ers take up the discussion and, in many points, argue against Berlin & Kay.

Kamp (1975) and Wallace (1972) discuss the question, whether the comparative should be the semantic primitive for relative adjectives or whether it should be the positive. Bartsch & Vennemann (1972) provide a valuable overview over the problems of comparison and the research of the sixties. Bierwisch (1967) suggests a decomposition analysis for dimension adjectives, and Wunderlich (1973) discusses the problems of dependence on a norm and comparison class.

Cresswell (1976) analyses the comparative in the framework of a Montague Grammar and introduces Alternative 1. Hamann et al. (1980) take his approach a bit further in demonstrating how the scales have to be altered in order to accomodate the difference of dimension and value adjectives. They also treat the minimal requirements on a mapping between scales for "mixed" comparisons. Hamann (1982) makes it plausible to treat a scale as a simple model of an order relation with or without a smallest element — without having to accept the structure of the real numbers lock stock and barrel.

Seuren (1973) and (1985) introduce and comment on Alternative 2, Hellan (1981) introduces Alternative 3. Von Stechow (1985) compares those three approaches. Bierwisch (1986) gives a recent and comprehensive treatment of the whole subject, which not only combines the methods of formal logic with the concepts of GB (as found in Chomsky 1981), but treats many more phenomena than could be listed and discussed here.

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32. Comparatives

1. Linguistic Phenomenology
1.1 A Comparative Look at Comparatives
1.2 Further Syntactic Considerations
2. Measurement
3. Degree Ontologies
3.1 The Degree Parameter
3.2 Degrees as Equivalence Classes
3.3 Numerical Degrees
3.4 Delineation Theories of Comparatives
3.5 Positive and Comparative
3.6 Commensurability
4. The Logical Form of Comparatives
4.1 The Comparative Complement
4.2 Connectives
4.3 Quantifiers
4.4 Modal Contexts
5. Concluding Remarks
6. Short Bibliography

1. Linguistic Phenomenology

1.1 A Comparative Look at Comparatives

It is usually fairly easy to recognize a comparative construction, but less easy to give a satisfactory general definition. Central to our discussion is the status of **gradable adjectives** such as *old*, *big*, and *generous* (also known as **degree** or **relative** adjectives). These have a number of interesting characteristics. The most important seems to be that they express properties which are **order inducing**, in the sense that we can impose an ordering (possibly incomplete) on objects according to whether one object possesses the relevant property to greater or less extent than another. It is difficult to disagree with Cresswell's (1976: 281) contention that our ability

Degree or Relative adjs: gradable

to draw comparisons has to be taken as "basic data", and that it is "the business of linguistics ... to tell us how we put the comparisons we do make into the linguistic forms into which we put them." From a cognitive point of view, it is highly plausible that the notion underlying a predicate such as *old* is a relational, intrinsically comparative notion. What is less clear is whether we should take this comparative relation as fundamental to the natural language semantics of gradable predicates. This is a topic of considerable and continued debate which we shall not attempt to resolve in this article; for discussion, cf. Bartsch & Vennemann (1972), van Benthem (1982, 1983 a, c), Bierwisch (1987/89), Cresswell (1976), Hoepelman (1983), Kamp (1975), Klein (1980), Sapir (1944), von Stechow (1983), Wallace (1972) and Wheeler (1972).

The second important characteristic of gradable predicates concerns the linguistic manifestation of their comparative nature, namely that they admit degree modification. Degree modifiers in English consist of expressions like *very, fairly, too*, and *so*, measure phrases such as *twenty five years*, *two metres* and *six kilograms*, and the comparative constructions themselves.

The third property of gradable predicates which we wish to draw attention to is that they typically come in pairs, standing in polar opposition; for example, *old* ~ *young*, *big* ~ *small*, and *generous* ~ *mean*. Following Hoepelman (1983), let us symbolize the polar opposite of a gradable predicate ζ as ζ°. For Lyons (1977: 272), the distinguishing logical characteristic of such pairs is that they validate inferences of type (1a), but not the converse (1b):

Wenig postulat?

(1) a. x is $\zeta \Rightarrow x$ is not ζ°
b. x is not $\zeta \Rightarrow x$ is ζ°

For further discussion of polarity, see also Hoepelman (1983), Lehrer & Lehrer (1982), Rusiecki (1985), Seuren (1978, 1984) and von Stechow (1984 d).

Traditionally, gradable predicates are said to allow four 'degrees of comparison':

(2)	a. positive:	*a is tall*
	b. equative:	*a is as tall as b*
	c. comparative:	*a is taller than b*
	d. superlative:	*a is the tallest of the children*

Types (b) and (c) are also sometimes classed as **comparatives of equality** and **inequality**, respectively. While equatives will receive a little attention in this article, comparatives will be the main focus.

In order to establish some grammatical terminology, let us briefly consider an English adjectival comparative. (3) illustrates a reasonably uncontroversial phrase structure analysis.

(3) *Sue is* $[_{AP}\ [_{AP}$ *taller*$]\ [_{PP}$ *than*
$[_{S}$ *Tom is* $[_{AP}$ *e*$]]]]$

The comparative construction itself is the constituent dominated by the topmost AP (Adjective Phrase) node. The head of this construction is the AP *taller*. The comparative complement of the head is the constituent *than Tom is*; following Larson (1985), we have classified this as a PP (Prepositional Phrase). Syntacticians are generally agreed that there is a grammatically controlled missing constituent in the comparative clause, which has here been indicated as a lexically empty AP node. The affix *-er* in *taller* is a degree modifier. While the comparative degree marker in English is realized as an inflection on monosyllabic and some disyllabic adjectives, it can also be expressed analytically by the modifier *more*; cf. also *less* and *as*.

It is useful to also give a 'notional' analysis of the comparative construction. Terminology in this area is somewhat confused; I shall largely follow that of Ultan (1972). The adjective *tall* expresses the **gradable property**. *Sue* is the **item of comparison**, while *Tom* is the **standard of comparison**. The **standard marker**, *than*, marks the degree relationship between the item and standard of comparison, and according to Ultan's analysis must belong to the same syntactic constituent as the standard of comparison. The **degree marker**, *-er*, is notionally characterized as the expression which marks the degree to which the item of comparison possesses the gradable property. (4) summarizes:

(4)	*Sue is*	*tall*	*-er*	
	ITEM	GRADABLE	DEG-MARKER	
	than		*Tom*	*is*
	STANDARD-MARKER		STANDARD	

The typological characteristics of comparative constructions have attracted a fair amount of attention; cf. particularly Andersen (1983), Stassen (1984), Ultan (1972). A major distinction can be drawn between **paratactic** and **hypotactic** constructions. A paratactic construction consists of two coordinate constituents, whereas a hypotactic construction consists of a complement embedded within a main clause. A second major distinction, crosscutting the first, can be drawn between **clausal** and **phrasal comparatives** (cf. Hankamer 1973), depending on whether the comparative complement is a phrase consisting of the standard of comparison, or whether it is a clause which contains the standard of comparison as a subconstituent. The examples in (5) (taken, like many others in this section, from Stassen 1984) illustrate paratactic clausal comparatives.

(5) a. NUER:
gáṯmàar díitɛ̀, kè díid nĕ γǎ̆n
my-brother is-big but big am I
'I am bigger than my brother'

b. NUER:
díid nè jín, kwíy nè γǎ̆n
big are you young am I
'You are older than I am'

c. HIXKARYANA:
Kaw-ohra naha Waraka,
tall-not he-is Waraka
kaw naha Kaywerye
tall he-is Kaywerye
'Kaywerye is taller than Waraka'

Typically in the paratactic clausal construction, each clause contains a gradable predicate and there is some feature which marks a contrast between the two clauses. The subtypes illustrated above are: (a) a conjunction of adversity ('but'), (b) polar opposition between the two predicates, and (c) negation of one of the two predicates.

Finally, Malay possesses a number of comparative constructions, one of which is semantically similar to the paratactic clausal structure, though hard to classify syntactically:

(6) MALAY (Kähler 1965: 165)
rumah saja dan rumah tuan,
house my and house your,
besar rumah tuan
big house your
'Of my house and your house,
yours is big'

Phrasal paratactic constructions have received relatively little discussion from a crosslinguistic perspective. Pinkham (1982) argues persuasively that examples like (6) and (7) are best analyzed as coordinate structures (cf. also Napoli 1983):

(7) a. FRENCH:
Plus d'hommes que de femmes
more of men than of women
sont venus
AUX come

b. More men than women came

(8) a. FRENCH:
Cet homme est plus avare qu' économe
this man is more greedy than frugal

b. This man is more greedy than frugal

Hypotactic clausal constructions seem often to be historically derived from paratactic structures (Seuren 1984, Stassen 1984: 175), and tend to involve a specialized comparative particle, such as *than* in English, *dan* in Dutch, or *quam* in Latin.

interesting

(9) DUTCH:
Ik zag hem eerder dan hij mij zag
I saw him earlier than he me saw
'I saw him earlier than he saw me'

(10) LATIN:
Haec verba sunt
these-NOM words-NOM are
Varronis, hominis doctioris
Varro-GEN man-GEN more-learned-GEN
quam fuit Claudius
than was Claudius-NOM
'These words are Varro's, a man more learned than Claudius was'

There has been some debate within generative grammar as to whether such particles are complementizers; the current weight of opinion tends to the view that they are not (cf. Chomsky & Lasnik 1977, den Besten 1978, Larson 1985). It is interesting to note that hypotactic comparative clauses frequently pattern like *wh*-constructions. Thus, in the case of English, Doherty & Schwarz (1967) have pointed out the possibility of inversion (though cf. Emonds 1976: 24):

(11) Politicians are friendlier than are statesmen.

evidence that than is not in C°.

Huddleston (1967) observed interesting similarities in scopal interaction with negatives; and Chomsky (1977) has drawn attention to the overt *wh*-expression in the (non-standard) construction (12):

(12) I am taller than what you are.

more evidence.

Languages such as Hungarian, Italian, Polish, Maltese and German show an analogous morphology; we illustrate from Hungarian:

(13) HUNGARIAN:
a. Mennyire magas János
how tall Janos
'How tall is John?'

b. János magas-abb mint a-mennyire
Janos tall-more than DEF-how
Vilmos volt
Vilmos was
'Janos is taller than Vilmos was'

It has also been noted, particularly by Seuren (1973, 1984), that hypotactic comparative clauses resemble paratactic ones in often containing a negative particle:

(14) FRENCH (Milner 1978: 686):
Pierre est plus gentil que tu ne disais qu' était Paul
Pierre is more nice than you NEG said that was Paul
'Pierre is nicer than you said that Paul was'

Interesting. Compare w/ Horn's observations.

However, Napoli and Nespor (1976) have argued that, at least in the case of Italian, the (optional) presence of negation is not a reflex of the underlying logical structure of comparatives (as supposed by Seuren), but instead conveys a rhetorical overtone of denying an existing assumption.

or does it have something to do w/ the ordering relation?

Phrasal hypotactic constructions are extremely widespread, and the major subtypes divide into what Stassen (1984: 149) calls **derived case** and **fixed case constructions**. We will return to derived case constructions later; the fixed case constructions are distinguished by the standard of comparison being formally marked by an invariable case assignment. Within this class, a further subdivision into direct object and adverbial comparatives can be drawn. The direct object construction employs a verb meaning 'to surpass' or 'to excel' whose subject is the item of comparison and whose object is the standard. Typical examples are illustrated in (15) and (16).

cf. Dakota

(15) a. DUALA:
Nin ndabo e kolo buka nine
this house it big exceed that
'This house is bigger than that'

b. VIETNAMESE:
Tiên này hon tiên cúa tôi
money this exceed money CLASS me
'This sum of money is greater than mine'

In adverbial comparatives, the standard of comparison is marked by an adposition or oblique case inflection. Within this category, we should probably place English examples like (16a) (cf. Hankamer 1973), and (16b) (cf. Huddleston 1967):

(16) a. Sue is taller than Tom.
b. The car was travelling faster than 90 mph.

However, across languages, the semantic content of the adverbial is overwhelmingly locational, and as such can be divided into three subclasses: (17a) separative ('from'), (17b) allative ('to'), and (17c) locative ('at/on').

(17) a. JAPANESE:
satowa kawa yori chikashi
village river *from* is-near
'the village is nearer than the river'

b. MASAI:
Sapuk ol-kondi to l-kibulekeny
is-big the-deer *to* the-waterbuck
'The deer is bigger than the waterbuck'

c. LATVIAN:
Anna smukaka aiz Trinas
Anna-NOM prettier-FEM *on* Trinas-GEN
'Anna is prettier than Trina'

We briefly alluded earlier to a type of phrasal construction which Stassen (1984: 149–150) calls derived case comparison. In such constructions, the case of the standard of comparison is parallel to, and thus determined by, the case of the item of comparison. This parallelism is neatly illustrated in the Latin *quam* construction:

(18) LATIN:
a. Brutum ego non minus amo quam tu
Brutus-ACC I-NOM not less love-1SG than you-NOM
'I love Brutus no less than you do'

b. Brutum ego non minus amo quam te
Brutus-ACC I-NOM not less love-1SG than you-ACC
'I love Brutus no less than I love you'

Latin does not have the ambiguity of English

Notice that the English counterpart of (18), *I love Brutus no less than you*, is ambiguous between the two readings made explicit in Latin. This factor provides some basis for the view that on at least one derivation, the phrase following the comparative particle is related by ellipsis to a clausal complement (Bresnan 1973, Hankamer 1973).

This brief overview of the major devices for expressing comparison has of course ignored much complicating detail. It has also omitted any discussion of equative constructions. According to Ultan (1972), the major type of equative construction involves a degree-like marker expressing similarity or identity, such as English *like*. He claims that of the different kinds of standard markers found in comparative, superlative and equative constructions, there is a marked similarity be-

Actually, English does the same thing: "She loves Brutus no less than I". "She loves Brutus no less than me"

Try to explain this.

Russian: INST case.

tween comparative and superlative markers, and generally dissimilarity between equative markers and either of the other type. A further interesting result of Ultan's survey concerned suppletive paradigms. Two thirds of his sample consistently exhibited shared suppletive bases for, on the one hand, comparatives and superlatives and, on the other hand, positives and equatives. This is illustrated by English *better* ~ *best* versus *good* ~ *as good*. (Suppletion in the field of gradable adjectives has also been studied by Wurzel 1985, 1987.)

Apart from the works already cited, the literature contains a variety of reports on comparative constructions in languages other than English: for example, Chinese (Arlotto 1975), Dutch (Bennis 1978, Hoeksema 1983), Eskimo (Mey 1976), French (Anscombre 1975, Milner 1973, 1978, Pinkham 1982), German (Wunderlich 1973), Japanese (Haig 1976), Italian (Bracco 1979, Napoli & Nespor 1976), Polish (Borsley 1981), Proto-Indo-European (Andersen 1980), Spanish (Rivero 1981), Swedish (Andersson 1973), and Turkish (Knecht 1976).

1.2 Further Syntactic Considerations

The syntax of English comparative constructions has been extensively studied within generative grammar; see, for example, Andrews (1974, 1975), Bresnan (1971, 1973, 1975, 1976 a, b), Bowers (1975), Chomsky (1965, 1977), Dieterich & Napoli (1982), Doherty & Schwarz (1967), Gazdar (1980), Hale (1970), Hellan (1981), Hendrick (1978), Heny (1978), Huddleston (1967), Jackendoff (1977), Kuno (1981), Lees (1961), McCawley (1973a), Napoli (1983 a), Pilch (1965), Rivara (1979), Smith (1961) and Williams (1976). Bresnan's (1975) analysis, according to which three rules are centrally involved, has provided a useful descriptive terminology which is widely accepted. **Comparative Deletion** (CD) was held to be responsible for deleting the phrases indicated in (19):

(19) a. You've written more articles than I've read __. (NP)
b. Bill is slimmer than he used to be __. (AP)
c. They ate more quickly than they drank __. (AdvP)

Bresnan points out that CD induces an unbounded dependency which respects familiar island constraints:

(20) a. He's not as successful as Mary claims to believe that he is __.
b. *He's not as successful as Mary believes the claims that he is __.

Comparative Subdeletion removed only part of the compared constituent, namely the degree modifier/quantifier:

(21) a. You've written more books than I've written __ articles.
b. Bill is as slim now as he was __ obese before.
c. My sister drives as carelessly as I drive __ carefully.

Finally, **Comparative Ellipsis** was regarded as optionally removing part or all of a verb phrase which had already undergone (CD):

(22) You've written more books than Bill (has) __.

Not surprisingly, subsequent work has called into question many of the details of Bresnan's proposals. For example, Chomsky (1977) has claimed that *wh*-Movement rather than an unbounded deletion rule is responsible for CD constructions; Bennis (1978) has argued that the only grammatically-determined gap in comparatives is caused by Subdeletion, and that CD constructions arise from pragmatically-determined ellipsis of the phrasal head; and Napoli (1983 a) has denied the existence of Comparative Ellipsis, arguing that the phenomena in question can be accounted for either by independently required mechanisms such as VP Ellipsis, Null Complement Anaphora and Gapping, or else by invoking a distinct construction which we earlier termed the Phrasal Comparative. (Additional discussion of Comparative Ellipsis can be found in Bach, Bresnan & Wasow (1974), Higgins (1973), Napoli (1983 b), Plann (1982), and Sag (1976).)

Despite the largely syntactic orientation of the above-mentioned studies, some observations of semantic interest can be found. Thus, Chomsky (1965: 180) remarks that (23a) entails (23b), though not (23c) (cf. also Bresnan 1973, Doherty & Schwarz 1967, McCawley 1979):

(23) a. John is a more clever man than Bill.
b. Bill is a man.
c. Bill is a clever man.

On the other hand, the entailment to (23b) is not licensed by (24):

(24) John is a man more clever than Bill.

Some attention has also been paid to constructions like (25) (cf. Bresnan 1973: 324–

327. McCawley 1976, Napoli 1983 a, Thompson 1972):

(25) Sue is more sad than angry.

Metalinguistic Comparison

This can be construed in two distinct ways. The most salient reading, which Thompson (1972) characterizes as 'denial of assumption', arises when (25) is interpreted as a negative answer to the question *Is Sue angry?* It also has a more normal 'degree' reading as an answer to the question *How sad is Sue?*

On the denial reading, such constructions have a number of distinctive properties. The inflected form of the adjective is not allowed (cf. also Andrews (1984), Huckin (1977), and Ross (1974) on this interaction between morphology and semantics):

(26) *Sue is sadder than angry.

They allow paraphrases of the form (27):

(27) a. Sue is sad, more than angry.
b. Sue is sad rather than angry.

And third, the adjective cannot occur as a prenominal modifier:

(28) *Sue is a more sad person than agry.

2. Measurement

Before considering proposals for the analysis of comparative constructions, it will be useful to review some of the basic mathematical ideas involved in comparison and measurement. For convenience, we adopt the framework developed by Krantz et al. (1971, especially Ch. 1). Following their exposition, we use the example of length measurement.

Is this begging the question? No: our ability to compare is basic; we're interested in it's linguistic expression (Cresswell)

We take as given a set A of straight rods whose length can be compared. If two rods a and b are placed side by side so that they are aligned at one end, then three situations may obtain: either a is longer than b, or b is longer than a, or a and b are equivalent in length. These cases are symbolized, respectively, as $a \succ b$, $b \succ a$, and $a \sim b$. As well as comparing rods, we can concatenate them, that is place two or more rods end to end in a straight line. The concatenation of a and b is symbolized $a \circ b$. Naturally, it is possible to compare the lengths of sets of concatenated rods, so for example $a \circ b \succ c \circ d$ expresses the proposition that the concatenation of a and b is longer than the concatenation of c and d.

It is convenient (in order to state connectedness) to take $\succsim$ as our basic empirical relation. In the present context, we can gloss it as 'at least as long as'. Ignoring concatenation for the moment, consider the structure $\langle A, \succsim \rangle$ consisting of a set A and the relation $\succsim$ on A. This is termed an **(empirical) relational structure**. It constitutes a *weak order* iff for all a, b and $c \in A$, the following two axioms are satisfied:

Weak Order:
(i) Connectedness: Either $a \succsim b$ or $b \succsim a$.
(ii) Transitivity: If $a \succsim b$ and $b \succsim c$, then $a \succsim c$.

To arrive at a system of ordinal length measurement, we must assign numbers to the rods in a way that preserves the empirical ordering induced by $\succsim$: the measure associated with a is greater than or equal to the measure associated with b just in case a is at least as long as b. That is, if φ is an assignment of numbers to rods, then the following condition must obtain:

(29) $\varphi(a) \geq \varphi(b)$ iff $a \succsim b$

This numerical assignment constitutes a homomorphism of an empirical relational structure into a numerical relational structure. For the latter, we take $\langle \mathbb{R}, \geq \rangle$, where $\mathbb{R}$ is the set of real numbers and $\geq$ is the usual 'greater than or equal to' relation. For φ to be a homorphism, it must send A into $\mathbb{R}$ and $\succsim$ into $\geq$ in a way which respects (i). The existence of the homorphism is guaranteed by a *representation theorem*; that is, a theorem which asserts that if a given relational structure satisfies certain axioms, then a homorphism into a certain relational structure can be constructed. There are of course many assignments of the required kind. A *uniqueness theorem* states that, under a certain class of permissible transformations, they are all equivalent. In the case at hand, two assignments φ and φ' are equivalent iff there is a monotone increasing function f such that for any $a \in A$, $\varphi'(a) = f(\varphi(a))$. Thus the permissible transformations for ordinal measurement is the set of all monotone increasing functions from $\mathbb{R}$ onto $\mathbb{R}$.

The relation $\geq$ on the reals is a weak order which is also anti-symmetric: if both $x \geq y$ and $y \geq x$, then $x = y$. We call this a *simple* order to distinguish it from the case of a weak order, where there can be distinct elements a and b such that $a \succsim b$ and $b \succsim a$.

Given the relation $\succsim$, we define two new relations as follows:

(30) $a \sim b$ iff $a \succsim b$ and $b \succsim a$
(31) $a \succ b$ iff $a \succsim b$ and $\neg\ (b \succsim a)$

If $\succsim$ is a weak order on A, then $\sim$ is an equivalence relation on A, and $\succ$ is transitive and asymmetric. The relation $\sim$ partitions A into a set of equivalence classes, where $\mathbf{a} = \{b \mid b \in A \wedge b \sim a\}$ is the equivalence class determined by a. The set of equivalence classes is denoted $A/\sim$. Suppose we define an ordering $\succsim$ on $A/\sim$ by

(32) $\mathbf{a} \succsim \mathbf{b}$ iff $a \succsim b$.

Right Then $\langle A/\sim, \succsim\rangle$ is a simple order, since if $\mathbf{a} \succsim \mathbf{b}$ and $\mathbf{b} \succsim \mathbf{a}$, then $\mathbf{a} = \mathbf{b}$.

We note in passing that since $\mathbf{a}$ is the set of objects which are exactly as long as a, by analogy with the Frege-Russell treatment of cardinal numbers, it might well be viewed as a formal reconstruction of the *length* of a. Indeed Cresswell (1976), amongst others, has proposed a general analysis of degrees of just this kind.

Consider a simple example. $A = \{a, b, c_1, c_2\}$, $a \succ b \succ c_1, c_2$, $A/\sim \; = \{\{a\}, \{b\}, \{c_1, c_2\}\} = \{\mathbf{a}, \mathbf{b}, \mathbf{c}_1\}$. The required representation theorem states that if $\langle A/\sim, \succsim\rangle$ is a simple order, then there is an assignment $\boldsymbol{\varphi}$ such that $\boldsymbol{\varphi}(\mathbf{a}) \geq \boldsymbol{\varphi}(\mathbf{b})$ iff $\mathbf{a} \succsim \mathbf{b}$. We show this by providing a method for constructing $\boldsymbol{\varphi}$. For each $\mathbf{a} \in A/\sim$, $\boldsymbol{\varphi}(\mathbf{a}) = \text{card}(\{\mathbf{b} \mid \mathbf{a} \succsim \mathbf{b}\})$. Thus, continuing OK our example, we have $\boldsymbol{\varphi}(\mathbf{c}_1) = 1$, $\boldsymbol{\varphi}(\mathbf{b}) = 2$, and $\boldsymbol{\varphi}(\mathbf{a}) = 3$. The assignment to $A/\sim$ carries over to A by setting $\varphi(a) = \boldsymbol{\varphi}(\mathbf{a})$.

Although this approach allows us to assign numbers to the rods, it is important to note that this induces an ordinal measure. The only thing we can do with the numbers is compare them under the $\geq$ relation; without more information about the structure $\langle A/\sim, \succsim\rangle$, and more constraints on the assignment φ, there is no guarantee that summing the values of φ will make any sense. Consequently, although we have constructed a formal notion of length as an equivalence class, it does not yet provide a basis for familiar systems of length measurement. To be concrete, given our example above, we cannot infer that the measure $\boldsymbol{\varphi}(\mathbf{b}) = \boldsymbol{\varphi}(\mathbf{c}_1) + \boldsymbol{\varphi}(\mathbf{c}_1)$. In fact, the assignment $\boldsymbol{\varphi}$ is compatible with c_1 having a length of 1 metre, and b having a length of 5 metres. In order to allow addition on the range of $\boldsymbol{\varphi}$, we must have the counterpart of addition on $\boldsymbol{\varphi}$'s domain. This, of course, is provided by the concatenation operation. Assuming, for example, that we take $\mathbf{c}$ as the unit measure, then we require that $\boldsymbol{\varphi}(\mathbf{b}) = \boldsymbol{\varphi}(\mathbf{c}) + \boldsymbol{\varphi}(\mathbf{c}) = 2\boldsymbol{\varphi}(\mathbf{c})$ only if $b \sim c_1 \circ c_2$. Notice that we assume that c_1 and c_2 are perfect copies of each other, and in general we assume that we can find indefinitely many such copies.

Suppose that a', a'', a''', … are perfect copies of the rod a. Krantz et al. (1971: 4) call the sequence a, $2a = a \circ a'$, $3a = (2a) \circ a''$, $4a$, $5a$, … a *standard sequence* based on a. Clearly, $\varphi(na) = n\varphi(a)$, while the value of $\varphi(a)$ will depend on the particular rod chosen to have unit length; if the unit rod is co-extensive with ma, then $\varphi(a) = 1/m$. If a rod b falls within an interval in the standard sequence, say $3a \succ b \succ 2a$, then b will be assigned some numerical value between $3\varphi(a)$ and $2\varphi(a)$. The interval can be made arbitrarily small by choosing finer and finer standard sequences.

For our purposes, the important property of standard sequences is that the numbers assigned are additive with respect to concatenation. That is, $\varphi(b \circ c) = \varphi(b) + \varphi(c)$. This holds because if b approximates na and c approximates $n'a$ then $(b \circ c)$ approximates $(n + n')a$. The additivity equation approaches exactness as for finer and finer standard sequences.

We close this section with one possible axiomatization of the conditions for extensive measurement (Krantz et al. 1971: 73). Let A be a nonempty set, $\succsim$ a binary relation on A, and $\circ$ a closed binary operation on A. Then $\langle A, \succsim, \circ\rangle$ is a **positive closed extensive structure** iff the following axioms are satisfied for all $a, b, c, d \in A$:

(33) i. Weak order: $\langle A, \succsim\rangle$ is a weak order.
ii. Weak associativity: $a \circ (b \circ c) \sim (a \circ b) \circ c$.
iii. Monotonicity: $a \succsim b$ iff $a \circ c \succsim b \circ c$ iff $c \circ a \succsim c \circ b$.
iv. Archimedean: If $a_1, \ldots, a_n, \ldots$ is a standard sequence, and there is some b such that for all a_n in the sequence, $b \succ a_n$, then the sequence is finite.
v. Positivity: $a \circ b \succ a$.

It can be shown that $\langle A, \succsim, \circ\rangle$ is a positive closed extensive structure iff there is a function φ from A to the positive reals such that for all $a, b \in A$,

(34) i. $a \succsim b$ iff $\varphi(a) \geq \varphi(b)$,
ii. $\varphi(a \circ b) = \varphi(a) + \varphi(b)$.

Another function φ' satisfies (i) and (ii) iff there is an $\alpha > 0$ such that $\varphi' = \alpha\varphi$.

3. Degree Ontologies

3.1 The Degree Parameter

Now that we have glanced at some foundational concepts in measurement, we turn to examine various proposals for analyzing natural language comparatives. Every approach to comparatives requires a means of expressing propositions like (35) (irrespective of the particular treatment of positive adjectives):

(35) Sue is tall to degree d

However, there is a lot of room for disagreement about what exactly is involved in the reference to degrees. It seems that we can ask at least the following questions:

(i) Should degrees be explicit parameters in the object language, or should they be regarded as contextual coordinates (e. g. Lewis 1970, Kamp 1975)?

(ii) Are gradable adjectives to be analysed as basically being noun modifiers (e. g. Cresswell 1976, Hoepelman 1983) or as vague predicates (e. g. Kamp 1975, Klein 1980)?

(iii) Is the semantics of the compared adjective a compositional function of the semantics of the positive adjective (e. g. Kamp 1975, Klein 1980), or are they both derived from some third, more abstract semantic structure (e. g. Cresswell 1976)?

(iv) Are degrees (a) equivalence classes under a relation of comparison (e. g. Cresswell 1976), (b) numbers closed under addition (e. g. Hellan 1981), or (c) delineations (or boundary specifications) for vague predicates (e. g. Kamp 1975)?

(v) If (35) is satisfied by some degree *d*, is it uniquely satisfied by *d* (e. g. Cresswell 1976), or is it also satisfied by each *d'* such that $d' \gtrsim d$ (e. g. Kamp 1975)?

Although the issues addressed by questions (i)–(iii) are of great interest, and involve some fairly knotty questions about the relation between context and content in natural language semantics, they are largely peripheral to my present concerns. For example, from our present perspective, the debate around (ii) can be viewed as largely a matter of notation; the noun modifier view has to invoke contextual parameters to supply a suitable property when adjectives are used predicatively, as in *Sue is tall*, whereas the one-place predicate view has to relativize the interpretation of adjectives to a suitable comparison class, which can be explicitly expressed by a modified nominal when adjectives are use attributively, as in *Sue is a tall woman*. At any rate, since we are not specifically concerned with the semantics of adjectives, we will not attempt to provide principled answers to (i)–(iii) here (though see also Siegel (1979), Beesley (1982) and von Stechow (1983) for further discussion).

As we have to make some choices, we answer (i) and (ii) in the manner that most simplifies exposition, namely gradable adjectives are predicates, parameterized for a degree. Consequently, we propose (36) as the object language representation of (35), where *tall* denotes a binary relation between degrees and individuals.

(36) *tall*(**d**, *Sue*)

We have little to say here about question (iii), but will return briefly to it later.

What we wish to focus on now are the issues raised in (iv) about the ontological status of degrees. It could be argued that the delineation approach makes weaker assumptions about the structure of the world than either of the other two approaches. In addition, we have already seen that the degrees-as-real-numbers approach presupposes the degrees-as-equivalence-classes approach, together with some assumptions about the behaviour of concatenation. In the next three subsections, we will try to find out whether any substantial benefits accrue from making these increasingly strong assumptions. We shall also have occasion to reflect on (v), since it has both formal and empirical consequences. As a last introductory comment, we note that because of our present concern with ontology, questions about the logical structure of comparatives will be kept in the wings for the time being, but will make their entrance on the stage in section 4.

3.2 Degrees as Equivalence Classes

Anyone who has glanced at the linguistics literature on comparatives will have encountered logical representations of (37) that resemble (38), (which we gloss as (39)).

(37) Sue is taller than Tom

(38) ι**d**[*tall*(**d**, *Sue*)] $\succ$ ι**d**[*tall*(**d**, *Tom*)]

(39) The degree to which Sue is tall exceeds the degree to which Tom is tall.

The notation provides an answer to our earlier question (v): it is assumed that each individual can be assigned a unique degree of height. However, it is left open what kind of thing the variable '**d**' ranges over. One possible answer is that a degree of height is a set of objects that are all exactly as high as each other. According to this view, proposed for example by Cresswell (1976), comparatives involve an ordering on degrees, where the latter are construed as equivalence classes.

In order to make this a little more precise, we will define a syntax and semantics for a language containing definite degree terms. As a simplifying assumption, the intended model will associate a weak order $\succsim_\zeta$ with each degree predicate ζ. We should not in general require this association to be one-one. On the one hand, two or more distinct predicates can be scaled along the same dimension; for example, we might allow that $\succsim_{tall} = \succsim_{short} = \succsim_{wide} = \succsim_{narrow}$. On the other hand, some predicates are 'multi-dimensional', in the sense that there may be several, possibly incompatible, criteria for their application. A simple example is *large*: city X may be larger than Y with respect to population, but less large with respect to surface area. We will ignore this kind of indeterminacy here (for discussion, see e. g. Kamp 1975, Pinkal 1983).

As before, we define some additional relations based on $\succsim_\zeta$:

(40) $a \sim_\zeta b$ iff $a \succsim_\zeta b$ and $b \succsim_\zeta a$.
(41) $a \succ_\zeta b$ iff $a \succsim_\zeta b$ and $\neg(b \succsim_\zeta a)$.
(42) $a \prec_\zeta b$ iff $b \succ_\zeta a$.

Since $\sim_\zeta$ is an equivalence relation on the universe A of individuals, the degree to which an individual a possesses the property expressed by ζ can be spelled out as the equivalence class $\{b \in A \mid b \sim_\zeta a\}$. As a shorthand, we also denote this class by $[a]_\zeta$. The set of all equivalence classes on A induced by $\sim_\zeta$ is the quotient algebra $A/\sim_\zeta$. So as to keep our language first-order, we do not use equivalence classes directly, but rather distinguish a subset $\mathfrak{D} \subseteq A$, each element of which corresponds to $[a]_\zeta$ for some a and ζ.

As suggested earlier, $\succ$ will be the object language relation on degree terms which represents comparatives of inequality. Analogous representations can be constructed for comparatives of equality and *less than* comparatives:

(43) a. Sue is (at least) as tall as Tom is.
b. Sue is less tall than Tom is.

(44) a. $\iota\mathbf{d}[tall(\mathbf{d}, Sue)] \succsim \iota\mathbf{d}[tall(\mathbf{d}, Tom)]$
b. $\iota\mathbf{d}(tall(\mathbf{d}, Sue)] \prec \iota\mathbf{d}[tall(\mathbf{d}, Tom)]$

Thus, let L be a first order language supplemented with the following nonlogical constants:

(45) A set *Term* of singular terms *Sue*, *Tom*, *Rob*, ...
(46) A set *DPred* of two-place degree predicates *tall*, *old*, *wise*, ...

In addition, we assume the iota operator, and a set $DVar = \{\mathbf{d}_0, \mathbf{d}_1, \mathbf{d}_2, \ldots\}$ of variables over degrees.

The set *Form* of well-formed formulae is defined in the usual way, with the addition of the following clauses:

(47) Every element of *DVar* belongs to *DTerm*.
(48) If $\zeta \in DPred$, $\delta \in DTerm$ and $\tau \in Term$, then $\zeta(\delta, \tau) \in Form$.
(49) If $\varphi \in Form$, and $\mathbf{d}_i \in DVar$, then $\iota\mathbf{d}_i[\varphi] \in DTerm$.
(50) If $\delta, \delta' \in DTerm$, then $\delta \succsim \delta'$, $\delta \succ \delta'$ and $\delta \prec \delta' \in Form$.

A **model for L** is a 4-tuple $\mathfrak{A} = \langle A, \mathfrak{D}, F, \succsim \rangle$, where

(i) A is a nonempty set.
(ii) $\mathfrak{D} \subseteq A$ is a nonempty set.
(iii) For each $\zeta \in DPred$, there is a partial function f_ζ from $\mathfrak{D}$ onto $(A - \mathfrak{D})/\sim_\zeta$; f_ζ is one-one on its domain; and $\mathfrak{D}_\zeta$ is the image of $(A - \mathfrak{D})/\sim_\zeta$ under f_ζ^{-1}.
(iv) F is a function on the nonlogical constants of L.
(v) If $\tau \in Term$, then $F(\tau) \in A - \mathfrak{D}$.
(vi) If $\mathbf{d}_i \in DVar$, then $\mathbf{d}_i$ is assigned a value in $\mathfrak{D}$.
(vii) $\succsim$: $DPred \to \mathfrak{P}(A \times A)$ is a function which assigns a weak order to each $\zeta \in DPred$.

Elements of $\mathfrak{D}_\zeta$ (degrees of ζ-ness) are a certain kind of abstract individual which possess useful properties with respect to the ordering $\succsim_\zeta$. In particular, we have the following facts:

But they're abstract individuals w/ internal structure – as sets of (ordinary) individuals ...

(51) If $d, d' \in \mathfrak{D}_\zeta$ and $d \sim_\zeta d'$, then $d = d'$.
(52) For all $a \in A - \mathfrak{D}$, and $d \in \mathfrak{D}_\zeta$, $a \sim_\zeta d$ iff $a \in f_\zeta(d)$.

According to (51), $\succsim_\zeta$ is an antisymmetric relation on $\mathfrak{D}$, while according to (52), an ordinary individual a is ζ-equivalent with a degree d just in case a belongs to the ζ-equivalence class to which d corresponds under f_ζ.

The truth definition for the logical part of L is standard, and we add the following clauses to deal with degree expressions:

(53) If $\zeta(\delta, \tau) \in Form$, then $[\![\zeta(\delta, \tau)]\!]^{\mathfrak{A}} = 1$ iff $[\![\tau]\!]^{\mathfrak{A}} \sim_\zeta [\![\delta]\!]^{\mathfrak{A}}$.
(54) If $\delta \succsim \delta' \in Form$, then $[\![\delta \succsim \delta']\!]^{\mathfrak{A}} = 1$ iff $[\![\delta]\!]^{\mathfrak{A}}, [\![\delta']\!]^{\mathfrak{A}} \in \mathfrak{D}_\zeta$ and $[\![\delta]\!]^{\mathfrak{A}} \succsim_\zeta [\![\delta']\!]^{\mathfrak{A}}$; similarly for $\succ$ and $\prec$.

for comparatives

We pointed out earlier that the approach under consideration imposes a uniqueness condition on degrees, in the sense that (55) holds:

right.

(55) $\forall\mathbf{x}\forall\mathbf{d}_0\forall\mathbf{d}_1[\zeta(\mathbf{d}_0, \mathbf{x}) \wedge \zeta(\mathbf{d}_1, \mathbf{x}) \to \mathbf{d}_0 = \mathbf{d}_1]$

This condition is valid under the class of intended models for L. For if the antecedent is true under an assignment $d_0/\mathbf{d}_0$, $d_1/\mathbf{d}_1$ and $a/\mathbf{x}$, then $a \sim_\zeta d_0$ and $a \sim_\zeta d_1$; since $\sim_\zeta$ is an equivalence relation, we have $d_0 \sim_\zeta d_1$ and thus $d_0 = d_1$.

It should be noticed that degrees do not play an essential semantic role in the analysis yet, since the comparatives in which they occur can always be reduced to comparisons between ordinary individuals. That is,

Right.

Generally, the crucial thing is that weak adjs are associated w/ any degree predicate.

(56) $[\![\iota\mathbf{d}[tall(\mathbf{d}, Sue)]]\!]^{\mathfrak{A}} \succsim_\zeta [\![\iota\mathbf{d}[tall(\mathbf{d}, Tom)]]\!]^{\mathfrak{A}}$ iff $d \succsim_\zeta d'$, where $d \sim_\zeta$ Sue and $d' \sim_\zeta$ Tom iff Sue $\succsim_\zeta$ Tom.

This should be an entailment.

3.3 Numerical Degrees

If we confine our attention to those instances where $\delta \succsim \delta'$ is defined, then $\succsim$ is connected, transitive and antisymmetric. Consequently, by a representation theorem of the kind discussed in Section 2, we know that we can assign real values to the degrees in $\mathfrak{D}_\zeta$, yielding an ordinal measurement. In order to implement this step, we augment L with an operator $^\#$ which maps a member of *DTerm* into an expression of category *Num* (or numeral):

(57) If $\delta \in DTerm$, then $^\#\delta \in Num$.

(58) If $\nu, \nu' \in Num$, then $\nu \succsim \nu' \in Form$.

We shall assume that the representation of *Sue is as tall as Tom* in L($^\#$) is the following:

(59) $^\#(\iota\mathrm{d}[tall(\mathrm{d}, Sue)]) \succsim {^\#}(\iota\mathrm{d}[tall(\mathrm{d}, Tom)])$

A **numerical model for L($^\#$)** is a 4-tuple $\mathfrak{A} = \langle \mathfrak{M}, \mathfrak{N}, F, \varphi \rangle$, where

(i) $\mathfrak{M} = \langle A, \mathfrak{D}, \succsim \rangle$, with A, $\mathfrak{D}$ and $\succsim$ as before.
(ii) $\mathfrak{N} = \langle \mathbb{R}, \geq \rangle$.
(iii) $\varphi : \mathfrak{D} \rightarrow \mathbb{R}$.
(iv) $\geq$ is a simple order on $\mathbb{R}$.
(v) For any $d \in \mathfrak{D}$, $F(^\#)(d) = \varphi(d)$.

As noted in the preceding section, the assignment φ must also satisfy the appropriate homomorphism condition:

HOM($\geq$): For all $d, d' \in \mathfrak{D}_\zeta$, $\varphi(d) \geq \varphi(d')$ iff $d \succsim_\zeta d'$.

Thus, the truth clause corresponding to (54) is the following:

(60) If $\nu \succsim \nu' \in Form$, then $[\![\nu \succsim \nu']\!]^{\mathfrak{A}} = 1$ iff $[\![\nu]\!]^{\mathfrak{A}} \geq [\![\nu']\!]^{\mathfrak{A}}$.

Tracing the various conditions on the interpretation of *Sue is as tall as Tom* yields the following list of equivalences:

(61) $[\![^\#(\iota\mathbf{d}[tall(\mathbf{d}, Sue)]) \succsim {^\#}(\iota\mathbf{d}[tall(\mathbf{d}, Tom)])]\!]^{\mathfrak{A}} = 1$ iff
$[\![^\#(\iota\mathbf{d}[tall(\mathbf{d}, Sue)])]\!]^{\mathfrak{A}} \geq$
$[\![^\#(\iota\mathbf{d}[tall(\mathbf{d}, Tom)])]\!]^{\mathfrak{A}}$ iff
$\varphi([\![\iota\mathbf{d}[tall(\mathbf{d}, Sue)]]\!]^{\mathfrak{A}}) \geq$
$\varphi([\![\iota\mathbf{d}[tall(\mathbf{d}, Tom)]]\!]^{\mathfrak{A}})$ iff
$[\![\iota\mathbf{d}[tall(\mathbf{d}, Sue)]]\!]^{\mathfrak{A}} \succsim$
$[\![\iota\mathbf{d}[tall(\mathbf{d}, Sue)]]\!]^{\mathfrak{A}}$ (by HOM($\geq$)) iff
Sue $\succsim_{tall}$ Tom (by (56)).

Again it turns out that the truth conditions of simple comparatives reduce to the primitive grading relation. While the mapping into real values has added an extra level of complexity in the interpretation rules, has it gained us any commensurate advantage? So far, the answer is 'no'. Although L($^\#$) allows us to represent numeric degrees, we cannot legitimately add, subtract or multiply such degrees, but only compare them. The numerical models for L($^\#$) induce an ordinal scale, and nothing more. Thus, we still have no way of representing sentences like (62), dubbed 'differential comparatives' by von Stechow (1984 a):

No way to represent (62a.b) — we need to do more than compare we need to be able to add/subtract.

(concatenate)

(62) a. Sue is twice as tall as Tom is.
b. Sue is 6cm taller than Tom is.

In order to deal with these, we need to supplement L with something like arithmetic addition on degrees. Let us therefore introduce a binary operator + on numerals, as follows:

(63) If $\nu, \nu' \in Num$, then $\nu + \nu' \in Num$.

A **model for L($^\#$, +)** is a 4-tuple $\mathfrak{A} = \langle \mathfrak{M}, \mathfrak{N}, F, \varphi \rangle$, where F and φ are as before, and

(i) $\mathfrak{M} = \langle A, \mathfrak{D}, \succsim, \circ \rangle$.
(ii) $\mathfrak{N} = \langle \mathbb{R}, \geq, + \rangle$.
(iii) $\circ : DPred \rightarrow (A \times A \rightarrow A)$ is a partial function which associates a concatenation with elements of *DPred*.

We assume that for each dimension $\succsim_\zeta$ there is a corresponding concatenation $\circ_\zeta$, and that each $\mathfrak{D}_\zeta$ is closed under $\circ_\zeta$. The homomorphism φ from $\mathfrak{M}$ to $\mathfrak{N}$ also obeys the following restriction:

HOM($\circ$): For all $d, d' \in \mathfrak{D}_\zeta$, $\varphi(d) + \varphi(d') = \varphi([d \circ_\zeta d']_\zeta)$

This says that the result of summing the numbers assigned to the degrees of d and d' is the same as the number assigned to the degree of the concatenation of d and d'.

Finally, it should be recalled that the relation between dimensions and concatenations is governed by certain axioms. In the preceding section, the axioms proposed were

How can you concatenate degrees of e.g. intelligence?
→ Max is {twice / three times} as intelligent as Mara.

Sue is smart the [illegible].

$\lambda x. \exists v > 0[smart(^{*}\iota d[smart(d, x)]) + v, x](sue)$

→ This should be false. ([illegible])

Weak Associativity, Monotonicity, the Archimedean axiom and Positivity.

Let's return now to the sentences in (62). The obvious way to deal with (62a) is to treat *twice* as a modifier of the comparative relation *as tall, as*. However, since $\gtrsim$ is introduced syncategorematically in L, it is easier to translate *twice* as though it applies directly to a numeral expression, as follows:

(64) If $v \in Num$, then $\mathbf{2}v, \mathbf{3}v, \ldots \in Num$.

The numerals **2**, **3**, ... can of course be defined in terms of the plus operator:

(65) $\forall v[\mathbf{2}v = v + v]$

Within this set of assumptions, the logical counterpart of *Sue is twice as tall as Tom is* will be (66):

(66) $^{*}(\iota \mathbf{d}[tall(\mathbf{d}, Sue)]) \gtrsim \mathbf{2}^{*}(\iota \mathbf{d}[tall(\mathbf{d}, Tom)])$

(This of course represents the reading on which *twice as tall* means *at least as twice as tall*. To get the *exactly as twice as tall* reading, $\gtrsim$ would be replaced by $\sim$.)

Again, it may be useful to work through the truth definition.

(67) $[\![^{*}(\iota \mathbf{d}[tall(\mathbf{d}, Sue)]) \gtrsim$
$\mathbf{2}^{*}(\iota \mathbf{d}[tall(\mathbf{d}, Tom)])]\!]^{\mathfrak{A}} = 1$ iff
$[\![^{*}(\iota \mathbf{d}[tall(\mathbf{d}, Sue)])]\!]^{\mathfrak{A}} \geq$
$[\![\mathbf{2}^{*}(\iota \mathbf{d}[tall(\mathbf{d}, Tom)])]\!]^{\mathfrak{A}}$ iff
$\varphi([\![\iota \mathbf{d}[tall(\mathbf{d}, Sue)]]\!]^{\mathfrak{A}}) \geq$
$\varphi([\![\iota \mathbf{d}[tall(\mathbf{d}, Tom)]]\!]^{\mathfrak{A}}) +$
$\varphi([\![\iota \mathbf{d}[tall(\mathbf{d}, Tom)]]\!]^{\mathfrak{A}})$ iff
$[\![\iota \mathbf{d}[tall(\mathbf{d}, Sue)]]\!]^{\mathfrak{A}} \gtrsim$
$[\![\iota \mathbf{d}[tall(\mathbf{d}, Tom)]]\!]^{\mathfrak{A}} \circ_{tall} [\![\iota \mathbf{d}[tall(\mathbf{d}, Tom)]]\!]^{\mathfrak{A}}$

Notice that the presence of degrees here allows us to avoid the embarrassment of trying to concatenate Tom with himself.

Let us turn now to sentences like (62b),

(62) b. Sue is 6 cm taller than Tom is.

These constitute more of a problem for the apparatus developed so far, since it is not immediately obvious how a measure phrase such as *6cm* should be integrated with the comparative relation expressed by *taller than*. One strategy, adopted by Hellan (1981) and von Stechow (1984a), is to express the comparative entirely in terms of the + operator. That is, we take (62b) to mean something like 'Sue is tall to [the degree to which Tom is tall plus 6cm]'. An analysis of this kind is readily expressed in our current language, assuming that we treat expressions like **6cm** as elements of *Num*.

(68) $tall(^{*}(\iota \mathbf{d}[tall(\mathbf{d}, Tom)]) + \mathbf{6cm}, Sue)$

→ This is starting to look more compositional also.

Notice that once this approach has been adopted for differential comparatives, it suggests a revision in the analysis of ordinary comparatives such as *Sue is taller than Tom is*. These can now be construed as the existential generalization of formulae like (68):

Right, Revise the approach altogether.

(69) $(\exists v \succ 0)[tall(^{*}(\iota \mathbf{d}[tall(\mathbf{d}, Tom)]) + v, Sue)]$

I think that this should be an entailment

3.4 Delineation Theories of Comparatives

The family of approaches that we have examined so far all adopted a degree-based analysis of comparatives. First, the item and standard of comparison were associated with degrees along some dimension, and this was formalized with iota terms over degree variables in the object language. Second, a comparative sentence was represented as a relation between two degrees, and this was formalized using either the $\gtrsim$ relation-symbol, or else by means of +.

In this section, we will examine a somewhat different approach, one which relies on what Lewis (1970) calls 'delineations'. A delineation is intended as a contextual parameter that plays a role in the evaluation of degree predicates. Just as the interpretation of *That is a sock* requires a specification of the object indexically invoked by *that*, so – according to this view – the interpretation of *Sue is tall* requires a specification of the standard according to which something is judged as tall. A delineation for *tall* determines where, along the dimension of height, the cut-off point between 'tall' and 'not-tall' is to be set, and it is claimed that this point can vary with context.

There are a number of ways in which contextual parameters can be formally captured. For our current purposes, the simplest strategy for dealing with delineations is to treat them like degrees, via an extra argument to the degree predicate. This argument will denote a **standard**, which in turn will determine a delineation of the predicate. Thus, we translate *Sue is tall* as (70), where **s** is a variable over standards.

(70) $tall(\mathbf{s}, Sue)$

Despite appearances, this step does not completely conflate the delineation approach with the degree-based one. In order to be faithful to the intuition underlying delineations, (70) should not be interpreted to mean that Sue belongs to an equivalence class associated with **s**, but rather that the standard determines a delineation according to which Sue is judged tall. In other words, (70) can be

Right – there aren't any abstract entities involved; rather, the standard is more like restrictive modification.

paraphrased as '*Sue* is at least as tall as standard *s*'. Notice that there is no unique standard of tallness possessed by an individual: if Sue is at least 2 metres tall, then she is at least 1.50 metres tall, and so on. Support for this position lies in the fact that a question like (71a) can be answered as (71b):

(71) a. Is Sue 1.50 metres tall?
b. Yes. In fact she's 2 metres tall.

Crucial interaction w/ →

By Grice's maxim of quantity, an utterance of *Sue is 1.50 metres tall* will conversationally implicate that Sue is at most 1.50 metres tall, unless there is explicit cancellation of the implicature as in (71b).

In order to formalize these notions, we might define a new first order language L(s); like L, it would contain a category *DPred* whose members denote binary degree relations between standards and individuals. We will not go into detail, but mention some of the most important points.

Models for L(**s**) will contain $\varphi_\zeta \subseteq A$, a set of standards for ζ. We assume, as with $\mathfrak{D}_\zeta$, that there is one-one correspondence between φ_ζ and $A/\sim_\zeta$. Every standard $s \in \varphi_\zeta$ generates a possible positive extension for ζ, namely the set $\{a : a \succsim_\zeta s\}$ of objects which are at least as ζ as *s*. We will call this set an ***s*-extension**. An *s*-extension is the principal filter generated by *s*, and we denote it by $[s)_\zeta$. We can now say that *tall*(**s**, *Sue*) is true in a model for L(**s**) just in case *Sue* belongs to $[s)_{tall}$.

Despite the fact that **s**'s value is allowed to vary from context to context, the admissible delineations must conform to the principle GRAD of Consistent Gradience:

Crucial

GRAD: For all $\zeta \in DPred$, $a, b \in A$, and $s \in \varphi_\zeta$
if $\langle s, a\rangle \in F(\zeta)$ and $b \succsim_\zeta a$,
then $\langle s, b\rangle \in F(\zeta)$, and
if $\langle s, a\rangle \notin F(\zeta)$ and $a \succsim_\zeta b$,
then $\langle s, b\rangle \notin F(\zeta)$.

That is, if Sue $\succsim_{tall}$ *Tom*, then any value of **s** which satisfies *tall*(**s**, *Tom*) must also satisfy *tall*(**s**, *Sue*). But once GRAD is imposed, then the converse regularity gives us a straightforward means of expressing comparatives. That is, if any value of **s** which satisfies *tall*(**s**, *Tom*) also satisfies *tall*(**s**, *Sue*), then it follows that *Sue* is at least as tall as *Tom*. As a result, the representation of comparative constructions in L(**s**) does not require the addition of a further relation such as $\succsim$, but only involves the quantification of **s**-variables. For example, *Tom is as tall as Sue is* has the translation (72):

The core of the analysis is Klein 1982. →

(72) $\forall$**s**[*tall*(**s**, *Sue*) $\rightarrow$ *tall*(**s**, *Tom*)]

Similarly, *Sue is taller than Tom is* holds if there is some standard which satisfies *tall*(**s**, *Sue*) but fails to satisfy *tall*(**s**, *Tom*):

(73) $\exists$**s**[*tall*(**s**, *Sue*) $\wedge$ $\neg$ *tall*(**s**, *Tom*)]

An attractive feature of this approach is that (73) is logically equivalent to the negation of (72), namely (74).

(74) $\neg\forall$**s**[*tall*(**s**, *Sue*) $\rightarrow$ *tall*(**s**, *Tom*)]

In other words, we get 'for free' the equivalence between (75) and (76):

(75) Sue is taller than Tom is.
(76) Tom is not as tall as Sue is.

But "Sue is not taller than Tom is" ≠ Sue is as tall as Tom is

The approach just sketched is notationally closest to the system presented in Klein (1982). However, the essential idea is to be found in several earlier studies, in particular Lewis (1970) (where it is attributed to unpublished work by David Kaplan), Kamp (1975), McConnell-Ginet (1973), and Seuren (1973). Related discussions can also be found in Klein (1980, 1981a, b) and van Benthem (1982, 1983a, c).

historical precedents of that analysis

At this point, let us briefly confront an objection against the delineation approach raised by von Stechow (1984a). As we have already seen, comparatives allow differential forms such as (62), repeated here.

(77) a. Sue is twice as tall as Tom is.
b. Sue is 6cm taller than Tom is.

How can these be captured? As a preliminary step, let us ask another question, namely how are sentences like (78) to be analyzed?

(78) Sue is 1m tall.

If we just focus attention on the measure phrase *1m*, a rather plausible step is to interpret it as a degree of height: it denotes the equivalence class of objects which are one metre in length. Instead of taking our earlier route of introducing a special category *Num* for numeral expressions, we might just as well assign *1m* to the category *DTerm* of degree terms. A second observation to be made about (78) is that *1m* appears to have the function of making explicit the appropriate standard for *tall*.

yes

Let us return to the problem of how to represent differential comparatives of inequality. Recall that (75) was represented as (73), repeated here:

(79) Sue is taller than Tom is.
(80) $\exists$**s**[*tall*(**s**, *Sue*) $\wedge$ $\neg$ *tall*(**s**, *Tom*)]

→ means

There is an equivalent way of expressing (80) which uses sets of standards; namely, that the set of standards which satisfy *tall*(**s**, *Sue*) but not *tall*(**s**, *Tom*) is non-empty. We can represent this in the object language by means of λ and an existential generalized quantifier ⋁:

(81) $\bigvee(\lambda \mathbf{s}[tall(\mathbf{s}, Sue) \wedge \neg\, tall(\mathbf{s}, Tom)])$

⋁, of course, is to be viewed as a second order predicate, true of just those sets which are non-empty. The set denoted by

(82) $\lambda \mathbf{s}[tall(\mathbf{s}, Sue) \wedge \neg\, tall(\mathbf{s}, Tom)]$

is the class of all those standards *s* such that Sue is tall according to *s* but Tom is not. Suppose for example that Sue is (exactly) 1.06 metres tall, Tom is 1 metre tall, and that standards of height correspond to centimetres. It follows from what we said earlier that Sue belongs not only to $[106\text{ cm})_{tall}$, but also to $[105\text{ cm})_{tall}$, $[104\text{ cm})_{tall}$, …, $[1\text{ cm})_{tall}$. Similarly, Tom belongs to $[100\text{ cm})_{tall}$, $[99\text{ cm})_{tall}$, …, $[1\text{ cm})_{tall}$; but, crucially, he does not belong to $[101\text{ cm})_{tall}$ nor to any higher *s*-extension. Consequently, (82) will denote the following set of standards in φ_{tall}:

(83) {106 cm, 105 cm, 104 cm, 103 cm, 102 cm, 101 cm}

It is exactly these six standards which make *Sue is tall* true but fail to make *Tom is tall* true. Moreover, it is fairly obvious that we can also associate a further standard with this set of standards, namely 6 cm. We conclude, therefore, that a differential comparative like (77b) is a special case of (82), one where the set of standards separating Sue from Tom is claimed not just to be nonempty, but equal to 6 cm. In order for this approach to succeed, we need to show in detail how an appropriate set of standards can form a 'standard sequence' which provides the basis for a metric. Roughly speaking, we have to say this: If *P* is a predicate of standards, then the higher order predicate *1 m** is true of *P* iff *P* is true of *1 m* and for any *s* > *1 m*, *P* is false of *s*. Thus, *Sue is 1 m tall* is analysed as *1 m** (λ**s**[*tall*(**s**, *Sue*)]). However, there is not space to develop this here in more detail.

for not exactly sure what this means.

hmm

3.5 Positive and Comparative

As noted earlier, there is still much debate as to the correct relation between positive and comparative gradable adjectives. One approach, put forward by Cresswell (1976) and supported by von Stechow (1984a), involves existential quantification of the degree argument by an operator **pos**, along the following lines:

(84) $\mathbf{pos}(\zeta)(x) \leftrightarrow \exists \mathbf{d}[\varphi(\mathbf{d}) \wedge \zeta(\mathbf{d}, x)]$
where φ means roughly 'is higher than average'

On a delineation approach, much the same effect can be gained by assuming that an appropriately high standard is picked out on the basis of contextual factors.

This makes more sense to me — standards are highly flexible.

Another important consideration, so far not discussed, is the fact that degree adjectives are interpreted relative to a **comparison (or reference) class** (see, for example, Hare 1952, Ross 1970b, Siegel 1979, von Stechow 1983, Wallace 1972, Wescoat 1984, Wheeler 1972, and Zwicky 1969). The comparison class can either be implicit, as when we interpret *Fergus is big* to mean that Fergus is big relative to the class of fleas, or explicit, as in (85).

(85) a. Fergus is a big flea.
b. Fergus is big for a flea.

A rather simple and attractive proposal is to analyse the comparison class as setting the domain of discourse relative to which the adjective is evaluated. On such an approach, we might interpret a degree adjective ζ as a function which, given a set *X*, acts very much as a one-place predicate restricted to *X*. Thus, the positive adjective *big* would denote a partial characteristic function which, when applied to the set of fleas, induces a partition into the set of big fleas and the set of small fleas, with possibly some residue of fleas which are neither definitely big nor small. We might then derive the comparative as a quantification over comparison classes:

(86) *x is bigger than y* is true iff there is some comparison class *X* such that *x is big* is true relative to *X* while *y is big* is false relative to *X*.

Indeed, the truth of (86) can be resolved on the basis of one particular comparison class, namely the set {*x*, *y*}. Given a few plausible axioms, it can be shown that the relation induced in this way is transitive and irreflexive and, on a quotient algebra, connected; cf. van Benthem (1982, 1983a), Hoepelman (1983) and Klein (1980).

From a structural point of view, the analysis sketched in (86) is a close variant of the delineation approach: just take **s** in (81) to range over comparison classes rather than standards. This is to be expected if we assume that every comparison class *X* for a predicate ζ is associated with a delineation; i.e. a divi-

right

I.e., the measure phrase makes reference to the difference.

sion of members of X into those objects which have the property ζ and those that lack it (or possess its polar opposite).

It is not clear however whether the degree/delineation parameter of gradable adjectives should be replaced or supplemented by a comparison class parameter. On the hypothesis that the head noun in an adjective-noun construction is taken to fill the comparison class slot, then a sentence like (87a) might argue for a representation like (87b):

(87) a. Blackie is a three year old horse.
b. *old(horse, 3yr, Blackie)*

Yet the oddness of (88) suggests that this hypothesis is inadequate, and that (87a) is more different from (87b) than often assumed:

(88) ?Blackie is three years old for a horse.

Further research is called for here.

3.6 Commensurability

In section 1 we briefly encountered Subdeletion constructions, such as (89):

(89) This table is longer than the door is wide.

Comparisons of this sort seem relatively straightforward inasmuch as the same dimension is common to the two predicates *long* and *wide*. Much less acceptable are examples where the properties in question appear to involve different scales:

notion of diff't scales — incompatible standards?

(90) ?This table is longer than it is heavy.
(91) ?Sue is thinner than Tom is rich.

The logical syntax of comparatives proposed earlier in this section would allow us to represent (89)–(91), but the semantics did not distinguish between the different cases. One approach, following Cresswell (1976), would be to render a Subdeletion comparative undefined if the degree predicates are associated with different scales, in the following manner:

(92) If $\delta \gtrsim \delta' \in Form$, then
(i) $[\![\delta \gtrsim \delta']\!]^{\mathfrak{A}} = 1$ if $[\![\delta]\!]^{\mathfrak{A}} \in \mathfrak{D}_\zeta$, $[\![\delta']\!]^{\mathfrak{A}} \in \mathfrak{D}_\xi$, $\gtrsim_\zeta = \gtrsim_\xi$, and $[\![\delta]\!]^{\mathfrak{A}} \gtrsim_\zeta [\![\delta']\!]^{\mathfrak{A}}$;
(ii) $[\![\delta \gtrsim \delta']\!]^{\mathfrak{A}} = 0$ if $[\![\delta]\!]^{\mathfrak{A}} \in \mathfrak{D}_\zeta$, $[\![\delta']\!]^{\mathfrak{A}} \in \mathfrak{D}_\xi$, $\gtrsim_\zeta = \gtrsim_\xi$, and $[\![\delta]\!]^{\mathfrak{A}} \gtrsim_\zeta [\![\delta']\!]^{\mathfrak{A}}$;
(iii) $[\![\delta \gtrsim \delta']\!]^{\mathfrak{A}}$ is undefined otherwise.

Yet it might be felt that this excludes too much. One way of making sense of a sentence like (91) is to compare relative positions on the respective scales; i.e. (91) could be paraphrased as saying that Sue is higher on the scale of thinness than Tom is on the scale of richness.

It should be pointed out that one of the inadequacies of the degree-as-numbers ontology is that it will always fail to predict any incommensurability. For example, when a degree of thinness has been mapped into a numerical value, it is qualitatively indistinguishable from a degree of richness, and thus the two should be readily comparable, despite the fact that we have seen this not to be so (cf. Cresswell 1984).

extract point

We briefly alluded earlier to the 'denial' interpretation of comparatives, illustrated by examples like the following ((b) and (c) are cited by Doherty and Schwarz (1967: 904)):

(93) a. Sue is more sad than angry.
b. This table is more decorative than it is useful.
c. His manner was more elegant than his matter was convincing.

On the denial reading, no condition of commensurability is required. Unfortunately, there is little discussion in the formal semantics literature about the interpretation of this type of comparative. As a first approximation (cf. Dieterich and Napoli 1982), one might say that

(94) x is more A than B

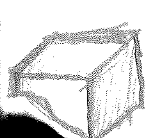
is true iff x *is* A is true and x *is* B is false. Perhaps slightly better would be: (94) is true iff x *is* A is true according to more criteria than x *is* B is true. This is motivated by the intuition that (94) involves a concession that x meets at least some of the prototypical criteria for being judged to be B; notice the oddness of **x is more tall than short*. However, the notion of 'true according to some criteria' remains to be explicated further.

METALINGUISTIC COMPARISON

4. The Logical Form of Comparatives

4.1 The Comparative Complement

Despite the expenditure of much effort, there is still little agreement about the appropriate logical representation of comparative constructions. The interaction of comparatives with quantifiers, logical connectives and opaque contexts presents a wealth of intricate puzzles. To date, the most detailed and comprehensive survey is von Stechow (1984a), and it is not possible to reproduce his discussion in the space of this article. However, we will take his proposed analysis as a basis for surveying the main issues and compare it to some of the main rival approaches.



Von Stechow agrees with a number of authors in interpreting gradable adjectives as relations between degrees and individuals, and interpreting comparative clauses as sets (or properties) of degrees (op cit: 54, 56). Thus, in the first instance, (95a) receives a representation like (95b) (where ζ is the appropriate adjective – von Stechow does not state exactly how this is determined).

(95) a. than Tom is
b. $\lambda\mathbf{d}\ \zeta(\mathbf{d}, \mathit{Tom})$

He also argues that comparative clauses, like other sentential complements, should be nominalized. This is achieved by a special rule of 'maximization' which maps (95b) into (96):

(96) $\mathbf{the}(\mathbf{max}(\lambda\mathbf{d}\ \zeta(\mathbf{d}, \mathit{Tom})))$

The **the** is essentially Russell's ι-operator, while **max** denotes a function on sets of degrees which yields as value the singleton set of degrees containing the maximal element of its argument; the definition is equivalent to the following (ignoring intensionality) (op cit: 37, 55):

(97) If D is a set of degrees and d is a degree, then $d \in [\![\mathbf{max}]\!](D)$ iff $d \in D$ and there is no d' such that $d' \succ d$ and $d' \in D$.

Von Stechow justifies the presence of the **max** operator by appealing to the context-dependence of definite descriptions; a different line of approach, which might be worth exploring in the light of our earlier remarks about hypotactic constructions in Section 1, would be to relate maximality to the exhaustiveness constraint which is typically associated with *wh*-questions.

Our sentence *Sue is taller than Tom is* is now represented by (98).

(98) $\lambda\mathbf{d}_1(\exists\mathbf{d}_2 \succ 0)[\mathit{tall}(\mathbf{d}_2 + \mathbf{d}_1, \mathit{Sue})]$ $(\mathbf{the}(\mathbf{max}(\lambda\mathbf{d}\zeta(\mathbf{d}, \mathit{Tom}))))$

By λ-conversion, this is equivalent to (99).

(99) $(\exists\mathbf{d}_2 \succ 0)[\mathit{tall}(\mathbf{d}_2 + \mathbf{the}(\mathbf{max}(\lambda\mathbf{d}\zeta(\mathbf{d}, \mathit{Tom}))), \mathit{Sue})]$

This can be paraphrased as saying that Sue is tall to a degree which is greater by some positive amount d_2 than the degree to which Tom is tall.

Equatives are treated in an analogous manner, with a quotient parameter instead of an additive one. So *Sue is as tall as Tom is* looks like (100):

(100) $\lambda\mathbf{d}_1[\mathit{tall}(1.\mathbf{d}_1, \mathit{Sue})]\ (\mathbf{the}(\mathbf{max}(\lambda\mathbf{d}\zeta(\mathbf{d}, \mathit{Tom}))))$

couldn't we stipulate that
$d \in$ the integers?

4.2 Connectives

Following Lakoff & Ross (1970) and Seuren (1973), it has often been observed that clausal complements of comparatives are negative polarity environments; cf. also Hoeksema (1983a), Klein (1982), Ladusaw (1979), McCawley (1981), and von Stechow (1984a). (101) illustrates some representative cases, where the polarity items are italicised:

(101) a. Sue was poorer than I would *ever care to be*.
b. John drives faster than he *need* do.
c. We bought more wine than we could *ever* drink.

According to the theory developed by Fauconnier (1975b) and Ladusaw (1979), the argument of an expression α is in a negative polarity environment only if α is Downward Entailing (DE). An informal characterization of DE is the following.

(102) A function f is called **downward entailing** iff for all X, Y in the domain of f, if X is more informative than Y, then $f(Y)$ is more informative than $f(X)$.

The expression 'more informative' is deliberately vague, but is meant to subsume relations like logical consequence and set inclusion. So, for example, (103) would be particular instances of a DE function f.

(103) a. If $X \models Y$, then $f(Y) \models f(X)$.
b. If $X \subseteq Y$, then $f(Y) \subseteq f(X)$.

Before examining the data, we need to briefly comment on the rather controversial and fundamental question as to whether there is a logical difference between the clausal form of the comparative, such as (104), and the phrasal form, such as (105).

(104) Sue is taller than Tom is.
(105) Sue is taller than Tom.

In particular, while it is generally thought that the clausal construction is DE, Hoeksema (1983, 1984) has argued that this is not the case for phrasal comparatives. We will review this question later, but in order not to prejudge the issue we will only use the clausal construction in examples for the time being, despite the sometimes un-idiomatic results.

Returning from our digression, let us now examine evidence for the claim that comparative clauses occur in a DE environment. An obvious starting place is the behaviour of *or* and *and*. Given the usual account of logical connectives, we would expect the following entailments to hold for a DE function f:

* what is "the exhaustiveness constraint typically associated w/wh-questions"?

(106) f(Tom is [tall] or Rob is [tall]) $\vDash$
f(Tom is [tall])

(107) $\vDash f$(Tom is [tall]) $\vDash$
f (Tom is [tall] and Rob is [tall])

Consider then the sentences in (108), where the putative DE context is *Sue is taller than*:

(108) Sue is taller than Tom is or Rob is $\vDash$
Sue is taller than Tom is

Despite the rather stilted style of the example, it seems clear that the inference is valid, and hence the hypothesis that comparatives induce a DE context gains support. On the other hand, the behaviour of *and* tends in the opposite direction; the following does not appear to be valid:

(109) Sue is taller than Tom is $\vDash$
Sue is taller than Tom is and Rob is

Nevertheless, the inference does seem legitimate if we add the additional premiss that Tom and Rob are the same height. Intuitions here seem delicate; they can perhaps be cajoled in the relevant direction if the conclusion of (109) is revised to (110).

(110) Sue is taller than Tom and Rob are.

The account of logical connectives and comparatives given by von Stechow (1984a) hinges on his analysis of *than* complements as definite NPs. We note first that the following analogue to (103b) holds:

(111) If $[\![D]\!]$, $[\![D']\!]$ are nonempty sets of degrees, and $[\![D]\!] \subseteq [\![D']\!]$, then
$[\![\mathbf{the}(\mathbf{max}(D'))]\!] \gtrsim [\![\mathbf{the}(\mathbf{max}(D))]\!]$.

For instance, we have (112).

(112)
$[\![\lambda\mathbf{d}[\mathit{tall}(\mathbf{d}, \mathit{Tom})]]\!] \subseteq$
$[\![\lambda\mathbf{d}[\mathit{tall}(\mathbf{d}, \mathit{Tom}) \vee \mathit{tall}(\mathbf{d}, \mathit{Rob})]]\!]$, so
$[\![\mathbf{the}(\mathbf{max}(\lambda\mathbf{d}[\mathit{tall}(\mathbf{d}, \mathit{Tom}) \vee \mathit{tall}(\mathbf{d}, \mathit{Rob})]))]\!] \gtrsim$
$[\![\mathbf{the}(\mathbf{max}(\lambda\mathbf{d}[\mathit{tall}(\mathbf{d}, \mathit{Tom})]))]\!]$

Second, we note the following obvious point:

(113) If $[\![\delta']\!]$, $[\![\delta]\!]$ are degrees, and $[\![\delta']\!] \gtrsim [\![\delta]\!]$, then for any assignment g to $\mathbf{x}$,
$[\![(\exists\mathbf{d} \succ 0)[\mathit{tall}(\mathbf{d} + \delta', \mathbf{x})]\!]_g \vDash$
$[\![(\exists\mathbf{d} \succ 0)[\mathit{tall}(\mathbf{d} + \delta, \mathbf{x})]\!]_g$.

By transitivity, we obtain the result (114).

(114) If $[\![D]\!]$, $[\![D']\!]$ are nonempty sets of degrees, and $[\![D]\!] \subseteq [\![D']\!]$, then
$[\![(\exists\mathbf{d} \succ 0)[\mathit{tall}(\mathbf{d} + \mathbf{the}(\mathbf{max}(D')), \mathbf{x})]\!]_g \vDash$
$[\![\,[\![(\exists\mathbf{d} \succ 0)[\mathit{tall}(\mathbf{d} + \mathbf{the}(\mathbf{max}(D)), \mathbf{x})]\!]_g$.

This explains how von Stechow's analysis correctly predicts the validity of entailments like (108) involving *or* in comparative complements.

The situation is slightly more complex when we turn to *and*. Since it is assumed that everyone is tall to some unique degree, the question arises as to whether the set denoted by an expression like (115) is empty or not:

(115) $\lambda\mathbf{d}[\mathit{tall}(\mathbf{d}, \mathit{Tom}) \wedge \mathit{tall}(\mathbf{d}, \mathit{Rob})]$

That is,
$\mathbf{the}(\mathbf{max}(\lambda\mathbf{d}[\mathit{tall}(\mathbf{d}, \mathit{Tom}) \wedge \mathit{tall}(\mathbf{d}, \mathit{Rob})]))$
denotes if Tom and Rob have the same height, and is undefined otherwise. Correspondingly, (116) – von Stechow's translation of (110) – also presupposes that Tom and Rob have the same height.

(116)
$\lambda\mathbf{d}_1(\exists\mathbf{d}_2 \succ 0)[\mathit{tall}(\mathbf{d}_2 + \mathbf{d}_1, \mathit{Sue})]$
$(\mathbf{the}\ (\mathbf{max}\ (\lambda\mathbf{d}[\mathit{tall}(\mathbf{d}, \mathit{Tom}) \wedge \mathit{tall}(\mathbf{d}, \mathit{Rob})])))$

This works also

This provides a rather persuasive explanation for the apparent invalidity of the inference (108) involving *and*. The reason why the conclusion fails to hold is that we do not know whether the *than* clause succeeds in denoting a degree; and this is because we do not know, from the premiss alone, whether Tom and Rob have the same height.

This doesn't work for we.

Let us briefly consider the presence of negation in comparative clauses.

(117) Sue is taller than Tom isn't.

Sentences such as (117) are usually felt to be anomalous. (Potential counterexamples to this claim noticed by Green (1970) should probably be considered as 'denial' comparatives.) The account supplied by von Stechow's framework again seems plausible. The *than* clause will have a representation like the following:

How does "Tom isn't tall." work?

(118) $(\mathbf{the}(\mathbf{max}(\lambda\mathbf{d}[\neg\ \mathit{tall}(\mathbf{d}, \mathit{Tom})])))$

If Tom is in fact 2 metres tall, then he is not 3 metres tall, nor 4 metres, nor …; i.e. there is no maximum degree in the set denoted by $\lambda\mathbf{d}[\neg\ \mathit{tall}\ (\mathbf{d}, \mathit{Tom})]$. As a result, (118) will fail to denote and the sentence as a whole will fail to express a proposition, giving rise to the perceived anomaly.

It has also been noted that the presence of a DE factive in the comparative clause has a similar effect, presumably for essentially the same reason:

(119) Sue is taller than she realizes/*regrets.

For discussion, see Carden (1977) and Vlach (1974).

What about
Sue is taller than Tom is short.
→ The degree approach should be able to get this

4.3 Quantifiers

Given the well-known equivalence of existential and universal quantification with (infinitary) disjunction and conjunction, respectively, we would expect the results of the previous section to extend to NPs with *some* and *every*. This is largely the case. However, the overall picture is complicated by two factors. First, quantifiers induce extra scope possibilities, and second, *some* has a negative polarity counterpart, namely *any*. The relevance of these consideration is illustrated in (120).

(120) a. Sue is taller than some boy is.
b. Sue is taller than any boy is.

The truth conditions of (120a, b) are brought out more clearly by the paraphrases in (121a, b), respectively.

(121) a. There is some boy such that Sue is taller than him.
b. Every boy is such that Sue is taller than him.

This is exactly what we would expect if both *some boy* and *any boy* corresponded to existential quantification in the logical representation, differing only with respect to scope: *some boy* is outside the scope of the DE context, while *any boy* is inside the scope. That is, within von Stechow's framework, (120) would correspond to the following two translations.

(122)
$(\exists \mathbf{x})[boy(\mathbf{x}) \wedge (\exists \mathbf{d}_1 \succ 0)$
$[tall\,(\mathbf{d}_1 + (\mathbf{the}\,(\mathbf{max}\,(\lambda \mathbf{d}\; tall(\mathbf{d}, \mathbf{x})))), Sue)]]$

(123) $(\exists \mathbf{d}_1 \succ 0)[tall\,(\mathbf{d}_1 + (\mathbf{the}\,(\mathbf{max}\,(\lambda \mathbf{d}$
$[(\exists \mathbf{x})\,[boy(\mathbf{x}) \wedge tall(\mathbf{d}, \mathbf{x})]]))), Sue)]]$.

The translation of the *than* clause in (123) will denote the maximum of the set containing the degrees of height of each boy in the domain. Consequently, (123) is equivalent to (124), which is the wide scope translation of (125).

(124)
$(\forall \mathbf{x})[boy(\mathbf{x}) \rightarrow (\exists \mathbf{d}_1 \succ 0)$
$[tall\,(\mathbf{d}_1 + (\mathbf{the}\,(\mathbf{max}\,(\lambda \mathbf{d}\; tall(\mathbf{d}, \mathbf{x})))), Sue)]]$

(125) Sue is taller than every boy is.

If we turn now to NP comparatives, the semantic facts appear to follow the same pattern. For example, (126a, b) parallel (120a, b):

(126) a. Sue is taller than some boy.
b. Sue is taller than any boy.

We surmise that both NPs correspond to existential quantifiers, differing only in that *some boy* is outside and *any boy* is inside the DE context. Some support for this view can be derived from the fact that further negative polarity items can appear in relatives modifying the second, but not the first NP:

Free choice any?

(127) a. *Sue is taller than some boy that I ever met.
b. Sue is taller than any boy that I ever met.

Hoeksema (1983:415) claims, contrary to what we have proposed so far, that NP comparatives induce upward entailing contexts. Part of his case rests on the observation that the following entailments hold, at least on one reading of the premisses.

(128) Sue is taller than Tom or Rob ⊨
Sue is taller than Tom or Sue is taller than Rob

(129) Sue is taller than Tom and Rob ⊨
Sue is taller than Tom and Sue is taller than Rob

Yet this data only shows that the readings in question are ones where the conjoined NPs have wider scope than the DE context. A further consideration adduced by Hoeksema involves the claim that the Dutch negative polarity item *ook maar* canot occur in NP comparatives; however, the data is disputed by Seuren (1984, FN. 15).

4.4 Modal Contexts

The interaction of comparatives with modal contexts has been a topic of long-standing interest in the semantics literature, apparently stimulated by Russell's (1905) famous example (130).

(130) I thought your yacht was larger than it was.

Russell adduced this in support of his proposal for assigning scope to definite descriptions. The logical structures in (131) indicate a Russellian rendering of the two relevant readings:

(131) a. $\iota\mathbf{d}[I\ thought(large(\mathbf{d}, y))] \succ$
$\iota\mathbf{d}\ [large\,(\mathbf{d}, y)]$
b. $I\ thought(\iota\mathbf{d}[large(\mathbf{d}, y)] \succ$
$\iota\mathbf{d}\ [large\,(\mathbf{d}, y)])$

The first structure represents the 'sensible', or **external reading** of (130), while the second represents the contradictory, or **internal reading** (cf. Larson 1985).

Since Russell, there have been a variety of analyses which share the goal of locating the *than* clause outside the scope of the propositional attitude verb. An early contribution within the Generative Semantics framework by Ross and Perlmutter (1970) derived the external reading of (130) by assigning it a deep structure analogous to (131a), and then transformationally lowering it into position; see also Lakoff (1970) and Bresnan (1971) for reactions. A non-transformational analysis was proposed by Hasegawa (1972), provoking a lengthy critique and counter-proposal by Postal (1974). Postal's paper was in turn met by a flurry of comment, most of it highly critical, including Abbott (1976), Dresher (1977), Horn (1981), Liddell (1975), Reinhart (1975), von Stechow (1984a) and Williams (1977).

Russell's analysis is maintained, with minor modifications, by von Stechow (1984a: 69), as illustrated in (132).

(132) a. $\lambda\mathbf{d}_2[I\ thought((\exists\mathbf{d}_1 \succ 0)\ [large\ (\mathbf{d}_1 + \mathbf{d}_2,\ y)])]\ (\mathbf{the}\ (\mathbf{max}\ (\lambda\mathbf{d}[large(\mathbf{d},\ y)])))$
b. $I\ thought((\exists\mathbf{d}_1 \succ 0)\ [large\ (\mathbf{d}_1 + (\mathbf{the}\ (\mathbf{max}\ (\lambda\mathbf{d}\ [large\ (\mathbf{d},\ y)]))),\ y)])$

One objection that might be levelled against (132a) is that it attributes a comparative component to the content of my thought: it claims that the size of your yacht is some degree **d** such that I thought your yacht was larger than **d** (see Larson 1985 for a variation of this point). To bring this out more clearly, let us change the example slightly so as to include an explicit differential measure:

(133) a. I thought that your yacht was 10 metres longer than it was.
b. $\lambda\mathbf{d}_2[I\ thought(long(10\text{m} + \mathbf{d}_2,\ y)])]\ (\mathbf{the}(\mathbf{max}(\lambda\mathbf{d}[long(\mathbf{d},\ y)])))$

Presumably (133a) would be a true belief-report if your yacht is in fact 20 metres long, and I thought it was 30 metres long. But, so the objection runs, it seems wrong in such a situation to represent the content of my belief as in (133b), namely by the proposition that your yacht is 10 metres more than 20 metres long. While this is logically equivalent to the proposition that your yacht is 30 metres long, yet on a sufficiently fine-grained view of propositions, we would presumably want to keep the two distinct.

A plausible alternative analysis of the scope ambiguity is suggested by Dresher (1977), inspired by Bresnan's (1973) syntactic analysis of comparatives (and earlier, Chomsky 1965, Lees 1961). On Dresher's approach, the discontinuous string *-er...than it was* in (130) forms a syntactic and semantic constituent, belonging to the category $\bar{Q}\bar{P}$ of degree modifiers. It is this constituent which can take wide or narrow scope with respect to the verb of propositional attitude. We could incorporate Dresher's idea easily into von Stechow's analysis of differential comparatives, since the phrase *10 metres ...-er...than it is* can be represented by the complex degree term $(10\text{m} + \mathbf{the}(\mathbf{max}(\lambda\mathbf{d}[long(\mathbf{d},\ y)])))$. This would yield (134) in place of (133b).

(134) $\lambda\mathbf{d}_0[I\ thought(long(\mathbf{d}_0,\ y)])]\ (10\text{m} + \mathbf{the}\ (\mathbf{max}(\lambda\mathbf{d}[long(\mathbf{d}, y)])))$

However, it is less clear how the existential quantification in non-differential comparatives should be accommodated.

a la Heim ?

5. Concluding Remarks

Within the confines of this article, it has not been possible to cover issues arising from nominal comparatives like (135) or adverbial comparatives like (136):

(135) a. Sue ate more apples than Tom did.
b. Tom ate two fewer apples than Sue did.
c. Sue found as much silver as gold.

(136) a. Sue eats apples as often as Tom does.
b. Sue likes Rob more than Tom.

To a large extent, the treatment of these constructions depends on a prior analysis of plural and mass determiners on the one hand, and of adverbs on the other. For some discussion, see in particular Cresswell (1976), Hellan (1981), Klein (1981a), and von Stechow (1984a).

A related topic is the observation that temporal prepositions such as *before* and *after* appear to be syntactically and semantically related to comparatives:

(137) a. Sue arrived before Bill (did)
b. *Sue arrived before Bill didn't
c. *Sue arrived an hour before Bill knows a man who did.

(138) a. Sue arrived earlier than Bill (did)
b. *Sue arrived earlier than Bill didn't
c. *Sue arrived an hour earlier than Bill knows a man who did.

For discussion, see Baker and Brame (1972), Geis (1970, 1973), Jayaseelan (1983), and Lakoff (1970b).

Let us turn now to briefly review the path we have taken. In Section 1, we surveyed various types of construction for expressing comparison. Many of these – particularly the paratactic forms and the "exceed" type – were **semantically transparent**, in the sense of Seuren (1984: 120); that is, the languages in question "make use of existing means to express what the comparative expresses in languages that have a special category for it". It also seems safe to say that the delineation family of approaches provides a plausible explanation of how the "existing means" of gradable adjectives, conjunction and negation give rise to a comparative ordering of the appropriate kind. Seuren (1984: 121–123) goes on to argue that the European *than*-comparatives derive historically from semantically more transparent constructions, and suggests that they involve a lexical encoding of logical structures of the kind that we saw in Section 3.3. In response, von Stechow (1984b) has argued that whatever the merits of such 'semantic archaeology', the empirical predictions of the delineation approach are simply incorrect when it comes to a detailed analysis of the logical form of English. The gist of Section 4 was an endorsement of von Stechow's conclusion.

Nevertheless, there are two, interlinked, questions which should be addressed at this stage in our research, before simply accepting von Stechow's theory as 'the right one'. First, is it really necessary to build so much mathematical structure into our models? As we saw in Section 2, treating degrees as real numbers leads us to make some uncomfortably strong assumptions about comparison and concatenation over our universe of discourse; and this holds not only with respect to gradable predicates like *tall*, but also *skilful*, *wise* and *generous*. We noted that there is also an empirical problem with this approach, in that there seems to be no way of accommodating incommensurability. Second, is it possible to explain how the more highly grammaticised constructions, replete with differential measure phrases, can be constructed out of the transparent constructions? Presumably the linguistic complexity of comparatives partially reflects the complexity of measurement devices, both conceptual and technological, that the linguistic community has at its disposal. A good theory should be able to show how both kinds of complexity are incrementally built up from our basic ability to draw comparisons.

good question!

Sounds like a good question to pose in LOW.

6. Short Bibliography

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