Vectors Across Spatial Domains:

From Place to Size, Orientation, Shape and Parts{1}

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0 Introduction

Every language has terms that are used to talk about space. The most prominent and most intensively studied of these spatial terms are the prepositions, the kind of words that in some sense have specialised in space, describing where something is or happens or where it is moving:

- (1) a. The milk is *in* the refrigerator
 - b. We met *behind* the statue
 - c. They moved *into* another appartment
 - d. I led him *across* the yard

Prepositions are one kind of what I will call place terms ((1a) and (1b)) and change of place terms ((1c) and (1d)). Members from other parts of speech can also be (change of) place terms:{2}

(2)	a.	He lives in the <i>vicinity</i> of a small town	(place noun)
	b.	How <i>far</i> do you live from town?	(place adjective)
	c.	They <i>entered</i> the bar	(change of place verb)
d. The airplane		The airplane went <i>down</i>	(change of place adverb)

Most of the literature about spatial language is restricted to place terms and prepositions.{3} However, the domain of place does certainly not exhaust the language of space. There are at least four other spatial domains: • The domain of size, with size terms ((3a) and (3b)) and change of size terms ((3c) and (3d)):{4}

- (3) a. a *thin* hair
 - b. the *height* of a tower
 - c. to *shorten* a rope

d. to grow

• The domain of orientation, with orientation terms ((4a) and (4b)) and change of orientation terms ((4c) and (4d)):

- (4) a. an *oblique* line
 - b. to stand *upright*
 - c. he kicked it *over*
 - d. two rotations

• The domain of shape, with shape terms ((5a) and (5b)) and change of shape terms ((5c) and (5d)):

(5) a. a *straight* stick

- b. a *circle*
- c. to straighten
- d. to *bend* a stick

• The domain of spatial parts, with nouns such as the following:

(6) top, backside, corner, centre, bump, notch

This variety of spatial terms (and there may be other domains not included in this brief overview) raises the following questions: How do these different classes of terms within the language of space fit together? Is the domain of space a collection of unrelated portions of vocabulary or is there an underlying system? If there is a system, then how can we best analyze that system?

O'Keefe (1996) and Zwarts (1997) and Zwarts and Winter (2000) have develop a semantics for spatial prepositions in which the position of a figure (also called located object, theme or trajector) relative to a ground (also reference object, relatum, or landmark) is represented by a *vector*. Vectors are also used in Marr's 3D models (1982) to represent the axes of objects and parts of objects and in this way they help to encode properties of size, orientation, and shape in object representations. This is also linguistically relevant as Landau and

Jackendoff (1993) and Jackendoff (1996b) show for some terms of size and spatial parts (the 'axial vocabulary'). The purpose of this paper is to adduce linguistic motivation for the integration of these two uses of vectors, the 'place use' and the 'axis use', by showing similarities and connections between the domain of place on the one hand and the domains of size, orientation, shape, and spatial parts on the other hand. The claim is that the relation between figure and ground in place terms is very similar to the notion of axis that plays a role in the meaning of terms of size, orientation, shape, and parts and that place relations and object axes are both represented by means of vectors.

I will first explain the vector-based approach to place terms in section 1. In section 2 I will show how the vector space model can be extended from the place domain to the domains of size, orientation, and spatial parts and what advantages that extension offers. In section 3 I will do the same thing for paths (sequences of vectors that represent change), which enables us to understand the language of shape and the language of change of size and orientation. Section 4 gives some final thoughts on the somewhat more complicated area of change of shape.

1 Place Terms and Vector Semantics

A place term is an expression that is used to locate an object or event. Place terms can be 'static' (7) or 'dynamic' (8) and they can be prepositions, adjectives, nouns, adverbs, or verbs:{5}

- (7) a. His wife was *out of* bed now, standing *beside* him *near* the window (preposition)
 - b. the big airbase at *nearby* Mildenhall (adjective)
 - c. He was *around* (adverb)
 - d. in the *vicinity* of the town (noun)
- (8) a. He moved *through* the half-daylight *over* the yard *to* the shed (preposition)
 - b. Gordon Butcher bent his head *closer and closer* still (adjective)
 - c. She went *downstairs* to make breakfast. (adverb)
 - d. They *reached* the forward point of it. (verb)

One reason for using vectors in the semantics of place terms comes from modified prepositional phrases (Zwarts 1997):

- (9) a. the plow, which lay *ten yards* behind the tractor
 - b. the plow, which lay *very far* behind the tractor
 - c. the plow, which lay *just* behind the tractor
 - d. the plow, which lay *straight* behind the tractor

The important thing about these modifiers is that they specify the distance or direction in which the located object (the plow) can be found relative to the reference object (the tractor). As a result they map the spatial region corresponding to the PP *behind the tractor* to a subregion. It is hard to imagine how this works when the region is a set of points or a 'blob' of space because all by themselves points or blobs do not have distances or directions. However, suppose that *behind the tractor* corresponds to a set of vectors pointing backwards from the tractor and that the plow is located at the end point of one of these, as indicated in the following figure (where the tractor and the plow are represented point-like and the shaded area corresponds to the end points of the vectors):



Figure 1: the plow behind the tractor

Modifiers can then be interpreted as mapping this set to a subset by imposing additional constraints on the length (*ten yards, very far,* and *just*) or orientation (*straight*) of the vectors. Furthermore, each preposition maps to a particular kind of region. *Near the tractor* denotes all those vectors that have a short length; *next to the tractor* corresponds to the vectors pointing sideways; *between the tractor and the barn* denotes the set of vectors pointing from the tractor towards the barn:



Figure 2: near the tractor, next to the tractor, between the tractor and the barn

The kind of regions denoted by PPs in this theory are not unstructured sets. The algebra of vectors (with its operations of vector addition, scalar multiplication, inversion, and rotation) endows regions with a rich structure that allows for the definition of semantically significant classes of PPs (see Zwarts 1997 and Zwarts and Winter 2000 for more details).

A few remarks about vectors and regions are in order here. First, for the purposes of this paper the precise formal or cognitive definition of vectors is of secondary importance. They can be taken as primitives, defined in terms of cartesian or polar coordinates, as pairs of points (Gambarotto & Muller this volume), bounded half-lines (Habel et al. this volume), or in terms of more complex neurological patterns (O'Keefe this volume). Of course, it does matter what a vector is, but not for the semantic phenomena under study in this paper. Second, the vector representations diagrammed above are admittedly too simple in that they fail to account for the important fact that PP regions are really graded and vaguely bounded, as shown in Carlson, Regier, & Covey (this volume) and the experiments cited there. They show how different factors operate on a vector representation to explain which parts of the region of a preposition receive higher acceptability ratings than others. For example, in an *above* region the points that are both close to the reference object and vertically aligned with it receive the highest ratings. Third, another simplification of the vector model presented here is the unwarranted exclusion of functional factors in the interpretation of prepositions, like 'containment' for in, 'support' for on and 'protection' for over (see Coventry this volume). How to represent these functional notions in a mildly formal way and how to integrate them with the vector representation is unclear at present, although I personally feel that the concepts of force dynamics (Talmy 1988) are crucial in understanding functions and relating them to spatial representations.

What about prepositions like those in (10) that do not just locate an object but that help to express a change of place (10a), an orientation (10b), or the distribution of an object over a stretch of space (10c)?

- (10) a. He went *into* the house
 - b. The sign points *away from* the village
 - c. The people were standing *along* the road

A widespread assumption, that I will follow here, is that the underlying concept for these kind of expressions is a *path* (Jackendoff 1983, Habel 1989, Nam 1995, Talmy 1996 and many others). It seems intuitively a quite obvious move to analyze paths simply as vectors, as Helmantel (1998) does, but the problem is that paths often do not describe straight lines. For instance, *around the tree* can describe a path that takes the shape of a circle (among other shapes, see Schulze 1991, Wunderlich 1993 and Taylor 1995). Another option would then be to represent such a curved path as a sequence of vectors that are connected head to tail, as in Bohnemeyer (this volume). See figure 3.



Figure 3: head-tail path

Each vector of the path represents the move that the theme makes over a particular interval of time. The shorter the vectors of the path, the better the actual shape of a path is approximated. A path consisting of one vector could be seen as the simplest instance of this kind of representation.

Even though there are potential applications for these kind of paths (see section 3.1), I think that, as far as the interpretation of directional prepositions is concerned, there are good reasons to analyze paths first of all as sequences of positions relative to a reference object (Langacker 1987, Jackendoff 1996a), that is, in our framework, as sequences of vectors that all have one and the same origin. For example, the PP *around the tree* would roughly correspond to a series of vectors like those in Figure 4, *over the house* could refer to the sequence of vectors in Figure 5 and *towards the city* corresponds to Figure 6:





Figure 5: over the house



Figure 6: towards the city

This representation captures in the most direct way the close relationship between position and motion and between locative prepositions and directional prepositions. It helps to understand more readily why a sequence of positions can be described by using a directional PP (Regier 1995) and why entailments like the following are valid:

- (11) a. The bird will fly *over* the house
 - b. The bird will be *above* the house

Sentence (11a) entails sentence (11b) because the position above the house is just one of the positions that is part of the path over the house. There are also systematic mappings between locative and directional PPs:

- (12) a. The mouse went under the cupboard
 - b. The hotel is over the bridge

In (12a) a locative PP *under the cupboard* is used to describe a path that ends under the cupboard. In (12b) a path *over the bridge* is used to specify a location as being the end point of the path. Representing paths as sequences of vector positions allows us to describe these mappings in a straightforward way.

Another reason for representing paths as sequences of vectors is that the length of the vectors of the path can sometimes be specified by a modifier, as in the following Dutch examples:

(13) a. De auto reed *vlak* langs de gracht

The car drove close along the canal

b. Het vliegtuig vloog *hoog* over de stadThe airplane flew high over the city

Notice that the distance modifiers *vlak* 'close, right' and *hoog* 'high' do not specify a distance along the direction of the path, but a distance perpendicular to it.

All directional PPs receive the same type of path representation; there are not different kinds of representations for prepositions like *to* and *from* and prepositions like *around* and *along* (pace Bohnemeyer this volume). One important reason for having just one kind of path (apart from theoretical parsimony) is that different directional prepositions can be combined in a sentence to describe one path, as in our example (8a) above:

(14) He moved *through* the half-daylight *over* the yard *to* the shed

There is only one path here, but it is described by three different directional PPs that specify different spatial positions of it.{6}

It is important that paths are atemporal entities, because they can be used in different ways, as we already saw in (10): for motion (real or 'virtual'), orientation, extension, distribution. However, this notion of path can be extended to include time by relating the primarily non-temporal sequence to a sequence of moments of time, an operation that is necessary for describing motion. (See Jackendoff (1983,1996a) and Talmy (1996) for more details and also sections 3 and 4 of this paper. See furthermore Zwarts and Winter (2000) for more details on the vector analysis of paths.)

2 Place Vectors and Axis Vectors

2.0 Preliminaries

The claim of this paper is that terms from the domains of place, size, orientation, shape, and spatial parts are all interpreted with respect to one and the same spatial structure, which has the form of a three-dimensional vector space \mathbf{V} .{7} There is no need to assume different 'ontological categories', 'sorts', or 'primitives' for places, sizes, orientations, shapes, or spatial parts. Vectors are the only primitive spatial objects and each spatial domain constructs its meanings on the basis of vectors. Even if this claim will turn out to be too strong (which is quite likely), it still provides a fruitful perspective for studying the language of space, as I hope to show.

I assume that a vector \mathbf{v} has two uses. It can either represent an axis of an object x, as in Marr (1982) or it can represent the relative position of an object x with respect to an object or point of reference y, as in O'Keefe (1996) and Zwarts (1997). This is illustrated in the following figures:



Figure 7: place vector

Figure 8: axis vector

I will use the term *place vector* for the vector in Figure 7 and *axis vector* for the vector in Figure 8. Even though the vectors are used in different ways, they come from one and the same vector space **V**. It is how they relate to objects that differs. I will represent this by means of two relations: **place** and **axis**.

An object x can relate to a place vector \mathbf{v} in two different ways, depending on whether x functions as the figure or the ground of a spatial relation. Both of these functions are illustrated in Figure 7: **place**(x,**v**) holds ('x is placed at vector \mathbf{v} ') as well as **place**(\mathbf{v} , \mathbf{y}) ('vector \mathbf{v} is placed at y'). An object can be placed at more than one vector (as example (7a) shows) and there can be many vectors starting in one object, but there is always one unique vector for objects x and y (pointing from y to x). I will sometimes use **place**(x, \mathbf{v} , \mathbf{y}) to abbreviate the conjunction **place**(x, \mathbf{v}) & **place**(\mathbf{v} , \mathbf{y}). In Figure 8 the relation axis(x,v) holds ('x has an axis v'). The assignment of axis vectors to objects is far more complicated than the assignment of place vectors and the treatment of axes can only be very rudimentary in this paper. I will restrict the discussion to what are called *primary*, *major*, or *generating* axes in the literature (Marr 1982, Lang 1990, Jackendoff 1991, Levinson 1992, Landau and Jackendoff 1993) and ignore the existence of *secondary*, *subsidiary*, or *orienting* axes. This restricts the domain of application to those objects that can be conceptualized as having only one axis, like sticks, arrows, poles, and it excludes objects with more than one axis, like windows, doors and cylinders, as well as objects that do not seem to have axes, like disks and balls. (Even with this restriction there still remains a lot to be said.) When an axis is assigned to an object it runs from one end to the other end and its direction is either arbitrarily chosen (as with a stick), or determined by the shape or function of the object (as with an arrow or spoon) (see Levinson 1992 for discussion about this issue).

I will now show with concrete examples how this dual role of vectors as places and axes helps us to understand the underlying unity of the terminologies of place, size, orientation, and spatial parts. The treatment of terms referring to change of size or orientation, and the treatment of shape terms will have to wait until after I will have explained the use of paths more fully later in this paper.

2.1 Place and Size

As indicated already by Clark (1973) there is a close similarity between the place terms *close* and *far* on the one hand and *short* and *long* on the other hand, which is shown for instance by paraphrase relations like those in (15):

(15) x is close to $y \approx$ the distance between x and y is short x is long \approx one end of x is far from the other end

This similarity is brought out in the definitions for these expressions:

(16) x is close (to y): there is a v such that place(x,v,y) and |v| < rx is short: there is a v such that axis(x,v) and |v| < rx is far (from y): there is a v such that place(x,v,y) and |v| > rx is long: there is a v such that axis(x,v) and |v| > r In these definitions | | gives the length of a vector **v** and r is some sort of context-dependent average for vector length. The y can be made explicit (*far from the house*) or left implicit (like the contextually given position of the speaker). Note that *close* and *short* both involve the same set of vectors, i.e.those vectors that are relatively small in length, pointing in all directions. The difference only consists in what those vectors are used to represent, place or axis. The fact that both place and size terms are based on vectors makes it easy to switch between place and size descriptions of the same situation, as in (15).

It has also been observed that the adjectives *high*, *low*, and *deep* (and the corresponding nouns *height* and *depth*) are ambiguous between a place and a size meaning (Clark 1973, Lang 1991, Taylor 1995):

(17) a. a high window

b. a deep hole

The place reading of (17a) expresses that the window is at a high position relative to the ground; the size reading makes a statement about its vertical dimension. Similarly, in (17b) either the hole is at a deep position, or it extends deep into the ground. What these terms have in common, both in their place and size use, is that the vectors involved have a vertical orientation, either upward (*high and low*) or downward (*deep*).{8} I will represent the upward direction **UP** as the set of vectors pointing upwards, and the downward direction as -UP (the inversion of all vectors in **UP**).{9} It is useful to have the concept of *projection* of a vector on a direction. As shown in Figure 9, vector **v** has a unique projection **v**' on the vertical direction:



Figure 9: projection of a vector on the upward direction

This makes it possible to define a 'weaker' notion of upward and downward. A vector is upward if its projection on the **UP** direction is not the zero vector. This more inclusive region includes all vectors in the upper two quadrants, including the vectors of **UP** but excluding the horizontal ones. Similarly, a vector is downward if its projection on the $-\mathbf{UP}$ axis is not **0**, which gives us the two lower quadrants. The definitions are then as follows: {10}

(18)	x is low (w.r.t. y):	there is an upward ${\bf v}$ such that $\textbf{place}(x, {\bf v}, y)$ and $ {\bf v} < r$	
	x is low:	there is an upward v such that $axis(x, v)$ and $ v < r$	
	x is high (w.r.t. y):	there is an upward v such that $place(x, v, y)$ and $ v > r$	
	x is high:	there is an upward v such that $axis(x, v)$ and $ v > r$	
	x is deep (w.r.t. y):	there is a downward v such that $place(x, v, y)$ and $ v > r$	
	x is deep:	there is a downward v such that $axis(x, v)$ and $ v > r$	

(The implicit reference point y of the place terms is a contextually given ground level.) The ambiguity of these terms is restricted to the **place** vs. **axis** relations.

The vertical dimension with its two opposite directions UP and -UP is not limited to adjectives, as shown by the following prepositions:

(19) x above y: there is an upward v such that place(x,v,y)
 x below y: there is a downward v such that place(x,v,y)

If place and size terms are based on the same vector structure, it comes as no surprise that they share the same system of measure phrases.

- (20) a. The stone was *twelve inches* deep
 - b. The rope was *twelve inches* long

There are no separate systems of unit nouns for distances on the one hand and for sizes on the other hand, but the same inches, meters, yards, etc. are used in both domains. This shows clearly that place and size are basically the same domain, because different domains (like time, temperature, or weight) are usually characterized by their own system of units (e.g. seconds, degrees Celcius, kilograms). Measure phrases can be treated uniformly as additional restrictions on the vectors of place and size terms:

(21) x is <u>twelve inches</u> deep: there is a downward **v** such that **place**(x,**v**,y) <u>and $|\mathbf{v}| = 12i$ x is <u>twelve inches</u> long there is a **v** such that **axis**(x,**v**) and $|\mathbf{v}| = 12i$ </u>

2.2 Place and Orientation

The major reason for building the semantics of place terms and orientation terms on a common foundation of vectors is the role that the vertical and horizontal directions play in both areas. Most orientation terms are interpreted relative to the framework of the vertical directions and the horizontal plane orthogonal to them. The framework sketched above for place and size terms extends naturally to orientation terms without much additional machinery. The simplest orientation terms state that an axis of x is pointing upward (22a), either downward or upward (22b), or that it is horizontal (22c): $\{12\}$

- (22) a. x is standing, upright: there is a v such that axis(x,v) and $v \in UP$
 - b. x is vertical, plumb: there is a v such that axis(x,v) and $v \in VERT$
 - c. x is horizontal, level, lying etc.: there is a v such that axis(x, v) and $v \in \perp VERT$

Other orientations can be can be described as falling between the vertical or the horizontal direction (23a) or as having a big or small angle with those directions (23b) and (23c):

- (23) a. x is diagonal, inclined, sloping: there is a v such that axis(x,v) and v is between VERT and \perp VERT
 - b. x is steep, precipitous: there is a v such that axis(x,v) and the angle between v and $\perp VERT > r$
 - c. x is gentle, gradual: there is a v such that axis(x,v) and the angle between v and $\perp VERT < r$

In other cases the axis of an object is reversed with respect to its canonical orientation vector c:

(24) x is reversed: axis(x,-c)

x is upside down: axis(x,-c) and $c \in UP$

There are also orientation terms that take other directions \mathbf{R} beside the vertical and horizontal as their 'reference direction', like *straight* or *oblique*:{13}

(25) x is straight: there is a v such that axis(x,v) and $v \in \mathbf{R}$

x is oblique: there is v such that axis(x,v) and v is between R and $\perp R$

The precise definitions are not so important. What is relevant here is that all of the concepts used (axis, verticality, orthogonality, inversion, and even angle) are part of the same vector universe on which the domains of place and size are based.

Another indication of the intimate relation of place and orientation is that some adjectives of orientation can be used to modify prepositional phrases:

- (26) a. *straight* above the window
 - b. *diagonally* under the painting

This would be surprising if place and orientation were completely separate domains. In a vector approach however the modifiers *straight* and *diagonally* contribute one and the same vector condition either to axis vectors or to the place vectors of the prepositional phrases:

(27) x is <u>straight</u> above the window: there is a v such that v is upward and place(x,v,the-window) and $\underline{v \in UP}$

x is <u>diagonally</u> under the painting: there is a **v** such that **v** is downward and **place**(x,**v**,the-painting) <u>and</u> <u>**v** is between $-\mathbf{UP}$ and $\perp -\mathbf{UP}$ </u>

Habel et al. (this volume) represent orientation and axes in terms of half-lines, that are like vectors except that they are unbounded in the direction in which they are pointing. As far as the domain of orientation is concerned their formalism covers the same ground as vector space semantics. However, because of their unbounded nature half-lines do not represent the length of object axes, even though this is relevant for dimensional adjectives. Either Habel et al. need two different representations for axes, one for orientation aspects and for one size aspects, or they need to add a bounding point on the half-line, which is basically a way of defining vectors.

2.3 Place and Parts

That there is a close connection between terms for parts and place terms has often been observed, especially in relation to body parts (see for instance Heine at al. 1991). Many languages use terms that are basically for parts of bodies or objects to talk about places adjacent to those parts. For instance, in English, the part noun *front* (like in *the front of the car*) forms the basis of the place term *in front of the car* and other part terms (like *back, top*, and *side*) can also be used to denote places (*at the back of, on top of, to the side of*). In this case the intrinsic axes of objects are used to define regions outside those objects. Parts of objects can also be named on the basis of properties of the surrounding space. For example, a tree, an object without intrinsic horizontal axes, can be assigned a front and a back from the point of view of an observer and the top and bottom of an object can be determined by the direction of gravity.

It is intuitively clear that place and part terms share basic spatial concepts. For example, the opposition between *top* and *bottom* is of the same kind as the opposition between *above/over* on the one hand and *below/under* on the other hand. This is also illustrated by the fact that we can say that the top of an object is above the bottom and the bottom is below the top (if the object is oriented in its normal, 'canonical' way).

The general semantics of spatial parts requires a combination of the two vector relations, **place** and **axis**. Figure 10 shows the schematic picture of a stick (y), with one end indicated by x.



Figure 10: a stick and one of its ends

The stick y has an axis represented by the vector \mathbf{v} and x is a part of y located at the end point of this vector. This can be represented as follows: (28) x is an end of y: x is a part of y and there is a v such that axis(y,v) and place(x,v)

Many spatial parts will be parts that are located at a particular position relative to the axis of the whole. This is usually an extremity, but it can also be the middle:

(29) x is the middle of y: x is a part of y and there is a v such that axis(y,v) and $place(x, \sqrt[1]{2}v)$

The middle of the axis is derived by multiplying the axis vector by a scalar ¹/₂, which yields a vector with half the length. The top and bottom of an object are defined as ends, but with an extra directional condition on the axis vector:

(30) x is the top of y: x is a part of y and there is an upward v such that axis(y,v) and place(x,v)
 x is the bottom of y: x is a part of y and there is a downward v such that axis(y,v) and place(x,v)

These definitions are still an approximation because they do not distinguish between *intrinsic* tops and bottoms and *absolute* or *environmental* tops and bottoms (Jackendoff 1996b, Levinson 1996). A candle has an intrinsic top and bottom, because its canonical position is upright with certain defining features at each end. Even when a candle has fallen over, we can still talk about its top and bottom. A stick that happens to be in an upright position has a top and bottom in virtue of its position at that time with respect to the direction of gravity, but not an intrinsic top and bottom.

The front and back parts of an object can also be intrinsic (defined by its shape, function, or motion) or *relative* or *deictic* (defined by the position of an observer watching it). The following definitions will have to suffice here:

(31) x is the front of y: x is a part of y and there is a vector v such that axis(y,v) and f(v) and place(x,v)x is the back of y: x is a part of y and there is a vector v such that axis(y,v) and -f(v) and place(x,v)

The front and back of y are ends of y, but with an additional condition on the direction of the axis that captures the nature of the front-back axis (**f** for the front and $-\mathbf{f}$ for the back). Some possibilities: **v** can represent the

direction of growth (Leyton 1992), the direction of motion, it can point to the side where the object is most commonly used or encountered (like the front of a television, Clark 1973) or to an observer. For more details, see Herskovits (1986), Vandeloise (1991), Van der Zee (1996), Van der Zee & Eshuis (this volume), among others.

How are places defined in terms of spatial parts? A possible way of doing this would be to take scalar multiplications of the axis vector. The range of numbers that are used for the multiplication determines whether the resulting place is external, in contact, or internal:

- (32) a. x is in front of y: there is a vector sv with s > 1 such that axis(y,v) and f(v) and place(x,sv)
 - b. x is on the front of y: there is a vector sv with s = 1 such that axis(y,v) and f(v) and place(x,sv)
 - c. x is in the front of y: there is a vector sv with $\frac{1}{2} < s < 1$ such that axis(y,v) and f(v) and place(x,sv)

These three situations are illustrated in Figure 11a-c:



Figure 11a: x in front of y, Figure 11b: x is on the front of y, Figure 11c: x is in the front of y

3 Place Paths and Axis Paths

3.0 Preliminaries

Vectors alone are sufficient to represent the meaning of *static* terms of place, size, orientation, and parts. For the representation of change or variation (of place, size, and orientation) we need paths, as we already saw in section 1 for the prepositional phrase *around the tree*. We also need paths, as I will show, to provide a semantics for shape terminology.

A path can be defined as a sequence of vectors, or, more formally, a mapping from an interval [0,k] of real or natural numbers to vectors. Following Jackendoff (1983) and others I will take a path as a non-temporal

entity, which means that the numbers used to define a path are an ordering device and do not represent moments of time. Figure 12 gives a simplified example of a path that describes a circle:{14}



Figure 12: a circular path

Like a vector, a path is an abstract spatial entity that can be used in different ways. The most obvious use is to describe the trajectory of an object in motion. The vectors function as place vectors, indicating the relative position of an object x moving through time relative to a point or object y, as illustrated in Figure 13 (*x circles y*):



Figure 13: x circles y

I will call this a *change of place path*. Instead of functioning as place vectors the vectors of the path can also function as axis vectors, which represents that the axis of an object changes through time, a situation that is illustrated in Figure 14 (*x rotates*) and that I will call a *change of axis path*:



Figure 14: x rotates

I propose to interpret these two verbs in the following way:

(33) to circle y { e: there is a p such that place-path(e,p,y) and p is circular } to rotate { e: there is a p such that axis-path(e,p) and p is circular }

The verbs (and the verb phrases and sentences built around them) are interpreted as sets of events, which is what the 'e' stands for. An event can relate to a path in two ways: (i) by means of the relation **place-path** that relates an event to the path that that event describes through space (relative to y), what is sometimes called the spatial 'trace' of the event (Krifka 1995, Link 1998), (ii) by means of a relation **axis-path** that associates an event to the path of a changing axis. Both functions are defined in terms of the more basic relations **place** and **axis** of section 2. If an event e has a particular path **p** as its path of motion, then for each moment of the event, its theme (the thing that is moving, the subject of *to circle*) is located at the corresponding vector of the path. And for change of axis, a path **p** represents the way an axis changes if and only if for each moment of the event the axis of the theme is identical to the corresponding vector of the path. $\{15\}$ A path is circular if it is the result of rotating the initial vector **p**(0) of the path the full 360° . $\{16\}$

In addition to using a path to represent properties of motion or rotation events we can also use paths to represent shape properties of contours or linear objects. The path of Figure 12 can be used to represent the contour of a disk or the circular axis of a ring, as illustrated in Figure 15 and Figure 16, respectively:



Figure 15: a round disk



Figure 16: a round ring

The adjective *round* can then be defined as follows:

(34) round

{ x: there is a **p** such that **place-path**(x,**p**,y) and **p** is circular }

The same function **place-path** that was used for *to circle* is used here to relate an *object* x to a path **p** (relative to y, which is here the centre of the circle). In this case there is not an association between temporal parts of the event to vectors on which to locate the theme, but there is an association between spatial parts of the object x and vectors on which those parts are located. When applied to the situation in Figure 15 the parts of the contour of the disk are related to the vectors of the path. In Figure 16, the parts of the ring are related to those vectors.{17} (Of course, the path describing a circle would need more than just the five vectors drawn here for the sake of the presentation.) I will call this kind of path a *shape path*.{18}

Can **axis-path** also apply to objects? What we would need in this case is an object that can be divided into a sequence of parts each of which has an axis and these axes show a variation in direction or length. For circular paths, one example might be a spiral staircase, where the stairs wind round a central pilar. Each stair has a vector that points from this centre and (when seen from above) the vectors together form a circular path, as illustrated in Figure 17.



Figure 17: spiral staircase

This situation can be described in the following way:

(35) there is a **p** such that **axis-path**(x,**p**) and **p** is circular

An addition complication of the spiral staircase is its vertical dimension. A more complete representation involves two paths: the circular path defined in (35) and a path of vertical vectors representing the heights of the stairs either from the ground up or from the top down. Each stair is then characterized by two vectors: a place vector for its height and an axis vector for its length and orientation.

Recall from section 1 that there are different ways of representing paths in terms of vectors. Instead of describing a path as the endpoints of vectors coming from one central origin, we can also construct a path by putting vectors head to tail. For 'circular' expressions like *round*, *around*, *to circle*, *to rotate*, and *spiral* a path with a central origin is required, but for other terms, a 'head-tail path' seems more appropriate. Which representation is appropriate depends on the lexical semantics of the expression under consideration, and maybe on other factors. We might even be able to switch between the two modes of representation.{19}

With this background concerning paths let us turn to the various spatial domains.

3.1 Paths in Place and Size

Size cannot always be represented on the basis of vectors, for the simple reason that many things that have size are not straight, but curved in various ways. The dimensional adjectives *long* and *short* apply also to ropes and rivers just as much as to sticks and streets and the size nouns *circumference* and *perimeter* measure something that excludes any idea of straightness. We saw that the contour or axis of a curved object can be represented by a shape path, which is illustrated in Figure 18 and Figure 19 for a part of a river:



Figure 18: centered path of a river

Figure 19: head-tail path of a river

Although any point could be taken as the origin of the vectors describing the centered path, it seems most natural to take one end of the river as the origin, as in Figure 18. Alternatively, we can describe the shape of the river by means of a head-tail path, as in Figure 19. In both cases, we need to choose an appropriate scale or grain size of our representation by fixing the number of vectors we use to represent the shape. Since *long* and *short* can apply to straight and curved objects, I assume that their definitions come in two varieties, for axis vectors in (36a), and for shape paths in (36b):

(36) a. x is long: there is a v such that axis(x,v) and |v| > rx is short: there is a v such that axis(x,v) and |v| < r

> b. x is long: there is a **p** such that place-path(x,p) and |p| > rx is short: there is **p** such that place-path(x,p) and |p| < r

The length function | | would have to be defined for paths in an appropriate way.

There are cases where a size is described in terms of the directional prepositions around and across:

- (37) a The crater was two miles round(i.e. The circumference of the crater was two miles)
 - b. The crater was two miles across

(i.e. The diameter of the crater was two miles)

The circumference of the crater, a size concept, is described in terms of the length of a movement around it and its diameter in terms of the length of a movement across. Notice also that the nouns *circumference*, *perimeter*, and the Dutch *omtrek* are all based on prepositions meaning 'around': *circum* in Latin, *peri* in Greek, *om* in Dutch. The word *diameter* and the corresponding Dutch word *doorsnede* are based on prepositions meaning 'through, across', *dia* in Greek and *door* in Dutch. This is not surprising if the same kind of paths are used in motion and shape description: *circumference* and *round* are both based on circular paths, and *diameter* and *across* on paths that go from one side of an object to the other side. The measure phrases in (36) specify the length of these paths.

Paths can be relevant for size in another way. When the size of an object changes, it is the axis vector that becomes longer (in the case of *to grow, to extend*) or shorter (in the case of *to shrink, to reduce*). The interesting thing is that the *direction* of change of size can be specified by means of a phrase that is also used for motion:

- (38) a. The town is extending to the east
 - b. John is growing <u>up</u>

I interpret these sentences as sets of events:

- (39) a. { e: there is a p such that axis-path(e,p) and theme(e,the-town) and the length of the vectors of p is increasing over its domain and the vectors of p are pointing to the east }
 - b. { e: there is a p such that axis-path(e,p) and theme(e,john) and the length of the vectors of p is increasing over its domain and the vectors of p are pointing upward }

The underlined expressions in (38) correspond to the underlined parts in (39).

3.2 Paths in Place and Orientation

The terminology for change of orientation (i.e. rotation) overlaps with terms in the place domain in several ways. Words like *around* and *turn* can be used to describe the rotation of an object that stays in one place (40a) but they can also be used to describe the path of a moving object (40b) and (40c):

- (40) a. He turned around to see what had happened
 - b. The car turned into a small street
 - c. She ran around

The ambiguity of the verb *turn* and the adverb *around* depends on whether the underlying circular path is used as a change of *axis* path (in (40a)) or as a change of *place* path (in (40b) and (40c)).{20}

On the other hand, the adverbs *up* and *down* are not only used for motion up and down, as in (41a) and (41b) but also for rotation (from horizontal to vertical or vice versa), as in (41c) and (41d):

- (41) a. The ball fell down
 - b. The bird flew up
 - c. The tree fell down
 - d. The mast was pulled up

The tree in (41c) and the mast in (41d) describes paths that are one quarter of a circular path, but what is more important here is the projection of their axis on the vertical directions. The projection of the tree's axis on the upward direction is decreasing, the projection of the mast's axis is increasing.

It has been observed (Jackendoff 1983, Talmy 1996) that verbs of orientation can be modified by path PPs (42a) and (42b) and also that the orientation of an object in relation to another object can be described by means of path PPs (42c) and (42d):

- (42) a. The stick is aligned toward the east
 - b. The arrow is pointing across the lake
 - c. The tree is lying along the railway
 - d. There is a scratch diagonally across the door

This is another instance of terms of orientation and terms of place interacting. The simplest spatial representation of the straight objects in (42) is a single vector. However, in these examples the orientation is described by means of directional PPs, which I assumed to be interpreted in terms of paths (sequences of vectors). Hence, there is a mismatch between the subject (one vector) and the predicate (sequence of vectors). In order to resolve this mismatch I assume that axes of straight objects can be represented in two ways: by means of single vectors, but also by means of straight shape paths (i.e. all the vectors point in the same direction). This captures the idea of Talmy (1996) that we can ascribe 'fictive motion' to elongated stationary objects, for example by our gaze moving from one end to the other, and it makes it possible to understand how the PPs can apply to these non-motion cases:{21}

- (43) a. there is a **p** such that **place-path**(the-stick,**p**,y) and **p** is toward the east
 - b. there is a **p** such that **place-path**(the-arrow,**p**,y) and there is an extension **p**' of **p** such that **p**' is across the lake
 - c. there is a **p** such that **place-path**(the-tree,**p**,y) and **p** is along the railway
 - d. there is a **p** such that **place-path**(a-scratch,**p**,y) and **p** is diagonal and **p** is across the door

The y in these definitions is one end of the object, chosen as the starting point of the shape path. Whether the directional PPs in (42) are used in this kind of sentences or in real motion sentences makes no difference for their path denotation. In motion sentences the same paths are not related to an object but to an event:

- (44) a. The bus drove toward the east
 - b. there is an e and a **p** such that **place-path**(e,**p**,**y**) and **p** is toward the east

Both in (42a) and in (44a) the denotation of the PP *toward the east* is a sequence of vectors all pointing to the east and increasing in length.

3.3 Paths in Place and Shape

In the domain of shape we see more examples of ambiguities and interactions. Several basic shape terms can also be used with place meaning. The shape adjective *round* forms the basis of the adverb/preposition (*a*)round and the noun *circle* for the verb *to circle*. All these terms are based on circular paths that are either associated to events (*to circle around*) or to objects (*a round circle*).

Path PPs can also be used to modify shape terms:

- (45) a. The coast curves *outward into the sea*
 - b. The Chinese wall zigzags *through the hills*

The coast and the wall are not moving along the paths denoted by the italicized PPs, of course. What happens is that there is an underlying path \mathbf{p} in each sentence associated with the coast and the wall respectively that is further restricted by the verb (*to curve*, *to zigzag*) and by the PP:

- (46) a. there is a **p** such that **place-path**(the-coast,**p**,y) and **p** is curving and the vectors of **p** are pointing outward and into the sea
 - b. there is a p such that place-path(the-Chinese-wall,p,y) and p is zigzagging and the vectors of p are pointing to positions in the hills

This is illustrated in the following figures:





Figure 20: the coast

Figure 21: the Chinese wall

Even though it might not be immediately obvious how to define curving and zigzagging of paths and 'outward', 'into', and 'through', still the rough definitions and figures demonstrate that an abstract notion of path, generalizing over the domains of place and shape help us to understand the semantic structure of sentences like those in (45). Note that for zigzagging paths a head-tail path might be more appropriate to capture the 'zigzagging' shape than a centered path:



Figure 22: a zigzag path

In this representation a head-tail path **p** is zigzagging iff every two subsequent vectors make an acute angle. Whether we choose the representation in Figure 21 or Figure 22, the important thing is that zigzag paths are made up of component vectors that represent the individual back and forth motions characteristic for zigzagging. Modifiers can specify the direction of these individual vectors, as in the following example (*in and out, back and forth, across the wake of the motorboat*):

(47) In the slalom event, the towing boat speeds straight through a field of anchored buoys while the contestant, riding on a single ski or a kneeboard, pursues a zigzag course in and out of the buoys, swinging back and forth across the wake of the motorboat. {22}

3.4 Other Interactions

Until now we have seen the interactions of the domain of place with various other domains, but there are also connections among the domains of size, orientation, shape, and spatial parts involving paths:

- (48) a. The tree is straight
 - b. De boom groeit scheef/krom
 The tree grows askew/curved
 - c. the circumference of the disk

The word *straight* in (48a) is ambiguous between an orientation and a shape meaning. It can be the opposite of *askew/oblique* or it can be the opposite of *curved*. These two meanings are closely related. The orientation meaning applies basically to one vector, saying that that vector makes a zero angle with a reference direction, as shown in Figure 23, the shape meaning applies to a path of vectors, roughly saying that each vector $\mathbf{p}(i)$ of the path points in the same direction, as shown in Figure 24.



Figure 23: orientation straight



Figure 24: shape straight

In other words, some basic shape concepts, like *straight* and *curved* can be defined in terms of the relative orientation of the vectors making up the shape paths of the objects to which these terms apply.

In the Dutch example (48b), the change of size verb *groeien* 'to grow' can be modified by an orientation term *scheef* 'askew' or a shape term *krom* 'curved'. The central part of the sentence states that there is a path \mathbf{p} associated with the tree and the vectors of this path are increasing in length (growing). The adjective *scheef* adds the information that all these vectors are making a particular angle with the vertical direction. The adjective *krom* says that the vectors of the path make an increasing angle with the vertical direction. The following two pictures show the two processes of growing:



Figure 25: 'growing askew'



Figure 26: 'growing curved'

Finally, (48c) shows an ambiguity between a part meaning and a size meaning. The circumference of the disk can be either a part of the disk, i.e. the rim, or it can be the length of that rim. Underlying both of these meanings is the circular path that was discussed earlier. The size meaning of *circumference* focuses on the length of this path, the part meaning takes the whole of all the parts of the disk located on this path.

4 Change of Shape

The basic entity of the model I am describing is the vector. Vectors can be used to capture the meaning of static terms of place, size, orientation, and spatial parts, insofar as these terms are based on a relative position or axis that can be conceived as a straight line. Paths are constructed out of vectors and are used to represent change of place, size, and orientation, as well as sizes, shapes, and parts that can not be represented by means of a vector. However, a more complicated object seems to be necessary to represent *change of shape*. Suppose we want to represent the meaning of sentence (49):

(49) The man bent the iron bar

At the beginning of the bending event the shape of the bar can be represented by means of a straight path, as explained a few paragraphs back. At the end of the event we have a bar with a curved path. Each stage of the event is associated with a path representing the axis of the bar, or, maybe, the event as a whole is associated with a sequence of paths, a 'hyper path'.

One thing that should be accounted for is the possibility of modifying the direction of shape change by means of directional PPs:

(50) The bar was bent to the left

One way to tackle this example is to say that the shape path at the end of the event is a path that is in the denotation of the PP *to the left*. This is shown in Figure 27 by means of a sequence of paths:



Figure 27: to bend to the left

I also mention in this respect, without providing an analysis, that motion and static shape can also be described in terms of curving or bending:

- (51) a. The jet fighter curved back to its formation
 - b. The road bends to the left

Even though the analysis is not obvious, these examples again illustrate the basic claim of this paper, that various spatial domains are intimately related.

Conclusion

One way of evaluating the explanatory value of a theoretical notion is to extend its domain of application as far as possible. If vectors are really a fundamental concept in spatial language and spatial cognition, then we expect them to be relevant to the analysis of more phenomena than, let's say, modified prepositional phrases. I have tried to show that there are in fact good linguistic reasons to treat vectors as the 'backbone' of spatial representation across the spatial domains of place, size, orientation, shape, and spatial parts. Even though some of the analyses are still a bit sketchy the general direction of research seems promising.

One critical conclusion that one might draw from the wide range of applications for vectors in spatial language is that vector space semantics is really too powerful. There seems to be no limit on what you can do with it. My conclusion, however, is that the comprehensiveness of vector semantics does not so much demonstrate the power of the formalism, but rather the conceptual economy of the spatial domain. A vector is really a very simple thing: a combination of a length and a direction. If so much of our spatial language can be analyzed in terms of vectors, then my conclusion would be that our conceptualization of space is almost exclusively based on these two notions.

On the other hand, as shown in Zwarts (1997) and Zwarts and Winter (2000), this does not mean that any mathematical construction based on vectors (any vector set or vector path) is a possible meaning for a spatial term. The denotations of prepositional phrases are subject to strong algebraic constraints and I would expect the same for terms of size, orientation, and shape. One of the aims for future research is to find the algebraic properties that are characteristic of meanings in the spatial domain as a whole as well as for meanings in specific subdomains. Obviously, such a program only makes sense when a model is used in which the right algebraic properties can be formulated in the first place.

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Footnotes

1 I gratefully acknowledge the comments that I received from the audience at the workshop and from one anonymous editor. Postal adress: SIL/BTL, P.O.Box 6645, Eldoret, Kenya. Email address: joost_zwarts@sil.org

2 A class of place nouns that will not be discussed here are those referring to places in an absolute way, like *place*, *site*, *area*, *spot*, etc.

But see for example Lang (1990), Levinson (1992), Landau & Jackendoff (1993), Bierwisch (1996), Cienki (1998), Van der Zee (1997,1998), and Habel et al. (this volume) for recent discussions of spatial terminology besides place terms or prepositions.

4 The size adjectives, like *thin* and *high*, are usually called dimensional adjectives (Bierwisch & Lang 1989). See also Bierwisch (1967), Fillmore (1971), and Teller (1969).

5 The examples in (7) and (8) are taken from Dahl (1977).

6 Formally, each directional PP is interpreted as a set of paths and the asyndetic conjunction of PPs in this example as the intersection of the conjuncts.

Actually, since many of the spatial terms will be interpreted on the basis of *located vectors* (vectors that have their starting point anchored in one particular spatial location) we need a vector space V_P for every point P, since each point is the origin of a vector space.

8 This is a simplification of the meaning of *deep*. A hole which extends horizontally into a wall can also be described as deep, so it is more appropriate to define the direction of *deep* as inward from a surface.

Possibly **UP** should be defined on the basis of a vector **b** that represents the canonical posture of the human body: **UP** = { s**b**: s > 0 }, where s**b** is the vector that results when vector **b** is multiplied by a real number s. In this way the biological origin (Clark 1973) or embodiment (Lakoff 1987) of the vertical dimension can be accounted for. $-\mathbf{UP} = \{-\mathbf{v}: \mathbf{v} \in \mathbf{UP}\} = \{ s\mathbf{b}: s < 0 \}.$

10 The definitions of the size adjectives *high*, *low*, and *deep* assume that the objects involved have a canonical upward position. The mast of a ship can be high, because its canonical position is upright, but arms are always long (or short), even if they are reaching up, because they do not have a canonical upward position.

11 As a reviewer observed, this may be true for one-dimensional size, but not always for two-dimensional and three-dimensional size, where we have special unit nouns like *acre* and *liter* (in addition to unit nouns based on the words *square* and *cubic*).

12 Like UP, VERT can also be defined on the basis of the 'body vector' **b** mentioned in footnote 7: VERT = { sv: s is a real number }. The horizontal plane \perp VERT is defined as the so-called 'orthogonal complement' to VERT, i.e. the set of vectors orthogonal to the vectors in VERT.

13 Note that straight has an orientation meaning (with the opposite *oblique*) and a curvature meaning (with the opposite *curved*) that will be discussed in section 3.4.

Depending on what is most convenient or clear, I will diagram paths either as a sequence of 'pictures' (e.g. Figure 12) or as a constellation of numbered vectors in one picture (e.g. Figure 15). This is just a matter of visual presentation and does not change the nature of paths as functions from numbers to vectors.

15 If we assume (following Krifka 1992 and Jackendoff 1996a among others) that an event e corresponds to a temporal interval time(e), that the theme of the event is given by theme(e), and that the relations **place** and **axis** have a temporal parameter t, then **place-path** and **axis-path** are defined as follows:

place-path(e,**p**,y) iff for every $t \in time(e)$ **place**_t(theme(e), **p**(μ (t)),y)

axis-path(e,p) iff for every $t \in time(e) axis_t(theme(e), p(\mu(t)))$

with μ an isomorphism from the interval time(e) to the domain of the path.

16 More precisely, the set of circular paths would be { $\mathbf{p}: \exists \rho \forall i \mathbf{p}(i) = \rho(i)\mathbf{p}(0)$ } where ρ is a function that maps every element i of the domain [0,k] of the path to a rotation $\rho(i)$ which is then applied to the initial vector $\mathbf{p}(0)$. In the 'prototypical' instances of around, ρ is a continuous, monotone increasing function with $\rho(0)$ the zero rotation and $\rho(k)\mathbf{p}(k) = \mathbf{p}(0)$, i.e. the path goes all the way round once with a constant radius. For many terms based on circular paths (like *around*, to *surround*, etc.) we need a more flexible definition.

17 For **place-path** to apply to objects we need a function called extent that map contours and extended objects to the ordered set of their paths:

place-path(x,**p**,y) iff for every $x' \in extent(x)$ **place**(x',**p**($\mu(x')$),y)

Again there is an isomorphism μ that maps the parts of x onto the domain of the path (see also Jackendoff 1996a).

18 Describing the shape of a three-dimensional object, like an apple, by means of a path is a bit more complicated. I assume that a three-dimensional object can be called 'round' because each of its cross-sections can be called 'round' as defined in (34).

Any 'centered' path \mathbf{p} can be mapped to a head-tail path \mathbf{p} ' in a systematic way, by connecting the end points of \mathbf{p} : if \mathbf{p} is a centered path over a domain of integers [0,k], then the corresponding head-tail path \mathbf{p} ' with domain [0,k–1] assigns to every i the vector that connects the end points of $\mathbf{p}(i)$ and $\mathbf{p}(i+1)$. Conversely, given a central point O we can turn a head-tail path **p** with domain [0,k] into a centered path **p**' with domain [0,k+1]: for every $i \in [0,k]$, **p**'(i) is the vector that connects O and the starting point of **p**(i) and **p**'(k+1) is the vector that connects O with the end point of **p**'(k).

20 The notion of circular path needs to be extended for these cases. The paths of these three sentences can take the shape of a semi-circle and in (40c) the path is most likely rotating in a more 'random' way in combination with strong variation of the length of the vectors.

In these definitions and elsewhere in the paper I leave PPs like *toward the east* undefined because the focus is not on the lexical semantics of directional prepositions. See Zwarts and Winter (2000) for a proposal on how to define various directional prepositions. Note that *toward the east* is a bit special because there is not a specific point called *the east* that we can take as the origin of the vectors of the path, like we did with *toward the city*. In some sense, *toward the east* expresses a direction, rather than a destination, and it seems more reasonable to represent it as a path of vectors of increasing length pointing east. How to unify the semantics of *toward* in such a way that it will cover both uses is a topic for future research.

22 Taken from "Waterskiing," Microsoft® Encarta® 98 Encyclopedia. © 1993-1997 Microsoft Corporation. All rights reserved.