

1 Translations into predicate logic

Translate the following English sentences into predicate logic, providing a key for the translation. If the sentence is ambiguous, be sure to give all possible translations. Provide unambiguous English paraphrases for the different meanings, and say which expressions of predicate logic correspond to which meanings. You do not need to worry about tense; in (1c), for example, you can translate *will like* as a single predicate.

- (1) a. Mo didn't see any students.
- b. No student read a book by Dostoevsky or Tolstoy except Lou. (*Be careful!*)
- c. If a student reads *Crime and Punishment*, he will like it.

Remember that one of the goals of translating sentences into predicate logic is to capture as much detail as possible. That means that you should translate as many parts of the sentence into appropriate terms of logic as possible, avoiding abbreviations. For example, even though the verb phrase *eat the bagel* could be translated as a single one-place predicate EAT-THE-BAGEL, it is more accurate to translate the verb as a two-place predicate EAT and the direct object as an argument term *b*.

2 Answers

NOTE: I will use *italics* instead of CAPS to represent predicates, I will use the symbol \wedge instead of & to indicate logical 'and', and I will use the symbol \neg instead of the tilde to represent negation.

- (1a) $\neg\exists x[student(x) \wedge see(m, x)]$
 $m = \text{Mo}$
 student = a 1-place predicate that is true of an object *x* if it is a student
 see = a 2-place predicate that is true of objects *x, y* if *x* sees *y*

- (1b) Interpretation 1: No student read any book by Dostoevsky or Tolstoy except Lee.
 $\forall x[(student(x) \wedge \neg is(x, l)) \rightarrow \neg\exists y[(book(y) \wedge (by(y, d) \vee by(y, t))) \wedge read(x, y)]] \wedge$
 $\exists y[(book(y) \wedge (by(y, d) \vee by(y, t))) \wedge read(l, y)]$

Interpretation 2: There is a book by Dostoevsky or Tolstoy that no student except Lee read.
 $\exists y[(book(y) \wedge (by(y, d) \vee by(y, t))) \wedge \forall x[(student(x) \wedge \neg is(x, l)) \rightarrow \neg read(x, y)] \wedge$
 $read(l, y)]$

$l = \text{Lee}$
 $d = \text{Dostoevsky}$
 $t = \text{Tolstoy}$
student = a 1-place predicate that is true of an object *x* if it is a student
is = a 2-place predicate that is true of objects *x, y* if *x* is *y*
book = a 1-place predicate that is true of an object *x* if it is a book
by = a 2-place predicate that is true of objects *x, y* if *x* is written by *y*
read = a 2-place predicate that is true of objects *x, y* if *x* reads *y*

Note that in both translations, part of the meaning of *except Lee* is the extra information that the universal quantifier doesn't just range over students, but over students who are not Lee. (This is the $student(x) \wedge \neg is(x, l)$ part.) Furthermore, both translations explicitly indicate that Lee read at least one book: in interpretation 1, these is any book by Dostoevsky or Tolstoy; in interpretation 2, this is the particular book by Dostoevsky or Tolstoy that no student (other than Lee) read.

Otherwise, the crucial difference between the two interpretations is the relative scope of the existential and universal/negative quantifiers.

(1c) $\forall x[(student(x) \wedge read(x, c)) \rightarrow like(x, c)]$

$c = Crime\ and\ Punishment$

$read =$ a 2-place predicate that is true of objects x, y if x reads y

$like =$ a 2-place predicate that is true of objects x, y if x likes y