Aspectual composition with degrees

Christopher Piñón*
Hungarian Academy of Sciences

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1 Introduction

Approaches to aspectual composition (most notably Krifka 1989, 1992; Verkuyl 1993) have generally focused on how to account for contrasts such as those in (1) and (2), where compatibility with an temporal *in*-adverbial is taken to indicate a *telic* interpretation, and compatibility with a temporal *for*-adverbial to signal an *atelic* interpretation, of the phrase that the adverbial attaches to. Nothing hinges on the use of the terms *telic* and *atelic* here, and for present purposes, *bounded* and *unbounded* would do equally well.

(1) a. Rebecca ate an apple in five minutes.
b. #Rebecca ate an apple for five minutes.

(2) a. *Rebecca ate apples in thirty minutes.
b. Rebecca ate apples for thirty minutes.

(3) a. Rebecca ate a bowl of applesauce in five minutes.
b. #Rebecca ate a bowl of applesauce for five minutes.

(4) a. *Rebecca ate applesauce in five minutes.
b. Rebecca ate applesauce for five minutes.

Although the prevailing view has been that sentences such as those in (1b) and (3b) are unacceptable, it is also clear that the corresponding sentences in (2a) and (4a) are less acceptable by comparison. The present view, in agreement with Smollett (2005), is that the sentences in (1b) and (3b) are acceptable but require more contextual support than the corresponding ones in (1a) and (3a). Thus, out of the blue (1a) requires no special effort, whereas acceptance of (1b) might lead one to imagine that Rebecca is a small child who hardly ever finishes the apples she is given to eat.[1] In contrast, the sentences in (2a) and (4a) are considered to be unacceptable.

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[1] Observe that judgments improve if the object NP is definite:

(i) Rebecca ate the apple for five minutes (before dropping it on the floor).

(ii) Rebecca ate her apple for five minutes (before dropping it on the floor).

One task of an aspectual theory is to determine the content of terms such as *telic* and *atelic*. In Krifka's (1989; 1992) approach, a VP is telic if the corresponding event predicate is *quantized*, whereas it is atelic if the corresponding event predicate is *cumulative*:

(5) $\mathsf{qua}(P) \stackrel{\text{def}}{=} \forall a \forall b(P(a) \wedge P(b) \rightarrow \neg(a \sqsubset b))$ $\triangleright$ P is quantized

(6) $\mathsf{nuniq}(P) \stackrel{\text{def}}{=} \exists a \exists b(P(a) \wedge P(b) \wedge \neg(a = b))$ $\triangleright$ P is nonunique

(7) $\mathsf{cum}(P) \stackrel{\text{def}}{=} \mathsf{nuniq}(P) \wedge \forall a \forall b((P(a) \wedge P(b)) \rightarrow P(a \oplus b))$ $\triangleright$ P is cumulative

In these definitions, P is a one-place predicate of *events* or *ordinary individuals* (thus a, b stand for events or ordinary individuals), $\sqsubset$ denotes the *proper part* relation, and $\oplus$ designates the *sum* operation. In prose, P is quantized just in case it never applies both to an event or an individual and to a proper part of that event or individual, P is nonunique only if it applies to at least two events or individuals, and P is cumulative just in case it is nonunique and applies to the sum of two events or individuals whenever it applies to each of the two events or individuals independently. Observe that quantization and cumulativity form contraries, hence if P is quantized, it is not cumulative, but P may be neither quantized nor cumulative. Moreover, if P is not quantized, then it is nonunique.[2]

To illustrate how quantization and cumulativity are applied, consider the examples in (1) and (2). In general, tense will be ignored, because it is not crucially relevant to the issues discussed in this paper. The telic VP *eat an apple* in (1a) is taken to denote the set of (minimal) events in which an apple is (completely) eaten. Since no event in which an apple is eaten properly contains an event in which an apple is eaten, the denotation of this VP is quantized (therefore not cumulative). In contrast, the atelic VP *eat apples* in (2b) is assumed to denote the set of events in which one or more apples are eaten. Since the sum of any two events in which one or more apples are eaten is also an event in which one or more apples are eaten, the denotation of this VP is cumulative (hence not quantized). The reasoning is analogous for the telic VP *eat a bowl of applesauce* in (3a) (quantized) versus the atelic VP *eat applesauce* in (4b) (cumulative). According to this line of thinking, the atelic interpretation of *eat an apple* in (1b) should be cumulative, which is the case if the VP on this reading denotes the set of events in which a specific apple is partly eaten.[3] This holds because the sum of two events in which a specific apple is partly eaten is again an event in which that apple is partly eaten: since a specific apple is at issue, it is kept constant.[4]

(ii), in particular, seems unobjectionable. Arguably, the difficulty in (1b) and (3b) is that we need a *specific* apple or bowl of applesauce that is repeatedly partially affected, and yet an existential reading of the object NP does not (automatically) yield this. Once a specific reading is forced (cf. *a certain apple*), the judgements pattern more readily like those of (i) and (ii).

[2]The definition of cumulativity in (7) is not quite equivalent to either the notion of *cumulativity* or that of *strict cumulativity* in Krifka (1989, 1992). The present definition of cumulativity, which appeals to nonuniqueness, ensures that quantization and cumulativity form contraries, even if the denotation of P is empty.

[3]Since Krifka's approach takes sentences such as those in (1a) and (3b) to be unacceptable, this extension is mine.

[4]See fn. 1 in this connection. For the paraphrase to work, *partly* should be understood in the sense of *improper part*, thus a partial eating of an apple is compatible with a complete eating of it. Observe also that if *an apple* were interpreted existentially, the VP would not be cumulative, even if it meant 'partly eat an apple', because in this case the choice of apple might well vary between the two events initially selected.

Beyond characterizing sentences such as those in (1)–(4) in terms of telicity and atelicity and clarifying that these two notions correspond to quantization and cumulativity, respectively, another task of an aspectual theory is to show how these results are achieved compositionally. For example, how does the telicity of *eat an apple* follow from the meaning of *eat* and that of *an apple*? The same may be asked about the atelicity of *eat apples*. Moreover, why does *eat apples* not allow for a telic reading, whereas *eat an apple* does allow for an atelic reading? In Krifka's approach, these results—again, with the exception of examples such as (1b) and (3b), which are considered to be unacceptable—depend on essentially two factors:

(i) whether the NP representing the internal argument of the verb is quantized (e.g., *an apple*) or cumulative (e.g., *apples*);

(ii) whether or not the internal argument of the verb is an *incremental theme* (Dowty 1991; e.g., *eat* versus *like*).

The factor mentioned in (i) is reasonably straightforward, once given the definitions of quantization and cumulativity and an appropriate analysis of the NPs in question. In contrast, the criterium described in (ii), namely, the characterization of an incremental theme, is somewhat involved and is formulated in terms of certain properties of *thematic relations*, where a thematic relation is treated as a two-place relation between ordinary individuals and events. The three central properties of incremental themes for Krifka are *uniqueness of objects*, *mapping to objects* and *mapping to events*. Without going into the formal details, uniqueness of objects states that if x is an incremental theme of e, then x is the unique incremental theme of e. Mapping to objects says that if x is an incremental theme of e, then every subevent e' of e has a part x' of x as its own incremental theme. Conversely, mapping to events states that if x is an incremental theme of e, then every part x' of x is an incremental theme of a subevent e' of e. These three properties specify the core of what it means for a thematic relation to be an incremental theme.[5] Assuming this setup and excluding an iterative interpretation, it can be shown that a VP is telic (or its corresponding event predicate is quantized) if the verb takes an incremental theme and the NP corresponding to the incremental theme is quantized, as in (1a) and (3a). Furthermore, a VP is atelic (or its corresponding event predicate is cumulative) if the verb takes an incremental theme and the NP corresponding to the incremental theme is cumulative, as in (2b) and (4b).[6]

Another aspectual problem, first succinctly described by Dowty (1979, sect. 2.3.5) and then freshly addressed by Hay, Kennedy, and Levin (1999) and Kennedy and Levin (2002), is posed by *degree achievements*:[7]

(8) a. Rebecca lengthened the rope for twenty minutes.
b. The soup cooled for ten minutes.
c. The boat sank for forty minutes.
d. The submarine ascended for thirty minutes.

[5] The incremental theme of verbs of consumption (*eat*) and creation (*write*) additionally satisfy *uniqueness of events*, which states that if x is an incremental theme of e, then x is an incremental theme of no other event but e.

[6] This is a high-level summary of Krifka's account—see Krifka (1989, 1992) for the details.

[7] The term 'degree achievement' is Dowty's. It is retained for the sake of tradition, but degree achievements are actually a kind of accomplishments and not achievements.

For Dowty, the problem posed by these examples was how to treat vagueness and gradual change with his sharp and instantaneous become predicate, a puzzle that he never really managed to resolve. Hay et al. and Kennedy and Levin, equipped with an analysis of gradable adjectives (that Dowty lacked), propose a treatment of degree achievements that aims to do justice to their deadjectival character and to account for their telic uses as well:[8]

(9) a. Rebecca lengthened the rope in twenty minutes.
b. The soup cooled in ten minutes.
c. The boat sank in forty minutes.
d. The submarine ascended in thirty minutes.

Although the account that Hay et al. and Kennedy and Levin propose will be discussed in detail in Section 2.1, their idea is that degree achievements have a 'degree of change' argument d that measures a change in the extent to which an individual has a certain gradable property. Depending how d is specified, the resulting VP is either atelic or telic. For instance, in (8b) there is an unspecified degree of change in the extent to which the soup becomes cool (this corresponds to the existential binding of d), which yields an atelic (cumulative) interpretation, whereas in (9b) the degree of change in the extent to which the soup becomes cool is contextually determined (e.g., the value of d is large enough so that the soup becomes cool enough to eat without the risk of burning one's mouth), which results in a telic (quantized) interpretation.

At first glance, there does not appear to be so much in common between the data in (1)–(4) and those in (8)–(9). More strikingly, perhaps, there seems to be even less in common between Krifka's account and the one proposed by Hay et al. and Kennedy and Levin. Nevertheless, Hay et al. and Kennedy and Levin claim that their degree-based account can be naturally extended to deal with data such as those in (1)–(4) and that it can even do so without the mapping properties that Krifka appeals to in order to characterize incremental themes. As Kennedy and Levin (2002, p. 2) put it, 'In our analysis, quantization/telicity follows completely from the structure of the degree of change argument.' In Section 2.1, I set out to evaluate Kennedy and Levin's claim and conclude that their account is at best incomplete as an analysis of aspectual composition. The question then arises about how to remedy this, and in Section 3 I offer a degree-based alternative that fills in some of the missing details. However, along the way, in Section 2.2, I take a brief look at Caudal and Nicolas's (2005) degree-based account and conclude that it is both formally and conceptually problematic and therefore not yet ready as a viable alternative.

Viewing the matter more broadly, the advantage of a unified degree-based account of aspectual composition would not merely reside in providing a common framework for the treatment of data such as those in (1)–(4) and (8)–(9), but it would also serve to capture more explicitly the intuition that verbs with an incremental theme are *gradable*. This intuition is present in Krifka's analysis, but it is expressed in a way that tends to *conceal* rather than to reveal how this sort of gradability is related to gradability in the adjectival domain. Furthermore, an advantage of having an explicit representation of degrees is that it would make it easier to talk about the degree

[8]Kennedy and Levin distinguish verbs of directed motion (*sink*, *ascend*) from degree achievements (*lengthen*, *cool*), whereas Dowty, as far as I can tell, would regard them all as degree achievements. For present purposes, I take verbs of directed motion to be a species of degree achievements and do not discuss them as a separate class, though they could certainly receive separate attention in a more elaborate treatment.

to which an event type is realized, something that is tricky to formulate in Krifka's approach without introducing degrees in the first place.[9]

In addition to degree achievements and verbs with an incremental theme, degrees have other potential applications in the verbal domain. To mention a few, Kiparsky (2005) argues that the choice of structural case in Finnish (Accusative versus Partitive) depends on the gradability of the verbal predicate. Tamm (2004) proposes that the choice between Total and Partitive case in Estonian is also sensitive to the gradability of the verbal predicate (though Estonian and Finnish differ in certain details). Martin (2006, chap. 8.2) considers the possibility of treating the intriguing difference between French *convaincre* 'convince' and *persuader* 'persuade' in terms of gradability. In previous work (Piñón 2000, 2005), I appealed to degrees to account for *gradually* and adverbs of completion (see also Kennedy and McNally 1999 for the latter). Finally, it should be acknowledged that the idea of using degrees to analyze verbs of gradual change in a formal semantics goes back at least to Ballweg and Frosch (1979) (but, interestingly, Ballweg and Frosch were shy about having degrees represented in their logical language, yet they had them in their model).

2 Two previous accounts: Kennedy and Levin (2002) and Caudal and Nicolas (2005)

2.1 Kennedy and Levin (2002)

In presenting Kennedy and Levin's account,[10] I make use of a four-sorted type-logical language, with sorts for *ordinary individuals* (x, x'), *events* (e, e'), *times* (t, t'), and *degrees* (d, d'). Degrees for Kennedy and Levin are positive or negative intervals on a scale, where a scale is modeled as the set of real numbers between 0 and 1. More precisely, a scale S may be closed, open, closed at 0 (and open at 1), or closed at 1 (and open at 0):

(10) a. $S_{[]} \stackrel{\text{def}}{=} [0,1]$ ▷ S is closed

b. $S_{()} \stackrel{\text{def}}{=} (0,1)$ ▷ S is open

c. $S_{[)} \stackrel{\text{def}}{=} [0,1)$ ▷ S is closed at 0 and open at 1

d. $S_{(]} \stackrel{\text{def}}{=} (0,1]$ ▷ S is closed at 1 and open at 0

Kennedy and Levin speak explicitly only of closed and open scales, i.e., of the cases in (10a) and (10b). However, it seems that half-closed scales play a more important role in their analyses

[9]In Piñón (2000, 2005), my strategy was to show how degrees could be introduced in a Krifka-style analysis without assuming that verbs with an incremental theme have a degree argument to begin with. Although I still think that there is merit in this strategy, the approach that I propose in Section 3 takes such verbs to have a degree argument from the outset, in agreement with Kennedy and Levin in this respect (though the details differ).

[10]My presentation is largely based on Kennedy and Levin (2002), which has its roots in Hay et al. (1999), Kennedy and McNally (1999), and Kennedy (2001). Note that I do not always recite Kennedy and Levin's formulations verbatim and often take the liberty of reformulating certain points. Kennedy and Levin (2007) update their account of degree achievements. Although, for reasons of timing, I do not discuss their updated account here, I believe that the essence of my evaluation largely applies to their updated account as well, given that my points concern more the adequacy of their analysis as an approach to aspectual composition and less the details of how to treat degree achievements.

than open scales (e.g., the scale of length is closed at 0 and open at 1). Indeed, it is unclear whether open scales or scales closed at 1 and open at 0 are ever really needed in their framework. Accordingly, I will restrict my attention to closed scales and scales closed at 0 and open at 1 in the following discussion.

Positive and negative degrees for closed scales and scales closed at 0 and open at 1 are defined as follows, where p is a chosen point on the scale in question:

(11) a. If S is closed or closed at 0 and open at 1: ▷ positive degrees of S

$$\mathsf{pos}(S) \stackrel{\text{def}}{=} \{[0,p] \subseteq S | 0 \leq p\}$$

b. If S is closed: ▷ negative degrees of S

$$\mathsf{neg}(S) \stackrel{\text{def}}{=} \{[p,1] \subseteq S | p \leq 1\}$$

and if S is closed at 0 and open at 1:

$$\mathsf{neg}(S) \stackrel{\text{def}}{=} \{[p,1) \subseteq S | p < 1\}$$

According to the definition in (11a), the minimal positive degree is $[0,0]$ (i.e., 0) if S is closed or closed at 0 and open at 1, the maximal positive degree is $[0,1]$ if S is closed, and there is no maximal positive degree if S is closed at 0 and open at 1, because S does not include 1 in this case. In contrast, the definition in (11b) states that the minimal negative degree is $[1,1]$ (i.e., 1) if S is closed, but there is no minimal negative degree if S is closed at 0 and open at 1. Finally, the maximal negative degree is $[0,1]$ if S is closed, but there is no maximal negative degree if S is closed at 0 and open at 1. Note that negative degrees have nothing to do with negative numbers—the essential difference between positive and negative degrees depends on whether the degrees (as intervals) begin at the bottom of the scale and go upwards (positive degrees) or begin at the end of the scale and go downwards (negative degrees).

As long as S is closed or closed at 0 and open at 1, positive degrees are closed at the right and negative degrees are closed at the left by definition. We can then say that the maximal point of a positive degree is its rightmost point, whereas the maximal point of a negative degree is its leftmost point:

(12) If d is positive: ▷ maximal point of d

$$\mathsf{max}(d) \stackrel{\text{def}}{=} \iota p(p \in d \wedge \neg\exists p'(p' \in d \wedge p < p'))$$

and if d is negative:

$$\mathsf{max}(d) \stackrel{\text{def}}{=} \iota p(p \in d \wedge \neg\exists p'(p' \in d \wedge p' < p))$$

As Kennedy and Levin point out, this model of degrees allows for the 'addition' ($+'$) of degrees to be expressed. In the case of two positive degrees, the idea is that the lengths of the two degrees are added together to yield a greater positive degree. Note that $+$ in (13) stands for arithmetic addition.[11]

[11] This definition differs from Kennedy and Levin's in that it makes explicit the condition that the sum of the two maximal points should not be greater than 1 if S is closed and less than 1 if S is closed at 0 and open at 1, for otherwise the maximal point of the resulting degree would fall 'off the scale', so to speak, which should be avoided. This at once brings out an intuitive difficulty with the formal notion of degree addition appealed to by Kennedy and Levin, namely, that it is neither as general nor as innocent as it initially appears.

(13) For all $d, d' \in \mathsf{pos}(S)$: $\triangleright$ addition of two positive degrees

if S is closed and $\mathsf{max}(d) + \mathsf{max}(d') \leq 1$:

$d +' d' \stackrel{\text{def}}{=} \iota d''(d'' = [0, \mathsf{max}(d) + \mathsf{max}(d')])$

and if S is closed at 0 and open at 1 and $\mathsf{max}(d) + \mathsf{max}(d') < 1$:

$d +' d' \stackrel{\text{def}}{=} \iota d''(d'' = [0, \mathsf{max}(d) + \mathsf{max}(d')])$

For example, if S is closed at 0 and open at 1, d is $[0, .4]$, and d' is $[0, .2]$, then $d +' d'$ is $[0, .6]$, given that $.6 = .4 + .2$ and the condition that .6 is less than 1 is fulfilled.

The addition of a negative degree and a positive degree is less straightforward. However, the intuitive strategy is to increase the length of the negative degree by the length of the positive degree to arrive at a potentially greater negative degree. Formally, this amounts to substracting the maximal point of the positive degree from the maximal point of the negative degree:[12]

(14) $\triangleright$ addition of a negative and a positive degree

For all $d \in \mathsf{neg}(S)$ and $d' \in \mathsf{pos}(S)$ and $0 \leq \mathsf{max}(d) - \mathsf{max}(d')$:

if S is closed:

$d +' d' \stackrel{\text{def}}{=} \iota d''(d'' = [\mathsf{max}(d) - \mathsf{max}(d'), 1])$

and if S is closed at 0 and open at 1:

$d +' d' \stackrel{\text{def}}{=} \iota d''(d'' = [\mathsf{max}(d) - \mathsf{max}(d'), 1))$

As an illustration, if S is closed, d is $[.5, 1]$ (a negative degree), and d' is $[0, .3]$ (a positive degree), then $d +' d'$ is $[.2, 1]$, because $.2 = .5 - .3$ and the condition that 0 is less than or equal to .2 is satisfied.[13]

An advantage of Kennedy and Levin's approach is that it offers an insightful analysis of pairs of gradable adjectives such as *long*/*short*. Kennedy and Levin take such adjectives to denote functions from individuals and times to degrees. For example, *long* is analyzed as the function long, where $\mathsf{long}(x)(t)$ is read as 'the degree to which individual x is long at time t'. To get things off the ground, it also needs to be postulated both that the scale of length is closed at 0 and open at 1 and that degrees of length are positive:

(15) a. $\forall x \forall t \forall d($ $\triangleright$ scale of length is closed at 0 and open at 1
$\mathsf{long}(x)(t) = d \rightarrow d \subseteq [0, 1))$

b. $\forall x \forall t \forall d(\mathsf{long}(x)(t) = d \rightarrow \exists p(d = [0, p]))$ $\triangleright$ degrees of length are positive

[12]This definition differs in two respects from Kennedy and Levin's. Firstly, and less importantly, it has two subcases, depending on whether whether S is closed or closed at 0 and open at 1, whereas Kennedy and Levin's assumes that S is closed (and the definition in Hay et al. 1999 assumes that S is closed at 0 and open at 1). Secondly, and more importantly, it makes explicit the condition that the difference of the maximal point of the negative degree and the maximal point of the positive degree should be at least 0, for if it were not, the maximal point of the resulting negative degree would also fall 'off the scale', though this time to the left, similarly to be avoided. Again, this reaffirms the intuitive difficulty mentioned in fn. 11.

[13]Hay et al. (1999) claim that the addition of two negative degrees is undefined. Although this may be desirable for empirical reasons, the technical apparatus would certainly allow for the addition of two negative degrees to be defined, even if it would have to be restricted in a way similar to how the addition of other degrees has to be (see fns. 11 and 12).

The meaning of *short* can then be defined in terms of the meaning of *long*: the degree to which x is short at t is identical to the negative degree d whose maximal point is equal to the maximal point of the (positive) degree d' to which x is long at t, as shown in (16).

(16) $\triangleright$ degrees of shortness are negative degrees of length

$$\mathsf{short}(x)(t) \stackrel{\text{def}}{=} \iota d(\exists p(d = [p,1)) \wedge \mathsf{max}(d) = \mathsf{max}(\iota d'(\mathsf{long}(x)(t) = d')))$$

For example, suppose that the (positive) degree to which x is long at t is $[0,.4]$. Then, according to this definition, the (negative) degree to which x is short at t is $[.4,1)$, because the maximal point of $[0,.4]$ is .4. In general, a hallmark of Kennedy and Levin's approach is to model what appear to be lesser degrees (e.g., 'x is shorter than y (at t)') and decreases in degrees (e.g., 'x is shortened') with respect to some property (e.g., length) as in fact greater negative degrees and increases in negative degrees, respectively, with respect to that property.

2.1.1 Kennedy and Levin's aspectual account

Kennedy and Levin propose that verbs of gradual change, i.e., degree achievements and verbs with an incremental theme, be analyzed with the help of a certain three-place relation between individuals, degrees, and events that is composed of a predicate $\mathsf{increase}$ and a gradable predicate constant G, as seen in (17a). The definition of $\mathsf{increase}$, a four-place relation between gradable predicates, individuals, degrees, and events, is given in (17b).

(17) a. $\lambda x \lambda d \lambda e.\mathsf{increase}(\mathsf{G}(x))(d)(e)$ $\triangleright$ format for verbs of gradual change

b. $\mathsf{increase}(\mathsf{G}(x))(d)(e) \stackrel{\text{def}}{=} (\mathsf{G}(x)(\mathsf{end}(e)) = \mathsf{G}(x)(\mathsf{beg}(e)) +' d)$

The gradable predicate G denotes a function that takes an individual x and a time t and yields the degree d to which G holds of x at t. This can be made more explicit with the help of the iota operator:[14]

(18) $\lambda x \lambda t.\iota d(\mathsf{G}(x)(t) = d)$ $\triangleright$ format for gradable predicates G

This, in turn, allows for a more explicit rendition of the formulas in (17):

(19) a. $\lambda x \lambda d \lambda e.\mathsf{increase}(\lambda t.\iota d'(\mathsf{G}(x)(t) = d'))(d)(e)$ $\triangleright$ cf. (17a)

b. $\mathsf{increase}(\lambda t.\iota d'(\mathsf{G}(x)(t) = d'))(d)(e) \stackrel{\text{def}}{=}$ $\triangleright$ cf. (17b)
$\iota d'(\mathsf{G}(x)(\mathsf{end}(e)) = d') = \iota d''(\mathsf{G}(x)(\mathsf{beg}(e)) = d'') +' d$

In prose, the definition in (19b) states that the degree to which the gradable predicate G holds of the individual x increases by the degree d in the event e just in case the degree to which G holds of x at the end of e is equal to the degree to which G holds of x at the beginning of e plus d. In other words, d, the so-called degree of change, signals the increase in the degree to which G holds of x in e. Note that the degree of change is always a positive degree.

[14] Kennedy and Levin do not make use of the iota operator. However, although a syntactic addition, the iota operator does not add anything on the semantic side that they are not already committed to. Once the logical language contains function symbols (e.g., G in (17b)), issues of definedness arise, and so it is not the use of the iota operator in combination with function symbols that raises them.

Kennedy and Levin's idea is that the relation in (19a) constitutes the common semantic element between degree achievements and verbs with an incremental theme—what differs is merely how G is instantiated. In each case, the meaning of the verb of gradual change is based on the meaning of a corresponding adjective that instantiates G. As an illustration, two VPs containing a degree achievement are analyzed in (20) and (21), and two containing a verb with an incremental theme are represented in (22) and (23).[15]

(20) a. long $\rightsquigarrow \lambda x \lambda t.\iota d(\mathsf{long}(x)(t) = d)$ ▷ gradable adjective
b. $[_{\mathrm{VP}}$ lengthen x (by d-much)$]$ $\rightsquigarrow$ ▷ degree achievement
$\lambda e.\mathsf{increase}(\lambda t.\iota d'(\mathsf{long}(x)(t) = d'))(d)(e)$

(21) a. short $\rightsquigarrow \lambda x \lambda t.\iota d(\mathsf{short}(x)(t) = d)$ ▷ gradable adjective; cf. (16)
b. $[_{\mathrm{VP}}$ shorten x (by d-much)$]$ $\rightsquigarrow$ ▷ degree achievement
$\lambda e.\mathsf{increase}(\lambda t.\iota d'(\mathsf{short}(x)(t) = d'))(d)(e)$

(22) a. written $\rightsquigarrow \lambda x \lambda t.\iota d(\mathsf{written}(x)(t) = d)$ ▷ gradable adjective
b. $[_{\mathrm{VP}}$ write (d-much) of $x]$ $\rightsquigarrow$ ▷ verb of creation
$\lambda e.\mathsf{increase}(\lambda t.\iota d'(\mathsf{written}(x)(t) = d'))(d)(e)$

(23) a. eaten $\rightsquigarrow \lambda x \lambda t.\iota d(\mathsf{eaten}(x)(t) = d)$ ▷ gradable adjective
b. $[_{\mathrm{VP}}$ eat (d-much) of $x]$ $\rightsquigarrow$ ▷ verb of destruction
$\lambda e.\mathsf{increase}(\lambda t.\iota d'(\mathsf{eaten}(x)(t) = d'))(d)(e)$

Notice that just as the meaning of *shorten* is based on the meaning of *short*, which denotes a function from individuals and times to (negative) degrees ($\mathsf{short}(x)(t)$ 'the degree to which x is short at t'—recall (15)), the meaning of *eat* is based on the meaning of *eaten*, which likewise denotes a function from individuals and times to (negative) degrees ($\mathsf{eaten}(x)(t)$ 'the degree to which x is eaten at t'). Furthermore, given the definition in (19b), the formulas in (20b), (21b), (22b), and (23b) reduce to the following:

(24) a. $\lambda e.\iota d'(\mathsf{long}(x)(\mathsf{end}(e)) = d') =$ ▷ reduction of (20b)
$\iota d''(\mathsf{long}(x)(\mathsf{beg}(e)) = d'') +' d$
b. $\lambda e.\iota d'(\mathsf{short}(x)(\mathsf{end}(e)) = d') =$ ▷ reduction of (21b)
$\iota d''(\mathsf{short}(x)(\mathsf{beg}(e)) = d'') +' d$
c. $\lambda e.\iota d'(\mathsf{written}(x)(\mathsf{end}(e)) = d') =$ ▷ reduction of (22b)
$\iota d''(\mathsf{written}(x)(\mathsf{beg}(e)) = d'') +' d$
d. $\lambda e.\iota d'(\mathsf{eaten}(x)(\mathsf{end}(e)) = d') =$ ▷ reduction of (23b)
$\iota d''(\mathsf{eaten}(x)(\mathsf{beg}(e)) = d'') +' d$

As Kennedy and Levin observe, the degree of change is not always syntactically expressed. In the case of degree achievements, it may be expressed by an explicit measure expression, as in

[15] Technically, the meaning of *lengthen* could be derived via the functional application of increase to long in (20a):

(i) $[\lambda G \lambda x \lambda d \lambda e.\mathsf{increase}(G(x))(d)(e)](\mathsf{long}) =$ ▷ by functional conversion
$\lambda x \lambda d \lambda e.\mathsf{increase}(\lambda t.\iota d'(\mathsf{long}(x)(t) = d'))(d)(e)$

The meanings of the verbs in (21b), (22b), and (23b) could be derived in a similar fashion.

(25a)[16] and (26a), but it may also remain implicit, as in (25b) and (26b), where it is existentially bound inside the VP.[17]

(25) a. [$_{\text{VP}}$ lengthen the rope (by) ten centimeters] $\rightsquigarrow$
$\lambda e.\mathsf{increase}(\lambda t.\iota d'(\mathsf{long}(\mathsf{the\text{-}rope})(t) = d'))(\mathsf{10\text{-}cm})(e)$

b. [$_{\text{VP}}$ lengthen the rope]^{atel} $\rightsquigarrow$
$\lambda e.\exists d(\mathsf{increase}(\lambda t.\iota d'(\mathsf{long}(\mathsf{the\text{-}rope})(t) = d'))(d)(e))$

(26) a. [$_{\text{VP}}$ eat half of the apple] $\rightsquigarrow$
$\lambda e.\mathsf{increase}(\lambda t.\iota d'(\mathsf{eaten}(\mathsf{the\text{-}apple})(t) = d'))(.5)(e)$

b. [$_{\text{VP}}$ eat the apple]^{atel} $\rightsquigarrow$
$\lambda e.\exists d(\mathsf{increase}(\lambda t.\iota d'(\mathsf{eaten}(\mathsf{the\text{-}apple})(t) = d'))(d)(e))$

Kennedy and Levin also allow for the degree argument to remain free inside the VP (and possibly existentially bound from outside it), in contrast to the setup in (25b) and (26b), where it is existentially bound inside the VP. The degree argument remains free within the VP in the following variations on (25b) and (26b):

(27) a. [$_{\text{VP}}$ lengthen the rope]^{tel} $\rightsquigarrow$
$\lambda e.\mathsf{increase}(\lambda t.\iota d'(\mathsf{long}(\mathsf{the\text{-}rope})(t) = d'))(d)(e)$

b. [$_{\text{VP}}$ eat the apple]^{tel} $\rightsquigarrow$
$\lambda e.\mathsf{increase}(\lambda t.\iota d'(\mathsf{eaten}(\mathsf{the\text{-}apple})(t) = d'))(d)(e)$

Observe that the event predicate representing the VP is quantized if the value of the degree argument is fixed within the VP:

(28) a. $\mathsf{qua}(\lambda e.\mathsf{increase}(\lambda t.\iota d'(\mathsf{long}(\mathsf{the\text{-}rope})(t) = d'))(\mathsf{10\text{-}cm})(e))$ ▷ cf. (25a)
$\mathsf{qua}(\lambda e.\mathsf{increase}(\lambda t.\iota d'(\mathsf{long}(\mathsf{the\text{-}rope})(t) = d'))(d)(e))$ ▷ cf. (27a)

b. $\mathsf{nuniq}(\lambda e.\exists d($ ▷ cf. (25b)
$\mathsf{increase}(\lambda t.\iota d'(\mathsf{long}(\mathsf{the\text{-}rope})(t) = d'))(d)(e))) \rightarrow$
$\mathsf{cum}(\lambda e.\exists d(\mathsf{increase}(\lambda t.\iota d'(\mathsf{long}(\mathsf{the\text{-}rope})(t) = d'))(d)(e)))$

(29) a. $\mathsf{qua}(\lambda e.\mathsf{increase}(\lambda t.\iota d'(\mathsf{eaten}(\mathsf{the\text{-}apple})(t) = d'))(.5)(e))$ ▷ cf. (26a)
$\mathsf{qua}(\lambda e.\mathsf{increase}(\lambda t.\iota d'(\mathsf{eaten}(\mathsf{the\text{-}apple})(t) = d'))(d)(e))$ ▷ cf. (27b)

b. $\mathsf{nuniq}(\lambda e.\exists d($ ▷ cf. (26b)
$\mathsf{increase}(\lambda t.\iota d'(\mathsf{eaten}(\mathsf{the\text{-}apple})(t) = d'))(d)(e))) \rightarrow$
$\mathsf{cum}(\lambda e.\exists d(\mathsf{increase}(\lambda t.\iota d'(\mathsf{eaten}(\mathsf{the\text{-}apple})(t) = d'))(d)(e)))$

[16] A background worry is how the meaning of the term 10-cm for *(by) ten centimeters* in (25a) relates to the scale of length, which is closed at 0 and open at 1. Strictly speaking, positive degrees in this case should be intervals in $[0,1)$ (see (11a) and (15)) and not lengths measured in terms of centimeters. For this to be intelligible, the length of ten centimeters should correspond to a positive degree in $[0,1)$, yet it is not evident which degree this should be. There may be a straightforward reply to this worry; at the same time, it is easy to suspect that degrees in Kennedy and Levin's account actually play a double role, as (i) indicators of degree of realization and (ii) measurements of particular extents. The question of degree addition (see fns. 11 and 12) is less problematic if the latter is the intended role for degrees. The resolution of this worry in the approach that I propose in Section 3 consists in sharply distinguishing these two roles.

[17] The superscript '*atel*' in (25b) and (26b) simply serves to mark the atelic interpretation of the VPs, thereby distinguishing them from those in (27a) and (27b) below, which are telic (and are marked with '*tel*').

For instance, consider the statements in (28). The event predicate in (25a) is quantized because any event in which the rope is lengthened by ten centimeters lacks a proper subevent in which it is lengthened by ten centimeters—in any proper subevent, it is at most lengthened by less than ten centimeters. Similarly, the event predicate in (27a) is quantized because any event in which the rope is lengthened by d-much lacks a proper subevent in which it is lengthened by d-much, where the value of d is implicit but fixed for the VP. In contrast, the event predicate in (25b) is cumulative—provided that it is nonunique—because the sum of any two events in which the rope is lengthened by some amount d is also an event in which the rope is lengthened by some (greater) amount d, where the value of d may vary with each event chosen. The reasoning behind the statements in (29) is analogous.

2.1.2 Evaluation

From the present perspective, the central question is to what extent Kennedy and Levin's account includes an analysis of aspectual composition in terms of gradability. Or, to recall their claim cited in Section 1, to what extent does quantization/telicity follow completely from the structure of the degree of change argument? At first glance, their account seems to fare well, because the characterizations in (28) and (29) are unobjectionable. However, as mentioned in Section 1, it is also desirable for an aspectual theory to derive these results, and here their account leaves something to be desired.

Consider how the following two telic VPs (ignoring the *in*-PPs) might be derived in Kennedy and Levin's framework:

(30) a. $[_{\text{VP}}$ eat the apple$]^{tel}$ (in five minutes)
b. $[_{\text{VP}}$ write a letter$]^{tel}$ (in twenty minutes)

Beginning with (30a), since the definite object NP *the apple* may be treated as a term, the meaning of *eat* can be applied to the meaning of *the apple*:[18]

(31) a. eat $\rightsquigarrow \lambda x \lambda d \lambda e.\iota d'(\mathsf{eaten}(x)(\mathsf{end}(e)) = d') =$
$\iota d''(\mathsf{eaten}(x)(\mathsf{beg}(e)) = d'') +' d$
b. the apple $\rightsquigarrow$ $\mathsf{the\text{-}apple}$
c. $[_{\text{VP}}$ eat the apple$]$ $\rightsquigarrow$
$\lambda d \lambda e.\iota d'(\mathsf{eaten}(\mathsf{the\text{-}apple})(\mathsf{end}(e)) = d') =$
$\iota d''(\mathsf{eaten}(\mathsf{the\text{-}apple})(\mathsf{beg}(e)) = d'') +' d$

A technical issue with the result in (31c) is that the degree argument has not yet been discharged. Allowing for a default mechanism of existential binding of the degree argument at the VP level (as Kennedy and Levin in fact do), we obtain the following event predicate:

(32) $[_{\text{VP}}$ eat the apple$]^{atel}$ $\rightsquigarrow$ ▷ atelic reading (cf. (1b), (3b))
$\lambda e.\exists d(\iota d'(\mathsf{eaten}(\mathsf{the\text{-}apple})(\mathsf{end}(e)) = d') =$

[18] For present purposes, nothing depends on whether *the apple* is analyzed as a term, a predicate (namely, $\lambda x.x =$ $\mathsf{the\text{-}apple}$), or even a generalized quantifier. Accordingly, nothing requires the verb to be treated as a functor for the object NP—cf. (35) below.

$\iota d''(\mathsf{eaten}(\mathsf{the\text{-}apple})(\mathsf{beg}(e)) = d'') +' d)$

Observe that this event predicate is cumulative as long as it is nonunique (cf. (29b)): the events in its denotation are events in which some amount d of the apple is eaten, and the sum of any two of these is also an event in which some (larger) amount d of the apple is eaten. This corresponds to the atelic reading of the VP, which is also the reading characteristic of (1b) and (3b) in Section 1.

Yet the issue is still how to derive the telic interpretion of (30a), for which there appear to be two potential strategies. The first would be to have the degree argument remain free within the VP, because this would yield a quantized event predicate (cf. (29a)):

(33) $[_{\text{VP}}$ eat the apple$]^{tel} \rightsquigarrow$ ▷ telic reading, first version (cf. (27b))

$$\lambda e.\iota d'(\mathsf{eaten}(\mathsf{the\text{-}apple})(\mathsf{end}(e)) = d') =$$
$$\iota d''(\mathsf{eaten}(\mathsf{the\text{-}apple})(\mathsf{beg}(e)) = d'') +' d$$

Although this event predicate is indeed quantized, it does not capture the intuitive meaning of the VP, because the events denoted in (33) are those in which d-much of the apple is eaten, for a fixed value of d, but nothing forces the value of d to be maximal (or nearly maximal, allowing for a certain vagueness). Yet, intuitively, the meaning of the VP applies to events in which the apple is wholly eaten. Consequently, the first potential strategy, according to which the degree argument is free and implicitly specified, as in (33), would be in general too weak.[19]

The second strategy would be to set the value of the degree of change to be maximal, which would yield the interpretation on which all of the apple is eaten. Technically, Kennedy and Levin treat eaten as a 'negative property', which means that the possible values are negative degrees. Furthermore, although Kennedy and Levin do not state this explicitly, the scale defined by eaten is evidently closed. Consequently, degrees of 'being eaten' have the form $[p, 1]$, for a choice of p (recall (11b)), and the maximal degree is $[0, 1]$. However, to get this as the result, the degree of change (which is always a positive degree) should also be set to $[0, 1]$:[20]

(34) $[_{\text{VP}}$ eat the apple$]^{tel} \rightsquigarrow$ ▷ telic reading, final version

$$\lambda e.\iota d'(\mathsf{eaten}(\mathsf{the\text{-}apple})(\mathsf{end}(e)) = d') =$$
$$\iota d''(\mathsf{eaten}(\mathsf{the\text{-}apple})(\mathsf{beg}(e)) = d'') +' [0, 1]$$

Since the degree of change in (34) is specified as $[0, 1]$, it follows that the degree to which the apple is eaten at the beginning of an event in the denotation of this predicate is $[1, 1]$ (or 1), which is the minimal negative degree. In other words, none of the apple is eaten at the beginning of such an event, and it is fully eaten by the end.

Although the second strategy would yield the desired result, it is a bit unclear how to obtain it in Kennedy and Levin's framework. More precisely, how does d, which is an argument of the verb, get specified as $[0, 1]$? More generally, if the value of d is not explicitly specified, Kennedy and Levin suggest that it may be inferred on the basis of the lexical semantics of the verb or its

[19]Note that this is not a problem in the case of degree achievements. For example, the meaning of *lengthen the rope* (cf. (27a)) does not require the degree of change d to be maximal—indeed, given that the scale of length is closed at 0 and open at 1 (cf. (15)), d could not have a maximal value anyway.

[20]Then the first clause in (14) would apply, where d is $[1, 1]$ (the minimal negative degree) and d' is $[0, 1]$ (the maximal positive degree of change), therefore $d +' d' = [1, 1] +' [0, 1] = [0, 1]$ (the maximal negative degree).

arguments, or even on the basis of extralinguistic knowledge. In the case of (31c), then, the idea seems to be that since *the apple* is quantized, it is natural to infer that the value of d is maximal. However, unless more is said about how this inference mechanism works in conjunction with representations like that in (31c), the account is missing something.

The telic VP in (30b) poses the same problem, though here the object NP is indefinite. If *a letter* is analyzed as a generalized quantifier, as in (35b), it can apply to the meaning of *write* in (35a) to yield the relation between events and degrees in (35c).

(35) a. write $\rightsquigarrow \lambda x \lambda d \lambda e. \iota d'(\mathsf{written}(x)(\mathsf{end}(e)) = d') =$
$\iota d''(\mathsf{written}(x)(\mathsf{beg}(e)) = d'') +' d$

b. a letter $\rightsquigarrow \lambda R \lambda d \lambda e. \exists x(R(x)(d)(e) \wedge \mathsf{letter}(x))$

c. [$_{\mathrm{VP}}$ write a letter] $\rightsquigarrow \lambda d \lambda e. \exists x(\iota d'(\mathsf{written}(x)(\mathsf{end}(e)) = d') =$
$\iota d''(\mathsf{written}(x)(\mathsf{beg}(e)) = d'') +' d \wedge \mathsf{letter}(x))$

As before, the problem is how to discharge d and to restrict its value to be maximal in the absence of an explicit degree expression. Moreover, as seen earlier in connection with (32), it would not suffice simply to existentially bind d at the VP level, because this would yield an atelic VP.

Suppose that we have a mechanism for setting the degree of change to be maximal. Then nothing would restrain it from applying in the case of atelic VPs, illustrated in (36).

(36) a. [$_{\mathrm{VP}}$ eat applesauce] (for five minutes)

b. [$_{\mathrm{VP}}$ write letters] (for two hours)

Consider how the VP in (36a) might be derived, treating the object NP *applesauce* as a generalized quantifier (cf. (35b)):

(37) a. eat $\rightsquigarrow \lambda x \lambda d \lambda e. \iota d'(\mathsf{eaten}(x)(\mathsf{end}(e)) = d') =$
$\iota d''(\mathsf{eaten}(x)(\mathsf{beg}(e)) = d'') +' d$

b. applesauce $\rightsquigarrow \lambda R \lambda d \lambda e. \exists x(R(x)(d)(e) \wedge \mathsf{applesauce}(x))$

c. [$_{\mathrm{VP}}$ eat applesauce] $\rightsquigarrow$
$\lambda d \lambda e. \exists x(\iota d'(\mathsf{eaten}(x)(\mathsf{end}(e)) = d') =$
$\iota d''(\mathsf{eaten}(x)(\mathsf{beg}(e)) = d'') +' d \wedge \mathsf{applesauce}(x))$

Given the relation between events e and degrees d in (37c), one possibility would be to existentially bind d at the VP level:

(38) [$_{\mathrm{VP}}$ eat applesauce] $\rightsquigarrow$ ▷ first version
$\lambda e. \exists d \exists x(\iota d'(\mathsf{eaten}(x)(\mathsf{end}(e)) = d') =$
$\iota d''(\mathsf{eaten}(x)(\mathsf{beg}(e)) = d'') +' d \wedge \mathsf{applesauce}(x))$

This event predicate is cumulative (provided that it is nonunique), hence this analysis would successfully capture the fact that the VP *eat applesauce* is atelic. In prose, this predicate denotes events in which some amount d of some applesauce is eaten, and the sum of any two such events is also an event in which some (greater) amount d of some (greater quantity of) applesauce is eaten. However, once a mechanism for fixing the value of d to be $[0,1]$ is available, it could apply to the relation in (37c) to yield the following event predicate (cf. (34)):

(39) $[_{\text{VP}}$ eat applesauce$] \rightsquigarrow$ ▷ second version

$$\lambda e.\exists x(\iota d'(\mathsf{eaten}(x)(\mathsf{end}(e)) = d') = \iota d''(\mathsf{eaten}(x)(\mathsf{beg}(e)) = d'') +' [0,1] \wedge \mathsf{applesauce}(x))$$

Perhaps somewhat surprisingly, this event predicate is also cumulative (provided that it is non-unique): it denotes events in which some applesauce is maximally eaten, but the sum of any two events in which some applesauce is maximally eaten is also an event in which some (greater quantity of) applesauce is maximally eaten. Of course, the quantity of applesauce is not held constant here, but nor does it have to be, given that x is existentially bound in (39).

The contrast between the event predicate in (34), which is quantized (telic) and specifies a maximal degree of change, on the one hand, and the event predicate in (39), which is cumulative (atelic) and specifies a maximal degree of change, on the other, sharply demonstrates that quantization/telicity does *not* follow completely from the structure of the degree of change argument, contrary to Kennedy and Levin's initial claim. Clearly, the choice of object NP is a crucial factor here.[21] In the case of *the apple*, which is quantized, the way in which the degree of change is specified *does* matter, as seen in the contrast between (32) (cumulative, atelic) and (34) (quantized, telic). However, in the case of *applesauce*, which is cumulative, the way in which the degree of change is specified does *not* matter, as witnessed in the *lack* of contrast between (38) and (39) (both cumulative, atelic).

The same issue arises for the atelic VP in (36b), which contains the cumulative object NP *letters*. No matter how the degree of change is specified, the corresponding event predicate is cumulative, which differs from the VP in (35c) with the quantized object NP *a letter*, for here the way in which the degree of change is specified does determine whether the VP is telic or atelic.

In sum, Kennedy and Levin's account does not quite succeed in offering an analysis of aspectual composition in terms of gradability, contrary to initial claims and appearances, because it does not relate the value of the degree of change argument to the quantized/cumulative character of the object NP. Related to this is the point that their analysis also does not allow characterizations such as those in (28) and (29) to be strictly deduced. Naturally, this conclusion does not entail that their account is on the wrong track, but it does mean that as presently formulated it is incomplete in an important respect.

2.2 Caudal and Nicolas (2005)

Caudal and Nicolas propose a degree-based aspectual alternative to the approaches by Kennedy and Levin, Krifka, and Verkuyl, hence it is relevant to the present discussion. Although they make a number of useful observations and aim to account for a wide range of facts, their proposed alternative is problematic in terms of its conceptual and formal development. My comments will be relatively brief, and I will focus on their analysis of *John eat an apple* (again,

[21] Rothstein (2004, p. 118) makes a similar point in a critique of Kennedy and Levin (2002) based on examples such as *The tailor lengthened skirts five centimeters for three months*, claiming that Kennedy and Levin's account is not a theory of telicity. Although I basically agree with Rothstein on this point, it is also evident that she has little sympathy for a degree-based account to begin with, given her own non-degree-based approach to promote. Moreover, as far as I can tell, she does not actually show how degree achievements would be treated in her framework.

ignoring tense), because the treatment of this kind of example brings their discussion closest to the present one:

(40) John eat an apple $\rightsquigarrow$ $\triangleright$ based on Caudal and Nicolas's (55)
$\exists x \exists e \exists d(\mathsf{eat}(e)(d) \wedge \mathsf{become}(\mathsf{eat}) \wedge \mathsf{quantity}(x)(d) \wedge \mathsf{agent}(\mathsf{john})(e) \wedge \mathsf{patient}(x)(e) \wedge \mathsf{apple}(x))$

This formula states that there is an individual x, an event e, and a degree d such that e is an eating to degree d, the event type 'eat' is a becoming, the quantity of x is d, the agent of e is John, and the patient of e is x, which is an apple. Setting aside the two thematic relations agent and patient,[22] the crux of the matter comes down to the interpretation of eat, become and quantity in order to understand the formula in (40). In this connection, it is also vital to ask about Caudal and Nicolas's conception of telicity and how the formula in (40) captures the telic reading of the sentence in question.

Beginning with eat, how does this predicate relate events to degrees? For Caudal and Nicolas (p. 287), in the case of nonatomic predicates such as eat, degrees are taken from the set of positive real numbers. However, Caudal and Nicolas do not say what the maximal degree is in this case, and even more crucially they do not indicate how degrees are supposed to be assigned to eating events.[23] Yet suppose that we have an eating event e: how is it determined what degree is assigned to e? For example, when would the degree assigned to e be 1 and when would it be 2 and when would it be 102? If there is any eating at all, why is the degree not maximal? Unfortunately, since Caudal and Nicolas do not address such questions, it is unclear what eat actually measures in the end.[24] Alternatively, if the degree assigned to e also depends on what is eaten in e, then it would make much more sense to treat tfeat as a *two-place* function on events and individuals, but this is not what Caudal and Nicolas do.

The clause $\mathsf{become}(\mathsf{eat})$ in (40) is unproblematic once eat is accepted. In prose, it says that there is a one-to-one mapping between initial subevents of eating events and degrees such that if eat yields d for e, then every initial subevent of e is mapped to a unique degree lower than d (but higher than 0), and every degree lower than d (but higher than 0) is mapped to a unique initial subevent of e. In other words, degrees steadily increase in the course of eating events. Although there is something intuitively correct about this, it remains unclear what eat is measuring.

Turning to quantity, the problem is similar as for eat: it is unclear what is being measured. Does quantity count atomic individuals, or does it measure the mass of an individual with respect to some unit of measure? And what is the maximal degree for quantity?[25] It is easy to imagine that the clause $\mathsf{quantity}(x)(d)$ in (40) counts apples, but then why not specify d to be 1 in the case? It is also unclear what the motivation is for identifying the degree argument of quantity

[22] At the same time, it is unclear what principles Caudal and Nicolas assume for patient. Do Krifka-style mapping properties hold for patient and, if not, how are parts of the patient argument related to parts of the event? Caudal and Nicolas recite (fn. 15) two of Krifka's (1998) mapping properties, but it is unclear to what extent they are committed to them.

[23] Although Caudal and Nicolas do not state explicitly that eat is functional with respect to its degree argument, I assume that it is.

[24] A reasonable but unintended interpretation for eat would be that it measures temporal length with respect to some unit of measurement (e.g., seconds), but Caudal and Nicolas clearly do not have this interpretation in mind.

[25] Caudal and Nicolas's axiom for quantity in their (56) presupposes a maximal degree.

with that of eat in (40).

Caudal and Nicolas do not model telicity in terms of quantization but instead offer a new definition:

(41) Caudal and Nicolas's definition of telicity (based on their (56)):
A predication is telic iff (i) it has an associated set of degrees with (ii) a specified maximal degree and (iii) its verbal predicate satisfies become.

Not immediately obvious is that this notion of telicity applies to relations between degrees and events and *not* to one-place event predicates, as quantization does. Consequently, this definition has to be applied to the relation between degrees and events underlying the formula in (40):

(42) $\lambda d \lambda e.\exists x(\mathsf{eat}(e)(d) \wedge \mathsf{become}(\mathsf{eat}) \wedge \mathsf{quantity}(x)(d) \wedge \mathsf{agent}(\mathsf{john})(e) \wedge \mathsf{patient}(x)(e) \wedge \mathsf{apple}(x))$ ▷ cf. (40)

Firstly, however, it is not evident that this relation specifies a maximal degree, and even if d were specified as 1 (assuming that individual apples are counted), it is unclear why this would be a maximal degree (how about the case of two apples?). But even if it were deemed that d is maximal in this case, it is hard to see why d should not also be maximal in the following formula (which presumably underlies the analysis of *John eat applesauce*), because presumably the particular quantity x of applesauce at issue also has a maximal value:

(43) $\lambda d \lambda e.\exists x(\mathsf{eat}(e)(d) \wedge \mathsf{become}(\mathsf{eat}) \wedge \mathsf{quantity}(x)(d) \wedge \mathsf{agent}(\mathsf{john})(e) \wedge \mathsf{patient}(x)(e) \wedge \mathsf{applesauce}(x))$

However, this formula should be atelic, in contrast to the formula in (42), which should be telic.

Secondly, the definition in (41) is global in an uncanny way, because it presupposes that we can tell, given an arbitrary relation between degrees and events, whether the verbal predicate buried inside (if there is one) satisfies become. However, on the usual assumptions of a compositional semantics, this information will in general no longer be accessible.[26]

In sum, it is difficult to view Caudal and Nicolas's account in its present state as a serious contender to either Kennedy and Levin's or Krifka's. Although they suggest a number of ideas which are intuitively attractive (e.g., a monotonic increase of degrees as a component of incrementality—even if they do not quite put it this way), their account needs significant work to cohere as it should.

3 A new account: degrees and descriptions

If the double aim is to treat verbs with an incremental theme as gradable *and* to provide an analysis of aspectual composition in terms of their gradability, then such verbs will need more than just a supplementary degree argument. But this claim implies that any approach that simply

[26]Caudal and Nicolas may have in mind a kind of representational approach, but then they should clarify this up front, and needless to say it would place their analysis in a different ballpark and accordingly make direct comparisons between their account and those by Kennedy and Levin, Krifka, and Verkuyl (not to mention the present account) more difficult to make.

adds a degree argument to verbs with an incremental theme will fall short of being able to provide an analysis of aspectual composition in terms of gradability. However, if a degree argument is not sufficient, what more is needed? What has been lacking thus far is a tighter connection between the degrees and how the incremental theme is *described*. If correct, then the description of the incremental theme has to be integrated more tightly into the gradable property that a verb with an incremental theme denotes. In what follows, I will show how this integration may be envisioned, beginning with a detailed treatment of verbs with an incremental theme in Section 3.1, and then returning briefly to degree achievements in Section 3.2.

3.1 Grading verbs with an incremental theme

The basic gradable properties that underlie the semantics of verbs with an incremental theme are measure functions from ordinary individuals x, descriptions O, and events e to degrees, as illustrated in (44).

(44)
a. $\mathsf{eat}_\delta(x)(O)(e)$ 'the degree to which x qua type O is eaten in e'
b. $\mathsf{write}_\delta(x)(O)(e)$ 'the degree to which x qua type O is written in e'
c. $\mathsf{read}_\delta(x)(O)(e)$ 'the degree to which x qua type O is read in e'

Note that O is just a one-place predicate of individuals (where 'O' is mnemonic for 'object' or 'ordinary individual'). The subscript δ is merely a discreet reminder that the respective predicate is a function symbol, yielding degrees as values.[27] For convenience, I will call such functions *incremental degree functions*.[28]

Strictly speaking, verbs with an incremental theme are taken to denote four-place relations between individuals x, descriptions O, degrees d, and events e, as illustrated in (45), obtained by abstracting over the output degrees of the respective incremental degree functions:

(45)
a. eat $\rightsquigarrow \lambda x \lambda O \lambda d \lambda e.\mathsf{eat}_\delta(x)(O)(e) = d$
b. write $\rightsquigarrow \lambda x \lambda O \lambda d \lambda e.\mathsf{write}_\delta(x)(O)(e) = d$
c. read $\rightsquigarrow \lambda x \lambda O \lambda d \lambda e.\mathsf{read}_\delta(x)(O)(e) = d$

It is vital to emphasize at the outset that what is measured by degree functions is the degree to which x qua type O is affected (or effected) in e with respect to the verbal property in question. Thus, the incremental degree function eat_δ in (45a) does *not* measure the degree to which x as a 'bare individual' or quantity gets eaten in e but *instead* measures the degree to which x as an individual of type O gets eaten in e. This is the essential, fundamental difference between the present approach and Kennedy and Levin's—all of the other differences (e.g., whether verbs should be further decomposed or not) are ultimately less basic and more cosmetic.

[27] Thus, despite appearances, δ here is not the same as the degree function δ employed in Piñón (2000, 2005), which was not a mere subscript.

[28] The general background framework that I have in mind is that of *fuzzy set theory* (Zadeh 1987). In particular, incremental degree functions can be viewed as a sort of *fuzzy relations*. Lack of space prevent me from comparisons, but I remark that ordinary fuzzy sets are much simpler than such fuzzy relations. Moreover, degrees here are not construed as degrees of truth.

In the next section, I present an axiomatic treatment of verbs with an incremental theme on the assumption that their meanings are based on incremental degree functions. I use V_δ in the axioms as a predicate constant instantiating such functions (namely, $\mathtt{eat}_\delta$, $\mathtt{write}_\delta$, etc.). In addition, the conception of degrees adopted here is simpler than Kennedy and Levin's: a degree is simply a rational number (i.e., a point and not an interval) from 0 to 1. Although degrees could be modeled as real numbers from 0 to 1, it seems questionable whether irrational numbers are needed for semantic applications. Moreover, there is no distinction made here between positive and negative degrees, which would also be rather tricky to implement if degrees are just points. At any rate, negative degrees are not essential for present purposes.[29]

3.1.1 An axiomatic treatment

The first axiom for incremental degree functions simply makes explicit the claim that O holds of x (the incremental theme), for otherwise there would be no sense in speaking of 'x qua type O':

(46) $\forall x \forall O \forall d \forall e($ $\quad\triangleright$ AXIOM: application of O to x

$$\mathsf{V}_\delta(x)(O)(e) = d \rightarrow O(x))$$

The second axiom affirms thematic uniqueness with respect to x:

(47) $\forall x \forall x' \forall O \forall d \forall e($ $\quad\triangleright$ AXIOM: thematic uniqueness of x

$$(\mathsf{V}_\delta(x)(O)(e) = d \wedge \mathsf{V}_\delta(x')(O)(e) = d) \rightarrow x = x')$$

For example, if x qua type O and x' qua type O are eaten to degree d in the same event e, then x and x' are identical.[30]

The third axiom states that if x qua type O and x' qua type O are affected in e and e', respectively, with respect to V_δ, where e' is a proper part of e, then x' is a proper part of x:

(48) $\forall x \forall x' \forall O \forall d \forall d' \forall e \forall e'($ $\quad\triangleright$ AXIOM: mapping from events to ordinary individuals

$$(\mathsf{V}_\delta(x)(O)(e) = d \wedge \mathsf{V}_\delta(x')(O)(e') = d' \wedge e' \sqsubset e) \rightarrow x' \sqsubset x)$$

This axiom is similar in spirit to Krifka's 'mapping to objects' (see Section 1), though it is actually weaker because it does not simply require every part of e to correspond to a part of x.[31]

The fourth axiom asserts that incremental degree functions are *summative* with respect to their incremental theme and the event argument, provided that O is cumulative:[32]

(49) $\forall x \forall x' \forall O \forall d \forall d' \forall e \forall e'($ $\quad\triangleright$ AXIOM: summativity of V_δ with cumulative O

[29] This is not intended as a criticism of Kennedy and Levin's conception of degrees—rather, it is simply a practical decision of implementation on my part. Needless to say, the question of negative degrees would be a topic in its own right.

[30] The English paraphrase with 'in the same event' is potentially misleading, because x' may be a proper part of x as long as x' is eaten in a *proper subevent* of e.

[31] Although Kennedy and Levin claim to be able to dispense with mappings between argument structure and events for the treatment of verbs with an incremental theme, their claim is not so easy to verify until their account is more fully formalized.

[32] Krifka employs a notion of summativity for (two-place) thematic relations. However, the matter is more complex in the present case, because the predicate argument O and the degree argument d of V_δ also have to be considered.

$$\mathsf{V}_\delta(x)(O)(e) = d \wedge \mathsf{V}_\delta(x')(O)(e') = d' \wedge d' \leq d \wedge \mathsf{cum}(O) \rightarrow$$
$$\mathsf{V}_\delta(x \oplus x')(O)(e \oplus e') = d)$$

Observe that the degrees in this case are not summed or added together but instead the higher of the two degrees is selected for the value of the summed event. For instance, if an event in which some x qua type 'applesauce' is eaten to degree 1 is summed with another event in which x' qua type 'applesauce' is eaten to degree 1, then the sum is an event in some applesauce is eaten to degree 1. It would make no sense to add the two degrees to get 2, because 2 is not a possible degree to begin with.[33]

The fifth axiom affirms that if x qua type O is affected to degree d in e with respect to V_δ, d is positive, and O is cumulative, then d is 1:

(50) $\forall x \forall O \forall d \forall e($ $\qquad \triangleright$ AXIOM: cumulative O yields $d = 1$ if $d > 0$
$$(\mathsf{V}_\delta(x)(O)(e) = d \wedge d > 0 \wedge \mathsf{cum}(O)) \rightarrow d = 1)$$

The idea behind this axiom is that if x qua type O is affected in e and O is cumulative, then x qua type O is fully affected in e—there is no hedging, there is no halfway house. More precisely, incremental degree functions are *defective* in this kind of situation, for they crucially depend on O for distinguishing degrees of change, and yet if O is cumulative, it does not provide the kind of property against which degrees of change can reasonably be distinguished. Another way of putting this is that if O is cumulative, then incremental degree functions are *ungradable*.

As an illustration, consider an event e in which some applesauce x is (possibly partly) eaten. In this case, O is the type 'applesauce', which is cumulative, thus the question is to what degree x qua type 'applesauce' is eaten in e. Suppose that in fact half of x is eaten in e. Neveretheless, bear in mind, as emphasized earlier, that the incremental degree function eat_δ does *not* measure the degree to which x as a 'bare individual' or quantity is eaten in e. Instead, it measures the degree to which x qua type 'applesauce' is eaten in e, and so a sensible answer seems to be that if x qua type 'applesauce' is eaten at all in e, then the degree to which x qua type 'applesauce' is eaten in e is 1. Although, naturally, eating more of x would mean that a larger quantity of applesauce is eaten, it would not change the degree to which x qua type 'applesauce' is eaten. To harp on this point, we are not measuring quantities of applesauce that are eaten—we are measuring the degree to which the event type 'eat applesauce' is realized.

The next two axioms play for quantized O a role parallel to those in (49) and (50) for cumulative O. The first of these axioms determines, for a given x and quantized O, whether V_δ applies to the sum of e and e' if it applies to e and e' independently. In (51), $\wr$ stands for the *discreteness* relation (i.e., no overlap), whereas $\oslash$ designates the *proper overlap* relation (i.e., overlap in a proper part only).

(51) $\forall x \forall O \forall d \forall d' \forall e \forall e'($ $\qquad \triangleright$ AXIOM: summativity with quantized O and fixed x
$$\mathsf{V}_\delta(x)(O)(e) = d \wedge \mathsf{V}_\delta(x)(O)(e') = d' \wedge \mathsf{qua}(O) \rightarrow$$
$$e' \wr e \rightarrow$$
$$d + d' \leq 1 \rightarrow \mathsf{V}_\delta(x)(O)(e \oplus e') = d + d' \wedge$$

[33] In the present framework, there is no addition of degrees in isolation, in contrast to Kennedy and Levin's account (recall fns. 11, 12, and 16 in this connection). How degrees are combined or 'added' depends on the arguments of the degree functions in question. In this connection, see also (51) below.

$$d + d' > 1 \rightarrow \neg\exists d''(\mathsf{V}_\delta(x)(O)(e \oplus e') = d'') \wedge$$
$$e' \oslash e \rightarrow \neg\exists d''(\mathsf{V}_\delta(x)(O)(e \oplus e') = d''))$$

This axiom guarantees that if e and e' are discrete and the arithmetic sum of d and d' is less than or equal to 1, then V_δ applies to the sum of e and e' with the value $d + d'$. However, if $d + d'$ is greater than 1, then V_δ does not yield a degree for $e \oplus e'$—this is an instance in which V_δ is undefined on its input. Furthermore, if e and e' properly overlap, then V_δ also does not yield a degree for $e \oplus e'$, which is another instance of undefinedness. This axiom does not mention the case in which e' is a part of e (note that the part relation is incompatible with the proper overlap relation), because the result is independently derivable: if $e' \sqsubseteq e$ holds, then $e' \oplus e$ is just e and so the degree is d, as already determined by the premise. Clearly, as seen in (49)–(50), the matter is less complex when O is cumulative, because in this case the degrees are never added together.

The seventh axiom, which parallels the one in (50), states that the value of the degree argument strictly increases in the course of an event if O is quantized, thereby encoding a strict notion of incrementality:

(52) $\forall x \forall O \forall d \forall d' \forall e \forall e'($ ▷ AXIOM: strict incrementality
$(\mathsf{V}_\delta(x)(O)(e) = d \wedge \mathsf{V}_\delta(x)(O)(e') = d' \wedge e' \sqsubset e \wedge \mathsf{qua}(O)) \rightarrow d' < d)$

For example, consider an event e in which x qua type '(an) apple' is eaten to degree d. Since '(an) apple' is quantized, this axiom applies and requires the degree to which x qua type '(an) apple' is eaten in any proper subevent of e to be less than d.

The eighth and final axiom determines a kind of initial condition on incremental degree functions, asserting that if x qua type O is affected to degree d in e with respect to V_δ, then x qua type O is affected to degree 0 in the very beginning of e with respect to V_δ:

(53) $\forall x \forall O \forall d \forall e($ ▷ AXIOM: $d = 0$ in the left boundary of e
$\mathsf{V}_\delta(x)(O)(e) = d \rightarrow \mathsf{V}_\delta(x)(O)(\mathsf{left\text{-}bound}(e)) = 0)$

The predicate left-bound is a function that yields the instantaneous beginning or *left boundary* of an event. Note that the left boundary of an event is a sort of event (albeit instantaneous) and not a time; as a subtle reminder of this, I write '*in* the left boundary of e' as opposed to '*at* the left boundary of e'.[34]

At first glance, the axiom in (53) may appear undesirable in light of the following kind of example. Suppose that x qua type 'half an apple' is eaten to degree d in e. This axiom would then require x qua type 'half an apple' to be eaten to degree 0 in the left boundary of e. However, it is easy to imagine that the other half of the apple had already been eaten before the beginning of e, hence it would seem wrong to require that no part of the apple be eaten before the beginning of e. This would indeed be wrong, but this is also not what the axiom in (53) enforces, because it leaves completely open what may have happened to the other half ot the apple prior to e. It restricts itself to what happens in e, stating that x qua type 'half an apple' is eaten to degree 0 in the left boundary of e. Again, we are not measuring the degree to which x is already (statively) eaten by the time that e begins—we are measuring the degree to which x qua type 'half an apple'

[34] See Piñón (1997) for a proposal making use of events and boundary events in an analysis of achievements.

gets eaten in the left boundary of e, which is another matter altogether.

The eight axioms in (46)–(53) specify the core of what it means for V_δ to be an incremental degree function. What follows are various facts that are pertinent to the semantic analyses in the next section.

At the outset, it is useful to make explicit a notion of *iterativity* for an individual or event a with respect to a one-place predicate P. We say that a is iterative with respect to P just in case P applies to a and to at least two parts of a (where, in the limiting case, one part could be a itself).

(54) $\mathsf{iter}(a)(P) \stackrel{\text{def}}{=}$ ▷ a is iterative with respect to P

$$P(a) \wedge \exists b \exists c(b \sqsubseteq a \wedge c \sqsubseteq a \wedge \neg(b = c) \wedge P(b) \wedge P(c))$$

If P is cumulative, then there is always some a with respect to which P is iterative:

(55) $\forall P($ ▷ FACT: cumulativity implies iterativity for some a

$$\mathsf{cum}(P) \rightarrow \exists a(\mathsf{iter}(a)(P)))$$

(56) $\forall P($ ▷ FACT: quantization excludes iterativity for any a

$$\mathsf{qua}(P) \rightarrow \neg\exists a(\mathsf{iter}(a)(P)))$$

The proofs are immediate from the respective definitions.

Turning to incremental degree functions more specifically, if O is quantized and the value of d is fixed, then V_δ is not iterative for any e:

(57) $\forall x \forall O \forall d \forall e($ ▷ FACT: no iterativity with quantized O and fixed d

$$(\mathsf{V}_\delta(x)(O)(e) = d \wedge \mathsf{qua}(O)) \rightarrow \neg\mathsf{iter}(e)(\lambda e'.\mathsf{V}_\delta(x)(O)(e') = d))$$

PROOF. Abbreviating $\lambda e'.\mathsf{V}_\delta(x)(O)(e') = d$ as $\varepsilon(x)(O)(d)$, suppose to the contrary that e is iterative with respect to $\varepsilon(x)(O)(d)$. Then by the definition of iterativity in (54) there is at least one proper subevent e' of e such that $\varepsilon(x)(O)(d)(e')$ holds. But this is ruled out by strict incrementality in (52), which requires a lower value of the degree argument in this case. Consequently, $\varepsilon(x)(O)(d)(e')$ does not hold, and neither does $\mathsf{iter}(e)(\varepsilon(x)(O)(d))$.

Observe that the value of the degree argument is fixed in (57). If it is allowed to vary, then the corresponding event predicate may be iterative with respect to some event. This is the case whenever the antecedent of the axiom of strict incrementality in (52) is satisfied:

(58) ▷ FACT: iterativity with quantized O and existentially bound d

$$\forall x \forall O \forall d \forall d' \forall e \forall e'(\mathsf{V}_\delta(x)(O)(e) = d \wedge \mathsf{V}_\delta(x)(O)(e') = d' \wedge e' \sqsubset e \wedge \mathsf{qua}(O) \rightarrow \mathsf{iter}(e)(\lambda e'.\exists d(\mathsf{V}_\delta(x)(O)(e') = d)))$$

The proof is immediate from the definition of iterativity in (54).

A comment is in order about the result in (57), because it may appear unduly restrictive at first. There are verbs with an incremental theme that seem to allow for iterativity even assuming a fixed degree. For example, it is certainly possible to (completely) read a certain letter more than once, though this appears to be excluded by the fact in (57). Note, though, that the fact in (57) does not prohibit the possibility of multiple readings of a certain letter—what it says is that incremental degree functions are not iterative for any event if O is quantized and the value of the degree argument is fixed. For concreteness, imagine an event e that is the sum of two discrete events e' and e'' in each of which one and the same letter is read to degree 1. The fact in (57) affirms that the incremental degree function read_δ cannot apply both to e' and e'' as well as to e. But what this means is that in order to describe e we need something more than just read_δ—we need to form a (new) event predicate based on read_δ that can apply to (iterative) events like e, e.g., by means of an iterative operator. Although I do not (for lack of space) define such an iterative operator here, the point is precisely to restrict the intended interpretation of incremental degree functions to exclude iterativity of this kind in the case of a quantized O and a fixed d.

A relaxed notion of incrementality for either a quantized or cumulative O is captured by the following fact, which states that the value of the degree argument does not decrease in the course of an event:

(59) $\forall x \forall O \forall d \forall d' \forall e \forall e'($ ▷ FACT: incrementality
$$\mathsf{V}_\delta(x)(O)(e) = d \wedge \mathsf{V}_\delta(x)(O)(e') = d' \wedge e' \sqsubseteq e \wedge (\mathsf{qua}(O) \vee \mathsf{cum}(O)) \rightarrow d' \leq d)$$

PROOF. There are two cases to consider, depending on whether O is quantized or cumulative. If O is quantized, it suffices to point out that strict incrementality in (52) implies incrementality. If O is cumulative, then by the axiom in (50) the only positive value for the degree argument is 1, which trivially satisfies the consequent.

The next result confirms that if an event predicate based on a degree function is nonunique, d is existentially bound, and O is cumulative, then the event predicate is cumulative:

(60) $\forall O($ ▷ FACT: cumulativity with cumulative O and existentially bound d
$$\mathsf{nuniq}(\lambda e.\exists d \exists x(\mathsf{V}_\delta(x)(O)(e) = d)) \wedge \mathsf{cum}(O) \rightarrow \mathsf{cum}(\lambda e.\exists d \exists x(\mathsf{V}_\delta(x)(O)(e) = d)))$$

PROOF. Abbreviating $\lambda e.\exists d \exists x(\mathsf{V}_\delta(x)(O)(e) = d)$ as $\varepsilon(O)$, it has to be shown that for any e, e' such that $\varepsilon(O)(e)$ and $\varepsilon(O)(e')$ hold, it follows that $\varepsilon(O)(e \oplus e')$ also holds. Given the definition of $\varepsilon(O)$, there are (possibly identical) x, x' and (possibly identical) d, d' such that $\varepsilon(x)(O)(d)(e)$ and $\varepsilon(x')(O)(d')(e')$ hold. By summativity in (49), if d' is less than or equal to d, then $\varepsilon(x \oplus x')(O)(d)(e \oplus e')$ holds, otherwise $\varepsilon(x \oplus x')(O)(d')(e \oplus e')$. In either case it follows that $\varepsilon(O)(e \oplus e')$ holds, which is what needed to be shown.

As a variation on the event predicate in (60), if O is cumulative, existentially binding d and restricting its value to be greater than 0 is tantamount to fixing its value to be 1, hence there is no need to consider the case of a fixed d separately:

(61) $\forall O($ ▷ FACT: no difference between $d > 0$ and $d = 1$ with cumulative O
$$\mathsf{cum}(O) \rightarrow \lambda e.\exists d \exists x(\mathsf{V}_\delta(x)(O)(e) = d \wedge d > 0) \leftrightarrow \lambda e.\exists x(\mathsf{V}_\delta(x)(O)(e) = 1))$$

The proof is immediate from the axiom in (50).

If an event predicate based on an incremental degree function takes a quantized O and a fixed d, then it is quantized:

(62) $\forall O \forall d($ $\quad\triangleright$ FACT: quantization with quantized O and fixed d

$$\mathsf{qua}(O) \rightarrow \mathsf{qua}(\lambda e.\exists x(\mathsf{V}_\delta(x)(O)(e) = d)))$$

PROOF. Abbreviating $\lambda e.\exists x(\mathsf{V}_\delta(x)(O)(e) = d)$ as $\varepsilon(O)(d)$, suppose to the contrary that $\varepsilon(O)(d)$ is *not* quantized. Then there are e, e' such that $e' \sqsubset e$, $\varepsilon(O)(d)(e)$, and $\varepsilon(O)(d)(e')$ hold. But then (by the definition of $\varepsilon(O)(d)$) there are also x, x' such that $\varepsilon(x)(O)(d)(e)$ and $\varepsilon(x')(O)(d)(e')$ hold. By the axiom in (48) it follows that $x' \sqsubseteq x$ holds, which in turn means that either $x' \sqsubset x$ or $x' = x$ holds. Yet the former is ruled out by the quantization of O, and the latter is excluded by strict incrementality in (52) (since d is fixed). Hence there is no such x', and $\varepsilon(O)(d)$ is quantized after all.

As a special case of the fact in (62), the resulting event predicate is also quantized if x is fixed:

(63) $\forall x \forall O \forall d($ $\quad\triangleright$ FACT: quantization with quantized O, fixed x, and fixed d

$$\mathsf{qua}(O) \rightarrow \mathsf{qua}(\lambda e.\mathsf{V}_\delta(x)(O)(e) = d))$$

The proof is similar to the one for (62), only there is no need to appeal to the axiom in (48).

As a variation on the event predicate in (62), if O is quantized and d is existentially bound, the resulting event predicate quantized as long as the value of the incremental theme varies:

(64) $\forall O($ $\quad\triangleright$ FACT: quantization with quantized O and existentially bound d

$$\begin{array}{l} \mathsf{qua}(O) \rightarrow \\ \quad \forall x \forall x' \forall d \forall d' \forall e \forall e'(\\ \quad\quad (\mathsf{V}_\delta(x)(O)(e) = d \wedge \mathsf{V}_\delta(x')(O)(e') = d' \wedge \neg(e = e')) \rightarrow \neg(x = x') \rightarrow \\ \quad\quad\quad \mathsf{qua}(\lambda e''.\exists d'' \exists x''(\mathsf{V}_\delta(x'')(O)(e'') = d'')))) \end{array}$$

PROOF. Abbreviating $\lambda e.\exists d \exists x(\mathsf{V}_\delta(x)(O)(e) = d)$ as $\varepsilon(O)$, suppose to the contrary that $\varepsilon(O)$ is *not* quantized. Then there are e, e' such that $e' \sqsubset e$, $\varepsilon(O)(e)$, and $\varepsilon(O)(e')$ hold. But then (by the definition of $\varepsilon(O)(d)$) there are also d, d' and x, x' such that $\varepsilon(x)(O)(d)(e)$ and $\varepsilon(x')(O)(d)(e')$ hold. By the axiom in (48) it follows that $x' \sqsubseteq x$ holds, which in turn means that either $x' \sqsubset x$ or $x' = x$ holds. However, the former is ruled out by the quantization of O, and the latter is excluded by the premise that value of the incremental theme varies. Hence there is no such x', and $\varepsilon(O)$ is quantized after all.

As a final result, an event predicate based on an incremental degree function that takes a quantized O, a fixed x, and an existentially bound d is cumulative if it is nonunique and is restricted to pairs of events that are discrete and whose degrees added together are less than or equal to 1 or to pairs of events one of which stands in the part relation to the other, as formalized in (65). This fact is crucial in accounting for the possible atelicity of examples such as those in (1b) and (3b).

(65) $\quad\triangleright$ FACT: cumulativity with quantized O, fixed x, and existentially bound d

$$\begin{array}{l} \forall x \forall O(\\ \quad \mathsf{qua}(O) \wedge \mathsf{nuniq}(\lambda e.\exists d(\mathsf{V}_\delta(x)(O)(e) = d)) \wedge \\ \quad\quad \forall d \forall d' \forall e \forall e'(\\ \quad\quad\quad \mathsf{V}_\delta(x)(O)(e) = d \wedge \mathsf{V}_\delta(x')(O)(e') = d' \rightarrow \end{array}$$

$$(e \wr e' \wedge d + d' \leq 1) \vee (e \sqsubseteq e' \vee e' \sqsubseteq e) \rightarrow$$
$$\mathsf{cum}(\lambda e''.\exists d''(\mathsf{V}_\delta(x)(O)(e'') = d''))))$$

PROOF. Abbreviating $\lambda e.\exists d(\mathsf{V}_\delta(x)(O)(e) = d)$ as $\varepsilon(x)(O)$, there are two cases to consider. The first case is to show that for any e, e' such that $\varepsilon(x)(O)(e)$, $\varepsilon(x)(O)(e')$, $e \wr e'$, and $d + d' \leq 1$ hold, it follows that $\varepsilon(x)(O)(e \oplus e')$ holds. But this follows by the axiom in (51), with the value of $d + d'$ for the degree argument. The second case is to show the same for any e, e' such that $\varepsilon(x)(O)(e)$, $\varepsilon(x)(O)(e')$ and $e \sqsubseteq e'$ or $e' \sqsubseteq e$ hold. If $e \sqsubseteq e'$ holds, then $e \oplus e'$ is e' and the degree argument is d', otherwise $e \oplus e'$ is e and the degree argument is d. Hence $\varepsilon(x)(O)(e \oplus e')$ holds here as well.

3.1.2 Applications

The theory of incremental degree functions presented in the previous sections is may be a bit hard to digest at first and may even seem somewhat removed from the down-to-earth aspectual issues posed by data such as those in (1)–(4). The task now is to show how to bring the two together.

As previewed in the opening of Section 3.1, the idea is to treat the meaning of verbs with an incremental theme as based on incremental degree functions. More precisely, such verbs denote four-place relations between individuals x, descriptions O, degrees d, and events e:

(66) a. $[_{\text{VP}}\ [_{\text{V}}\ \text{eat}]\ [_{\text{NP}}\ \alpha]]$
eat $\rightsquigarrow \lambda x \lambda O \lambda d \lambda e.\mathsf{eat}_\delta(x)(O)(e) = d$ ▷ cf. (45a)
b. $[_{\text{VP}}\ [_{\text{V}}\ \text{write}]\ [_{\text{NP}}\ \alpha]]$
write $\rightsquigarrow \lambda x \lambda O \lambda d \lambda e.\mathsf{write}_\delta(x)(O)(e) = d$ ▷ cf. (45b)
c. $[_{\text{VP}}\ [_{\text{V}}\ \text{read}]\ [_{\text{NP}}\ \alpha]]$
read $\rightsquigarrow \lambda x \lambda O \lambda d \lambda e.\mathsf{read}_\delta(x)(O)(e) = d$ ▷ cf. (45c)

However, if incremental degree functions form the semantic core of verbs with an incremental theme, there is a small price to pay when it comes to treating semantic composition. The technical obstacle is that since these verbs take both an individual argument x and a predicate argument O for what is syntactically a single incremental theme argument, it is hard to see how type-driven functional application could be employed to combine the verb with the object NP without first resorting to fancy type-shifting manuevers. The strategy adopted here is to invoke a special rule of semantic composition for VPs headed by verbs with an incremental theme, formulated as follows:

(67) ▷ VP semantic rule
1: If γ is of the form $[_{\text{VP}}\ [_{\text{V}}\ \alpha]\ [_{\text{NP}}\ \beta]]$,
2: where $[\alpha \rightsquigarrow]$ is of the type $\langle e_O, \langle \langle e_O, t \rangle, \langle e_D, \langle e_E, t \rangle \rangle \rangle \rangle$:
3: if $[\beta \rightsquigarrow]$ is of the type $\langle e_O, t \rangle$,
4: then $\gamma \rightsquigarrow \lambda d \lambda e.\exists x([\alpha \rightsquigarrow](x)([\beta \rightsquigarrow])(d)(e))$
5: and if $[\beta \rightsquigarrow]$ is of the type e_O,
6: then $\gamma \rightsquigarrow \lambda d \lambda e.[\alpha \rightsquigarrow](x)(\lambda x'.x' = [\beta \rightsquigarrow])(d)(e)$

The types e_O, e_D, and e_E serve to make clear whether the type e is of the sort of ordinary individuals, degrees, or events. Moreover, a bracketed form such as $[\alpha \rightsquigarrow]$ designates the translation of α in the logical representation language.

Although the rule in (67) may appear involved at first, there is in fact nothing mysterious about it. According to lines 3–4, if the object NP is analyzed as a one-place nominal predicate, then it is substituted for O and x is existentially bound. According to lines 5–6, if the object NP is analyzed as a term, then it is substituted for one argument of the identity relation, the result of which is in turn substituted for O. In this case, x is free, but this does not pose a concern, because the axiom in (46) has the effect of identifying x with the term representing the object NP.[35]

Since a two-place relation between degrees and events is derived in (67) as the meaning of the VP, a means for discharging the degree argument is necessary. I assume that there are at least two covert ways of achieving this. One is to set value of the degree argument to 1, thereby maximizing it; another is to existential quantify over the degree argument, restricting its value to be greater than 0:[36]

(68) a. $[_{\text{VP}}\ \alpha]^{/1}$
$^{/1} \rightsquigarrow \lambda R \lambda e.R(1)(e)$ ▷ degree maximizing operator

b. $[_{\text{VP}}\ \alpha]^{/+}$
$^{/+} \rightsquigarrow \lambda R \lambda e.\exists d(R(d)(e) \wedge d > 0)$ ▷ positive degree binding operator

Other things being equal, the use of the degree maximizing operator seems to be preferred to that of the positive degree binding operator. Presumably, this is because the use of the former yields a stronger meaning than that of the latter, and so unless there is information to the contrary, the stronger meaning is to be preferred.[37]

The first derivation in line is that of the telic VP *eat an apple* in (1a). The object NP *an apple* is treated as a quantized nominal predicate, and the degree maximizing operator is applied to the ensuing relation between degrees and events. The resulting event predicate is then demonstrably quantized.

(69)
1: $[_{\text{NP}}$ an apple$] \rightsquigarrow \lambda x.\mathsf{apple}(x)$ ▷ NP meaning
2: $\mathsf{qua}(\mathsf{apple})$ ▷ related axiom
3: $[_{\text{VP}}\ [_{\text{V}}$ eat$]\ [_{\text{NP}}$ an apple$]] \rightsquigarrow$ ▷ by (66a), (67), and functional conversion
4: $\lambda d \lambda e.\exists x(\mathsf{eat}_\delta(x)(\mathsf{apple})(d)(e))$
5: $[_{\text{VP}}\ [_{\text{V}}$ eat$]\ [_{\text{NP}}$ an apple$]]^{/1} \rightsquigarrow$ ▷ apply degree maximizer
6: $\lambda e.\exists x(\mathsf{eat}_\delta(x)(\mathsf{apple})(1)(e))$
7: $\mathsf{qua}(\lambda e.\exists x(\mathsf{eat}_\delta(x)(\mathsf{apple})(1)(e)))$ ▷ by fact in (62)

[35] For lack of space, I do not consider the case where the object NP is quantificational, because the treatment of quantifiers in an event semantics raises a number of issues in its own right.

[36] A third way would be to set the value of the degree argument to be greater than or equal to some contextually fixed value. Indeed, the degree maximizing operator in (68a) might be considered a special instance of the third way, yet for simplicity I treat it as a separate case here.

[37] Kennedy and Levin (2007) make a similar point in connection with the preferred telicity of certain degree achievements. It is feasible to view this preference as an instance of the pragmatic principle of maximal informativeness.

If, instead, on line 5 the positive degree binding operator were applied, by the fact in (64) the resulting event predicate would still be quantized, provided that a different apple is involved in each event in the denotation of the predicate. The derivation of the telic VP *eat a bowl of applesauce* is analogous, assuming that the object NP *a bowl of applesauce* is also analyzed as a quantized nominal predicate.

The derivation of the atelic VP *eat apples* in (2b) is shown in (70), where the object NP *apples* is treated as a cumulative nominal predicate. Although the degree maximizer operator is applied here as well, recall that by the fact in (61) it would make absolutely no difference if the positive degree binding operator were applied instead: in either case, the resulting event predicate is cumulative, provided that it is nonunique.

(70)
1: $[_{\text{NP}}\ \text{apples}] \rightsquigarrow \lambda x.\mathsf{apples}(x)$ ▷ NP meaning
2: $\mathsf{cum}(\mathsf{apples})$ ▷ related axiom
3: $[_{\text{VP}}\ [_{\text{V}}\ \text{eat}]\ [_{\text{NP}}\ \text{apples}]] \rightsquigarrow$ ▷ by (66a), (67), and functional conversion
4: $\quad \lambda d\lambda e.\exists x(\mathsf{eat}_\delta(x)(\mathsf{apples})(d)(e))$
5: $[_{\text{VP}}\ [_{\text{V}}\ \text{eat}]\ [_{\text{NP}}\ \text{apples}]]^{/1} \rightsquigarrow$ ▷ apply degree maximizer
6: $\quad \lambda e.\exists x(\mathsf{eat}_\delta(x)(\mathsf{apples})(1)(e))$
7: $\mathsf{nuniq}(\lambda e.\exists x(\mathsf{eat}_\delta(x)(\mathsf{apples})(1)(e))) \rightarrow$ ▷ by facts in (60) and (61)
8: $\quad \mathsf{cum}(\lambda e.\exists x(\mathsf{eat}_\delta(x)(\mathsf{apples})(1)(e)))$

Evidently, the analysis of the atelic VP *eat applesauce* from (4b) would be similar. Note, moreover, that since there is no way to derive a telic reading of such VPs, the examples in (2a) and (4a) are unacceptable.

The derivation of the atelic VP *ate an apple* in (1b) is displayed in (71). Recall that the atelic interpretation of this VP requires a specific reading of the object NP *an apple* (see fn. 1), in contrast to the telic interpretation, which does not (cf. (69)). For present purposes, all that I need to assume is that the specific reading is individual-denoting, akin to definite NPs (e.g., *the apple*) in this respect. As a useful approximation, I take the specific reading of *an apple* to mean 'a certain apple', where *certain* remains unanalyzed. Furthermore, it is also crucial that the positive degree binding operator and not the degree maximizing operator be used, for otherwise by the fact in (63) the resulting event predicate would be quantized.

(71)
1: $[_{\text{NP}}\ \text{a (certain) apple}] \rightsquigarrow \mathsf{a\text{-}certain\text{-}apple}\quad (\text{type } e_O)$ ▷ NP meaning
2: $[_{\text{VP}}\ [_{\text{V}}\ \text{eat}]$ ▷ by (66a), (67), and functional conversion
3: $\qquad [_{\text{NP}}\ \text{a (certain) apple}]] \rightsquigarrow$
4: $\quad \lambda d\lambda e.\mathsf{eat}_\delta(x)(\lambda x'.x' = \mathsf{a\text{-}certain\text{-}apple})(d)(e)$
5: $[_{\text{VP}}\ [_{\text{V}}\ \text{eat}]\ [_{\text{NP}}\ \text{a (certain) apple}]]^{/+} \rightsquigarrow$ ▷ apply positive degree binder
6: $\quad \lambda e.\exists d(\mathsf{eat}_\delta(x)(\lambda x'.x' = \mathsf{a\text{-}certain\text{-}apple})(d)(e) \wedge d > 0)$
7: $\mathsf{nuniq}(\lambda e.\exists d($ ▷ by fact in (65)
8: $\qquad \mathsf{eat}_\delta(x)(\lambda x'.x' = \mathsf{a\text{-}certain\text{-}apple})(d)(e) \wedge d > 0)) \wedge$
9: $\quad \forall d \forall d' \forall e \forall e'($
10: $\qquad \mathsf{eat}_\delta(x)(\lambda x'.x' = \mathsf{a\text{-}certain\text{-}apple})(e) = d \wedge$
11: $\qquad \mathsf{eat}_\delta(x')(\lambda x'.x' = \mathsf{a\text{-}certain\text{-}apple})(e') = d' \rightarrow$
12: $\qquad\quad (e \wr e' \wedge (d + d') \leq 1) \vee (e \sqsubseteq e' \vee e' \sqsubseteq e) \rightarrow$

13: $\mathsf{cum}(\lambda e.\exists d(\mathsf{eat}_\delta(x)(\lambda x'.x' = \mathsf{a\text{-}certain\text{-}apple})(d)(e) \wedge d > 0)))$

The ensuing event predicate is cumulative provided that it is nonunique and that its denotation is restricted to discrete eating events whose combined degrees are less than or equal to 1 or to eating events that are contained in one another. Informally, this means that everything is okay as long as we restrict our attention to discrete events in which the apple in question is partly eaten or to 'growing events' in which it is partly eaten. Clearly, the derivation of the atelic reading of the VP *ate a bowl of applesauce* in (3b) would be analogous.

It is an advantage of the present approach that it can treat the aspectual difference between pairs of examples such as (1a)/(1b) and (3a)/(3b) without ambiguity at the lexical level. On Krifka's approach, as far as I can tell, the difference would have to be due to a lexical difference between the thematic relations for the internal argument.

As a final application, I briefly point out that the present theory allows for a straightforward analysis of adverbs of completion in combination with verbs with an incremental theme—at any rate, it is more straightforward than the one offered in Piñón (2005). The basic observation is that adverbs of completion such as *completely*, *partly*, and *half* are acceptable with telic VPs but unacceptable with atelic VPs headed by verbs with an incremental theme:

(72) a. Rebecca completely ate an apple (in five minutes).
b. *Rebecca completely ate apples (for thirty minutes).

(73) a. Rebecca partly ate a bowl of applesauce (in five minutes).
b. *Rebecca partly ate applesauce (for five minutes).

Focusing on *completely*, the idea is to view it as a kind of overt counterpart of the degree maximizing operator in (68a), but with the difference that it presupposes that the relation which it applies to may yield degrees greater than 0 but less than 1:[38]

(74) $[_{\mathrm{VP}}$ completely $[_{\mathrm{VP}}\ \alpha]]$
completely $\rightsquigarrow \lambda R\lambda e.R(1)(e) \wedge \exists e'\exists d'(R(d')(e') \wedge d' > 0 \wedge d' < 1) \wedge$ ▷ cf. (68a)

The presupposition is readily satisfied when the relation that the meaning of *completely* applies to allows for a variety of degrees. This is the case in (72a) and (73a), where a quantized nominal predicate instantiates O of the incremental degree function (recall the axiom of strict incrementality in (52)). In contrast, the presupposition is not satisfied in (72b) or (73b), where the nominal predicate instantiating O of the incremental degree function is cumulative, due to the fact in (61), since the only positive degree is 1. The analysis of the adverbs *partly* and *half* would involve the same presupposition.

[38]Caudal and Nicolas (2005, p. 287) state a similar intuition about *completely*. Incidentally, it would be more accurate to embed the presupposition in (74) under a possibility operator, but for simplicity I keep matters extensional here. In Piñón (2005) I argue that adverbs of completion are verb modifiers, whereas *completely* in (74) is treated as a VP modifier. Frankly, it was more crucial to treat adverbs of completion as verb modifiers in my earlier approach than it is now, though the present analysis could be revised to accommodate this view if desired.

3.2 Back to degree achievements

At this point, it would be nice to be able to simply apply the analysis developed for verbs with an incremental theme to degree achievements. However, degree achievements differ in a significant respect from verbs with an incremental theme, and so a simple application of the theory developed so far to degree achievements is not feasible. The way in which degree achievements differ from verbs with an incremental theme is not that the former have a degree argument which the latter lack (because the latter also have a degree argument), but rather that the former have (what I will call) an *extent argument* that the latter lack. It is this additional argument, the extent argument, that complicates things in the sense that the degree functions underlying degree achievements need to take it into account. Although it would be feasible to systematically extend the axiomatic treatment developed in Section 3.1.1 to degree achievements, lack of space prevents me from doing so here. Consequently, the aim of this section is merely to sketch the outlines of such an extension (and so the discussion is significantly less formal), taking *lengthen* as a canonical example.

The degree achievement *lengthen* is analyzed as five-place relation between individuals x, descriptions O, extents n, degrees d, and events e:

(75) $[_{\mathrm{VP}}\ [_{\mathrm{V}}\ \text{lengthen}]\ [_{\mathrm{NP}}\ \alpha]\ ((\text{by})\ [_{\mathrm{NP}}\ \beta])]$
$\text{lengthen} \rightsquigarrow \lambda x \lambda O \lambda n \lambda d \lambda e.\mathsf{lengthen}_{\delta}(x)(O)(n)(e) = d$

In order to keep the potentially confusing terminology straight, I emphasize that the extent argument n in (75) corresponds to Kennedy and Levin's degree of change argument (cf. (20b), (24a)) and that the degree argument d in (75) has no correspondent in Kennedy and Levin's analysis. Naturally, there is nothing sacred about this terminology and so the extent argument n could well be called a 'degree of change argument', but then it would be much easier to confound the extent argument n with the degree argument d, which is precisely a distinction that should be kept clear (as previewed in fn. 16). With this said, the degree function underlying the meaning of *lengthen* in (75) is a function that determines the degree d to which x qua type O is lengthened by extent n in e.

Since the meaning of *lengthen* is not lexically decomposed in (75), it needs to be ensured that the extent argument n indeed measures the difference in the length of x, which Kennedy and Levin guarantee via the definition of $\mathsf{increase}$ in (19b) (see also (20b) and (24a)):

(76) $\forall x \forall O \forall n \forall e($ $\qquad \triangleright$ AXIOM: difference in length is n if $d = 1$
$\quad \mathsf{lengthen}_{\delta}(x)(O)(n)(e) = 1 \rightarrow$
$\qquad \iota n'(\mathsf{long}(x)(n')(\mathsf{end}(e))) = \iota n''(\mathsf{long}(x)(n'')(\mathsf{beg}(e))) + n$

In prose, if the degree to which x qua type O is lengthened by n in e is 1, then the extent to which x is long at the end of e is equal to the extent to which x is long at the beginning of e plus n. For example, if the degree to which a rope is lengthened by ten centimeters in e is 1, then the extent to which the rope is long at the end of e is equal to the extent to which the rope is long at the beginning of e plus ten centimeters. Note that this axiom does not apply if d is less than 1, because in this case x is lengthened to an extent less than n in e.

The contrasts to be treated below are illustrated in (77)–(79).

(77) a. Rebecca lengthened the rope for twenty minutes. ▷ cf. (8a)
b. Rebecca lengthened the rope in twenty minutes. ▷ cf. (9a)

(78) a. *Rebecca lengthened the rope (by) ten centimeters for twenty minutes.
b. Rebecca lengthened the rope (by) ten centimeters in twenty minutes.

(79) a. Rebecca lengthened ropes (by) ten centimeters for twenty minutes.
b. *Rebecca lengthened ropes (by) ten centimeters in twenty minutes.

As seen in (77), if the extent argument is not overtly expressed, the VP *lengthen the rope* allows for both an atelic and a telic interpretation. In (78), in contrast, if the extent argument is overtly specified, only a telic reading of the VP is acceptable (excluding an iterative interpretation). Finally, as shown in (79), if the object NP is the bare plural *ropes*, then the VP is atelic even if the extent argument is overtly specified (as expected, it is also atelic if the extent argument is not overtly expressed).

The atelic VP in (77a) is analyzed as follows:

(80) a. $[_{\text{VP}}$ lengthen the rope$]^{atel} \rightsquigarrow$ ▷ cf. (77a)
$\lambda e.\exists n(\mathsf{lengthen}_\delta(x)(\lambda x'.x' = \mathsf{the\text{-}rope})(n)(e) = 1)$
b. $\mathsf{nuniq}(\lambda e.\exists n(\mathsf{lengthen}_\delta(x)(\lambda x'.x' = \mathsf{the\text{-}rope})(n)(e) = 1)) \rightarrow$ ▷ FACT
$\forall e' \forall e''($
$\exists n'(\mathsf{lengthen}_\delta(x)(\lambda x'.x' = \mathsf{the\text{-}rope})(n')(e') = 1) \wedge$
$\exists n''(\mathsf{lengthen}_\delta(x)(\lambda x'.x' = \mathsf{the\text{-}rope})(n'')(e'') = 1) \rightarrow$
$e' \wr e'' \rightarrow$
$\mathsf{cum}(\lambda e.\exists n(\mathsf{lengthen}_\delta(x)(\lambda x'.x' = \mathsf{the\text{-}rope})(n)(e) = 1))$

The event predicate in (80a) denotes the set of events in which the degree to which the rope is lengthened by some extent is 1. Since n existentially quantified over, its value may vary with the event chosen. This predicate is cumulative as long as it is nonunique and its denotation is restricted to discrete events.[39] Accordingly, the sum of two such events is also an event in which the degree to which the rope is lengthened by some extent is 1.[40]

The analysis of the telic VP in (77b) differs from the previous one in that the value of the extent argument is implicitly fixed:

(81) a. $[_{\text{VP}}$ lengthen the rope$]^{tel} \rightsquigarrow$ ▷ cf. (77b)
$\lambda e.\mathsf{lengthen}_\delta(x)(\lambda x'.x' = \mathsf{the\text{-}rope})(n)(e) = 1$
b. $\mathsf{qua}(\lambda e.\mathsf{lengthen}_\delta(x)(\lambda x'.x' = \mathsf{the\text{-}rope})(n)(e) = 1)$ ▷ FACT

The event predicate in (81a) denotes the set of events in which the degree to which the rope is lengthened by a particular extent n is 1. This predicate is quantized because the value of n is implicitly fixed and hence no proper subevent of such events is also an event in which the degree to which the rope is lengthened by n is 1. Rather, in any proper subevent either the extent is less

[39] The restriction to discrete events ultimately makes matters easier because the counting of any extent more than once needs to be avoided.

[40] The proof of this fact and those in (81b) and (82b) requires certain axioms not introduced here. Even so, the hope is that the results are clear enough on intuitive grounds.

than n or the degree is less than 1 (or both).

The VP in (78b) differs from the one in (77b) in that the extent argument is overtly specified, but this naturally also gives rise to a telic interpretation:

(82) a. [$_{\text{VP}}$ lengthen the rope (by) ten centimeters] $\rightsquigarrow$ ▷ cf. (78b)
$\lambda e.\mathsf{lengthen}_\delta(x)(\lambda x'.x' = \mathsf{the\text{-}rope})(\mathsf{10\text{-}cm})(e) = 1$

b. $\mathsf{qua}(\lambda e.\mathsf{lengthen}_\delta(x)(\lambda x'.x' = \mathsf{the\text{-}rope})(\mathsf{10\text{-}cm})(e) = 1)$ ▷ FACT

The event predicate in (82a) is quantized and denotes the set of events in which the degree to which the rope is lengthened by ten centimeters is 1. This analysis excludes an atelic interpretation of the VP, ruling out the sentence in (78a).

Finally, the VP in (79a) receives the following analysis:

(83) a. [$_{\text{VP}}$ lengthen ropes (by) ten centimeters] $\rightsquigarrow$ ▷ cf. (79a)
$\lambda e.\exists x(\mathsf{lengthen}_\delta(x)(\mathsf{ropes})(\mathsf{10\text{-}cm})(e) = 1)$

b. $\mathsf{nuniq}(\lambda e.\exists x(\mathsf{lengthen}_\delta(x)(\mathsf{ropes})(\mathsf{10\text{-}cm})(e) = 1)) \rightarrow$ ▷ FACT
$\forall x'\forall x''\forall e'\forall e''($
$\mathsf{lengthen}_\delta(x')(\mathsf{ropes})(\mathsf{10\text{-}cm})(e') = 1 \wedge$
$\mathsf{lengthen}_\delta(x'')(\mathsf{ropes})(\mathsf{10\text{-}cm})(e'') = 1 \wedge e' \wr e'' \rightarrow$
$x' \wr x'' \rightarrow$
$\mathsf{cum}(\lambda e.\exists x(\mathsf{lengthen}_\delta(x)(\mathsf{ropes})(\mathsf{10\text{-}cm})(e) = 1))$

The event predicate in (83a) denotes the set of events in which the degree to which ropes are lengthened by ten centimeters is 1. Interestingly, this predicate is cumulative as long as it is nonunique and a restriction to discrete events implies that no ropes are lengthened more than once. Since this analysis does not allow for a telic reading of the VP, the sentence in (79b) is excluded.

4 Conclusion

In speaking of 'aspectual composition with degrees', I allude to an aspectual approach in which the notion of the degree of realization of an event type plays a central role. In this paper, I have proposed how such an approach might look in the context of an event semantics, applying it in greater detail to verbs with an incremental theme and in lesser detail to degree achievements. In a nutshell, it is an attempt to take seriously the idea that such verbs are gradable. The present account differs from Krifka's in that the latter lacks degrees altogether and as a result can express the notion of partial realization in a roundabout way at best. Perhaps a bit ironically, although the present account shares a degree-based spirit with Kennedy and Levin's approach, it mischievously recasts their degrees as extents, hence it also ends up having degrees where the latter lacks them. Even so, the main contrast with Kennedy and Levin's approach is undoubtedly that the present account makes the degree functions underlying the semantics of verbs with an incremental theme and degree achievements sensitive to the description of the internal argument as well.

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