one can imagine to respond to this problem, and in other work we discuss them in detail (Kennedy and Stanley 2008). The end result of our discussion is that there is no compositional repair for this problem, short of adopting something very close to our own semantics.

The second problem for Carlson and Pelletier’s theory is that it does not respect the generality of the phenomenon. The abstract interpretation of *average* appears in several construction types in addition to the adjectival form. Two additional (and more colloquial) forms of abstract average are illustrated in (5):

(5) NYU has reported that the 53 teens have lost an average of half of their excess weight over the past year, and that’s truly excellent, considering that [their average weight was 297 pounds] at the beginning!

The examples in (6a-e) illustrate the different ways of expressing the content of the bracketed part of (5) (other word orders are also possible). That these are all instances of abstract average is shown by the fact that the numeral in each example can be felicitously modified by *exactly* and by the fact that each of these examples could be true even if no individual student among the group of 53 weighed 297 lbs

(6) a. The average weight of the teens in the study was 297 lbs.
b. The teens in the study averaged 297 lbs in weight.
c. The teens in the study weighed an average of 297 lbs.
d. The teens in the study weighed on average 297 lbs.
e. The average teen in the study weighed 297 lbs.

However, Carlson and Pelletier's theory is a theory only of the use in (6e). Thus even if it were compositional, it would not be sufficiently general. It tells us nothing about the relation between this use and e.g. the use of *average* as a verb, as in (6b).

Higginbotham's (1985) classic discussion of *average* DPs gets around this problem to some extent by suggesting that prenominal *average* can function as an adverb as well as an adjective, where the former corresponds to its abstract interpretation and the latter to its concrete one. On this view, (1a) should be understand as equivalent to (7):

(7) Americans, on average, have 2.3 children.

Since (7) does not contain a definite description that has to be analyzed as making reference to odd entities, an analysis of (1a) in terms of this structure would bypass Chomsky's metaphysical worries.

But Higginbotham's suggestion does not in fact provide us with a compositional semantics for *average* sentences, and as a result, does not provide us with a response to the worries such constructions pose for referential semantics. The problem becomes clear when we take a closer look at the verb phrase in this example, have 2.3 children. If we assume that numbers are of the category D and combine with NPs to form generalized quantifiers (Barwise and Cooper 1981, Keenan and Stavi 1986), then 2.3 denotes the relation between sets in (8).

(8) $[\![[_D \text{ 2.3}]]\!] = \lambda P \lambda Q. \mid P \cap Q \mid = 2.3$

This meaning will give us the generalized quantifier denotation in (9a) for the DP *2.3 children* (assuming for simplicity that NPs denote sets), which will in turn give us the property in (9b) as the denotation for the VP *have 2.3 children*:

(9) a. $[\![[_{DP} \text{ 2.3 children}]]\!] = \lambda Q. \mid children' \cap Q \mid = 2.3$
b. $[\![[_{VP} \text{ have 2.3 children}]]\!] = \lambda x. \mid children' \cap \{y \mid have'(y)(x)\} \mid = 2.3$

According to (9b), *have 2.3 children* is true of an object x if and only if the cardinality of the intersection of the set of children and the set of things that x has equals 2.3. If this property is a constituent of the logical form of (7), then any way of spelling out the meaning of *on average* that involves predicating the VP of actual individuals is going to entail a commitment to the existence of individuals with fractional children. This is clearly the wrong result, since (7) does not carry such an entailment. Note also that scoping the DP *2.3 children* out of the VP will not help. Such a move would have consequences for the denotation of the second (Q) argument of 2.3, but it would not eliminate the fractional child entailment. The relational semantics for the numeral in (8) requires the cardinality of the intersection of Q, no matter what set it defines, and the set of children to equal 2.3, which will be the case only if there are some fractional children.

In short, merely assuming that pronominal *average* can be interpreted adverbially does not eliminate our problem; we also need to say what the semantic contribution of *have 2.3 children* is, and why this surface constituent doesn't end up introducing a property such as (9b). So Higginbotham's proposal does not in fact amount to a compositional semantics for *average* sentences.

3. Our analysis

3.1. Overview

Consider again the range of contexts in which *average* occurs, illustrated in (6). The generalization that can be drawn from these examples is that, independent of its grammatical category and syntactic position, abstract average requires three semantic arguments: a measure function (here based on the meaning of *weight/weighed*), a domain (provided by the DP *the teens in the study*), and an average, the result of dividing the sum of the values derived by applying the measure function to each object in the domain by the set's cardinality (297 lbs.). In other words, all of the examples in (6) convey the information in (10) (possibly along with other, construction-specific aspects of meaning that we abstract away from here), where **weight** is a function from objects to their weights, T is the set of teens in this particular study, and $297lbs$ is a degree of weight.

(10) $$\frac{\sum_{x \in T} \mathbf{weight}(x)}{|T|} = 297lbs$$

In prose: the sum of the weights we get by applying the **weight** function to all of the objects in T, divided by the number of elements in T is $297lbs$.

Our challenge is to show that we can get from each of the different syntactic forms in (6) to truth conditions equivalent to (10) — and in particular, that we can get from (6d) and (6e) to (10) — without doing violence to generally accepted assumptions about the nature of semantic composition. Our strategy will be to assume that one of the forms in (6) is basic, provide it with a denotation that derives the truth conditions in (10), and show how this basic denotation, together independently justified assumptions about possible Logical Forms and compositional operations on them, can be used to derive appropriate truth conditions for all forms of *average*.

Before proceeding, we want to make explicit two assumptions that we will adopt in order to maximize the clarity of the following exposition. The first is a simplifying assumption: we will treat the domain argument of *average* in all cases as a set, ignoring the fact that the linguistic expression that provides this argument may take different forms (a bare plural, a definite plural, a conjunction structure, a bare noun, etc.) and also ignoring the potentially important contribution of verbal particles like *each, per year* and so forth. It is quite likely that a proper analysis will need to assume that the different forms of *average* actually include mappings from different kinds of expressions to sets (or possibly to more structured objects, such as pluralities), but since it is straightforward to define such mappings and since our more general proposals are consistent with different analytical options here, we will talk in terms of sets in what follows.

Second, although we assume that one of the crucial semantic components of averaging is a measure function (type $\langle e,d\rangle$), as described above, in all of the constructions we examine the actual linguistic terms that provide this component denote degree relations, either type $\langle e,dt\rangle$ (such as the noun *weight*) or type $\langle d,et\rangle$ (such as the verb *weigh*):

(11) a. $[\![weight_N]\!] = \lambda x \lambda d.\mathbf{weight}(x) = d$
b. $[\![weigh_V]\!] = \lambda d \lambda x.\mathbf{weight}(x) = d$

Degree relations (either lexical or derived) can easily be converted into measure functions, however, so we will use the following abbreviatory conventions in our semantic representations to simplify the notation:

(12) a. If $f \in D_{\langle e,dt\rangle}$, then $f_{meas} = \lambda x.max\{d \mid f(x)(d)\}$
b. If $f \in D_{\langle d,et\rangle}$, then $f_{meas} = \lambda x.max\{d \mid f(d)(x)\}$

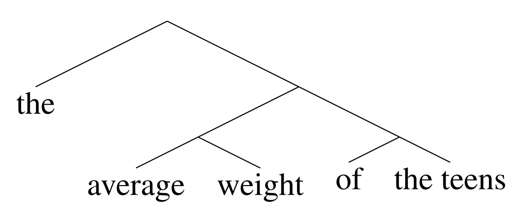
See e.g., Cresswell (1977), Heim (1985), Klein (1991), Carpenter (1997) and Kennedy (to appear) for the use of such conversions in the semantic analysis of comparatives.

3.2. Basic cases

We begin with the assumption that the form of *average* in (6a), which combines directly with a measure noun, reflects the basic meaning of the term. This assumption, while arbitrary, is based on an informal search of the British National Corpus for collocations of *the average, an average* and *on average*, which suggests that the measure noun-modifying form in (6a) is by far the most frequent. Nothing hinges on this particular assumption, however, and our analysis is completely consistent with another (or a more abstract, category-neutral) form being basic.

The structure of a noun phrase containing this form of *average* is as shown in (13) for *the average weight of the teens*.

(13)



Assuming that the noun *weight* denotes the degree relation in (11a) and that the plural DP *the teens* introduces a set as discussed above (which we will abbreviate throughout as $teens'$), this structure indicates that the core meaning of *average* is the function **average** in (14) (where f_{meas} is the measure function based on f, as defined in (12)).

(14) $\textbf{average} = \lambda f \lambda S \lambda d. \dfrac{\sum_{x \in S} f_{meas}(x)}{|S|} = d$

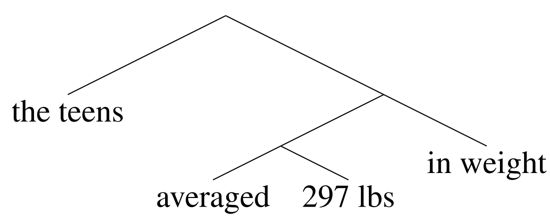
Composition of the nominal portion of (13) gives us (15a), which spells out as the property of degrees in (15b) after lexical insertion and λ-conversion.

(15) a. $\textbf{average}([\![\text{weight}]\!])([\![\text{the teens}]\!])$
b. $\lambda d \left[\dfrac{\sum_{x \in teens'} \textbf{weight}(x)}{|teens'|} = d \right]$

This property is true of a degree if it equals the average weight of the teens, and further composition with the definite article will result in a definite description that picks out the unique degree that satisfies this property. The net result is that (6a) is predicted to be true just in case the average weight of the teens equals the degree denoted by *297 lbs.*, which is exactly what we want.

The verbal form of *average* in (6b) can be analyzed in much the same terms, the only difference being the order of argument composition. Taking the surface syntax as a guide, the verbal form differs from the basic form in selecting the degree argument first, then the measure argument (which can also be implicit if the context is rich enough, as in as in *The teens averaged 297 lbs.*), and finally the domain argument, as shown in (16).

(16)



An appropriate meaning for the verb can then be defined in terms of **average**:

(17) $[\![[_V \text{ average}]]\!] = \lambda d \lambda f \lambda S. \textbf{average}(f)(S)(d)$

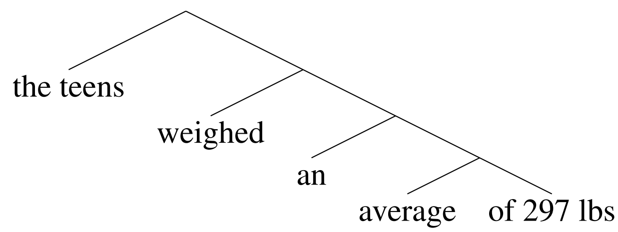
Composition of the various constituents in (16) gives (18a), which maps onto (18b) after lexical insertion and λ-conversion, which is in turn equivalent to (18c).

(18) a. $[\![\text{average}_V]\!]([\![297 \text{ lbs}]\!])([\![\text{weight}]\!])([\![\text{the teens}]\!])$
b. $\textbf{average}([\![\text{weight}]\!])([\![\text{the teens}]\!])([\![297 \text{ lbs}]\!])$
c. $\dfrac{\sum_{x \in teens'} \textbf{weight}(x)}{|teens'|} = 297 lbs$

3.3. Derived degree relations

We now turn to the nominal form of *average* in (6c), which can be analyzed semantically in exactly the same way as verbal *average*, even though its syntactic properties are different. Assuming the structure of (6c) is as shown in (19), the denotation we want is the one in (20).

(19)



(20) $[\![[_N \text{ average}]]\!] = \lambda d \lambda f \lambda S.\mathbf{average}(f)(S)(d)$

Composition is then straightforward: *average* combines first with the measure phrase *297 lbs*, then with the measure verb *weigh*, and finally with the subject, resulting in (21a). (We assume for simplicity here that *an* and *of* are semantically vacuous.)

(21) a. $[\![\text{average}_N]\!]([\![\text{297 lbs}]\!])([\![\text{weigh}]\!])([\![\text{the teens}]\!])$
b. $\mathbf{average}([\![\text{weigh}]\!])([\![\text{the teens}]\!])([\![\text{297 lbs}]\!])$
c. $\dfrac{\sum_{y \in teens'} \mathbf{weight}(y)}{|teens'|} = 297 lbs$

Given the denotation in (20), (21a) is equivalent to (21b), which spells out as (21c) after lexical insertion and λ-conversion. (6c) is thus correctly predicted to be truth-conditionally equivalent to (6a) and (6b).

In examples like (6c), the degree relation that **average** converts into a measure function is lexical, provided directly by the verb *weigh*. However, in many other constructions involving *an average of*, the degree relation is not lexical but instead must be derived in the syntax. (22) is an example of such a construction.

(22) The teens ate an average of 17.5 hamburgers each.

The degree relation we want in order to get the right truth conditions for this example is the relation between quantities *n* and individuals *x* that is true just in case the number of hamburgers that *x* ate equals *n*, which we represent informally in (23).

(23) $\lambda n \lambda x.x \textit{ ate n hamburgers}$

If (23) is supplied as the second argument of nominal *average*, and the plural subject as the third argument, the truth conditions we will ultimately end up with are those represented in (24).

(24) $$\frac{\sum_{y \in teens'} \lambda x.max\{n \mid x \textit{ ate n hamburgers}\}(y)}{|teens'|} = 17.5$$

Given that (24) correctly characterizes the meaning of (22), the question is how we get from the verb phrase *eat an average of 17.5 hamburgers* to the degree relation in (23). In fact, this is exactly the question that we kept running up against in our discussion of previous approaches to *the average NP* in section 2. Recall from that discussion that the problem we confronted was how to avoid interpreting a verb phrase like *have 2.3 children* in a way that didn't entail of any entity that it has 2.3 children, an entailment made by any approach that assumes that numerals are quantificational determiners that combine with nominals to yield generalized quantifiers. In order to derive a degree relation like (23), and avoid these problems, we need to give up this assumption. Instead, we need to recognize that number terms lead a dual life. In addition to their use as quantificational determiners and corresponding relational meanings, they also occur as *singular terms*. As such, they can saturate a degree/quantity position inside the noun phrase, and take scope independently of the rest of the noun phrase in which they occur. (See Kennedy and Stanley (2008) for discussion and defense.)

The hypothesis that numbers saturate an amount/degree argument even in constructions in which they superficially appear to be determiners is developed in great detail by Manfred Krifka (1989, 1992) and used to account for a range of facts involving aspectual composition and the relation between nominal and verbal reference. (See also Cresswell (1977), and for a different implementation of the same idea, see Hackl (2001).) What is important for our purposes is Krifka's analysis of plural count nouns as two place relations between numbers (or degrees/amounts — we do not draw a distinction between these things here) and plural individuals. The plural noun *hamburgers*, on this view, has the denotation in (25) (where the variable x ranges over plural rather than atomic individuals; see Link (1983)).

(25) $[\![hamburgers]\!] = \lambda n \lambda x.hamburgers'(x) \wedge |\, x \,| = n$

Composition with a number returns a property that is true of pluralities of hamburgers whose cardinality is equal to that number. The denotation of *three hamburgers*, for example, is (26).

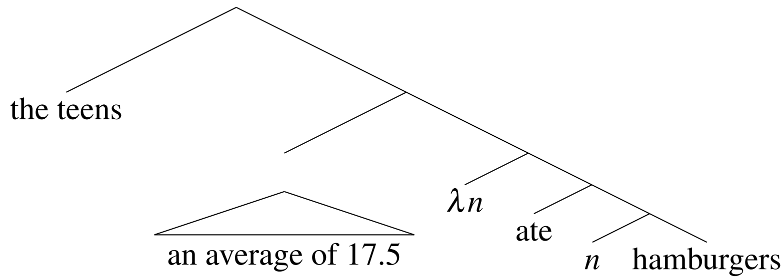
(26) $\lambda x.hamburgers'(x) \wedge |\, x \,| = 3$

This property may then compose with a verb meaning, saturating an open argument, and the variable corresponding to this argument will ultimately be bound by a default existential quantifier, deriving truth conditions that are equivalent to what we get on a standard generalized quantifier semantics.

What is important for our purposes is that on this analysis, a number or other amount term saturates the degree argument of a plural noun, and so can in principle take scope independently of the rest of the noun phrase, leaving a degree variable in its place. This provides us with a straightforward means of deriving the degree relation in (23) and providing a compositional analysis of (22). The analysis runs as follows.

First, we assume that *an average of 17.5* is a constituent in this example that occupies the same syntactic position as a simple number. As such, it may take scope independently of the rest of the noun phrase. A Logical Form that returns the desired truth conditions can be derived by raising *an average of 17* to adjoin to VP:

(27)



Assuming existential closure over the variable introduced by the object, the denotation of the sister of *an average of 17.5* is (28), which is a more precise characterization of the degree relation that we posited earlier in (23).

(28) $\lambda n \lambda x.\exists y[ate'(y)(x) \wedge hamburgers'(y) \wedge \mid y \mid = n]$

Composition may then proceed as described above, deriving the truth conditions in (24). In effect, the LF we are positing for (22) is a variant of the synonymous sentence in (29), which uses verbal *average*.

(29) The teens each averaged 17.5 in number of hamburgers eaten.

Before moving to the next section, we should say a few words about our assumption that *an average of 17* — and by extension, numbers in general — can undergo quantifier raising. While our assumptions about semantic type certainly allow for this option, one might object that the syntax of English does not allow for such structures, pointing to the impossibility of overt extraction of number terms in examples like (30a-b).

(30) a. *How many did they eat *t* hamburgers?
b. *It was 17 that they ate *t* hamburgers.

However, there are other kinds of examples which suggest that English syntax does allow for such structures. One case involves quantity comparisons like (31a), where it is generally assumed that the comparative clause has the structure in (31b) (see e.g., Bresnan 1973, Chomsky 1977, Heim 1985, Hackl 2001, Kennedy 2002).

(31) a. Miller has hit more big shots in playoff games than O'Neal has hit free throws. (*Chicago Tribune, June 3, 2000*)
b. [*wh* O'Neal has hit [*t* free throws]]

Another piece of evidence that the syntax-semantics interface allows an amount term to scope independently of the rest of the DP comes from so-called "reconstruction effects" in *how many* questions (Heycock 1995) like (32), which has the two interpretations paraphrased in (32a) and (32b).

(32) How many people did Jones decide to hire?

a. What is the number of people such that Jones decided to hire them?
b. What is the number such that Jones decided to hire that many people?

Different syntactic and semantic mechanisms have been proposed to derive this ambiguity (see Fox (1999) for discussion of alternatives); what is crucial for us is that the reading in (32b) involves scoping only the amount quantifier above the intensional verb *decide* and interpreting the rest of the nominal in its base position in the embedded clause, which is exactly what we are suggesting for numerals.

3.4. Parasitic scope

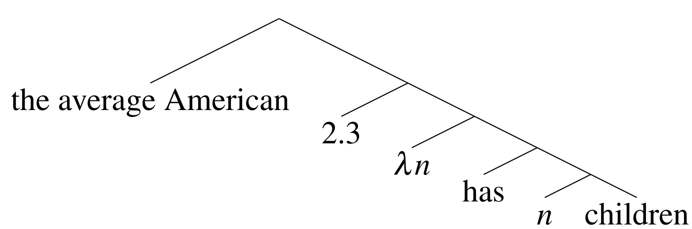
We are now ready to tackle the final two *average* sentences: the adjectival and adverbial forms shown in (33a-b).

(33) a. The average American has 2.3 children.
b. Americans have 2.3 children on average.

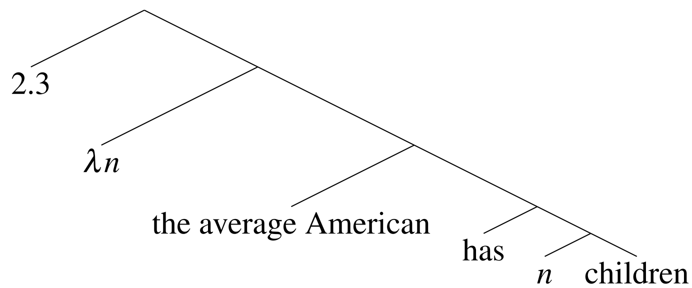
Recall from our discussion in section 2 that a central failing of previous analyses is that they fail to provide a compositional semantics that explains why the verb phrases in these examples do not denote the property of having 2.3 children. Our discussion in the previous section provides an answer to this question: since numbers may take scope independently of the noun phrases in which they occur, these examples can be mapped onto LFs in which the number has been raised out of the VP, leaving behind a constituent of the form *have n children*. We have already seen that this type of constituent can end up supplying the content of a measure function mapping individuals to the number of children they have, which both avoids the problematic entailments created by leaving the number in place, and moreover is exactly what we will need to compute the desired truth conditions for (33a-b).

There is a complication, however, which we illustrate with a discussion of (33a). (The same considerations apply to (33b).) If the number takes scope, then either of (34a-b) are possible LFs for this example.

(34) a.



b.



Unfortunately, neither LF is consistent with either of the possible denotations for adjectival *average* based on the underlying function **average** (both of which assume that the nominal provides the domain argument, as reflected by surface syntax).

(35) a. $[\![[_A \text{ average }]]\!] = \lambda S \lambda f \lambda d.\textbf{average}(f)(S)(d)$
b. $[\![[_A \text{ average }]]\!] = \lambda S \lambda d \lambda f.\textbf{average}(f)(S)(d)$

The problem is that these denotations presume that the average term and the degree relation are provided by distinct syntactic constituents. However, given the standard syntactic definition of QR, the term that corresponds to the former (the raised number) and the term that corresponds to the later (the constituent marked by λn in (34a-b)) form a syntactic constituent exclusive of *the average American*. Compositionality therefore dictates that these two elements also form a semantic constituent, and indeed, this is the normal result of QR. The Logical Forms for (33b) will be identical in the relevant respects, as the number will end up scoping either below the adverb, as in (34a), or above it, as in (34b). What we need is some principle that allows us to "split apart" the average and the degree relation and provide each as separate arguments to *the average American* or *on average*.

Fortunately for us, it turns out that exactly such a mechanism has been independently invoked in order to account for the interpretation of comparative constructions (Heim 1985, Bhatt and Takahashi 2007, Kennedy to appear), distributive interpretations of plural DPs (Sauerland 1998), and noun-modifying uses of *same* and *different* (Barker 2007). Specifically, we need to allow for the possibility that a third constituent can intervene between a scope-taking constituent and the function-denoting constituent that normally serves as its scope, a configuration that Barker (2007) dubs "parasitic scope".

Comparatives like (36) provide a good illustration of parasitic scope.

(36) More people live in New York than Chicago.

Most work on comparatives assumes that *more* and the *than*-phrase form a constituent in Logical Form. There is also a substantial amount of syntactic evidence that the "standard" constituent in an example like (36) (the complement of *than*) can be a simple noun phrase, rather than an underlyingly clausal structure (Hankamer 1973). Such a structure requires the denotation for *more* in (37), which is looking for two individual arguments — a standard of comparison y and a "target" of comparison x — and a degree relation (Heim 1985, Hoeksema 1984, Bhatt and Takahashi 2007, Kennedy 1999, to appear).

(37) $[\![more]\!] = \lambda y \lambda f_{\langle d,et \rangle} \lambda x. f_{meas}(x) \succ f_{meas}(y)$

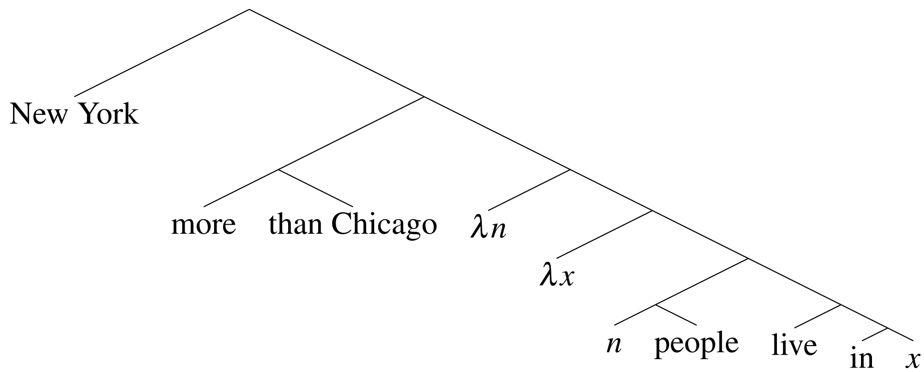
The degree relation is converted into a measure function and applied to the target and standard, so that the truth conditions of a comparative are ultimately stated in terms of an asymmetric ordering between two degrees.

In (36), the standard argument is directly provided by *Chicago* (*than* is typically assumed to be vacuous). In order to derive the right truth conditions, the target argument should be *New York*, and the degree relation should be the one in (38).

(38) $\lambda n \lambda x.n$ *people live in x*

We can derive such a relation by raising both *more than Chicago*, which saturates the degree argument of the plural DP (the same slot occupied by a number), and *New York*. But in order to derive a Logical Form that is interpretable in just the right way, we crucially need to close off the scope of the latter below the former, as shown in (39).

(39)

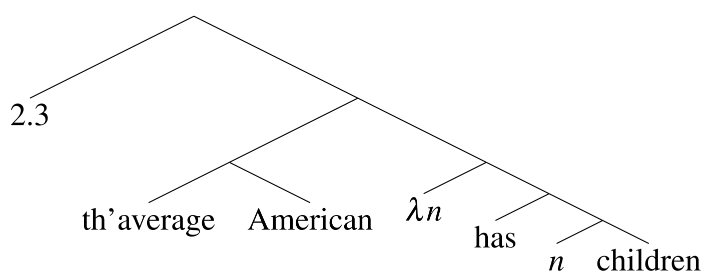


In other words, we need a representation in which the comparative constituent is "parasitic" on the scope term created by QR of the DP *New York*. The resulting LF is fully interpretable, and gives us exactly the truth conditions we want, as shown in the following derivation:

(40) a. $[\![\mathit{more}]\!]([\![\mathit{Chicago}]\!])([\![\lambda n\lambda x.n \text{ } \mathit{people\ live\ in\ x}]\!])([\![\mathit{NY}]\!])$
b. $\lambda x.max\{n \mid n \text{ } \mathit{people\ live\ in\ x}\}(NY') \succ$
$\lambda x.max\{n \mid n \text{ } \mathit{people\ live\ in\ x}\}(Chicago')$

Returning now to *the average American*, the Logical Form we need in order to derive the correct truth conditions for (33a) is one that is parallel in the relevant respects to (39). Specifically, we need a representation in which the number is raised to a position just above *the average American*, while the variable it leaves behind is bound off just below this DP. In other words, we need *the average American* to be parasitic on the scope of the number, as shown in (41).

(41)



Assuming with Carlson and Pelletier (2002) that the definite article in DPs with abstract average is vacuous (indicated in (41) by *th'average*; see Kennedy and Stanley (2008) for a possible explanation of why this is so), composition in (41) gives us (42a), which maps onto (42b) when we plug in the denotation for *average* in (35a), and ultimately spells out as the truth conditions in (42c).

(42) a. $[\![[_A \text{ average}]]\!]([\![\text{American}]\!])([\![\lambda n\lambda x.x \text{ has } n \text{ children}]\!])([\![2.3]\!])$

b. $\textbf{average}([\![\lambda n\lambda x.x \text{ has } n \text{ children}]\!])([\![\text{American}]\!])([\![2.3]\!])$

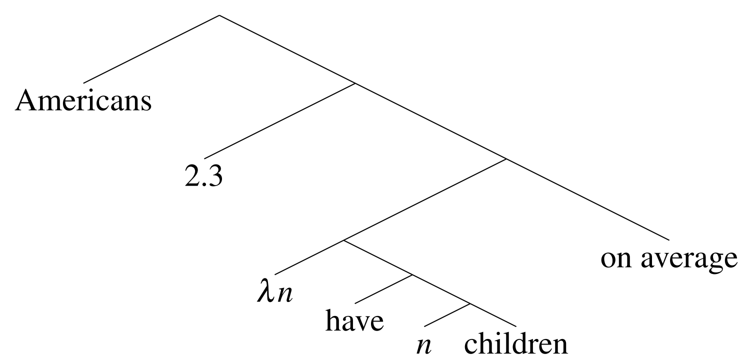
c. $$\frac{\sum_{y\in American'}\lambda z.max\{n \mid z\ has\ n\ children\}(y)}{|American'|} = 2.3$$

Note that this derivation assumes a non-quantificational type d for the number. If we want to assume that numbers undergo QR only if they are assigned a quantificational type, then the derivation would be one in which *2.3* has the type $\langle dt,t\rangle$ denotation $\lambda D_{\langle d,t\rangle}.D(2.3)$ and takes the rest of the sentence (which is type $\langle d,t\rangle$ after composition) as its argument.

Constructions with adverbial *on average*, such as (33b), are analyzed in exactly the same way. Assuming that *on average* attaches to VP (it can be preposed and included in VP-ellipsis), we can analyze its meaning as in (43) and posit the LF in (44) for (33b). (Alternatively, if numbers are type $\langle dt,t\rangle$ when they raise, then *on average* should be assigned a denotation in which the domain comes second and the average term comes third, and *2.3* should take scope above the subject.)

(43) $[\![_{PP} \text{ on average}]\!] = \lambda f\lambda d\lambda S.\textbf{average}(f)(S)(d)$

(44)



(45) shows the derivation of the truth conditions of this LF; once again, we end up with a meaning that is equivalent to what we get in the other *average* constructions.

(45) a. $[\![[_{PP} \text{ on average}]]\!](\lambda n\lambda x.x\ has\ n \text{ children})(2.3)([\![\text{Americans}]\!])$

b. $\textbf{average}(\lambda n\lambda x.x\ has\ n \text{ children})([\![\text{Americans}]\!])(2.3)$

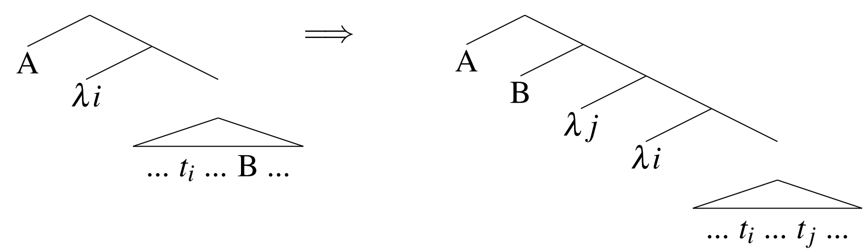
c. $$\frac{\sum_{y\in Americans'}\lambda z.max\{n \mid z\ has\ n\ children\}(y)}{|Americans'|} = 2.3$$

We have thus accounted for our two most difficult cases — *the average NP* and *on average* — without positing any special interpretive mechanisms beyond those that are independently necessary to account for other constructions. The crucial final piece of the analysis is the assumption that natural language allows for the possibility that some expressions can take another expression's nuclear scope as an argument: in Barker's terms, they may be "parasitic" on the scope of another term. The literature on comparatives, plurals and *same/different* that we have cited indicates that such an option must be available to the interpretive system; *average* can be viewed as further evidence for this conclusion.

We have presented our analysis here in terms of the semantic framework developed in Heim and Kratzer (1998), in which scope relations are encoded in a syntactic representation of Logical Form. However, however, our proposals are consistent with "directly compositional" alternatives that reproduce syntactic scope through type- and category-shifting rules. In fact, when we take a closer look at how parasitic scope is implemented in different frameworks, we see that *average* sentences may actually provide an argument in favor of preferring one of these alternative theories of the syntax-semantics interface over an LF-based approach.

To see why let us consider how parasitic scope LFs like (41) and (44) can actually be derived. At first glance, it appears that such representations are a consequence of the grammar of quantifier raising. As pointed out by Sauerland (1998) (see also Bhatt and Takahashi (2007)), given Heim and Kratzer's (1998) formulation of QR, which results in the "index-adjunction" structures that we have represented using as adjoined λs, parasitic scope representations can be derived by two instances of QR, such that the second targets the position just below the first. This is illustrated in (46).

(46)



A feature of this approach to parasitic scope is that it necessarily involves two instances of QR: the first creates the scope term that the second is parasitic on. In the case of *the average NP* sentences, this is not a problem: let the number correspond to the A term in (46) and let *the average NP* correspond to the B term. It is not so clear how to derive the right LFs for *on average*, however, if (as generally assumed) adverbs have fixed positions in the syntax. In order to generate a representation with the structural relations shown in (44), we would need to first raise the number, then insert the adverb. But this creates an ordering paradox: if QR is part of the covert (post-spellout) part of the derivation, then the fact that the adverb is pronounced means that it must be inserted after the number is raised.

Fortunately, there are other ways to derive parasitic scope that do not run into this problem. One option would be to allow for numbers and other terms to appear in scope positions at a level of Logical Form, but to set up the system so that every constituent between the scope-taking term and its base trace is a function over the denotation of the trace. The simplest way of implementing this would be to adopt Jacobson's (1999) variable-free analysis of bound pronouns, and traces as identity functions. In this approach, the principle in (47) (the "Geach rule") has the effect of binding off an unsaturated argument at each higher node.

(47) a. SYNTAX: $\mathbf{g}_{cat}(\mathrm{B}/\mathrm{A}) = \mathrm{B}^{cat}/\mathrm{A}^{cat}$

b. SEMANTICS: If f is a function of type $\langle a,b\rangle$ then $\mathbf{g}_c(f)$ is a function of type $\langle\langle c,a\rangle,\langle c,b\rangle\rangle$, where $\mathbf{g}_c(f) = \lambda h \lambda i.f(h(i))$

Normally, (47) applies successively, so that an argument position occupied by a pronoun remains unsaturated, until a special "binding" rule **z** kicks in. However, nothing rules out the possibility that certain terms are actually looking for a Geached constituent as a matter of lexical category/meaning; such terms would be parasitic on the scope of another term in exactly the sense we need. Assigning the lexical entry in (48) to *on average*, for example, makes it parasitic on the scope of a number:

(48) $$\begin{bmatrix} \textit{on average} & \\ \text{CAT} & (\text{NumP}\backslash(\text{DP}\backslash\text{S}))/(\text{DP}\backslash\text{S})^{NumP} \\ \text{SEM} & \lambda f_{\langle d,et \rangle}\lambda d \lambda S.\textbf{average}(f)(S)(d) \end{bmatrix}$$

This approach clearly needs closer scrutiny, in particular for problems of overgeneration. However, the fact that the status of a particular expression as a "scope parasite" is a matter of lexical specification may provide a means of appropriately restricting the range of constructions in which parasitic scope applies.

Alternatively, we could simply adopt the analysis of parasitic scope developed by Barker (2007), which straightforwardly handles the examples under consideration here. The whole theory can be boiled down to a single structural postulate ("λ" in section 7 of Barker's paper), which can be added to an off-the-shelf type logical grammar and forms the basis of Barker's more general continuation-based theory of scope. We confess to not yet have full enough command of the details of the theory to explore all its predictions, but the fact that it derives parasitic scope as a by product of a more general theory of quantification (and is directly compositional), is certainly a significant point in its favor.

4. Conclusion

In this paper, we have provided a semantics of *average* sentences according to which (morphosyntactically) definite noun phrases of the form *the average NP* are semantically not referring expressions, but rather what we might call "averaging expressions". As such, they do not involve reference to bizarre individuals, and as we have seen, they do not involve predication of impossible properties (like the property of having 2.3 children) of any individuals. Crucially, our analysis is fully compositional, and has an empirical advantage over all previous analyses in extending beyond the adjectival and adverbial forms of *average* and explaining how the various other ways of expressing averages illustrated in (6) give rise to the same core truth conditions. Finally, our account of the interpretation of *the average NP* and its adverbial cousin *on average* provides new evidence that the grammar must include principles that allow for the possibility of parasitic scope.

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