Polar Opposition and the Ontology of 'Degrees'

Chris Kennedy, Northwestern University

Lectures on the Syntax and Semantics of Gradability and Comparison
Kyushu University, August 2000

1 Comparison and Polar Opposition

1.1 Cross-Polar Anomaly

The empirical starting point for today's class is a phenomenon that I will refer to as *cross-polar anomaly* (CPA), which is exemplified by the sentences in (1) (see Hale (1970), Bierwisch (1989), and Kennedy (1997a, to appear)).

(1) a. ? Alice is shorter than Carmen is tall.
b. ? *The Brothers Karamazov* is longer than *The Idiot* is short.
c. ? The Mars Pathfinder mission was cheaper than the Viking mission was expensive.
d. ? New York is dirtier than Chicago is clean.
e. ? A Volvo is safer than a Fiat is dangerous.

These sentences demonstrate that comparatives formed out of so-called 'positive' and 'negative' pairs of adjectives are semantically anomalous.

This anomaly cannot be accounted for in terms of syntactic ill-formedness: structurally identical examples of 'comparative subdeletion' in which both adjectives have the same polarity, such as those in (2), are perfectly well-formed.

(2) a. The space telescope is longer than it is wide.
b. After she swallowed the drink, Alice discovered that she was shorter than the doorway was low.

The factors underlying cross-polar anomaly must therefore involve the interaction of the semantics of positive and negative adjectives and the semantics of the comparative construction.

The classification of gradable adjectives as positive or negative can be made based on a number of empirical characteristics (see Seuren (1978) for general discussion of this issue):

1. Negative adjectives license downward entailments and negative polarity items in clausal complements, but positive adjectives do not (see Seuren (1978), Ladusaw (1979), Linebarger (1980), Sánchez-Valencia (1994), Kennedy to appear a).
2. Positive but not negative adjectives can appear with measure phrases (compare '2 meters long' with '?2 meters short').
3. A large class of antonymous positive and negative pairs make (3) valid.

(3) *x is more ϕ_{pos} than y if and only if y is more ϕ_{neg} than x.*

(4) a. Carmen is taller than Alice if and only if Alice is shorter than Carmen.

b. A Volvo is safer than a Fiat if and only if a Fiat is more dangerous than a Volvo.

Initial assumptions

- gradable adjectives map objects to abstract representations of measurement (scales), which are sets of points (degrees) that are totally ordered along some dimension (e.g. *height, weight, temporal precedence*, etc.)
- comparatives define ordering relations between degrees
- antonymous pairs of adjectives such map identical arguments onto the same degrees (and are therefore associated with the same scales), but introduce the opposite ordering relations (Rullmann (1995)).

On this view, antonymous pairs of adjectives are *duals*:

(5) For all antonymous adjectives ϕ_{pos}, ϕ_{neg} that map their arguments onto a shared scale S, and for all $d_1, d_2 \in S$:
$d_1 \succ_{\phi_{pos}} d_2 \Leftrightarrow d_2 \succ_{\phi_{neg}} d_1$

The truth conditions of comparatives:

(6) a. *x is more ϕ than y*
b. *the degree to which x is $\phi \succ_{\phi}$ the degree to which y is ϕ*

The truth conditions of e.g. (4a):

(7) *the degree to which Carmen is tall $\succ_{tall}$ the degree to which Alice is tall $\Leftrightarrow$ the degree to which Alice is short $\succ_{short}$ the degree to which Carmen is short*

Since the degree of Alice's tallness is the same point on the scale of height as her degree of shortness (and likewise for Carmen), the two conjuncts in (7) are correctly predicted to be logically equivalent.

The very same reasoning that provides an elegant account of the validity of (3) makes exactly the wrong predictions about cross-polar anomaly. An example like (1a) should have the interpretation in (8).

(8) *the degree to which Alice is short $\succ_{short}$ the degree to which Carmen is tall*

The difference between the logical representation in (8) and that of the first conjunct in (7), for example, is that in the former, only one of the arguments of the ordering relation is a degree whose polarity agrees with the adjective that determines the type of ordering relation.

This suggests a plausible explanation of CPA:

(9) The ordering relation in comparatives is defined only for degrees of the same polarity.

Although (9) is exactly the analsys that I will develop in section 4.1, within the set of assumptions outlined so far, it is unavailable.

- The problem is that if degrees correspond to points in an ordered set, and if positive and negative adjectives map their arguments onto the same degrees—an assumption that is necessary to account for the validity of constructions with the form in (3)—then (8) is equivalent to (10).

(10) *the degree to which Alice is short* $\succ_{short}$ *the degree to which Carmen is short*

The result is that (1a) should not only not be anomalous, it should be logically equivalent to 'Alice is shorter than Carmen' (and likewise 'Carmen is taller than Alice').

1.2 Comparisons of Divergence and Deviation

The picture presented in the previous section is complicated by two sets of facts that appear at first glance to counterexemplify the claim that comparatives constructed out of positive and negative pairs of adjectives are anomalous.

1.2.1 Comparison of Divergence

The first set of facts, brought to my attention by Chris Barker (personal communication), is illustrated by the sentences in (11).

(11) a. The A-string on your guitar is sharper than the D-string is flat.
b. My watch is faster than your watch is slow.
c. She was earlier than I was late.

There is good reason to believe that the adjectives in the comparatives in (11) are not antonymous in the same way as the adjectives involved in cross-polar anomaly, however.

1. All of the adjectives in (11) can take measure phrases; a characteristic of positive adjectives, but not negative adjectives:

(12) a. Your A-string is 30 Hz flat/sharp.
b. My watch is 10 minutes fast/slow.
c. She was an hour early/late.

2. Statements using the adjectives in (11) that appear to be substitution instances of (3) are not valid.

(13) The A-string is sharper than the D-string if and only if the D-string is flatter than the A-string.

The first conjunct of (13), would be true, but the second conjunct would be false, in a context in which the A-string is farther above its proper pitch than the D-string is, but the D-string is higher in absolute pitch than the A-string (the normal situation for a properly tuned guitar).

3. The sentences in (11) are non-anomalous only on a very specific interpretation: one in which the adjectives measure divergence from some common point of reference, rather than the 'absolute' degree to which an object has some gradable property.

When absolute degrees are compared, the adjectives in (11) trigger cross-polar anomaly. This is shown by the minimal pair (11b) and (14).

(14) ? My car is faster than your car is slow.

Conclusion

Both members of the pairs of adjectives in (11) have the same polarity (and are in fact positive), in which case they do not represent counterexamples to the generalization established in section 1.1.

These facts do, however, raise larger questions about the nature of antonymy and polar opposition: the adjectives 'fast' and 'slow' are opposites in both (11b) and (14), for example, yet the nature of the opposition is different in the two cases.

1.2.2 Comparison of Deviation

A second set of apparent counterexamples to the generalization that comparatives constructed out of antonymous adjectives are anomalous is illustrated by the naturally occurring examples in (15).

(15) a. [The Red Sox] will be scrutinized as closely as the Orioles to see whether they are any more legitimate than the Orioles are fraudulent. [*New York Times*, Summer 1998 (exact date unknown)]
b. Grace especially had a forgettable playoff series that won't soon be forgotten. Grace was as cold as he was hot in the 1989 playoffs. [*Chicago Tribune*, October 4, 1998, Section 3, p. 6]
c. I can still remember the sound it made, a lovely special sound, as light and thin as the clothes were solid and heavy. [Dibdin, M.: 1989, *Ratking*, Bantam Crime, New York, p. 178]

These "Comparison of Deviation" (COD) constructions have two characteristics that distinguish them from standard comparatives.

1. COD constructions compare the relative extents to which two objects deviate from some standard value associated with the adjective (cf. Bierwisch (1989, p. 220). The meaning of (15a) is paraphrased as in (16).

(16) The degree to which the Red Sox exceed a standard of legitimacy is greater than the degree to which the Orioles exceed a standard of fraudulence.

Standard comparatives and equatives compare the absolute projections of two objects on a scale. The most natural paraphrase of the equative construction in (17), for example, is (18).

(17) It was a squarish hole, as deep as a ten-story building is tall, cut down into the hard and uncooperative earth. [Reynolds, W.J., 'The Lost Boys', in Hillerman, T.: 1994, *The Mysterious West*, Harper Collins, New York, p. 223]

(18) The depth of the hole is at least as great as the height of a ten-story building.

2. Unlike typical comparatives, COD constructions entail that the properties predicated of the compared objects are true in the absolute sense. (19a) is contradictory, but (19b) is not.

(19) a. The Red Sox are more legitimate than the Orioles are fraudulent, but they're not legitimate.

b. The hole is deeper than a two-year old is tall, but it's not deep.

This property is clearly related to the interpretation of COD. Since the truth of an expression of the form '*x is* ϕ' is determined by checking whether the degree to which x is ϕ exceeds an appropriate standard value, the fact that COD constructions compare the degrees to which two objects exceed their respective standard values derives the observed entailment patterns.

Two important observations about comparison of deviation

1. Interpretations of this type are not restricted to comparatives involving positive and negative pairs of adjectives. This is most clearly illustrated by (20), which can have either the 'standard' interpretation paraphrased in (21a), which is false, or the COD interpretation paraphrased in (21b), which is (arguably) true.

(20) The San Francisco Bay Bridge is as long as the Sears Tower is tall.

(21) a. The length of the San Francisco Bay Bridge is at least as great as the height of the Sears Tower.

b. The degree to which the San Francisco Bay Bridge exceeds a standard of length (for bridges) is at least as great as the degree to which the Sears Tower exceeds a standard of tallness (for buildings).

It must therefore be the case that comparison of deviation interpretations represent a freely available option in comparatives (cf. Bierwisch (1989, pp. 220-221), where the availability of such interpretations is claimed to be a marked option).

2. In comparatives that are in fact constructed out of positive and negative pairs of adjectives, the COD interpretation is the *only* interpretation available.

1.3 Summary

The observations so far are summarized in the descriptive generalization in (22).

(22) Comparatives are semantically well-formed only if they define ordering relations between the same sorts of degrees: between positive degrees, between negative degrees, or between degrees that measure divergence from a reference point.

The central question: How can we derive this result?

2 Polar Opposition and the Syntax-Semantics Interface

Before moving to a semantic analysis of cross-polar anomaly, however, a different type of approach to the problem should be considered, in which cross-polar anomaly is explained in terms of principles of the syntax–semantics interface, in particular, the principles governing when a particular type of adjective may appear in the comparative clause.

2.1 No negative adjectives in the comparative clause

Bierwisch (1989, 1998) develops an analysis in which negative adjectives are ruled out in the comparative clause across the board.

The starting point of this analysis is the observation that both of the German sentences in (23a) and (23b) are equally unacceptable (as are their English counterparts; see Bierwisch (1989, pp. 104-105)).

(23) a. ? Der Tisch ist 10cm höher als er schmal ist.
'The table is 10cm taller than it is narrow.'

b. ? Der Tisch ist 10cm niedriger als er schmal ist.
'The table is 10cm lower than it is narrow.'

The use of a measure phrase in these examples is important, as it rules out the possibility of a comparison of deviation interpretation: as Bierwisch observes, a COD interpretation is possible in both the German and English sentences in (24).

(24) a. Der Tisch ist niedriger als er schmal ist.

b. The table is lower than it is narrow.

Bierwisch 1989: all standard comparative interpretations are derived from representations in which the adjective in the comparative clause is positive. In other words, in Bierwisch's analysis, both (25a) and (25b) are derived from an underlying representation along the lines of (26), where the struck-through material is deleted or unpronounced in the surface form.

(25) a. The table is longer than the rug is.

b. The table is shorter than the rug is.

(26) The table is longer/shorter than the rug is ~~long~~.

Examples like (27) above contradict the claim that negative adjectives cannot appear in the comparative clause when the entire construction has a standard interpretation, since there is strong evidence that it is not a comparison of deviation construction.

(27) After she swallowed the drink, Alice discovered that she was shorter than the doorway was low.

1. As shown by the passage in (28), it is possible to insert a measure phrase into comparatives like (27) without anomaly in certain contexts.

(28) Alice came upon a very small, very low doorway, with a bottle of some kind of potion next to it marked DRINK ME. Alice swallowed the potion, and began to shrink. When she had stopped shrinking, Alice discovered that she was exactly two inches shorter than the doorway was low.

2. The entailments of (29) show that comparatives with negative adjectives in the comparative clause can have standard interpretations.

(29) After she swallowed the potion, Alice became as thin as the doorway was narrow.

(29) also has a standard comparative interpretation (in addition to a COD interpretation), which asserts that Alice's thinness is at least as great as the doorway's narrowness.

Given these observations, I conclude that the anomaly of Bierwisch's examples do not provide evidence that negative adjectives in the comparative clause can never have a standard interpretation. Instead, their anomaly reflects the general markedness of negative adjectives.

(30) a. How narrow is the table?
b. How wide is the table?

Like *how*-questions, comparatives typically make no claims about whether the gradable property that forms the basis of the comparison actually holds of the compared objects, because the basic goal of a comparison is only to establish an ordering relation between two objects along some dimension. It follows that in the default case, the unmarked (positive) form(s) of the adjective(s) should be used, and the marked (negative) form(s) should be reserved for contexts that are consistent with their presuppositions.

The crucial difference between e.g. (28) and (23b) is that the former provides just such a context. (28) describes a state of affairs that is part of a scenario in which Alice is reduced to a small size as a result of drinking a potion, a context that is perfectly compatible with (and arguably demands) the use of the negative forms.

2.2 Anti-redundancy

Faller (1998): prohibit the overt realization of an adjective in the comparative clause when it maps its argument onto the same scale as the adjective that heads the comparative construction (cf. Bierwisch (1989, p. 104)).

The intuition underlying this proposal—redundant information must be deleted from the comparative clause—definitely reflects a very deep property of comparatives. However, there are a number of reasons to doubt that this is the correct explanation of CPA.

1. Comparison of divergence and deviation constructions show that comparatives *can* be constructed out of antonymous adjectives as long their interpretations involve comparisons of similar sorts of measures (degrees of difference). Faller's proposal would require us to assume that e.g. *fast* and *slow* in (31) map their arguments onto different scales, an assumption that not only is at odds with the clear intuition that both adjectives locate their arguments on the same scale (the timeline), but also makes it accidental that they measure divergence from the same point (the 'correct time').

(31) My watch is faster than your watch is slow.

2. It is not only possible for antonymous adjectives associated with the same scale to appear together in comparisons of divergence and deviation, it is also possible for the adjective in the comparative clause to be *identical* to the adjective that heads the comparative when it has contrastive stress, as in the following examples.

(32) The ski poles are not only longer than the box is wide, they're also longer than the box is LONG.

(33) Watching the Cubs on his satellite dish has been almost as difficult for Beck as watching Beck close games has been difficult for the CUBS. [*Chicago Tribune*, June 6, 1999, section 4, p. 3 (emphasis added to indicate the required pronunciation)]

Bierwisch (1989) proposes a redundancy condition that requires deletion of a lexically identical adjective from the comparative clause when it is not prosodically contrastive with the adjective in the matrix.

Bierwisch (1998) extends this observation to claim that prosodic contrast induces comparison of deviation interpretations, pointing to the the contrasts in (34) and (35) (where capitalized lexical items are focused and items marked with a '↓' are deaccented).

(34) a. Kim is TALLer than Lee is SHORT.
b. ? Kim is taller than Lee is ↓short.

(35) a. Lee is SHORTer than Kim is TALL.
b. ? Lee is shorter than Kim is ↓tall.

Actuall, *all* instances of subdeletion require prosodic contrast.

(36) a. The table is LONGer than it/the carpet is WIDE.
b. ? The table is longer than it/the carpet is ↓wide.

This point is crucial, because it shows that the distribution and interpretation of antonymous adjectives in comparatives cannot be explained in terms of requirements on prosodic contrast any more than they can be explained in terms of requirements on 'scale contrast'. If all subdeletion constructions require prosodic contrast, then the impossibility of a standard comparative interpretation in CPA environments must be due to some other factor.

3 Intervals and Adjectival Polarity

(37) Comparatives are semantically well-formed only if they define ordering relations between the same sorts of degrees: between positive degrees, between negative degrees, or between degrees that measure divergence from a reference point.

The problem for the traditional model, in which degrees are formalized as points on a scale, is that the (independently necessary) assumption that positive and negative degrees are the same objects rules out the possibility of making the necessary sortal distinction between them.

This result can be achieved, however, if we make the assumption that degrees are *intervals* of a scale (Seuren 1978, 1984, von Stechow 1984b, Kennedy to appear; see also Löbner 1990).

3.1 Capturing Adjectival Polarity

Antonymous pairs of adjectives provide the same kind of information about an object (both 'tall' and 'short' characterize an object's height, for example), but they provide different perspectives on the projection of an object on a scale.

Capture this intuition in terms of a structural distinction between two sorts of degrees: *positive* and *negative degrees*.

Roughly speaking, positive degrees are intervals that range from the lower end of a scale to some point, and negative degrees are intervals that range from some point to the upper end of a scale.

Define scales and degrees as follows:

- A *scale S* is a linearly ordered, infinite set of points, associated with a dimension that indicates the type of measurement that the scale represents.[1]

[1] E.g., height, length, weight, brightness and so forth; the dimensional properties of scales are important for accounting for the anomaly of cross-scalar comparisons such as (i), sometimes referred to as 'incommensurability' (see Klein (1991) and Kennedy (1999a)).

(i) ? Alice is taller than she is clever.

- A *degree* d is a convex, nonempty subset of a scale, i.e., a subset of the scale with the following property: $\forall p_1, p_2 \in d \forall p_3 \in S[p_1 \prec p_3 \prec p_2 \rightarrow p_3 \in d]$

The set of positive and negative degrees for any scale S ($POS(S)$ and $NEG(S)$, respectively), can then be defined as in (38).

(38) a. $POS(S) = \{d \subseteq S \mid \exists p_1 \in d \forall p_2 \in S[p_2 \preceq p_1 \rightarrow p_2 \in d]\}$
b. $NEG(S) = \{d \subseteq S \mid \exists p_1 \in d \forall p_2 \in S[p_1 \preceq p_2 \rightarrow p_2 \in d]\}$

Finally, assume that for any object x, the positive and negative projection of x on a scale S ($pos_S(x)$ and $neg_S(x)$, respectively) are related as in (39), where MAX and MIN return the maximal and minimal elements of an ordered set.

(39) $\text{MAX}(pos_S(x)) = \text{MIN}(neg_S(x))$

The positive and negative projections of an object x on a scale S are (join) complementary intervals on the scale, as illustrated by the diagram in (40).

(40) S: 0 ——— $pos_S(x)$ ———•——— $neg_S(x)$ ———→∞

A result of this analysis is that the set of positive degrees on a scale S and the set of negative degrees on S are disjoint. Adjectival polarity can thus be characterized as a difference in the ranges of the functions denoted by positive and negative adjectives:

- Positive adjectives denote functions from individuals to positive degrees.
- Negative adjectives denote functions from individuals to negative degrees.

Antonymy holds when two adjectives have the same domains but different ranges, and they map identical arguments onto (join) complementary regions of the same scale.

3.2 Comparatives

Structures such as (41a) are mapped onto interpretations of the form in (41b), which defines a total ordering between degrees.

(41) a. *x is more* ϕ_1 *than y is* ϕ_2
b. $\phi_1(x) \succ \phi_2(y)$

Since degrees qua intervals are formalized set-theoretically, we can formulate the truth conditions of comparatives with 'more', 'less', and 'as', in terms of the Boolean definitions of ordering relations in (42a)-(42c), respectively, where d_1 and d_2 are elements of some set of degrees D on a scale S (i.e., $POS(S)$ or $NEG(S)$).

(42) $\forall d_1, d_2 \in D$:
a. $d_1 \succ d_2 \Leftrightarrow d_1 \cap d_2 = d_2 \wedge d_1 \neq d_2$
b. $d_1 \prec d_2 \Leftrightarrow d_1 \cap d_2 = d_1 \wedge d_1 \neq d_2$
c. $d_1 \succeq d_2 \Leftrightarrow d_1 \cap d_2 = d_2$

A simple comparative such as (43a) has the interpretation in (43b), which is true just in case the degree to which Alice is tall exceeds the degree to which Carmen is tall; i.e., just in case the positive projection of Alice on a scale of height stands in the relation defined in (42a) to the positive projection of Carmen on the height scale, as in the diagram in (44).

(43) a. Alice is taller than Carmen (is tall).
b. $tall(a) \succ tall(c)$

(44) *height*: 0 ——— $tall(a)$ ———•
height: 0 —— $tall(c)$ ——•

The analysis of comparatives with negative adjectives is exactly the same.

(45a) has the interpretation in (45b), which is true just in case the negative projection of Carmen on the height scale exceeds the negative projection of Alice. According to (42a), this will hold just in case $short(c)$ extends further down the scale (i.e., has a lower minimal element) than $short(a)$, as indicated in (46).

(45) a. Carmen is shorter than Alice (is short).
b. $short(c) \succ short(a)$

(46) *height*: 0 •—— $short(a)$ ——→ ∞
height: 0 •—— $short(c)$ ————→ ∞

This analysis explains the validity of (47).

(47) Alice is taller than Carmen if and only if Carmen is shorter than Alice.

The interpretation of (47) is a substitution instance of (48), where pos_S and neg_S are functions from objects to positive and negative degrees on a scale S, respectively, i.e., antonymous positive and negative gradable adjectives.

(48) $pos_S(x) \succ pos_S(y) \Leftrightarrow neg_S(y) \succ neg_S(x)$

The validity of the schema in (48) follows from the relation in (39). Using (39) as a starting point, the complements of positive and negative degrees can be defined as follows:

(49) a. $-neg_S(x) = pos_S(x) - \text{MIN}(neg_S(x))$
b. $-pos_S(x) = neg_S(x) - \text{MAX}(pos_S(x))$

If $pos_S(x) \succ pos_S(y)$, then $pos_S(x) - \text{MAX}(pos_S(x)) \succ pos_S(y) - \text{MAX}(pos_S(y))$, since $\text{MAX}(pos_S(x))$ and $\text{MAX}(pos_S(y))$ are the maximal elements of $pos_S(x)$ and $pos_S(y)$, respectively. It follows that $-neg_S(x) \succ -neg_S(y)$, by substitution, and finally that $neg_S(y) \succ neg_S(x)$, by contraposition. The other direction of the biconditional can be proved in exactly the same way.

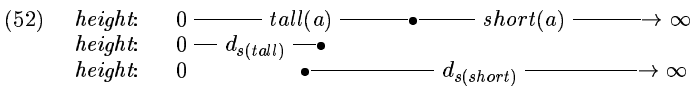
3.3 Non-comparative predications

The truth conditions of the examples in (50) can be analyzed as in (51), where $d_{s(tall)}$ and $d_{s(short)}$ are free variables over standard-denoting degrees whose values are determined contextually.

(50) a. Alice is tall.
b. Alice is short.

(51) a. $tall(a) \succeq d_{s(tall)}$
b. $short(a) \succeq d_{s(short)}$

In a context such as the one represented by the diagram in (52), 'Alice is tall' would be true, and 'Alice is short' would be false, since the requisite ordering relation between the two degrees in (51a) is satisfied, but the one in (51b) is not.

(52)


NB: All that is required to evaluate the truth conditions of an adjectival predication, for both positive and negative adjectives, is that a relation between two degrees can be established (the projections of two objects on a scale for comparatives, and the projection of one object on a scale plus the relevant standard-denoting degree in non-comparatives). The actual values of the relevant degrees are of secondary importance, and are only relevant insofar as they interact with the relevant ordering relation. This point is crucial, as it responds to a common criticism of the approach advocated here, which runs as follows: since all negative degrees are infinite (at least for scales without a maximal element, see below for discussion of this point), they are all equal, and the analysis makes the obviously wrong prediction that all objects have equal shortness, narrowness, slowness, etc.

In fact, this is not the case. While it is true that all negative degrees are infinite (a point that can be exploited to account for the distribution of measure phrases, as will be shown in section 4.2), it is not the case that they are all equal. The basic assumptions about scale structure (scales are totally ordered, infinite sets of points) allow negative degrees to differ in their minimal values, with the consequence that orderings of the sort in (42) can be evaluated. These orderings in turn impose a relational structure on the domain of a particular adjective, effectively capturing differences in shortness, narrowness, slowness, and so forth.[2]

4 Comparison and Polar Opposition Revisited

4.1 Cross-Polar Anomaly Explained

Comparatives constructed out of antonymous pairs of adjectives are anomalous because they involve comparison of different sorts of degrees. Within the system proposed here, this can be explained in terms of very general principles of ordering relations.

[2] A second objection to negative degrees is one of 'conceptual implausibility' (see e.g. Bierwisch (1989), p. 247ff). Since the strength of such an argument can only be evaluated in the context of psycholinguistic evidence, which has up to now not been presented, I will not address it here. However, it should be observed that psycholinguistic studies on adjectival polarity have demonstrated asymmetries in positive and negative adjectives. (For example, the latter take longer to process; see Clark (1970) for an overview.) Whether the inherent asymmetry between positive and negative degrees can be exploited to provide some account for the observed psycholinguistic facts is a question that should be a focus of future work.

1. The arguments of an ordering relation must be elements of the same ordered set.
2. Assume that this requirement is part of the meaning of expressions of ordering in natural language: the comparative morphemes 'more', 'less' and 'as' presuppose that their degree arguments are elements of the same ordered set.
3. If this requirement is not met, the relations denoted by comparative morphology are undefined, and a truth value cannot be computed.

It is precisely this type of breakdown that is responsible for cross-polar anomaly:

- The truth conditions for comparatives are formulated in terms ordering relations.
- A comparative constructed out of adjectives of opposite polarity defines an ordering relation between positive and negative degrees.
- Positive and negative degrees are elements of disjoint sets.
- Therefore, such a comparative fails to have a truth value.

The logical representation of (53a), in which the adjective in the main clause is negative and the adjective in the comparative clause is positive, is (53b).

(53) a. ? Alice is shorter than Carmen is tall.
b. $short(a) \succ tall(c)$

$short(a)$ and $tall(c)$ denote degrees in different ordered sets ($NEG(height)$ and $POS(height)$, respectively). The ordering relation introduced by the comparative morpheme is undefined for its two arguments, rendering the sentence anomalous. (Examples in which the adjectives are reversed are explained in exactly the same way.)

This analysis extends beyond examples involving pure antonyms like (53a) to any comparative in which the adjective in the main clause is of different polarity from the adjective in the comparative clause.

(54) a. ? Alice is shorter than the doorway is high.
b. $shorter(a) \prec high(d)$

Even if we assume that 'short' and 'high' map their arguments onto a shared scale—call it *physical extent* (see note 17)—the same problem that arises in examples like (53a) shows up here. The negative degree denoted by $short(a)$ and the positive degree denoted by $high(d)$ are objects in disjoint sets ($NEG(physical\ extent)$ and $POS(physical\ extent)$), so the comparative relation is undefined for its arguments, and the sentence is correctly predicted to be anomalous.

4.2 Measure Phrases

The analysis explains another well known class of anomalous adjectival predications in the same way as CPA:

(55) a. ? 'The Dream of a Ridiculous Man' is 21 pages short.
b. 'The Dream of a Ridiculous Man' is 21 pages long.

For any scale with a minimal element, positive degrees correspond to intervals that range from the minimal element of the scale (the zero point) to some positive value.

Assuming that scales typically have no maximal element (see von Stechow (1984a) and Rullmann (1995), but see also Kennedy and McNally (to appear) for arguments that some scales do have maximal values), it follows that positive degrees correspond to finite, closed intervals, but negative degrees correspond to infinite, open intervals.

If we further assume that nominals like 'meter', 'centimeter', 'pound', and 'kilometer per hour' refer to (conventionally determined) degrees that begin at the zero point of the scale (cf. Bierwisch (1989)), and that numerals induce concatenation of such measures (see Krantz *et al.* (1971) for discussion of concatenation of closed scale segments), then it follows that measure phrases introduce only positive degrees.

(56) $long(r) \succeq 21\ pages$

Since $long(r)$ and the degree denoted by '21 pages' are both positive degrees, the partial ordering in (56) can be evaluated.

(57) $short(r) \succeq 21\ pages$

(57) imposes a partial ordering relation on the negative degree $short(r)$ and the degree denoted by '21 pages'. The measure phrase cannot denote a negative degree, however, so the ordering relation cannot be evaluated.

4.3 Differential Comparatives

Although measure phrases are incompatible with negative adjectives in their non-comparative forms, this is not true of comparatives.

(58) a. Alice is 12 cm shorter than Carmen (is).
b. *The Brothers Karamazov* is 122 pages longer than *The Idiot* (is).

In (58a) and (58b), however, the phrases '12 cm' and '122 pages' denote degrees that measure the difference between the compared (positive or negative) degrees.

For any two degrees, it is possible to identify their difference as the interval that is contained in one but not the other:

(59) $\forall d_1, d_2 \subseteq S : d_1 - d_2 = \{p \in S \mid p \in d_1 \wedge p \notin d_2\}$

Crucially, such 'differential degrees', like positive degrees, correspond to finite, closed intervals. Given this we can build a semantics for differential comparatives by introducing a function that maps differential degrees onto degrees with minimal elements that correspond to the zero point of the scale in a structure-preserving way.

(60) $[\![\text{ZERO}]\!] = \{\langle d_1, d_2\rangle \mid$
i. $\forall p_1, p_2 \in d_1[p_1 \prec p_2 \rightarrow \exists p_1', p_2' \in d_2[p_1' \prec p_2']] \wedge$
ii. $\forall p_1', p_2' \in d_2[p_1' \prec p_2' \rightarrow \exists p_1, p_2 \in d_1[p_1 \prec p_2]] \wedge$
iii. $\text{MIN}(d_2) = \text{MIN}(S)\}$

A differential comparative with the form in (61a) (where 'MP' is a measure phrase) can be assigned the interpretation in (61b).

(61) a. *x is MP more* ϕ_1 *than y is* ϕ_2
b. $\phi_1(x) \succ \phi_2(y) \wedge \text{ZERO}(\phi_1(x) - \phi_2(y)) \succeq MP$

The logical representation of (58a), for example, is (62), which is true just in case the difference between $short(a)$ and $short(c)$ can be mapped onto a degree that is at least as great as the degree denoted by '12 cm'.

(62) $short(a) \succ short(c) \wedge \text{ZERO}(short(a) - short(c)) \succeq 12\ cm$

Crucially, the fact that this example involves a negative adjective is irrelevant. Since the difference between two negative (or two positive) degrees is a closed interval on the scale, a mapping to a degree in the set of degrees named by measure phrases can be established by the ZERO function. Since this degree and the degree denoted by '12 cm' are elements of the same set, the ordering relation between them is defined, and the entire expression in (61b) can be assigned a truth value.

This establishes that the characterization of adjectival polarity argued for in this paper supports a semantics for differential comparatives, but the story is far from complete. Of particular interest here are pairs like 'warm/cold', which accept measure phrases in differential comparatives, but *not* in the positive (or negative) non-comparative forms, as shown by the examples in (63).

(63) a. Today is 45 degrees warmer than yesterday.
b. Yesterday was 45 degrees colder than today.
c. ? Today is 70 degrees warm.
d. ? Yesterday was 25 degrees cold.

4.4 Comparisons of Divergence and Deviation Revisited

Consider first the case of comparison of deviation, an example of which is repeated in (64) (cf. (15a)).

(64) The Red Sox are more legitimate than the Orioles are fraudulent.

COD can be analyzed as comparison of differential degrees, specifically the intervals that correspond to the differences between the (positive and negative) projections of the compared objects on the scale and the corresponding standard values for the antonymous adjectives.

(65) $\text{ZERO}(legitimate(r) - d_{s(legit)}) \succ \text{ZERO}(fraudulent(o) - d_{s(fraud)})$

(64) is true just in case the degree to which the Red Sox deviate from a standard of legitimacy exceeds the degree to which the Orioles deviate from a standard of fraudulence.

This logical representation accurately captures the meaning of this sentence, and more importantly, since the degrees derived by applying the ZERO function to the differences in (65) are comparable, it correctly predicts (64) to be acceptable.

The representation in (65) also accounts for the entailment patterns observed in COD constructions. In order for the difference operator to return a degree, it must be the case that its first argument exceeds its second argument (recall from the definitions in section 3.1 that degrees must be *nonempty* convex subsets of a scale). In the case of (65), this means that the degree to which the Red Sox are legitimate must exceed the standard value for 'legitimate' in the context of utterance. Since the truth conditions for the non-comparative state that a sentence of the form 'x is ϕ' is true just in case $\phi(x)$ is at least as great as the standard for ϕ, the truth conditions for the non-comparative are satisfied whenever the truth conditions for the comparison of deviation interpretation are satisfied.

Comparisons of divergence such as (66) can be analyzed in much the same way, with important difference: the properties of the former suggest that the adjectives in these constructions map their arguments directly onto differential degrees.

(66) My watch is faster than your watch is slow.

Recall from the discussion in section 1.2.2 that one of the properties of the adjectives in these constructions is that both members of the 'antonymous' pair accept measure phrases, as illustrated by the examples in (67).

(67) a. My watch is 15 minutes fast.
b. Your watch is 10 minutes slow.

A second property that characterizes the adjectives in these constructions is that their interpretations differ from those associated with their standard uses. In (66) and (67), for example, 'fast' and 'slow' measure the degrees to which their arguments diverge from some arbitrary point (whatever counts as 'on time' in the context of utterance).

This can be explained by assuming that the difference between the 'measuring-from-a-reference point' interpretations of these adjectives and their standard interpretations is that in the former case, the adjectives map their arguments onto intervals that extend in different directions (depending on the polarity of the adjective) from some arbitrary point ('on time' in (66) and (67)).

For the positive member of the pair, this point provides the lower bound of the interval; for the negative member, this point provides its upper bound. The basic idea is illustrated in (68), where the subscript δ indicates the differential interpretation.

(68) *time*: 0 ——————[*on time*]——————→∞
•—$slow_\delta(w_y)$—•——$fast_\delta(w_m)$——•

The acceptability of (67a) and (67b) can be accounted for by making the additional assumption that on their differential interpretations, these adjectives include the ZERO function as part of their meanings, so that the logical representations of (67a)–(67b) are (69a) and (69b), respectively.

(69) a. $\text{ZERO}(fast_\delta(w_m)) \succeq 15\ minutes$
b. $\text{ZERO}(slow_\delta(w_y)) \succeq 10\ minutes$

Since the differential degrees onto which the adjectives map their arguments are bounded, closed intervals on the scale, they can be mapped by the ZERO function onto intervals in

the set of degrees that includes those denoted by the measure phrases '15 minutes' and '10 minutes', and the ordering relations in (69) can be evaluated.

If this characterization of the meanings of these adjectives is correct, the analysis of the comparative in (66) is straightforward. This sentence can be assigned the interpretation in (70), which is just like a standard interpretation except for the addition of the ZERO function, which, by hypothesis, is built in to the adjectives' meanings.

(70) $\text{ZERO}(fast_\delta(w_m)) \succ \text{ZERO}(slow_\delta(w_y))$

The result is a comparison between degrees of the same sort, which should be perfectly interpretable.

The third characteristic of antonymous adjectives in comparison of divergence constructions is that they fail to make substitution instances of (3) valid.

(71) a. My watch is faster than your watch if and only if your watch is slower than my watch. (INVALID)
b. My car is faster than your car if and only if your car is slower than my car. (VALID)

This difference follows from the fact that positive and negative degrees in instances of 'differential' antonymy are structurally distinct from positive and negative degrees in the more standard cases of 'complementary' antonymy in an important way: unlike the latter, the former do not (obligatorily) correspond to complementary regions of a scale.

4.5 Types of Polar Opposition

Even though differential antonyms like 'fast' and 'slow' do not show the same set of properties as complementary antonyms like 'tall' and 'short' (or 'fast' and 'slow' on complementary interpretations, as in **??**)—in particular, they do not give rise to cross-polar anomaly—they are clearly antonymous in some sense. The question is in what sense, and how can the underlying similarities between these two classes of antonyms be captured in a way that also makes the type of semantic distinctions necessary to account for their different distributions. The discussion of comparison of divergence in the previous section suggests an answer to this question.

One property in particular that is shared by both complementary and differential antonyms is that the positive member of the pair maps its argument onto a degree that ranges from some lower bound upwards to some point, while the negative member maps its argument onto a degree that ranges from some upper bound downwards to some point.[3] This indicates that the crucial feature underlying both types of antonymy is one of *directionality*: in both complementary and differential uses, positive adjectives map their arguments onto degrees that have a natural ordering towards the upper end of the scale, and negative adjectives map their arguments onto degrees that have a natural ordering towards the lower end of the scale. They idea that directionality underlies antonymy goes back at least to Sapir (1944), and it appears in generative grammar in both traditional degree-as-point analyses in which positive and negative adjectives are treated as duals (see e.g. Rullmann (1995) and

[3]The two classes of antonyms differ regarding the nature of the lower or upper bound, however. In the case of complementary antonyms, the lower and upper bounds for positive and negative degrees correspond to the lower and upper bounds of the scale itself. In the case of differential antonyms, however, the lower bound of the positive degree and the upper bound of the negative degree are literally the same point, which corresponds to some (conventionalized) value on the scale. This feature is related to the difference in complementarity discussed below.

the discussion in section 1.1), and in analyses that characterize the meaning of negative adjectives in terms of (some notion of) degree subtraction (see e.g. Bierwisch (1989), Faller (1998, to appear)).

The model of adjectival polarity that I have defended here differs from purely directional characterizations of polar opposition, however, by introducing an additional feature of *complementarity*. This feature provides a means of distinguishing between the two classes of antonyms under discussion here: complementary antonyms map their arguments onto complementary intervals of a scale; differential antonyms do not. As shown in the preceding sections, this difference in complementarity also plays a crucial role in the explanation of the distributions of antonymous adjectives in comparatives, thus making the crucial semantic distinction that the differing behavior of these two classes of antonyms demands.

References

Abney, S.: 1987, The English Noun Phrase in its Sentential Aspect, doctoral dissertation, Massachusetts Institute of Technology, Cambridge, Mass.

Atlas, J.: 1984, 'Comparative Adjectives and Adverbials of Degree: An Introduction to Radically Radical Pragmatics', Linguistics and Philosophy **7**, 347–377.

Bartsch, R. and T. Vennemann: 1973, Semantic Structures: A Study in the Relation between Syntax and Semantics, Atheänum Verlag, Frankfurt.

van Benthem, J.: 1983, The Logic of Time, Reidel, Dordrecht.

Bierwisch, M.: 1998, 'Adjectives that Cannot Surface in Comparative Constructions', paper presented at the Workshop on the Syntax and Semantics of Comparatives, Zentrum für Allgemeine Sprachwissenschaft, Berlin, November 27–28, 1998.

Bierwisch, M.: 1989, 'The Semantics of Gradation', in M. Bierwisch and E. Lang (eds.), Dimensional Adjectives, Springer-Verlag, Berlin.

Bresnan, J.: 1975, 'Comparative Deletion and Constraints on Transformations', Linguistic Analysis **1**, 25–74.

Clark, H.: 1970, 'Comprehending Comparatives', in Flores d'Arcais, G. and W. Levelt (eds.), Advances in Psycholinguistics, New York: Elseiver.

Corver, N.: 1990, The Syntax of Left Branch Constructions, Ph.D dissertation, Tilburg University.

Corver, N.: 1997, 'Much-support as a Last Resort', Linguistic Inquiry **28.1**, p. 19–164.

Cresswell, M.J. 1976, 'The Semantics of Degree', in B. Partee (ed.), Montague Grammar, Academic Press, New York.

Faller, M.: 1998, 'A Vector Space Semantics for Comparatives', unpublished ms., Stanford University.

Faller, M.: to appear, 'Dimensional Adjectives and Measure Phrases in Vector Space Semantics', in Faller, M., S. Kaufmann, and M. Pauly (eds.), Proceedings of the Seventh CSLI Workshop on Logic, Language, and Computation, CSLI Publications, Stanford.

Gawron, J-M. 1995, 'Comparatives, Superlatives, and Resolution', Linguistics and Philosophy **18**, 333–380.

Grimshaw, J.: 1991, 'Extended Projection', unpublished ms., Brandeis University.

Hale, A.: 1970, 'Conditions on English Comparative Clause Pairings', in R. Jacobs and P. Rosenbaum (eds.), Readings in English Transformational Grammar, Georgetown University Press, Washington.

Hay, J., C. Kennedy, and B. Levin: to appear, 'Scalar Structure Underlies Telicity in Degree Achievements', Proceedings of SALT 9, Cornell Linguistics Club Publications, Ithaca.

Heim, I.: 1985, 'Notes on Comparatives and Related Matters', unpublished ms., University of Texas, Austin.

Hellan, L.: 1981, Towards an Integrated Analysis of Comparatives, Narr, Tübingen.

Hendriks, P.: 1995, Comparatives and Categorial Grammar, Ph.D. dissertation, University of Groningen.

Hoeksema, J.: 1983, 'Negative Polarity and the Comparative', Natural Language and Linguistic Theory **1**, 403–434.

Izvorski, R.: 1995, 'A Solution to the Subcomparative Paradox', in Camacho, J., L. Couieri and M. Watanabe (eds.) The Proceedings of WCCFL XIV, Stanford, CSLI Publications.

Kamp, H.: 1975, 'Two Theories of Adjectives', in Keenan, E. (ed.) Formal Semantics of Natural Language, Cambridge University Press, Cambridge. 123–155.

Kennedy, C.: 1999a, Projecting the Adjective: The Syntax and Semantics of Gradability and Comparison, New York, Garland. (1997,Ph.D. thesis, University of California, Santa Cruz.)

Kennedy, C.: 1999b, 'Gradable Adjectives Denote Measure Functions, not Partial Functions', Studies in the Linguistic Sciences **29.1**, 65–80.

Kennedy, C.: 1997a, 'Comparison and Polar Opposition', in Lawson, A. (ed.), Semantics and Linguistic Theory 7, Ithaca, Cornell Linguistics Club Publications.

Kennedy, C.: 1997b, 'On the quantificational force of the comparative clause', In Austin, J. and A. Lawson (eds.), Proceedings of the 1997,Eastern States Conference on Linguistics, Ithaca, Cornell Linguistics Club Publications.

Kennedy, C.: to appear a, 'On the Monotonicity of Polar Adjectives', in J. Hoeksema, H.: Rullmann, V.: Sánchez-Valencia and T. van der Wouden (eds.), Perspectives on Negation and Polarity, Amsterdam, Benjamins.

Kennedy, C.: to appear b, 'Comparative (Sub)deletion: Evidence for Ranked, Violable Constraints in Syntax', Proceedings of NELS 30, Amherst, GSLA Publications.

Kennedy, C. and L. McNally: to appear, 'From Event Scales to Adjectival Scales: Degree Modification in Deverbal Adjectives', in Proceedings of SALT 9, Ithaca, Cornell Linguistics Club Publications.

Kennedy, C. and L. McNally: in preparation, 'Scale Structure and the Semantics of Degree Modification', ms., Northwestern University and Universitat Pompeu Fabra.

Klein, E.: 1991, ' Comparatives', in von Stechow, A. and D. Wunderlich (eds.) Semantik: Ein Internationales Handbuch der Zeitgenössischen Forschung, Walter de Gruyter, Berlin.

Klein, E.: 1982, 'The Interpretation of Adjectival Comparatives', Journal of Linguistics **18**, 113–136.

Klein, E.: 1980, 'A Semantics for Positive and Comparative Adjectives', Linguistics and Philosophy **4.1**, 1–45.

Krantz, D.: R. D. Luce, P. Suppes, and A. Tversky: 1971, Foundations of Measurement, Volume 1, Academic Press, New York.

Ladusaw, W.: 1979, Polarity as Inherent Scope Relations, Ph.D. dissertation, University of Texas, Austin.

Ladusaw, W.: 1986, 'Principles of Semantic Filtering', in M. Dalrymple et al. (eds.), Papers from the 5th Western Conference on Formal Linguistics, Stanford Linguistics Association, Stanford, Ca.

Landman, F.: 1991, Structures for Semantics, Kluwer, Dordrecht.

Larson, R.: 1988, 'Scope and Comparatives', Linguistics and Philosophy **11**, 1–26.

Lechner, W.: 1999, Comparatives and DP-Structure, Ph.D. dissertation, University of Massachusetts, Amherst.

Lehrer, A.: 1985, 'Markedness and Antonymy', Journal of Linguistics **21**, 397–429.

Lerner, J-Y. and M. Pinkal: 1995, 'Comparative Ellipsis and Variable Binding', Computerlinguistik an der Universität des Saarlandes Report No. 64. (Also in Semantics and Linguistic Theory 5).

Lerner, J-Y. and M. Pinkal: 1992, 'Comparatives and Nested Quantification', Computerlinguistik an der Universität des Saarlandes Report No. 21.

Linebarger, M.: 1980, The Grammar of Negative Polarity, Ph.D. dissertation, Massachusetts Institute of Technology.

Löbner, S.: 1990, Wahr neben Falsch: Duale Operatoren als die Quantoren natürlicher Sprache, Niemeyer, Tübingen.

Ludlow, P.: 1989, 'Implicit Comparison Classes', Linguistics and Philosophy **12**, 519–533.

McConnell-Ginet, S.: 1973, Comparative Constructions in English: A Syntactic and Semantic Analysis, Ph.D. dissertation, University of Rochester.

Moltmann, F.: 1992, Coordination and Comparatives, Ph.D. dissertation, Massachusetts Institute of Technology.

Pinkal, Manfred: 1989, 'On the Logical Structure of Comparatives', in Studer, Rudi (ed.), Lecture Notes in Artificial Intelligence, Springer-Verlag, Berlin, pp. 146–167.

Postal, P.: 1974, 'On Certain Ambiguities', Linguistic Inquiry **5.3**, 367–425.

Rullmann, H.: 1995, Maximality in the Semantics of WH-Constructions, Ph.D. dissertation, UMass., Amherst.

Russell, B.: 1905, 'On Denoting', Mind **14**, 479–493.

Sánchez-Valencia, V.: 1994, 'Polarity, Predicates, and Monotonicity', unpublished ms., University of Groningen.

Sapir, E.: 1944, 'Grading: A Study in Semantics', Philosophy of Science **11**, pp. 93–116.

Schwarzschild, R. and K. Wilkinson: 1999, 'Interval Semantics for Scalar Predication', unpublished ms. Rutgers University.

Seuren, P.A.M.: 1984, 'The Comparative Revisited', Journal of Semantics **3**, 109–141.

Seuren, P.A.M.: 1978, 'The Structure and Selection of Positive and Negative Gradable Adjectives', in Farkas, D., W. Jacobson and K. Todrys (eds.), Papers from the Parasession on the Lexicon, CLS 14, University of Chicago, Chicago, Il.

Seuren, P.A.M.: 1973, 'The Comparative', in F. Kiefer and N. Ruwet (eds.), Generative Grammar in Europe, Riedel, Dordrecht.

Siegel, M.: 1979, Capturing the Adjective, New York, Garland.

von Stechow, A.: 1998, 'Some Remarks on Ch. Kennedy's Syntax and Semantics of Comparison', paper presented at the Workshop on the Syntax and Semantics of Comparatives, Zentrum für Allgemeine Sprachwissenschaft, Berlin, November 27–28, 1998.

von Stechow, A.: 1984a, 'Comparing Semantic Theories of Comparison', Journal of Semantics **3**, 1–77.

von Stechow, A.: 1984b, 'My Reaction to Cresswell's, Hellan's, Hoeksema's, and Seuren's Comments', Journal of Semantics **3**, 183–199.

Williams, E.: 1977, 'Discourse and Logical Form', Linguistic Inquiry **8.1**, 101–139.

Zwarts, J.: 1995, 'The Semantics of Relative Position', in Proceedings of Semantics and Linguistic Theory 5.

Zwarts, J. and Y. Winter: 1997, 'A Semantic Characterization of Locative PPs', In Lawson, Aaron (ed.), Proceedings of Semantics and Linguistic Theory 7, 294–311.