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NEGATIVE POLARITY AND THE COMPARATIVE*

0. Introduction

In this paper, I will argue that the two comparative constructions exhibited in (1) and (2) below, differ not only syntactically, but also semantically.

(1) Moscow is older than Washington.
(2) The Sahara was hotter than I had expected it would be.

Let us call the construction in (1), where the comparative particle *than* is followed by a noun phrase, the NP-COMPARATIVE, and the construction in (2), where *than* is followed by a clausal complement, the S-COMPARATIVE. The NP-comparative is also known as the prepositional construction, since Hankamer (1973) has argued that the *than* in sentences like (1) is a preposition, whereas it is a complementizer in the clausal construction.

After reviewing some of the evidence presented in the literature for making this distinction (section 1), I will present new semantic evidence from the distribution of the Dutch negative polarity item **ook maar** (for which there does not seem to be a good English gloss, though it is related to modifiers like *at all* and *whatsoever*). The argument is based on the fact that **ook maar** may occur in the S-construction, but not in the NP-construction. Following Ladusaw (1979) and Zwarts (1981), I will assume that the distribution of negative polarity items is a semantic matter, to be accounted for in a precise formal semantics of the kind employed in Montague grammar. For arguments against syntactic approaches to the distribution of negative polarity items, such as that of Klima (1964), I refer the reader to Ladusaw (1979). After providing the facts about **ook maar** in section 2, I will proceed to give a semantics for the two comparative constructions, which explains the polarity facts and is motivated independently by judgments about the validity of inferences involving these constructions (sections 3 and 4).

I will not take a stand in the debate about the correct syntactic characterization of the S-comparative: whether it contains a gap created by *Wh*-movement, as argued by Chomsky (1977) for English and Den Besten (1978) for Dutch, or whether this gap is created by means of an unbounded

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deletion rule, as proposed by Bresnan (1973, 1977), or whether the S-comparative can be (and should be) handled by an enriched phrase structure grammar of the type discussed in Gazdar (1980). I believe that all these systems are compatible with what I am going to say here.

There is another limitation of scope: I will not treat adnominal comparatives, like the ones in (3) below, or adverbial comparatives, like the ones in (4).

(3) a. A better answer than this one is not to be found.
b. They know of no taller woman than we know of.

(4) a. The cheetah runs considerably faster than the lion.
b. Children can acquire a language more easily than adults.

Likewise, I will not treat comparatives without a *than*-complement, as in *Work harder*!.

These limitations are meant to make the main ideas of this paper stand out as clearly as possible.

1. Evidence for a Syntactic Distinction between NP- and S-comparatives

A sentence like (5a) below can be taken to be derived from a somewhat more elaborate construction involving the S-comparative (5b), by means of the rule of Comparative Ellipsis:

(5) a. Judy is stronger than Doris.
b. Judy is stronger than Doris is.

Invoking Comparative Ellipsis yields a considerable dividend. It enables one to explain the synonymy of (5a) and (5b). Furthermore it promises a unified account of superficially different sentences like those above. However, as Hankamer (1973) has demonstrated, although we may derive superficial NP-comparatives from underlying clausal sources by Comparative Ellipsis, there is good evidence that there is a real NP-comparative in English as well.

First of all, *Wh*-movement out of the NP-comparative is possible (though somewhat marginal), whereas it is impossible out of the S-comparative. Here are some of Hankamer's examples of movement out of NP-comparatives:

(6) a. ?You finally met somebody you're taller than.
b. ?Who does she eat faster than?

Compare these examples with the decidedly worse examples involving movement out of an S-comparative (the b-sentence is mine, the a-sentence is Hankamer's).

(7) a. *Who is John taller than—is?

b. *You finally met somebody who you are taller than I thought—was.

In Dutch, examples like the ones in (6) are ungrammatical, but this does not mean that Dutch lacks the NP-comparative. The reason that Dutch does not allow extraction from the NP-comparative is probably just that extraction from prepositional phrases is more restricted than it is in English (cf. Van Riemsdijk 1978).

Another piece of evidence[1] for distinguishing the NP-comparative and the S-comparative syntactically comes from the distribution of reflexives and reciprocals. It is possible to have a reflexive or reciprocal pronoun in the NP-comparative, whereas such pronouns are not allowed in the S-comparative, at least not when the antecedents are outside. Consider:

(8) a. No man is stronger than himself.
b. *No man is stronger than himself is.

(9) a. They cannot possibly be taller than each other.
b. *They cannot possibly be taller than each other are.

The impossibility of the b-sentences follows from the binding conditions for these expressions, as their antecedents are outside their minimal S-domains. So we have here a clear difference between S-comparatives and NP-comparatives, calling for a syntactic distinction between the two constructions.

It is possible to show that Dutch has an NP-comparative construction comparable to the one in English, for we find the same range of facts in regard to reflexive and reciprocal pronouns:

(10) a. Niemand is sterker dan zichzelf.
Nobody is stronger than himself.

b. *Niemand is sterker dan zichzelf is.
Nobody is stronger than himself is.

(11) a. Zij kunnen onmogelijk langer dan elkaar zijn.
they can not-possibly taller than each other be
They cannot possibly be taller than each other.

[1] Somewhat surprisingly, Hankamer does not mention the possibility of reflexives in the English NP-comparative, although he does give an example from classical Greek on p. 183 of his paper.

b. *Zij kunnen onmogelijk langer zijn dan elkaar zijn[2].
they can not-possibly taller be than each other are

They cannot possibly be taller than each other are.

The third and last argument to be presented for distinguishing the NP-comparative and the S-comparative in the syntax is taken from Pinkham (1982, p. 112 and p. 128–130), where it is argued that sentences like (12a) below must contain non-clausal comparatives, since a source like (12b) does not make sense, whereas a source like (12c) cannot be related to (12a) through the rule of Comparative Ellipsis, because in that case the identity condition on deletion would not be met.

(12) a. Mary ran faster than the world record.
b. Mary ran faster than the world record ran.
c. Mary ran faster than the world record is.

For Dutch, exactly the same argument can be given, so our conclusion will have to be that both Dutch and English have an NP-comparative, along with the S-comparative. (Of course it is not claimed that these are the only comparative constructions in either Dutch or English – in fact Pinkham (1982) argues for another phrasal comparative in constructions like *a more tasty than elegant dinner* and *more men than women.*)

2. The Distribution of Dutch **ook maar**

Now that we have seen some syntactic differences between NP-comparatives and S-comparatives, we may wonder whether these differences are reflected in the semantic properties of the two constructions. One indication that there is indeed a semantic difference between the two comparative constructions comes from the distribution of the Dutch negative polarity item **ook maar**, which is literally 'also but' or 'also only'. It is an idiomatic combination which can be prefixed to certain indefinite noun phrases, like **iemand** 'somebody, anybody', or **iets** 'something, anything'. It can be approximated in English by adding *at all*, *what(so)ever*, or *who(so)ever* to the English translation of the indefinite noun phrase. Where possible the English equivalents of the Dutch examples with **ook maar** are appropriately modified. Those readers who are familiar with German may compare it with the quite similar expression **auch nur**, which has the same properties, it seems, as **ook maar**. For a detailed discussion of the constructions **ook maar** may occur in, the reader is referred to Paardekooper (1979).

[2] The two **zijn**'s in this sentence are homophones: the first is the infinitival form ('be'), the second is the plural finite form ('are').

It turns out that **ook maar** is perfectly acceptable when embedded in an S-comparative, as in the following examples:

(13) a. Het feest duurde langer dan ook maar
the party lasted longer than at-all

iemand verwacht had.
anybody expected had

The party lasted longer than anybody at all had expected.

b. Ik wacht liever een uur, dan ook maar
I wait rather an hour, than even

een minuut te laat te zijn.
one minute too late to be

I would rather wait for an hour, than be even a minute late.

c. Wim was minder vervelend, dan ook maar iemand
W. was less obnoxious, than at-all anyone

voor hem was geweest.
before him (had) been

Wim was less obnoxious than anyone at all before him had been.

On the other hand, when occurring in an NP-comparative construction, **ook maar** causes ungrammaticality, as is illustrated by the following examples:

(14) a. *Wim is gevaarlijker dan ook maar iemand.
W. is more-dangerous than whosoever anyone.

Wim is more dangerous than anybody whosoever.

b. *Deze trein gaat sneller dan ook maar een auto.
this train goes faster than whatsoever a car

This train goes faster than any car whatsoever.

c. *Mijn tante is slimmer dan ook maar een speurder.
My aunt is smarter than whosoever a detective

My aunt is smarter than any detective whosoever.

Instead of the **ook maar** phrases, a related construction involving Wh-pronouns and the particle **ook** 'also, too' may be used in these cases[3]. The following sentences are the correct way to express the contents of the starred

[3] See Paardekooper (1978) for the discussion of this construction.

examples in (14):

(15) a. Wim is gevaarlijker dan wie ook.
W. is more-dangerous than who ever
Wim is more dangerous than anybody.

b. Deze trein gaat sneller dan welke auto dan ook.
this train goes faster than which car so-ever
This train goes faster than any car.

c. Mijn tante is slimmer dan welke speurder dan ook.
My aunt is smarter than which detective so-ever
My aunt is smarter than any detective.

The difference between the examples in (13) on the one hand, and the examples in (14) on the other is immediately explained by the semantic analyses of the two comparative constructions, which will be given in the next section. There it will be shown that the S-comparative is a negative polarity environment and that the NP-comparative is not. For English this does not, at first blush, appear to be quite right, since we find the polarity item *any* in both the NP-comparative and the S-comparative (witness the English translations of the Dutch examples given above). Does this mean that we must postulate a semantic difference between Dutch comparatives and English comparatives? If it did, that would surely be a rather unpleasant surprise, considering the striking similarities between Dutch and English comparatives. By far the more likely assumption is that the comparative constructions in these languages are entirely comparable.[4]

Now suppose that this is in fact the case and that the NP-comparative is not a negative polarity environment in English either. We would then have to assume that the occurrences of *any* in NP-comparatives are never occurrences of the negative polarity item *any*, but rather occurrences of so-called 'free choice' *any* (cf. Ladusaw 1979, where this distinction is argued for in chapter 4). This is the *any* we find in sentences like:

(16) a. Anybody could solve that problem.
b. I believe any answer will do.
c. I'll do anything you want me to.

[4] Pinkham (1982) also stresses this point in connection with the comparative in English and French. She argues for a syntactic theory which minimizes the differences between the two languages with respect to the comparative.

As pointed out by Horn (1972), free choice *any*, like other universal determiners, may be modified by adverbs like *almost* or *nearly*:[5]

(17) a. Almost all dogs like meat.
b. Almost any dog can bark.

Compare this with the impossibility of *almost* in front of existential determiners:[6]

(18) a. *Almost some boys were swimming.
b. *Almost a dog was barking.

If negative polarity *any* is taken to be an existential determiner, the counterpart to *some* in negative polarity environments, and not a universal determiner, as some have argued, then it is expected that it cannot be modified by *almost* or *nearly*. It appears that this is in fact the case. Consider the following examples:

(19) a. They didn't talk to (*almost) anyone.
b. The police doubt that (*nearly) anyone is in the bank safe.

These examples are fine without the offending adverbs *nearly* and *almost*. Now we have a means to check whether the *any* found in NP-comparatives is indeed free choice *any*, simply by prefixing *almost* or *nearly* and seeing whether the result is grammatical:

(20) a. This girl is smarter than almost any boy.
b. One diamond is more valuable than almost any number of bricks.
c. This movie is more important than nearly anything by Antonioni.

Since these examples are acceptable, we may conclude that the evidence from **ook maar** is not incompatible with the behaviour of *any*. So I will from now on assume that both Dutch and English have an NP-comparative that is

[5] McCawley (1977) notes that in imperatives free choice *any* does not allow modification by *almost*, cf.:

(i) *Take almost any of the apples.

McCawley goes on to argue that this is the result of a violation of the Gricean Cooperation Principle, for "if there are restrictions to your generosity, you must indicate what they are in order to make clear what you are offering".

[6] Sentences like the one below are not counterexamples to this claim, if we sharpen it a little, and require that *almost* modify the determiner:

(i) Almost a year went by.

It should be clear that *almost* does not modify the determiner *a* in this example, but rather the whole phrase *a year*.

not a negative polarity environment.[7] The main motivation, however, comes from Dutch, since **ook maar** behaves straight-forwardly. (Cf. (13) and (14).)

3. A Semantics for NP- and S-Comparatives

3.1. *Grading Relations*

For the semantics of both the NP-comparative construction and the S-comparative construction, a primitive grading relation will be used, which is assumed to be associated with each gradable adjective. For instance, if the adjective is *tall*, the grading relation associated with it tells us how the domain of discussion is ordered with respect to tallness. We will write this relation as $>_{tall}$. When no adjective is specified, we will simply use $>$.

At this point it should be mentioned that grading relations need not be primitive, but may be derived from the interpretation of the positive form of the adjective when a suitable theory of contextual interpretation is assumed for positive adjectives (cf. Klein 1980). I will not address this issue here, although it is clearly of interest for a general theory of the interpretation of adjectives.

Let me now state some desiderata as to the logical properties of the grading relations. The first property we need is *almost-connectedness*, which is defined as below:

DEFINITION 1.

$$\forall xyz[x > y \rightarrow (x > z \vee z > y)]$$

The motivation for this property is intuitively clear. Let $x =$ John, $y =$ Bill and $z =$ Mary. Suppose John is taller than Bill. Then, as in the definition, $x > y$. How might Mary relate in height to John and Bill? She might be (1) taller than both, (2) as tall as John, (3) not as tall as John, but taller than Bill, (4) as tall as Bill, or (5) shorter than both. In the first case $z > y$ and

[7] This does not mean, of course, that I am excluding the possibility of free choice *any* in S-comparatives. In fact, there is evidence that such occurrences exist. Consider a sentence like:

(i) John is taller than almost any of his friends think he is.

However, it is a reasonable assumption, that negative polarity *any* may also occur in the S-comparative, given the possibility of other negative polarity items in that construction – e.g. in Seuren's (1973: 534) examples:

(ii) The fifth glass was more than I *cared to* drink.
(iii) The sound of her voice was more than I *could stand.*

(The polarity items in question are italicised.) For the NP-comparative no such support exists for the possibility of negative polarity items, as far as I am aware.

$z > x$, but in the second and third cases, although $z > y$ still holds, $z > x$ does not (since Mary is not taller than John). If, as in case (4), Mary is as tall as Bill, then, given $x > y$, it follows that $x > z$. And of course, if Mary is even shorter than Bill, as in the last case, then $x > z$ still holds. So given $x > y$, the conclusion that $x > z$ or $z > y$, is warranted. The stronger requirement of connectedness would be too strong, for in that case we would have for any x, y: either $x > y$ or $y > x$, which is of course not correct, since x and y may possess the property in question to equal degrees.

The only other property of grading relations that we need to state is *asymmetry*:

DEFINITION 2.

$$\forall xy[x > y \rightarrow \neg(y > x)]$$

This requirement is even more straightforward than the previous one: of course, if John is taller than Bill, Bill is not taller than John.

These two requirements suffice for the logic of grading relations. Other properties of comparatives, like transitivity, can be shown to follow from them.

3.2. *The NP-comparative*

In this subsection, the semantics for the NP-comparative construction will be presented. In its definition, the grading relations postulated in the previous subsection will be employed, together with some notions from Barwise and Cooper's (1981) study of quantification in natural language, and, of course, from standard formal semantics.

In the Barwise and Cooper paper, quantifiers are the meanings of noun phrases. Quantifiers are modeled formally as sets of predicates, where a predicate in turn is simply a set, as is standard in Montague semantics. So the predicate *lion* is taken to be the set of lions in the domain of discussion. The quantifier associated with the noun phrase *all lions* is the set of all sets that contain all lions. For example, the set of all beasts of prey is a member of that quantifier, whereas the set of all animals living in a zoo is not, for the latter set does not contain all lions. Likewise, the meaning of the noun phrase *a lion* is the quantifier consisting of all sets containing a lion.

A sentence is just a subject-predicate combination. It is true iff the predicate, which, as we have seen, is a set, is a member of the interpretation of the subject. In other words: iff the predicate set is a member of the quantifier (i.e. the collection of sets), associated with the subject NP. Therefore the sentence *All lions are beasts of prey* is true, given that the set of beasts of prey is a member of the quantifier associated with *all lions*.

The quantifier with a noun phrase like *Jim* is the set of all sets containing Jim. It seems more natural to directly associate the name *Jim* with the person Jim, and not with the more elaborate set-theoretic construction, but the set-theoretic construction has the benefit of being more in line with the interpretations of other noun phrases. (A noun phrase like *a man* cannot be taken to refer to an individual in the same sense as the name *Jim* can.) At all events, the sentence *Jim walks* is true if and only if the set of people who walk is a member of the quantifier associated with *Jim*. This is the case if and only if the set of people walking contains Jim. So the construction does its job the way it should.

I will now introduce some notation and terminology. Let U be the domain of discussion (the 'universe'). A quantifier Q defined on U is a set of subsets of U. The denotation, or meaning, of an expression X is written as $[\![X]\!]$. For example, $[\![\text{all lions}]\!] = \{X \subseteq \mathrm{U} \mid [\![\text{lion}]\!] \subseteq X\}$. This example also illustrates my policy of using capital letters for sets, X and Y for arbitrary sets, and Q, Q_1, Q_2 for quantifiers. The expression $POW(X)$ will indicate the power set of X, that is, the set of all subsets of X. Every quantifier, being a set of subsets of U, is a subset of POW(U), the power set of the domain of discussion.

The NP-comparative *Adj-er than* is interpreted formally as a function from quantifiers to predicates, because it combines syntactically with a noun phrase to form a predicate. More precisely, it denotes a function from POW(POW(U)) into POW(U) defined according to the following definition:

DEFINITION 3.

$$[\![\mathit{Adj}\text{-er than}]\!](Q) =_{\mathrm{df}} \{x \in \mathrm{U} \mid \{y \in \mathrm{U} \mid x >_{adj} y\} \in Q\}$$

To see how this definition works, we will discuss an example. Notice first though that this definition is very different in intention from the two previous definitions. Whereas the previous ones gave precise formulations of some formal notions, this one embodies an empirical claim about the semantics of the NP-comparative. As is common practice in Montague grammar, a hypothesis is formulated as a definition. So it is important to know whether the definitions give rise to an arbitrary formal system, or whether they are meant to constitute a formal system representing a theory of natural language. Definition 3 certainly has empirical content.

Let us now consider an example. Suppose that Bill and Dave are members of U, our domain of discussion, and it turns out that Dave is taller than Bill. One thing we have to ensure now, is that Dave is in the set $[\![\text{taller than}]\!]$ $([\![\text{Bill}]\!])$. Definition 3 is designed to ensure that this is the case. Remember that *Bill* denotes the set of all subsets of the domain containing Bill, i.e.

$\{X \subseteq \mathrm{U} | \mathrm{Bill} \in X\}$. Therefore, by definition 3, $[\![$taller than$]\!]([\![$Bill$]\!]) = \{x \in \mathrm{U} | \{y \in \mathrm{U} | x >_{\mathrm{tall}} y\} \in \{X \subseteq \mathrm{U} | \mathrm{Bill} \in X\}\}$. This set is the same as the set $\{x \in \mathrm{U} | \mathrm{Bill} \in \{y \in \mathrm{U} | x >_{\mathrm{tall}} y\}\}$, which in turn is the same as $\{x \in \mathrm{U} | x >_{\mathrm{tall}} \mathrm{Bill}\}$, the set of all individuals in the domain taller than Bill. Hence, no matter who Dave is, he is in this set, given that he is taller than Bill, which is just what we want.

Definition 3 generalizes this procedure for any quantifier and any gradable adjective (non-gradable adjectives, like *wooden* or *Atlantic*, do not have comparative forms). The set corresponding to $[\![$*Adj*-er than$]\!]([\![NP]\!])$ will be the set of individuals such that the set of individuals that possess the property in question to a lesser degree, is a member of the quantifier $[\![NP]\!]$. So, to give another example, the set $[\![$bigger than$]\!]([\![$a lion$]\!])$ is the set to which some member of U belongs if and only if the set of all members of U smaller than it is a member of $[\![$a lion$]\!]$, that is, if and only if the latter set contains a lion (recall that a set is a member of the quantifier $[\![$a lion$]\!]$ iff it contains a lion). So you will be in the set $[\![$bigger than$]\!]([\![$a lion$]\!])$ iff there is a lion in the set of individuals smaller than you are, i.e. iff there is a lion smaller than you.

Let me close the discussion of the semantics for the NP-comparative as given by definition 3 by saying a little about positive adjectives. I will show how the interpretation of a comparative adjective can be derived from that of its positive counterpart. For the purposes of comparatives, we only need a grading relation. For the interpretation of positive adjectives themselves we need something in addition to that: the set of individuals to which the adjective applies truly.

We will view the interpretation of a positive adjective as a pair $(X, >)$, where X is a subset of U and $>$ some grading relation. X gives us the set of individuals that have the property in question, and $>$ gives us the ordering of the domain with respect to the extent that its members possess the property. So for *tall*, X will be the set of tall individuals, and $>$ will be the ordering of the domain with respect to tallness, i.e. $>_{\mathrm{tall}}$.

For computing the truth-value of *Mary is tall* we have to see whether Mary is in X, so we only need the left part of the pair here. However, for *Mary is (precisely) as tall as John* we need the second part: this sentence is true iff neither Mary $>_{\mathrm{tall}}$ John, nor John $>_{\mathrm{tall}}$ Mary. For the purposes of the comparative, the left part of the positive adjective is irrelevant: in order to know whether Dave is taller than Bill, we obviously do not have to know whether either Dave or Bill is tall. In fact both of them might be rather small. This is reflected in definition 3, where the grading relation is used, but not the set of individuals X.

We will return to the interpretation of positive adjectives in subsection 3.7., when a formalization of the notion *degree* is discussed.

3.4. *Boolean Properties of the NP-comparative*

In this subsection, the behaviour of NP-comparatives with respect to the Boolean operations of set intersection, union and complementation will be discussed. This discussion will show the correctness of the semantics given for the NP-comparative in the previous subsection.

In definition 3 we defined a function $[\![Adj\text{-er than}]\!]$ from POW(POW(U)) – that is, the set of all quantifiers – into POW(U), the set of all predicates. This mapping of quantifiers to predicates preserves the Boolean operations of complementation, union and intersection and for that reason it may be called a HOMOMORPHISM, i.e. a mapping preserving structure. More precisely, we say that a mapping is a homomorphism preserving Boolean structure if and only if that mapping obeys the following three conditions (where f is our name for the mapping):

(21) *Conditions for Boolean homomorphisms.*

(i) $\mathrm{f}(\mathrm{X} \cap \mathrm{Y}) = \mathrm{f}(\mathrm{X}) \cap \mathrm{f}(\mathrm{Y})$
(ii) $\mathrm{f}(\mathrm{X} \cup \mathrm{Y}) = \mathrm{f}(\mathrm{X}) \cup \mathrm{f}(\mathrm{Y})$
(iii) $\mathrm{f}(-\mathrm{X}) = -\mathrm{f}(\mathrm{X})$

In (21) (iii) $-X$ indicates complementation with respect to the universe. So if Q is any quantifier, $-Q$ is POW(U) $-Q$, and if X is a subset of U, i.e. if X is a predicate, $-X$ is $U - X$, the set of elements of U that are not in X.

If, as I have claimed above, the function $[\![Adj\text{-er than}]\!]$ defined in definition 3 is a Boolean homomorphism, then the following equations, corresponding to (i), (ii) and (iii) of (21), with $[\![Adj\text{-er than}]\!]$ substituted for f, should be valid:

(22) (i) $[\![Adj\text{-er than}]\!](Q_1 \cap Q_2) = [\![Adj\text{-er than}]\!](Q_1) \cap [\![Adj\text{-er than}]\!](Q_2)$
(ii) $[\![Adj\text{-er than}]\!](Q_1 \cup Q_2) = [\![Adj\text{-er than}]\!](Q_1) \cup [\![Adj\text{-er than}]\!](Q_2)$
(iii) $[\![Adj\text{-er than}]\!](-Q) = -[\![Adj\text{-er than}]\!](Q)$

Proof: the statements in (22) all follow in a straightforward way from definition 3. I will only show how to derive (i). As follows: $[\![Adj\text{-er than}]\!](Q_1 \cap Q_2) = \{x \in \mathrm{U} \mid \{y \in \mathrm{U} \mid x > y\} \in Q_1 \cap Q_2\}$ [by definition 3] $= \{x \in \mathrm{U} \mid \{y \in \mathrm{U} \mid x > y\} \in Q_1$ & $\{y \in \mathrm{U} \mid x > y\} \in Q_2\} = \{x \in \mathrm{U} \mid \{y \in \mathrm{U} \mid x > y\} \in Q_1\} \cap \{x \in \mathrm{U} \mid \{y \in \mathrm{U} \mid x > y\} \in Q_2 = [\![Adj\text{-er than}]\!](Q_1) \cap [\![Adj\text{-er than}]\!](Q_2)$ [again by definition 3]. □

By using the fact that intersection, union and complementation correspond with conjunction, disjunction and negation, respectively, we may now check the predictions made by definition 3 for the validity of natural language sentences involving the NP-comparative and the conjunction, disjunction and negation of noun phrases. If these predictions are correct, they are evidence for the correctness of the semantics presented for the NP-comparative. If the predictions are incorrect, the semantics is simply wrong, and definition 3 should be abandoned.

Let us first consider conjunction. We will construct English sentences reflecting (22) (i) as much as possible. Since (22) (i) is valid according to our semantics for the NP-comparative, the following sentences should be intuitively valid as well (that is, they can never be false):

(23) a. To be taller than Tom and Sam is to be taller than Tom and taller than Sam.
b. To be taller than all boys and all girls is to be taller than all boys and taller than all girls.
c. To be slower than some boy and many girls is to be slower than some boy and slower than many girls.

The reader will agree that these sentences are true, no matter the circumstances, or, in other words, that these sentences are valid.[8] So they confirm the present theory, or at least one of its theorems, (22i).

Next consider complementation. Now we have to find English counterparts to (22iii). For this, we need two noun phrases that denote each other's complement. Simply using negation for this will not do, unfortunately, as noun phrases introduced by *not* are virtually restricted to subject position (cf. the examples below):

(24) a. Not a minute was wasted.
b. *He wasted not a minute.

For our purposes, however, the pair *no man*–*any man* (where *any* is negative polarity *any*, i.e. existential) will do as well. The quantifier $[\![\text{no man}]\!]$ is the set of all subsets of U such that no man is a member, and is the complement with respect to POW(U) of the quantifier $[\![\text{any man}]\!]$, which is the set of all subsets of U containing a man. This is obvious, for if a predicate X is a

[8] There is a use of *and* (studied in Massey (1976), Hoeksema (1983)) which does not correspond to set intersection at all. This use is the collective use we find in examples like (i) below, which is equivalent to (ii):

(i) Carl is heavier than Tom and Sam.
(ii) Carl is heavier than Tom and Sam together.

On this reading, the phrase *heavier than* does not distribute over the conjuncts, since (i) is not equivalent to (iii):

(iii) Carl is heavier than Tom and heavier than Sam.

This non-equivalence is no problem for our present theory, since it only predicts such an equivalence if *and* corresponds to set intersection.

member of the first quantifier, it cannot be a member of the second one, and vice versa.

Let us now construct some appropriate examples:

(25) a. To be taller than no man is to be not taller than any man.
b. To be faster than no dog is to be not faster than any dog.

These examples may seem somewhat unnatural, but they do appear to be valid. So English sentences corresponding to case (iii) of (22) are valid. Again, this is predicted by our semantics for the NP-comparative, as formulated in definition 3.

We now have evidence that the denotation of *Adj-er than* does indeed preserve the Boolean operations of intersection and complementation. By the laws of Boolean algebra, any such mapping has to preserve unions as well.[9] Nevertheless, it is necessary to take a look at sentences involving disjunction (which corresponds to union), in order to see whether this is confirmed by the facts of English. It turns out that the facts are not as straightforward as in the cases of intersection and complementation. Consider the following counterparts to case (ii) of (22):

(26) a. To be taller than Tom or Mary is to be taller than Tom or taller than Mary.
b. To be faster than a lion or a tiger is to be faster than a lion or faster than a tiger.
c. To be slower than all birds or all creatures of the dark is to be slower than all birds or slower than all creatures of the dark.

These examples exhibit a systematic ambiguity, only one reading being consistent with case (ii) of (22). On one reading of the NP-comparative, the examples in (26) are indeed valid, but on the other they are not. Instead, we have:

(27) a. To be taller than Tom or Mary is to be taller than Tom and taller than Mary.
b. To be faster than a lion or a tiger is to be faster than a lion and faster than a tiger.
c. To be slower than all birds or all creatures of the dark is to be slower than all birds and slower than all creatures of the dark.

Examples like (27) are validated by Cresswell's (1976) semantics for the

[9] If a function f preserves intersection and complementation, then $f(X \cup Y) = f(-(-X \cap -Y))$, by one of De Morgan's laws, $= -f(-X \cap -Y) = -(f(-X) \cap f(-Y)) = f(X) \cup f(Y)$, again by De Morgan. So f preserves unions as well.

comparative, which he explicitly notes in footnote 10 of his paper. The other reading, however, on which (26) are valid, is not captured by his semantics, which generally fails to make the semantic distinction between NP- and S-comparatives. Cresswell appears to doubt the existence of this reading; at least he states that (28a) below does not mean ("at least not usually", he adds) the same as (28b):

(28) a. Bill is taller than Arabella or Clarissa.
b. Bill is taller than Arabella or Bill is taller than Clarissa.

Notice however, that it is possible to make this reading more prominent by adding something:

(29) Bill is taller than Arabella or Clarissa, but I'm not sure which one.

So we have two readings, whereas our theory of NP-comparatives can account for only one of them. (In Dutch, by the way, there is exactly the same ambiguity.) Does this mean that our semantics for the NP-comparative is incorrect?

That conclusion is, I believe, not necessary. It can be argued that the two readings are not due to the comparative construction itself, but rather to the disjunction element *or*. To see this, consider quantifiers that are semantically unions, although they correspond to noun phrases that are not disjunctions. A case in point is *some boy*, a noun phrase which does not contain a disjunction element, but which is equivalent to *John, or Bill, or*..., *Peter*, where John, Bill... Peter are the boys in the domain of discussion. To be taller than some boy is not the same, however, as being taller than John, and taller than Bill,... and taller than Peter. So it may well be that *or* is the culprit, causing the ambiguity of (26).[10] But in that case our semantics for the NP-comparative has nothing to do with it. Although the problem is not solved, it need not worry us too much.

3.5. *Necessity of the Semantics for the NP-comparative*

In this subsection I will show that the semantics for the NP-comparative as

[10] It is not entirely clear how convincing this point is. One anonymous reader has pointed out that (i) below can be read as (ii):

(i) Superman flies faster than a speeding train.
(ii) Superman flies faster than any speeding train.

I will assume here that the 'universal' reading of *a speeding train* is a matter of genericity. Whatever causes the universal reading in (iii) below, is probably responsible for that in (i).

(iii) A speeding train will frighten the cows.

formalized in definition 3 is not just *some* analysis compatible with the facts considered so far, but the *only* semantics available within the general framework adopted here. More specifically, I will show that the function $[\![Adj\text{-er than}]\!]$ defined in definition 3 is the only mapping from quantifiers to predicates preserving Boolean structure as well as the ordering relation $>$.

I will make clear what I mean by a mapping from quantifiers to predicates preserving $>$ in a moment. First recall that we have adopted the Barwise & Cooper approach to quantifiers; so quantifiers are taken to be sets of subsets of U, the domain of discussion. For every individual in U, there is a unique quantifier consisting of all the properties of that individual, i.e. of all the subsets of U it belongs to. Let a be any member of U; then Q_a will denote the quantifier $\{X \subseteq U \mid a \in X\}$. With this as background, we can provide a formal definition of what it means to preserve $>$:

DEFINITION 4.
Let f be a function mapping quantifiers into predicates. We say that f *preserves the ordering* $>$ *on* U iff $a > b$ is equivalent to $f(Q_b) \in Q_a$.

To see the motivation behind this definition, recall our treatment of names. Names can be interpreted in the Barwise and Cooper system as quantifiers, just like the other types of noun phrases. If *John* is the name of some individual a, then Q_a will be the interpretation of *John.* Now consider the sentence

(30) John is taller than Mary.

Obviously, this sentence is true if and only if John $>_{tall}$ Mary. In other words, $[\![$taller than Mary$]\!] \in [\![$John$]\!]$ if and only if John $>_{tall}$ Mary. (Recall that a sentence of the form NP VP is true in the system of Barwise and Cooper if and only if $[\![VP]\!] \in [\![NP]\!]$, that is, iff the property denoted by the VP is a member of the set of properties denoted by the NP.) Since $[\![$taller than Mary$]\!] = [\![$taller than$]\!]([\![$Mary$]\!])$, this requirement boils down to the condition '$[\![$taller than$]\!](Q_{Mary}) \in Q_{John}$', which is an instantiation of the condition in definition 4. So it is clear that $[\![Adj\text{-er than}]\!]$ should preserve $>$ in the sense of definition 4. Furthermore we have seen that this function should be a homomorphism preserving Boolean operations. Now I will show that there is only one function mapping quantifiers into predicates that has these two properties. Before doing so, I will first establish the following:

FACT 1.
Let f and g be any two functions from quantifiers into predicates, preserving $>$, then $f(Q_x) = g(Q_x)$ for any x in U.

In other words, if there were no quantifiers other than those consisting of the property set of some individual in U, f and g would be identical – i.e. there would be only one such function.

This fact follows from definition 4. Proof: suppose that $f(Q_a) \neq g(Q_a)$ for some a in U. I will show that this leads to contradiction. Recall that $f(Q_a)$ and $g(Q_a)$ are both predicates, i.e. subsets of U. If they are different, then one set should contain at least one member of U which is not in the other set. Let b be such a member of U, and $f(Q_a)$ be the set containing b. Given that f preserves $>$, and that $f(Q_a) \in Q_b$ (because every set containing b is a member of Q_b), we have, by definition 4, that $b > a$. On the other hand, we also have that $b \not> a$, because $g(Q_a) \notin Q_b$ (as $g(Q_a)$ does not contain b). A contradiction. So our initial assumption, that $f(Q_a) \neq g(Q_a)$ for some a in U is false. This proves fact 1.

Let's call those quantifiers that are the property set of some individual ATOMS. Any quantifier can be constructed out of atoms by using Boolean operations only.[11] For example, if a, b and c are the men in U, then the quantifier $[\![\text{all men}]\!]$ can be constructed out of Q_a, Q_b and Q_c by using the operation of intersection: $[\![\text{all men}]\!] = Q_a \cap Q_b \cap Q_c$. That is because every property of all men is a property of a and a property of b and a property of c. Likewise, the quantifier $[\![\text{no man}]\!]$ is constructed from the same atoms by using complementation and union: $[\![\text{no man}]\!] = -(Q_a \cup Q_b \cup Q_c)$. Again, the reason is obvious: every property of no man is not in Q_a, or in Q_b or in Q_c, so it is not in their union.

Let me now show the main claim of this subsection:

FACT 2.
Suppose f and g are arbitrary Boolean homomorphisms from quantifiers to predicates preserving $>$. Then we have that $f = g$, or, in other words, that there is only one such function.

Proof: Let f and g be as required. If f and g agree on two quantifiers Q_1 and Q_2, i.e. $f(Q_1) = f(Q_1)$ and $f(Q_2) = g(Q_2)$, then they agree on $Q_1 \cap Q_2$, on $Q_1 \cup Q_2$ and on $-Q_1$ and $-Q_2$. For example: $f(Q_1 \cup Q_2) = f(Q_1) \cup f(Q_2)$ [because f is a Boolean homomorphism] $= g(Q_1) \cup g(Q_2)$ [f and g agree on Q_1 and Q_2] $= g(Q_1 \cup Q_2)$. Therefore, if f and g agree on two quantifiers, they agree on Boolean combinations of these quantifiers as well. Since every quantifier is a Boolean combination of the atoms, and since f and g agree

[11] For a proof, see Keenan and Faltz (1978). On infinite domains of discussion, finite unions and intersections will not do. There we need infinite unions and intersections.

on all atoms, by the fact that they preserve $>$ (cf. fact 1), they must agree on all quantifiers. Therefore $f = g$. $\square$

With this proof, we complete the demonstration that $[\![Adj\text{-er than}]\!]$ as defined in definition 3 is the only function preserving both Boolean structure and the ordering relation $>$.

3.6. *The NP-comparative is Not a Negative Polarity Environment*

Now that we have a formal semantics for the NP-comparative construction, it is possible to show that this construction is not a negative polarity environment according to the theory of Ladusaw (1979). Ladusaw has argued that an element is in a negative polarity environment only if it is in the scope of an expression denoting a monotone decreasing function, where monotone decreasing is defined thus:

DEFINITION 5.
A function f is called MONOTONE DECREASING iff for all X, Y in the domain of f: if $X \subseteq Y$, then $f(X) \supseteq f(Y)$.

So a monotone decreasing function is a function sending sets to sets which reverses the subset ordering of its domain. A simple example of such a function is the complementation operation. Let A be any set, X a subset of A, and f the function such that $f(X) = A - X$. (In other words: $f(X)$ is the complement of X with respect to A.) Now let Y be any superset of X which is a subset of A (i.e. $X \subseteq Y \subseteq A$). It should be clear that $A - Y$ is a subset of $A - X$, because every member of $A - Y$ is in A, but not in Y, and therefore not in X, which in turn implies that it is in $A - X$. So we have that $f(Y) \subseteq f(X)$, whenever f is complementation with respect to some set and $X \subseteq Y$. Therefore, by definition 5, complementation with respect to some set is monotone decreasing.

Given Ladusaw's theory, mentioned earlier, that negative polarity items occur only in the scope of monotone decreasing functions, this explains why *not*, which is naturally interpreted as complementation with respect to a set, creates a negative polarity environment. Hence, for example, it is expected that negative polarity *any* or *ever* is acceptable in the scopal domain of *not*, which is of course the case:

(31) Joyce did not ever eat any cookies.

(It should be noted right away that although all negative polarity items must occur in the scope of a monotone decreasing expression, it is not true that all monotone decreasing expressions can trigger all negative polarity items.

In other words, being in the scope of a monotone decreasing expression is a necessary, but not a sufficient condition for grammatical occurrences of negative polarity items. More about this will be said in section 4, with appropriate examples.)

A notion related to that of a monotone decreasing function is its opposite, a monotone increasing function:

DEFINITION 6.
A function f is called MONOTONE INCREASING iff for all X, Y in the domain of f: if $X \subseteq Y$, then $f(X) \subseteq f(Y)$.

Notice that whereas monotone decreasing functions reverse the subset ordering on their domain, monotone increasing functions preserve it.

We will now show that the function $[\![Adj\text{-er than}]\!]$ defined by definition 3, is monotone increasing, and therefore does not create a negative polarity environment, according to Ladusaw's theory. In order to establish this, it is sufficient to recall that this function is a Boolean homomorphism, for then we can use the following fact:

FACT 3.
Every Boolean homomorphism is monotone increasing.

Proof: Let $A \subseteq B$, and f be any homomorphism. We then have:

(a) $A \subseteq B \Leftrightarrow A \cap B = A$
(b) $f(A) = f(A \cap B)$
(c) $f(A) = f(A) \cap f(B)$
(d) $f(A) \subseteq f(B)$ $\square$

In this proof, (a) is a simple fact of set theory, (b) follows from the assumption that f is a Boolean homomorphism, (c) follows from (b) and the same assumption, (d) is a consequence of (c) and (a) together. Hence, the NP-comparative, being a Boolean homomorphism, is monotone increasing.

Now recall the discussion of **ook maar** in section 2. There it was shown that this Dutch negative polarity item cannot occur in the NP-comparative construction. The reason for this non-occurrence is now clear: the NP-comparative is not a negative polarity environment at all, because the function $[\![Adj\text{-er than}]\!]$ is monotone increasing, not decreasing. In fact we predict that $[\![Adj\text{-er than}]\!]$ may not trigger any negative polarity item, because any trigger has to be monotone decreasing, according to the theory of Ladusaw. This seems to be borne out by the facts.

Now that we have an explanation for the lack of negative polarity items in the NP-comparative construction, it is time to consider the S-comparative.

3.7. *A Semantics for the S-comparative*

Before considering the semantics of the S-comparative, we need to take a look at some aspects of its syntax. No detailed analysis will be given here; only a few points will be high-lighted.

Many investigators have suggested that besides the many gaps created by ellipsis in the comparative clause, there is a systematic gap corresponding to a degree phrase.[12] (Degree phrases are expressions like *very*, *too*, *as*, *so* etc. which form a phrase together with an adjective; they should not be confused with *degrees*, which are semantic, not syntactic, entities, to be discussed shortly.) Consider the following sentence:

(32) Judy is taller than Grace is—.

Here we have a gap corresponding to the adjective that normally follows the copula. This gap is not a systematic gap in the sense that it can be found in all comparative clauses, however. This is shown by the following example:

(33) This table is longer than that one is—wide.

Here we find the adjective – but not a degree phrase. In fact no degree phrase may be inserted in the position of this gap:

(34) *This table is longer than that one is **very** wide.

(35) *Rover is more cowardly than Fido is **too** careful nowadays.

Example (35) is adapted from Gazdar's (1980, p. 177) example:

(36) Rover is more cowardly than Fido is careful nowadays.

So let us assume that all comparative clauses have a gap corresponding to a degree phrase. For the purposes of this paper, it does not matter how this gap is created – by *Wh*-movement, by means of a deletion rule, or by the base rules. More important is the fact that this gap is an 'unbounded-dependency'-gap, like the ones we find in *Wh*-questions, relative clauses and related constructions. It is fairly standard to treat this kind of gap as a variable on some level of logical form. In this respect, there is little difference between such diverse theories as Chomsky's Government and Binding theory and Montague grammar. For example, a sentence like (37), taken from Chomsky (1981, p. 44), is represented in LF, Chomsky's level of logical form, as (38) below.

(37) Who do you think would win?

(38) For which person x, you thought [x would win].

[12] For example, Bennis (1978).

Here the variable x is put in the position of the gap preceding *would*. Likewise representations like (40) below can be found in Montague grammar for the relative clause in example (39).[13]

(39) I found the man [you wanted—].

(40) λx[you wanted x]

Here the operator binding the variable is a lambda-operator. Formula (40) may be interpreted as: the set of individuals such that you wanted them. Again a variable corresponds to an 'unbounded dependency'-gap.

For comparative clauses we may do the same. We will let the systematic gap corresponding to a degree phrase translate by a lambda-bound variable ranging over degrees. For instance, the comparative clause in (33), *than that one is—wide*, will be represented as follows:[14]

(41) λx [that one is x wide]

This formula is to be interpreted as the set of degrees x such that x is the degree to which that one is wide. Since we may take for granted that a table may be wide to one degree only, this set will have to be a unit set. In general, comparative clauses will be interpreted as sets of degrees.

Before going on to present a formal semantics for the S-comparative, I must define what I mean by a degree. Intuitively, the notion may be clear, but a formal reconstruction of this notion is necessary in an explicit semantic theory. Remember that degrees are the values of the variables that correspond to degree phrases. Degree phrases combine with gradable adjectives to form predicates like *very big*, *far too tall*, *this much* etc. Gradable adjectives are interpreted in the present system, as the reader may recall from section 3.3, as ordered pairs $(X, >)$, and predicates are subsets X of U. Therefore degrees should be things that make predicates from ordered pairs $(X, >)$.[15] Formally:

DEFINITION 7.
A DEGREE is a function from ordered pairs $(X, >)$, where $X \subseteq \mathrm{U}$ and $>$ a grading relation on U, to subsets of U, such that
$\mathrm{d_a}(X, >) = \{x \in \mathrm{U} \mid \neg(x > \mathrm{a}) \;\&\; \neg(\mathrm{a} > x)\}$.

A few comments are in order here. Note that $\mathrm{d_a}$ is to be read as 'the degree

[13] I am leaving out all irrelevant details of Montague's intensional logic.
[14] The element *than* is not interpreted at all. I am assuming that *than* is just a meaningless marker of the comparative construction. My habit of writing ⟦*Adj*-er than⟧ should not be taken to imply anything about the proper interpretation of *than*. It is just an arbitrary convention.
[15] In Cresswell (1976), a different definition of degrees is given. Cresswell defines a degree as a pair (u, $>$), where $>$ is a grading relation, and u a member of its field.

of a'. For example, d_{John} is the function which maps the interpretation of some gradable adjective onto the set of elements of U which possess the property in question to an equal degree as John does. In other words, it maps any ordered set onto the equivalence class with respect to the ordering generated by John. The interpretation of *big* will be mapped by d_{John} onto the set of individuals that are exactly as big as John.
This definition captures the fact that anyone as big as John has the same degree of bigness. (For example: $d_{John}(X, >_{big}) = d_{Jim}(X, >_{big})$ iff $\neg$(John $>_{big}$ Jim) and $\neg$(Jim $>_{big}$ John).)

Let us now turn to the semantics of the S-comparative construction. One part of it has already been handled: the comparative clause is interpreted as a set of degrees. The other part is the interpretation of the comparative adjective. To obtain this, we will simply extend the definition of the function $[\![$*Adj*-er than$]\!]$ to sets of degrees in the following manner:[16]

DEFINITION 8.
Let D be any set of degrees. We define: $[\![Adj\text{-er than}]\!](D) =_{df} \{x \in U \mid \forall d_y \in D: d_x > d_y\}$

In this definition an ordering between degrees is used. This ordering is induced by the ordering relation associated with the adjective in the following way: $d_a > d_b$ iff $a > b$. For example: $[\![\text{bigger than}]\!](D)$ is the set $\{x \in U \mid \forall d_y \in D: d_x >_{big} d_y\}$. If D is $\{d_a, d_b\}$ then $[\![\text{bigger than}]\!](D) = \{x \in U \mid d_x >_{big} d_a$ and $d_x >_{big} d_b\} = \{x \in U \mid x >_{big} a$ and $x >_{big} b\}$.

To see how definition 8 works, together with the previous definitions, consider the following example:

(42) Susan is lovelier than anyone expected her to be.

First we interpret the comparative clause as the set of degrees d such that there is someone who expected Susan to be d lovely. Suppose that this set is $\{d_a, d_b\}$, where a and b are members of the domain of discussion. Now we apply the function $[\![\text{lovelier than}]\!]$ to this set, and we get the set $\{x \in U \mid x >_{lovely} a \;\&\; x >_{lovely} b\}$ (cf. the example below definition 8). This set is a member of the quantifier $[\![\text{Susan}]\!]$ iff Susan is a member of it, i.e. iff Susan $>_{lovely}$ a and Susan $>_{lovely}$ b. So if some people expected Susan to have the degree of loveliness of a, and some people expected her to have the degree of loveliness of b, then

[16] It does not make any difference if we consider $[\![Adj\text{-er than}]\!]$ as one function, defined differently on sets of degrees and quantifiers, or if we split it up in two different functions. The only thing in favor of the first approach is that it reflects the fact that the same form of the adjective is involved in both cases.

Susan must be more lovely than both a and b, if (42) is to be true. This, of course, is the appropriate truth-condition.

3.8. *The S-comparative is a Negative Polarity Environment*

As we have seen in section 2, the S-comparative is one of the places where we can find negative polarity items. Below, some additional English examples are given (the polarity item is italicised).

(43) a. Thanks to Star-drops, my teeth are brighter than they have *ever* been before.

b. Fido is more dangerous than *any* dog has *ever* been.

c. I cried more than I *could help.*

Now we have to check whether the fact that the S-comparative is a negative polarity environment, is indeed accounted for by the semantic definition 8. In other words, we have to show that the function $[\![Adj\text{-er than}]\!]$ is monotone decreasing for those arguments that are sets of degrees. More precisely, if D, D' are sets of degrees such that $D \subseteq D'$, we must show that $[\![Adj\text{-er than}]\!](D') \subseteq [\![Adj\text{-er than}]\!](D)$, by definition 8. This we can prove to be the case. First, by definition 8, $[\![Adj\text{-er than}]\!](D) = \{x \in U \mid \forall d_y \in D: d_x >_{adj} d_y\}$, and similarly, $[\![Adj\text{-er than}]\!](D') = \{x \in U \mid \forall d_y \in D': d_x >_{adj} d_y\}$. Of course, every x such that d_x is a higher degree than every d_y in D', is also an x such that d_x is a higher degree than every d_y in D, for D is a subset of D'. Therefore $\{x \in U \mid \forall d_y \in D': d_x >_{adj} d_y\} \subseteq \{x \in U \mid \forall d_y \in D: d_x >_{adj} d_y\}$. Finally, by using definition 8 again, we derive the conclusion we were looking for: $[\![Adj\text{-er than}]\!](D') \subseteq [\![Adj\text{-er than}]\!](D)$. □

The monotone decreasing character of $[\![Adj\text{-er than}]\!]$ in the S-comparative construction can be illustrated by certain inferences. Just as we may infer from *Nobody shouts or whispers* (which involves the monotone decreasing quantifier *nobody*), that nobody shouts, so we may infer from *Lilly is nastier than I expected or you expected* that Lilly is nastier than you expected. The semantics provided above validates this inference. This can be seen in the following way: let A be the set of degrees to which I expected Lilly to be nasty, and let B be the set of degrees to which you expected her to be nasty. Since $B \subseteq A \cup B$, and since $[\![Adj\text{-er than}]\!]$ is monotone decreasing, we have that $[\![Adj\text{-er than}]\!](A \cup B) \subseteq [\![Adj\text{-er than}]\!](B)$. So if Lilly is in the former set, she is certainly in the latter set as well.

Given Ladusaw's theory that monotone decreasing functions, and only they, may trigger negative polarity items in their scope, the comparative clause, which is in the scope of the comparative adjective, is a negative polarity environment. Ladusaw's theory is therefore confirmed by the facts about comparatives. Further motivation for definition 8 and the logical properties it assigns to the S-comparative construction will be given in section 4, where we will return to the discussion of the Dutch polarity item **ook maar**.

3.9. *Relating NP-comparatives to S-comparatives*

In this subsection, the connection between NP-comparatives and S-comparatives will be examined in more detail. In certain cases, a sentence containing an NP-comparative is equivalent to another containing an S-comparative, as is the case in the following pair:

(44) a. I am bigger than you.
b. I am bigger than you are.

Since two rather different definitions have been presented for the semantics of the two comparative constructions, it might be thought that this equivalence is not captured in the system. However, the equivalence in question follows from the present approach. Consider first (44b). The comparative clause *than you are* contains a gap corresponding to an adjective. Te gap can get a value in various ways, depending upon the syntactic system one wants to adopt. For example, if one adopts a deletion approach, the gap is interpreted as *big* by deriving the comparative clause in question from the underlying source *than you are big*. Another way to proceed is to use a copying transformation as an 'interpretative rule' of logical form (cf. Williams 1977), copying, in effect, the adjective of the main clause. (This would seem to commit one to the claim that *bigger* is always derived from *-er big*, for otherwise the comparative form would be copied; cf. Bresnan (1973) for arguments supporting this claim, and Heny (1978) for some problems with her account.) Still another way is proposed in Klein (1980), using the power of the lambda-calculus for interpreting the gap in a compositional fashion, while eschewing the use of syntactic transformations for interpretive purposes.

I will not choose a particular solution here, for the value of the gap in (44b) is in fact irrelevant for our purposes. Let A be any adjective; then there is only one degree satisfying λd_y[**you are** $d_y A$], to wit, the degree that *you* have (call this degree $\mathrm{d}_{\mathrm{you}}$). So we may equate the interpretation of the comparative clause in (44b) with the singleton set $\{\mathrm{d}_{\mathrm{you}}\}$. Now we can use definition 8, and we arrive at the following: $[\![\text{bigger than}]\!](\{\mathrm{d}_{\mathrm{you}}\}) = \{x \in \mathrm{U} \mid d_x >_{\mathrm{big}} \mathrm{d}_{\mathrm{you}}\}$. Because of the way we have defined the $>$-relation between degrees, this set is the same set as: $\{x \in \mathrm{U} \mid x >_{\mathrm{big}} \text{you}\}$.

Now consider (44a). There we have the NP-comparative. By definition 3, we have $[\![\text{bigger than}]\!]([\![\text{you}]\!]) = \{x \in \mathrm{U} \mid \{y \in \mathrm{U} \mid x >_{\mathrm{big}} y\} \in [\![\text{you}]\!]\}$. We define

$[\![\text{you}]\!]$ as: $\{X \subseteq \mathrm{U} \mid \text{you} \in X\}$. Now we can equate the interpretation of *bigger than you* with $\{x \in \mathrm{U} \mid \{y \in \mathrm{U} \mid x >_{\text{big}} y\} \in \{X \subseteq \mathrm{U} \mid \text{you} \in X\}\} = \{x \in \mathrm{U} \mid x >_{\text{big}} \text{you}\}$. So, finally, the predicates *bigger than you are* and *bigger than you* have precisely the same denotation. Since (44a) and (44b) are otherwise the same, this accounts for their equivalence.

A more systematic account of all the equivalences and non-equivalences between NP-comparatives and S-comparatives must be saved for another occasion.

4. A Strong Form of Negative Polarity

4.1. *Triggers for* **ook maar**

It was mentioned above that all triggers of negative polarity items denote monotone decreasing functions, but that not all such expressions may trigger all negative polarity items. Some of the latter put stronger requirements on their triggers. This fact, noted by Seuren (1976) and Zwarts (1981, to appear), implies that the set of negative polarity items does not form a homogeneous class. Therefore, in order to know whether an expression may trigger some negative polarity item X, we need sufficient conditions for triggers of X, to supplement the necessary condition that they denote monotone decreasing functions. For many negative polarity items, such sufficient conditions are simply not known, one reason being that a lot of descriptive work on the classification of negative polarity items still needs to be done.

To show that there are differences between individual polarity items, I will present some examples from Dutch. Consider the following contrast between **ook maar** and the negative polarity verb **hoeven** 'need':

(45) a. Niet iedere Nederlander **hoeft** te werken.
Not every Dutchman needs to work.

b. *Niet iedere Nederlander doet **ook maar** iets.
not every Dutchman does whatsoever anything
Not every Dutchman does anything whatsoever.

These examples show that the noun phrase **niet iedere Nederlander** 'not every Dutchman', which can be shown to denote a monotone decreasing function,[17] can trigger **hoeven**, but not **ook maar**. Why is this the case? In order to answer that question, we need to know what the necessary and

[17] Note that we must view quantifiers here as functions, not as collections of sets. See below, p. 429 for some discussion of this issue.

sufficient conditions are for **ook maar** to occur. Fortunately, these are known. They were first conjectured by Zwarts (1981) and can be formulated in the following succinct statement:

HYPOTHESIS
An expression A is a trigger of **ook maar** if and only if it denotes an anti-additive function.

Additive functions are functions that preserve the Boolean operation of union, so anti-additive functions are functions that map unions into their opposites, intersections.[18] More formally:

DEFINITION 9.
A function f is ANTI-ADDITIVE iff for all X, Y: $f(X \cup Y) = f(X) \cap f(Y)$.

A well-known example of an anti-additive function is the complementation operation, for $-(X \cup Y) = -(X) \cap -(Y)$, by one of De Morgan's laws. Negation, which corresponds with complementation, should therefore trigger **ook maar**. This prediction is correct:

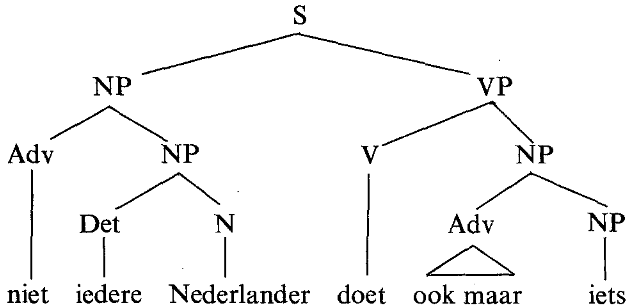
(46) Ik denk niet, dat hij ook maar iets doet.
I think not that he at-all anything does.
I don't think that he does anything at all.

Here the negation adverb **niet** 'not' triggers the occurrence of **ook maar**.

In order to explain the illformedness of example (45b), I have to show that the noun phrase **niet iedere Nederlander** is not interpreted as an anti-additive function. Furthermore I have to indicate why the adverb **niet**, which occurs in this noun phrase, may not trigger **ook maar** as it did in (46).

The latter point will be best illustrated by a tree-diagram:

(47)



[18] The term *anti-additive* is a suggestion of Zwarts. Chang & Keisler (1973: 307) use the term *additive*, but suggest that *anti-additive* would be the more natural name. Anti-additive functions are related to the notion *ideal* in Boolean algebra.

There can be no doubt about the fact that **niet** forms a constituent together with the subject, as indicated in (47), because the finite verb is always in second position in Dutch main clauses, so **niet iedere Nederlander** must be a constituent. The question that remains is whether in (47) (= (45b)) **ook maar** is in the scope of **niet**.

For the purposes of this paper, a standard definition of the notion 'scope' in terms of the structural relation *c-command* will do as an explanation for the fact that **niet** in (47) cannot trigger **ook maar**. The relation c-command (cf. Reinhart 1976) can be defined as follows: in a syntactic tree, a node A c-commands another node B if and only if every branching node dominating A dominates B as well. We will say that B is in the scope of A just in case A c-commands B. (In Klima's (1964) terminology: just in case B is in construction with A; Klima's relation *in construction with* is the inverse of *c-command*.) Ladusaw (1979) has given a semantic definition of scope, taking function-argument structure into account. Although ultimately Ladusaw's definition should perhaps be adopted, the present definition in terms of the more familiar structural relation *c-command* already suffices to show why **niet** cannot trigger **ook maar** in (47): **niet** does not c-command **ook maar**, so the latter expression is not in its scope. The complete noun phrase **niet iedere Nederlander**, on the other hand, does c-command the polarity item **ook maar**, so this might be expected to trigger it, if it could be shown to be an anti-additive function.

Now it is time to show that in fact the noun phrase **niet iedere Nederlander** does *not* denote an anti-additive function, thus accounting for the unacceptability of (45b). For this, we need a slight redefinition of our notion quantifier. Instead of saying that a quantifier is a collection of sets, we will now say that it is the characteristic function of such a collection of sets. A characteristic function is a function which 'describes' a set in the sense that it gives the value True for all arguments that are a member of the set in question, and False for all other arguments. So a quantifier is now a function from sets to truth-values, which assigns True to those sets which were a member of the quantifier under the previous definition of quantifiers, and False to all other sets. For example, the noun phrase *every farmer* denoted on the previous account the quantifier consisting of all sets containing every farmer. On the present account, this noun phrase denotes the function from subsets of U, the universe of discourse, to the truth-values, such that it assigns True to any set containing every farmer, and False to any other set.

Now we can say that a sentence $NP\ VP$ is true iff $[\![NP]\!]([\![VP]\!]) = \text{True}$. (This is of course equivalent to the condition that $[\![VP]\!] \in [\![NP]\!]$ under the previous definition of quantifiers.) A quantifier $[\![NP]\!]$ is anti-additive iff $[\![NP]\!]([\![VP_1]\!] \cup [\![VP_2]\!]) = [\![NP]\!]([\![VP_1]\!]) \cap [\![NP]\!](VP_2]\!])$.

By using again the correspondence between union and intersection on the one hand, and conjunction and disjunction, respectively, on the other hand, we may translate this into the following biconditional:

(48) $NP\ VP_1$ or $VP_2 \leftrightarrow NP\ VP_1$ and $NP\ VP_2$

Following Zwarts (1981), I will use (48) as a test for checking which noun phrases have the properties of anti-additive functions. If an NP denotes an anti-additive quantifier, then its substitution in schema (48) should make for a valid biconditional. If an NP does not denote an anti-additive function, then the resulting biconditional will be invalid. The difference in validity between (49) and (50) below clearly shows that *nobody* is interpreted as an anti-additive quantifier, whereas *everybody* is not:

(49) Nobody walks or talks $\leftrightarrow$ Nobody walks and nobody talks
(50) Everybody eats or drinks $\not\leftrightarrow$ Everybody eats and everybody drinks

Now consider **niet iedere Nederlander**, or its English translation *not every Dutchman*. We will see that the biconditional is invalid (both in Dutch and English), although it is valid in one direction, from left to right:

(51) a. Niet iedere Nederlander wandelt of praat. $\overset{?}{\leftrightarrow}$ Niet iedere Nederlander wandelt en niet iedere Nederlander praat.
b. Not every Dutchman walks or talks. $\overset{?}{\leftrightarrow}$ Not every Dutchman walks and not every Dutchman talks.

To see that (51a), or its English translation (51b), is valid only one way, consider a domain of discourse in which there are only two Dutchmen, Jan and Piet. Suppose that Jan is walking, but silent, while Piet is sitting and talking. In this setting it is certainly true that not every Dutchman is walking (since Piet is not walking), and that not every Dutchman is talking (since Jan is silent). So the second part of the biconditional is true. However, the first part is not, since in fact every Dutchman is either walking or talking. So the biconditional is invalid from right to left. Consequently the noun phrase *not every Dutchman*, or **niet iedere Nederlander**, is not interpreted as an anti-additive function. **Niet iedere Nederlander** is therefore predicted by Zwarts' hypothesis to be unable to trigger **ook maar**. Example (45b) shows that this prediction is correct. We see that Zwarts' hypothesis concerning **ook maar** can handle the ungrammaticality of example (45b) in a straightforward way.

At this point, one might wonder whether there are possible areas of conflict between Zwarts' hypothesis concerning the triggers for **ook maar**, and Ladusaw's conjecture that all triggers of negative polarity items denote monotone decreasing functions. Suppose that some expression denotes an anti-additive function which is not monotone decreasing. In a case like that, Zwarts' hypothesis predicts that it may trigger **ook maar**, whereas Ladusaw's predicts the opposite. Such conflicts, however, are ruled out by the following fact:

FACT 4.
All anti-additive functions are monotone decreasing.

Proof. Suppose f is an arbitrary anti-additive function, and A and B two sets in its domain such that $A \subseteq B$. We only have to show that $f(B)$ is a subset of $f(A)$ in order to establish that f is monotone decreasing (cf. definition 5). By anti-additivity of f, we have $f(A) \cap f(B) = f(A \cup B)$, which, in turn, is equal to $f(B)$, because $A \cup B = B$ by the fact that $A \subseteq B$. But if $f(A) \cap f(B) = f(B)$, then $f(B) \subseteq f(A)$. □

4.2. *Why* **ook maar** *May Occur in the S-comparative*

In section 2 some examples were presented showing that **ook maar** may occur in the S-comparative. In order to account for the possibility of such occurrences, we need to establish that the function $[\![Adj\text{-er than}]\!]$, as defined in definition 8 on sets of degrees, is an anti-additive function, given the hypothesis that only these may trigger **ook maar**.

FACT 5.
On sets of degrees, the function $[\![Adj\text{-er than}]\!]$ is anti-additive.

Proof. Let D and D' be any two sets of degrees. By definition 8, $[\![Adj\text{-er than}]\!](D) = \{x \in U \mid \forall d_y \in D : d_x >_{adj} d_y\}$ and, similarly, $[\![Adj\text{-er than}]\!](D') = \{x \in U \mid \forall d_y \in D' : d_x >_{adj} d_y\}$. So $[\![Adj\text{-er than}]\!](D) \cap [\![Adj\text{-er than}]\!](D') = \{x \in U \mid \forall d_y \in D[d_x >_{adj} d_y] \;\&\; \forall d_y \in D'[d_x >_{adj} d_y]\} = \{x \in U \mid \forall d_y \in D \cup D' : d_x >_{adj} d_y\}$, which, by definition 8, is equal to $[\![Adj\text{-er than}]\!](D \cup D')$. Therefore, by the definition of anti-additivity, $[\![Adj\text{-er than}]\!]$ is anti-additive on sets of degrees. □

Besides this direct proof, there is also an indirect way of showing the anti-additivity of the comparative adjective in the S-comparative construction (cf. our discussion of (48) above). Let C and C' be symbols standing for comparative clauses; then the following biconditional should be valid:

(52) *NP* is *Adj*-er than C and *Adj*-er than $C' \leftrightarrow$ *NP* is *Adj*-er than C or C'

The following instantiations of this schema appear to be valid, thus showing that fact 5 is not only a theorem of our semantics for the S-comparative, but also linguistically motivated.

(53) a. She is younger than Dave assumed and younger than Sam expected. $\leftrightarrow$ She is younger than Dave assumed or Sam expected.

b. Arnold is stronger than we are and stronger than they are. $\leftrightarrow$ Arnold is stronger than we are or they are.

c. Some butchers are more cruel than they need to be and more cruel than the regulations allow. $\leftrightarrow$ Some butchers are more cruel than they need to be or the regulations allow.[19]

Now we have two independent reasons why fact 5 is important: it explains the distribution of **ook maar**, and the validity of the equivalences in (53), both of which follow directly from it. Since definition 8 guarantees anti-additivity, we have rather strong evidence for the correctness of this definition.

5. Summary and Conclusions

In this paper I have given a rather simple and, I believe, elegant semantics for the comparative,[20] which explains the distribution of certain negative polarity items and correctly predicts the validity of inferences concerning comparatives. It was argued that a distinction has to be made between NP-comparatives and S-comparatives, in order to capture the very different semantic properties of these two constructions. In this respect the present paper differs from previous treatments of the semantics of the comparative, such as Cresswell (1976) and Klein (1980). According to the theory presented above, the comparative adjective denotes a Boolean homomorphism in the NP-comparative construction (just like all extensional transitive verbs), but it denotes an anti-additive function in the S-construction.

The semantics for the S-comparative shares some basic features with the semantic analysis of the comparative in Klein (1980). It differs, however, in that grading relations have been introduced here as primitive notions,

[19] There is a problem concerning conjunction here. As one of the anonymous readers noted, we may have conjunction instead of disjunction, without apparent change of meaning. Thus, instead of (i) below, we may have (ii):

(i) Diana is more attractive than Margaret is or Anne is.

(ii) Diana is more attractive than Margaret is and Anne is.

In Dutch the same obtains. In the present semantics, only the first sentence makes sense. Similar cases, where conjunction does not express intersection, but rather union, can be found in examples like *Every boy and girl is merry*, which means: everyone who is either a boy or a girl, is merry, and not: everyone who is both a boy and a girl, is happy.

For a general discussion of such cases, and a solution to the problem, see Partee & Rooth (forthcoming).

[20] No account has been given of NP-comparatives involving measure noun phrases as in *Susan is taller than five foot*. See Klein (1980: 27ff.) for an excellent discussion.

whereas Klein proposed to explain them away by a suitable theory of contextual interpretation of positive adjective. By introducing grading relations as primitive notions, I have been able to simplify the semantics for the S-comparative to a considerable degree. However, a Klein-style theory of context could be placed on top of the present theory, if one wanted to do so. The present paper also differs from the contributions by Cresswell and Klein in that it does not provide an explicit syntax to complement the semantics. I have refrained from giving a fragment in the style of Montague-grammar, because I felt that I did not have anything substantial to contribute there.

The semantics for the S-comparative provided here can account for the possibility of the Dutch negative polarity item **ook maar** occurring in the S-comparative (though not in the NP-comparative). There were two reasons for focussing on **ook maar**: first, it calls our attention to an important logical property of the S-comparative, viz. its anti-additivity, and, secondly, it points out an area where Ladusaw's theory of negative polarity needs further elaboration, to wit, the differences in distribution that are found among the various negative polarity items, as illustrated here by the Dutch examples **ook maar** and **hoeven** (cf. (45) above). Still a lot of work, even on the basic level of description, has to be done in this area, but the progress made so far by Ladusaw (1979) and Zwarts (1981, to appear) at least warrants the expectation that this work will not be without its rewards. These works also show that the model-theoretic approach to semantics is especially well-suited to giving an explanatory account of polarity phenomena. I hope that the present paper may help to convince those who still hold that formal semantics is irrelevant for linguistic purposes.

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