The Composition of Complex Cardinals

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Abstract

This paper proposes an analysis of the syntax and semantics of complex cardinal numerals, which involve multiplication (two hundred) and/or addition (twenty-three). It is proposed that simplex cardinals have the semantic type of modifiers ($\langle e, t \rangle, \langle e, t \rangle$). Complex cardinals are composed linguistically, using standard syntax (complementation, coordination) and standard principles of semantic composition. This analysis is supported by syntactic evidence (such as Case assignment) and semantic evidence (such as internal composition of complex cardinals). We present several alternative syntactic analyses of cardinals, and suggest that different languages may use different means to construct complex cardinals even though their lexical semantics remains the same. Further issues in the syntax of numerals (modified numerals and counting) are discussed and shown to be compatible with the proposed analysis of complex cardinals. Extra-linguistic constraints on the composition of complex cardinals are discussed and compared to similar restrictions in other domains.

1 INTRODUCTION

The goal of this paper is to propose an account of the cross-linguistic syntax and semantics of complex cardinals. While there has been much work examining the syntax and semantics of simplex cardinals such as *three*, complex cardinals, which involve multiplication (1) and/or addition (2), have not previously received much attention.

(1) a. five hundred thousand
b. quatre vingt
   four twenty
   ‘eighty’ (French)

(2) a. three hundred and five
b. twenty-seven
c. tri ar ddeg
   three on ten
   ‘thirteen’ (Welsh)
d. sto sem’
hundred seven
‘a hundred and seven’ (Russian)

We will argue that complex cardinals are composed entirely in syntax and interpreted by the regular rules of semantic composition (i.e., construction of complex cardinals is done exclusively by linguistic means). This analysis is independently motivated by syntactic transparency of complex cardinals and their compositional semantics.

The paper is organized as follows. In section 2 we argue that simplex cardinals have the semantic type of modifiers (⟨e, t⟩, ⟨e, t⟩) and show how this accounts for the internal composition of complex cardinals via iterative syntactic complementation. Existential force of cardinal-containing extended NPs (xNPs)¹ in argument positions is also treated in this section. Section 3 addresses the semantic atomicity requirement imposed by cardinals on their complements, and shows that such morphosyntactic operations as Case-assignment and number marking in cardinal-containing xNPs provide evidence that complex cardinals are built in the syntax; this section also discusses extra-linguistic factors in the composition of complex cardinals. Section 4 presents an analysis of complex cardinals like twenty-two in terms of coordination and discusses some ordering constraints. Section 5 concludes the paper and poses some questions for further research. The Appendices address some further issues in the syntax and semantics of cardinals.

Since our goal is to provide an analysis that works for complex cardinals cross-linguistically, we draw upon data from a variety of (typologically different) languages. While some empirical phenomena (e.g. articles, morphological Case assignment, etc.) are visible only in a subset of languages, we will extend the analysis based on these phenomena to other languages, unless there are empirical reasons for not doing so. One caveat is in order: we focus primarily on non-classifier languages in this paper; however, we show in section 3 that our analysis can be logically extended to classifier languages as well. Due to lack of space, we concentrate here on the semantics of complex cardinals, and discuss their syntax in a relatively superficial manner. More discussion of the thornier issues arising in our analysis can be found in Ionin & Matushansky (in preparation).

¹ We use the term xNP rather than NP or DP to indicate that it is irrelevant which functional layers are projected and which aren’t.
2 SEMANTICS OF CARDINALS

This section is dedicated to the semantics of complex cardinals involving multiplication. We ask how complex cardinals such as three hundred, four hundred thousand, etc., are composed semantically (on cardinals involving addition, such as forty-two, see section 4).

The background assumption we start with is that the semantics of cardinals is the same cross-linguistically, at least in languages that have complex cardinals. We follow the natural hypothesis that complex cardinals are derived from simplex ones: that four hundred should be semantically related to four as well as hundred.

(3) a. four hundred books
    b. four books

Furthermore, we strive to capture the basic intuition that the four in (3a) is semantically the same as the four in (3b). The meaning of a complex cardinal should be derived in such a way that each cardinal inside it is also semantically compatible with a lexical xNP: the same four should be able to combine with books as easily as with hundred books.

2.1 Semantic type of cardinals: cardinals are modifiers

The above intuition is captured straightforwardly if simplex cardinals have the semantic type of modifiers \(\langle\langle e, t\rangle, \langle e, t\rangle\rangle\). Although the proposal that cardinals are modifiers has been much discussed in the literature (see Link 1987; Verkuyl 1993; Carpenter 1995; Landman 2003, among others), no distinction has previously been made between simplex and complex cardinals. We propose that simplex cardinals are of type \(\langle\langle e, t\rangle, \langle e, t\rangle\rangle\), and derive the meaning of xNPs containing complex cardinals compositionally.

In order to derive the meaning of complex cardinals, we need full recursivity, which we derive from the semantic type \(\langle\langle e, t\rangle, \langle e, t\rangle\rangle\), as illustrated in the structure in (4), where the lexical xNP is the sister

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2 The semantics that we propose for simplex cardinals is necessary only for languages that have complex cardinals. The main motivation for the semantic type \(\langle\langle e, t\rangle, \langle e, t\rangle\rangle\) (see below) is the compositional semantics of complex cardinals; if a language has only simplex cardinals, they can be type \(\langle e, t\rangle\) and combine with the lexical xNP via Predicate Modification—see section 2.2.2. It is quite likely that cardinals historically developed from type \(\langle e, t\rangle\) (see also Hurford 2001); in some languages simplex cardinals originate as predicates synchronically as well, and are converted to the modifier type (see Ionin and Matushansky (in preparation)). Since the issue is orthogonal to our concerns, we will not address it here.

3 One option that we do not discuss here, in view of total lack of morpho-syntactic evidence for such a hypothesis, is that not all simplex cardinals inside a complex one have the same semantic type (i.e. that four is \(\langle\langle e, t\rangle, \langle e, t\rangle\rangle\) in (3a) and \(\langle e, t\rangle\) in (3b)).
of the innermost cardinal. (5) is a sample lexical entry for simplex cardinals.

\[
\begin{align*}
(4) & \\
\langle e, t \rangle & \\
\langle et, et \rangle & \text{two} \\
\langle e, t \rangle & \text{hundred} \\
\langle e, t \rangle & \text{books}
\end{align*}
\]

(5) \( [2] = \lambda P \in D_{\langle e, t \rangle} . \lambda x \in D_e . \exists S \in D_{\langle e, t \rangle} [\Pi(S)(x) \land |S| = 2 \land \forall s \in S P(s)] \)

S is a partition \( \Pi \) of an entity \( x \) if it is a cover of \( x \) and its cells do not overlap (cf. Higginbotham 1981: 110; Gillon 1984; Verkuyl & van der Does 1991; Schwarzschild 1994):

(6) \( \Pi(S)(x) = 1 \) iff

- S is a cover of \( x \), and
- \( \forall z, y \in S [z = y \lor \exists a [a \leq i z \land a \leq i y]] \) (Forbidding that cells of the partition overlap ensures that no element is counted twice.)

(7) A set of individuals \( C \) is a cover of a plural individual \( X \) iff

- \( X \) is the sum of all members of \( C \): \( \sqcup C = X \)

The most important part of (6) is the notion of partition (see Higginbotham 1981: 110 and Verkuyl & van der Does 1991): a cover of a plural individual \( X \) into the corresponding number (in the case of (5), two) of possibly plural individuals such that they do not share any parts (i.e., there is no overlap).\(^4\) This means that an xNP such as hundred books has the extension in (8a), stated informally in (8b).\(^5\)

(8) a. \( [[\text{hundred books}]] = \lambda x \in D_e . \exists S [\Pi(S)(x) \land |S| = 100 \land \forall s \in S [[\text{book}](s)]] \)

\(^4\) Defined in terms of sets (cf. Gillon 1984; Schwarzschild 1994), a family of sets \( C \) is a cover of the set \( X \) iff

- (i) \( C \) is a set of subsets of \( X \)
- (ii) Every member of \( X \) belongs to some set in \( C \)
- (iii) \( \emptyset \) is not in \( C \)

The first two conditions amount to claiming that \( X \) is the union of all members of \( C (\sqcup C = X) \). The last condition is superfluous for the definition in terms of (plural) individuals, since we do not assume empty individuals in the domain.

\(^5\) See section 3 for a discussion of the semantic and morphological plurality of the lexical xNP here.
b. $\lambda x \in D_e . x$ is a plural individual divisible into 100 non-overlapping individuals $p_i$ such that their sum is $x$ and each $p_i$ is a book

(8a) shows that *hundred books*, being of type $\langle e, t \rangle$, can be a sister of a cardinal, such as *two*, which has the denotation in (5) and the type $\langle \langle e, t \rangle, \langle e, t \rangle \rangle$. We have therefore achieved full compositionality—and this gives us the denotation for *two hundred books* in (9a), with its informal variant in (9b).

(9) a. $\left[\text{two hundred books}\right] = \lambda x \in D_e . \exists S \left[\Pi(S)(x) \land |S|=2 \land \forall s \in S \exists S' \left[\Pi(S')(s) \land |S'|=100 \land \forall s' \in S' \left[\text{book}\right](s')\right]\right]$

b. $\lambda x \in D_e . x$ is a plural individual divisible into 2 non-overlapping individuals $p_i$ such that their sum is $x$ and each $p_i$ is divisible into 100 non-overlapping individuals $p_k$ such their sum is $p_i$ and each $p_k$ is a book

Having the semantic type of modifiers, cardinals necessitate an argument of type $\langle e, t \rangle$. We have already shown that this can be an $xNP$ argument, as in *two books*. We predict that a cardinal can also take a PP argument, as in *two thirds of this book*. (On regular partitives, see Ladusaw 1982; Hoeksema 1984, 1996; Barker 1998; Cardinaletti & Giusti 2005; Gawron 2002 and Ionin, Matushansky and Ruys to appear, among others; see Martí Girbau, in press, on whether they contain a null NP.) We can therefore analyse fractions as fully compositional and built on the structure of regular partitives (see Ionin et al., to appear, where we extend our analysis of cardinals to cardinal, measure and fraction partitives).

### 2.2 Ruling out alternative semantic types

$xNP$-internal cardinals have also been treated as determiners (semantic type $\langle \langle e, t \rangle, \langle e, t \rangle, t \rangle$—see Bennett 1974; Scha 1981; van der Does 1992, 1993 among others) or as predicates (type $\langle e, t \rangle$—see Partee 1986). As shown below, these alternatives do not work for complex cardinals (for which they were never intended, to be fair).

#### 2.2.1 Ruling out the determiner theory of cardinals

If simplex cardinals have determiner type $\langle \langle e, t \rangle, \langle e, t \rangle, t \rangle$, then it is not possible to derive the semantics of complex cardinals, as shown by (10). If *hundred* is

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6 Note that *two/hundred* on our analysis means *exactly two/exactly hundred*, rather than *at least two/at least a hundred*, since otherwise *two hundred books* would mean, roughly, ‘at least two sets of at least a hundred books’. See Krifka (1999) for other problems with the *at least* analysis of cardinals.
combined with *books* first, as we have proposed, the resulting NP is a generalized quantifier (type \(\langle e, t \rangle, \langle e, t \rangle\)), which cannot then be combined with another cardinal of type \(\langle e, t \rangle, \langle e, t \rangle\).

\[
\begin{array}{c}
\langle \langle \rangle \rangle \\
\langle \langle e, t \rangle, \langle e, t \rangle \rangle \\
\langle e, t \rangle \\
\langle e, t \rangle
\end{array}
\]

The problem would not be solved if *hundred* combined with *two* before combining with *books*. This would mean combining two determiners of type \(\langle e, t \rangle, \langle e, t \rangle\). However, we have no semantic rules for combining two elements of type \(\langle e, t \rangle, \langle e, t \rangle\) and moreover, such combinations are independently disallowed—cf. *the every book*, *no these books*, etc.

Though the proposal that (simplex) cardinals are determiners is usually associated with a syntactic structure where they are projected as heads (Ritter 1991; Giusti 1991, 1997; Zamparelli 1995, 2002), semantically, this view is indistinguishable from the theory that they occupy the specifier of some functional projection, since in both approaches they form a unit to the exclusion of the lexical xNP (see section 3.1 for discussion).

2.2.2 *Ruling out the predicate theory of cardinals* The proposal that cardinals are predicates (type \(\langle e, t \rangle\)) faces the same problem as the proposal that cardinals are determiners: semantic composition of complex cardinals would fail, unless additional assumptions are made.

Unlike the proposal discussed in subsection 2.2.1, assuming that simplex cardinals are predicates does not lead to a type clash. The relevant interpretation rule for complex cardinals (Predicate Modification) does exist and treats the semantic composition of two predicates (type \(\langle e, t \rangle\)) as conjunction (Heim & Kratzer 1998). This results in an xNP of type \(\langle e, t \rangle\), as shown in (11).

\[
\begin{array}{c}
\langle e, t \rangle \\
\langle e, t \rangle \\
\langle e, t \rangle
\end{array}
\]
However, Predicate Modification would result in incorrect truth-conditions for complex cardinals, whatever semantics we assume for simplex cardinals. Suppose that we assume the very simple semantics in (12). The xNP *two hundred books* would then be self-contradictory, since nothing can simultaneously have the cardinality 100 (consist of 100 atoms) and the cardinality 2 (consist of 2 atoms).

(12) a. $[\text{two}] = \lambda x \in D_e . |x|=2$
   b. $[\text{hundred}] = \lambda x \in D_e . |x|=100$
   c. $[\text{two hundred books}] = \lambda x \in D_e . \text{[books]}(x) \land |x|=100 \land |x|=2$

Suppose that we assume instead the semantics in (5), modified so that cardinals have type $\langle e, t \rangle$, as shown in (13). The resulting reading for 200 books is also problematic: in order to simultaneously be divisible into 100 non-overlapping individuals and 2 non-overlapping individuals, it is sufficient for a plural individual to consist of just 100 books. This is clearly unsatisfactory.

(13) a. $[\text{two}] = \lambda x \in D_e . \exists S [\Pi(S)(x) \land |S|=2]$
   b. $[\text{hundred}] = \lambda x \in D_e . \exists S [\Pi(S)(x) \land |S|=100]$
   c. $[\text{200 books}] = \lambda x \in D_e . \exists S [\Pi(S)(x) \land |S|=2] \land \exists S' [\Pi(S')(x) \land |S'|=100 \land \text{[books]}(x)]$

Once again, the standard approaches to cardinals treating them as a unit to the exclusion of the lexical xNP, don’t help: the order in which *two, hundred*, and *books* combine is irrelevant, since on the view discussed here all three are predicates.

2.3 Syntax and semantics of existential quantification

If xNPs containing cardinals have the type of predicates $\langle e, t \rangle$ what happens to such xNPs in argument positions? We know that such xNPs may be part of definite or quantificational DPs, as in (14). The fact that cardinals can combine with determiners means that cardinals are not determiners, since semantic combination of two determiners is independently known to be impossible.

(14) the two birds/every two birds/those two birds

In cases like (14), the semantic composition is straightforward: the determiner (type $\langle (e, t), (e, t) \rangle$) combines with the xNP *two birds* (predicate type $\langle e, t \rangle$), resulting in a generalized quantifier (type $\langle (e, t), t \rangle$). (Alternatively, the definite article may map the predicate to a type $e$ R-expression).
In a sentence like (15a), the xNP *two birds* is associated with existential force, but there is no overt element that could be judged responsible for it, unlike in (15b).

\[ \text{(15) a. Two birds sang.} \]
\[ \text{b. A bird sang.} \]

The standard view that (simplex) cardinals are determiners (Montague 1974; Bennett 1974; Barwise & Cooper 1981; Scha 1981; van der Does 1992, 1993; among others) attributes existential force to them as part of their semantics. However, we argued in section 2.2.1 that such an approach is untenable for complex cardinals.

How is the existential force introduced in (15a)? We believe that any standard theory of indefinites can account for cardinals as well. One traditional view is that predicate xNPs can become generalized quantifiers (type \( \langle e, t \rangle \)) as a result of a type-shifting operation (see Partee 1986; Landman 2003). Another possible way of passing from the predicate reading of *three birds* to the generalized quantifier reading is suggested by Krifka (1999). While Krifka’s semantic analysis of cardinals, based on that of Link (1987), is different from ours, his proposal that the empty D head is interpreted as an existential quantifier is fully compatible with our view. Alternatively, the existential force is introduced via global existential closure, per Heim (1982).

In view of the above, our analysis clearly can shed no new light on the availability of long-distance scope readings (cf. Fodor and Sag 1982 and much subsequent literature). In the vast literature on indefinites (see, among many others, Farkas 1981; Ludlow & Neale 1991; Ruys 1992; Winter 1997, 2001a, 2005; Kratzer 1998), indefinites containing unmodified cardinals (*three birds, four books*, etc.) have been shown to behave much like indefinites headed by *a* or *some* with respect to exceptional scope-taking abilities. Like *a*– and *some*–indefinites, cardinal indefinites can therefore be analyzed as choice functions. We can follow the analysis of Winter (2001a, 2005), where the existential force of indefinites comes from a phonologically null choice function operator in \( D^0 \). Combined with our semantics this yields the structure in (16).

\[ \begin{array}{c}
\text{DP(\( e \))} \\
\text{\( D^0 \)} \\
\text{\( f_{\text{CF}}(\langle e, \emptyset, \emptyset \rangle, \emptyset) \)} \\
\text{\( \text{NP}(e, \emptyset) \)}
\end{array} \]

(16)

On this proposal, a choice function \( f \) applies to the set of all plural individuals \( x \), such that each \( x \) is divisible into two non-overlapping
individuals, each of which is a bird, and returns a single such x. A DP such as two birds thus has type e: it is a plural individual (consisting of two non-overlapping individuals, each of which is a bird), which is picked out by the choice function from the set of such plural individuals.

As a result, any standard theory of indefinites combined with the semantics in (5) yields the existential force of indefinite cardinal-containing xNPs. However, the question arises why cardinal-containing indefinite xNPs behave like a/some-indefinites rather than bare plurals.

Bare xNPs are known to generally have narrow-scope readings only (Carlson 1977; Chierchia 1998, etc.), which means that they cannot combine with either choice functions or existential quantification. If, as suggested above, the existential force of cardinal-containing indefinite xNPs comes from the null determiner, why is this not available for bare plurals?

We do not have a straightforward answer to this question. However, we note that the status of the determiner is not the same in bare xNPs as in cardinal-containing indefinite xNPs, as shown by the fact that the latter but not the former are compatible with the indefinite article in English. The indefinite article appears obligatorily when the leftmost cardinal is one of the so-called ‘semi-lexical’ cardinals (hundred, dozen, etc.), exemplified in (17), and in the modified cardinal construction as in (18) (for more discussion of this construction see Jackendoff 1977; Babby 1985; Gawron 2002; Ionin & Matushansky 2004, in preparation). Nothing comparable happens with English bare plurals (but see Bennis et al. 1998 for indefinite article insertion in Dutch plurals).

(17) a. *(a) hundred/thousand/million/ dozen books
   b. (a) twenty/thirty/five/twelve/one thousand books

7 In addition to the presence of an article, semi-lexical cardinals are characterized by their compatibility with plural morphology, unlike other cardinals (Jackendoff 1977):

(i) a. hundreds *(of) books
    b. * twenties of books
    c. three hundred *(of) books

We believe that semi-lexical cardinals are semantic modifiers (type ⟨e, t⟩, ⟨e, t⟩) whatever the lexical category of their complement. The correlation of overt plural marking with the inability to assign Case (and hence the appearance of an of-PP rather than an xNP complement) is language-specific, as discussed in section 3.2. It should also be noted that though the presence of an article correlates with the ability to be a multiple in English, it does not do so in French (cent livres ‘one hundred books’).

8 Note that the possibility of adjectival modification provides additional evidence in favour of treating the combination of a cardinal and an NP as a predicate.
We hypothesize that the appearance of the indefinite article does not correspond to any semantic operation but marks the fact that a semantic operation introducing existential force has applied (since it distinguishes between bare plurals and cardinal-containing indefinite xNPs). We leave aside here the question why modification triggers indefinite article insertion (see Ionin & Matushansky, in preparation), but note that a similar effect occurs in the post-copular position in French (Matushansky & Spector 2005) and in Dutch (de Swart et al. 2005) and in a number of languages with proper names (Sloat 1969; Matushansky, to appear).

To sum up, we have argued that the existential force of cardinal-containing indefinite xNPs in argument positions does not come from the cardinal involved. Instead, any standard theory introducing existential force (existential closure, type-shifting, choice functions, etc.) is applicable here. We have observed that cardinal-containing indefinite xNPs group with some and a indefinites rather than with bare plurals, and noted that this grouping correlates with the possibility of indefinite article insertion. While we can offer no simple explanation for when and why an indefinite article becomes obligatory, we believe that its presence marks rather than introduces existential force.

2.4 Summary

The compositional semantics of complex cardinals necessitates that simplex cardinals have the semantic type of modifiers, rather than determiners or predicates. If cardinals were determiners or predicates, complex cardinals would have to be treated as un-analysable units (i.e. the entire cardinal two hundred would have to have the semantic type of a determiner or predicate), and we would lose the intuition that complex cardinals are semantically related to simplex cardinals. Importantly, though the hypothesis that cardinals are semantic modifiers requires a special mechanism introducing existential force for cardinal-containing indefinite xNPs in argument positions, this mechanism is provided by any standard view of indefinites.

An alternative hypothesis is that complex cardinals are constructed extra-linguistically and thus belong to a completely separate system (see Wiese 2003 for arguments in favour of this hypothesis). Our objection to this theory is twofold. Firstly, if syntactic composition and semantic interpretation of complex cardinals can be seamlessly incorporated into
standardly assumed syntax and semantics, such an appeal to an extra-linguistic system is unnecessary. Secondly, in the next section we will show that complex cardinals are transparent to such morphosyntactic operations as Case-assignment and number marking, which means that they are constructed by regular linguistic mechanisms. Even though we have to make an appeal to extra-linguistic factors in order to explain certain ordering constraints in complex cardinals, we will argue that similar factors come into play in complex measure phrases, such as five feet five inches, for which an extra-linguistic analysis seems unwarranted.

3 SYNTAX OF CARDINALS

A member of the set denoted by two books is a plural individual consisting of two atomic books. This is why for the semantics in (5) to work, the lexical complement of a cardinal has to be atomic—otherwise, a member of the set denoted by two books could have denoted a plural individual divisible into two sets of books (and thus a plural individual of unknown cardinality), which is not what we want. Likewise, two hundred books has the extension in (19b), where, crucially, each pk needs to be a single book rather than a set of books:

(19) b. λx ∈ D_e . x is a plural individual divisible into two non-overlapping individuals p_i such that their sum is x and each p_i is divisible into 100 non-overlapping individuals p_k such their sum is p_i and each p_k is a book

Importantly, our compositional analysis of complex cardinals requires that the lexical xNP that a cardinal combines with denote a set of atoms. This permits us to immediately account for languages where a lexical xNP combining with a cardinal must be morphologically singular, despite the availability of plural morphology. This is illustrated by the Finnish examples from Nelson & Toivonen (2000) in (20).

(20) Yhdeksän omena-a puto-si maa-han.
nine-NOM apple-PART.SG fall-PAST.SG earth-ILL
‘Nine apples fell to earth.’ (Finnish)

From our point of view, Finnish morphology conforms completely to what we expect: the lexical xNP is morphologically singular,
obligatorily so. The same phenomenon can be observed in Hungarian (Farkas & de Swart 2003), Welsh ((21) from Mittendorf & Sadler 2005), and Turkish.

(21) y tair cath ddu hynny the.PL three.F cat.ESG black.SG that.PL
   ‘those three black cats’ (Welsh)

If Welsh, Finnish, Turkish and Hungarian morphology correctly reflects the semantic atomicity of the lexical xNP, what happens in languages like English and Russian, where the lexical xNP in two books is morphologically plural?

In section 3.1, we will discuss the two possible answers to this question: (1) the plural marking reflects semantic plurality (books in two books denotes a set of plural individuals) and some operation extracts atoms out of the plural xNP; or (2) the plural marking here is misleading: books in this context really means book (i.e. its extension consists of singular individuals (atoms)) and an additional operation marks the lexical xNP as plural.

In section 3.2, we will show that number marking favours the first option and therefore a particular syntax of cardinals, and in section 3.3 we demonstrate that Case-assignment argues for a different syntax and thus for the second option. We will argue that the second option is preferable to the first (except in classifier languages), and propose how it can account for both the number marking and Case assignment facts. Finally, section 3.4 addresses the role of various extra-linguistic conventions in the composition of complex cardinals.

### 3.1 The atomicity requirement

Our semantics of cardinals requires that the lexical xNP complement of a cardinal denote an atomic set. We see two possibilities for ensuring this requirement: (1) a (null) classifier, and (2) a special constraint that would exclude plural lexical xNPs as complements of cardinals. Depending on the implementation of this atomicity requirement, different syntactic structures must be adopted.

### 3.1.1 Classifiers

One standard assumption intended to distinguish between mass and count nouns is that only atoms can be counted

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10 Note that the verb is singular in (20) and the determiner is plural in (21). For the effects of cardinals on number marking inside and outside the xNP, see Ionin & Matushansky (in preparation).
(Kratzer 1989; Chierchia 1998). Under most standard approaches, a cardinal combines with an xNP denoting a set of entities and returns a subset of this set: those members of it that have the relevant cardinality (cf. (12)), i.e. contain the relevant number of atoms. Since mass individuals have an undefined number of atoms, the notion of cardinality is not defined for them.

This approach is not enough for our semantics because the lexical entry in (5) requires the sister of a cardinal to be a singular count noun (which denotes an atomic set, i.e. a set of entities of cardinality 1), rather than just a count noun (plural or singular). On the surface, however, a cardinal might have a morphologically plural complement, as in *two books*. If *books* in *two books* is a semantically plural predicate, we need to convert it into a semantically singular predicate (denoting an atomic set). Such an atomizing conversion is usually associated with classifiers (see Chierchia 1998; Kobuchi-Philip 2003, and references cited therein). The atomicity requirement imposed by cardinals on their lexical xNP sister can now be explained by positing a null classifier in languages like English, which exhibit plural marking inside cardinal-containing xNPs (see Borer 2005 for a similar proposal). In languages like Finnish, Welsh, Turkish or Hungarian, no such operation would be necessary.

The question arises where in the structure this classifier appears. One possibility is that it appears between the lexical xNP and the lowest (simplex) cardinal, as in (22a) (cf. Cheng & Sybesma 1999).\(^{11}\) In this configuration the classifier must have the semantic type of a modifier (\((e, t), (e, t)\)). The alternative is that the classifier is a complement of the innermost simplex cardinal and the lexical xNP is merged as the sister of the entire complex cardinal, as in (22b).\(^{12}\) In this case, the classifier is a predicate (type \(\langle e, t \rangle\)).\(^{13}\)

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\(^{11}\) Chierchia (2004) proposes a variant of the structure in (4), where the atomization operation is written directly into the lexical entry of each cardinal. This makes it necessary to relativize the definition of an atom to include such entities as *hundred books*. However, the property of being a ‘non-plural atom’ (necessary, for example, for the quantifier *each* or for plural/singular marking) then has to be redefined. This approach also has to deal with the number mismatches discussed in section 3.2 for the structure in (22a).

\(^{12}\) It could be argued that a third structure is available, where the classifier is a head, as in (22a), but the cardinal is a specifier, as in (22b). Such a structure can be excluded on the grounds of semantic redundancy: the cardinal in a specifier position requires a complement denoting an atomic set, i.e. another classifier.

\(^{13}\) The idea that cardinals occupy [Spec, NumP/QP], while Num\(^0\)/Q\(^0\) holds number features can be found, with some variation, in Selkirk (1977), Jackendoff (1977), Li (1999), Haegeman & Guéron (1999), Gawron (2002) and Gärtnert (2004), among others. Zabbal (2005) proposes that though complex cardinals are interpreted by dedicated semantic principles, they are nonetheless constructed as maximal projections in regular syntax; Num\(^0\) contains an operator mediating between the lexical xNP and the numeral.
Both structures result in roughly the same semantics as in (9) in section 2.1, though the order of composition is not the same. Importantly, the assumption that the interpretation of the lexical xNP as an atomic set is achieved via a classifier does not distinguish between the iterative complementation structure in (22a) and the specifier structure in (22b) in languages without overt classifiers.\footnote{We will not attempt here the discussion of the syntax of cardinals in classifier languages. Although the classifier generally follows the cardinal, the cardinal-classifier sequence combines with the lexical xNP in a variety of ways (see Downing 1984, 1996; Muromatsu 1998, among others, for Japanese facts; Cheng & Sybesma 1999 for Chinese facts; and Simpson 2005 for cross-linguistic facts). This suggests that more than one structure may be available in a given language and cross-linguistically.}

We believe, nonetheless, that the classifier analyses should not be applied to such languages. The reason is that the presence of overt classifiers in a language has been linked to the lack of a singular/plural and mass/count distinction (Sanches & Slobin 1973; Chierchia 1998; Borer 2005, among others; see Cheng & Sybesma 1999 for objections). This connection is weakened if languages with plural morphology have null classifiers.

From the semantic point of view, our analysis can be easily extended to classifier languages. Following Chierchia (1998, 2004), we assume that lexical xNPs in classifier languages are mass-denoting. For the structure in (22a) this means that a classifier maps the denotation of a mass-denoting xNP onto a set of atoms. In addition, an overt classifier in a language like Chinese or Japanese also has additional constraints on which atoms to consider (e.g. humans, groups of humans, long cylindrical objects, etc.—see Downing 1984). The reason that cardinals in classifier languages cannot combine directly with an xNP is that cardinals require atomicity of their complements and thus cannot take mass-denoting xNPs as complements (see Chierchia 2004). Once a classifier has converted the mass xNP denotation into an atomic one, combination with a cardinal is possible. For the structure in (22b), the combination of the lexical xNP and the cardinal (via Predicate Modification) yields the intersection of the denotation of the lexical xNP with the set of plural individuals of a particular cardinality.
3.1.2 Countability If we don’t assume the presence of classifiers in complex cardinals, how can we ensure that the lexical xNP denotes an atomic set? We propose modifying not the definition of an atom (cf. Chierchia 2004) but the constraint on what can be counted: informally, only individuals of the same (known) cardinality can be counted. Formally, this means that the complement of a cardinal can only denote a set of individuals x such that there exists a number n such that for every x, |x|=n:

(23) \[ \lfloor 2 \rfloor = \lambda P \in D(c, v) \cdot \lambda x \in D_e \cdot \exists S [\Pi(S)(x) \land |S|=2 \land \forall s \in S P(s)] \]
\[ \lfloor 2 \rfloor (P)(x) \text{ is defined only if } \exists n \forall z [P(z) \rightarrow |z|=n] \]

The constraint in (23) ensures that true plurals cannot combine with cardinals: a plural such as books denotes a set of individuals x where each x is a plurality of books, and such pluralities do not necessarily have the same cardinality. This means that to be interpretable, the lexical xNP in two books has to be semantically singular, despite appearances. (We come back to where the number marking on the lexical xNP comes from in section 3.2.) Cardinals can of course combine with singular xNPs: the singular xNP book denotes a set of atomic individuals, which by definition all have the same cardinality. At the same time, a cardinal can combine with an xNP headed by another cardinal, such as hundred books: since such an xNP denotes a set of plural individuals divisible into one hundred books, all members of the set have the same cardinality.

The presupposition in (23) is probably due to pragmatics, since counting pluralities of an unknown size is pointless. A similar constraint prohibits counting of dissimilar entities:

(24) a. # There are 17 people and chairs in this room.

b. There are 17 women and children in this room.

One remaining issue is that of dual and trial marking. The approach suggested here incorrectly predicts that a cardinal should be able to combine not only with a singular xNP or an xNP headed by another cardinal (a base), but that the lexical xNP complement of a cardinal could also be a dual or a trial (since a dual or a trial xNP denotes a set of individuals of the same cardinality). A simple way to exclude this issue is to suggest that (non-singular) syntactic number, including dual and trial, is projected as Num\(^0\), while a cardinal combines with a bare xNP. However, this idea would not be able to explain the plural marking on intermediate cardinals in languages that have it (see section 3.2), which should be due to the presence of a NumP. We therefore hypothesize that a dual or a trial xNP cannot be a complement of a cardinal for the same reason a cardinal such as seven cannot: it is a not a multiple. A simple morpho-syntactic realization of this hypothesis is to assume that the dual/trial is a head taking the lexical xNP as a complement, exactly as a nominal two or three does.

We note that duals and trials are as much of a problem for the null classifier approach in section 3.1.1 and for Chierchia’s analysis mentioned in footnote 11: on both analyses, there is an operation extracting atoms from a set, and there is no reason why atoms cannot be extracted from the set denoted by a dual or trial xNP.
While (24a) is odd, (24b) is rescued by the fact that women and children saliently qualify as human beings. It is tempting to hypothesize that plural individuals with different cardinalities cannot be similar in the relevant respect, but given that a single shared property (e.g. being a human being) suffices, it is not clear whether the presupposition in (23) can be derived from the same source as the contrast in (24).

3.2 Number morphology

In the previous sub-section we have proposed the semantic means of accommodating the fact that our semantics requires the sister of the innermost cardinal to denote an atomic set. Three syntactic structures are compatible with this constraint: the simple iterative complementation structure in (4), the classifier structure of (22a), which is identical to (4) with the exception of a classifier between the innermost cardinal and the lexical xNP, and the classifier structure of (22b), where (4) contains a classifier and resides in the specifier of the functional projection NumP. In all three structures, it is necessary to ensure the correct number marking, ruling out number mismatches as in (25), which are compatible with our semantics. We will show that it is easy to do so in the specifier structure in (22b), while the complementation structures, (4) and (22a), require additional stipulations.

(25) a. *one CL books
   b. *four book

In the specifier structure in (22b), the number marking on the xNP can be attributed straightforwardly to Spec-head agreement. The only possible source of singular marking in a cardinal-containing xNP would then be the cardinal one; otherwise plural marking has to be used. Thus both (25a) and (25b) would be impossible. A minor problem with this theory is that if number marking is due to agreement, there is no apparent need to project a classifier.

This solution is not available for the configurations in (4) and (22a). Even if agreement between a head and its complement is available, in (22a) a classifier intervenes between the lowest cardinal and the lexical xNP, and so agreement is blocked syntactically. Conversely, if plural marking in cardinal-containing xNPs reflects semantic plurality, then to exclude (25a) it is necessary to assume that one takes a singular NP rather than a CIP and to exclude (25b) it becomes necessary to stipulate that a cardinal can combine only with another cardinal or CIP.

In (4), on the other hand, in order for the agreement to take place it is necessary to show that cardinals are syntactically plural—and this is
not at all obviously the case: (a) they don’t have to bear plural marking (cf. (26)), and (b) it is not clear whether a semantic notion of plurality can be defined for objects of the semantic type $\langle e, t \rangle$, $\langle e, t \rangle$.

(26) a. two dozen(*s) books  
    b. three hundred(*s) people  
    c. four score(*s) and seven years ago

An alternative, also suggested by Farkas & de Swart (2003: 47), is applicable to both (4) and (22a). Suppose that the source of the plural marking on the lexical xNP is the semantic plurality of the entire cardinal-containing xNP, which would denote a plural individual with all cardinals but one. The number marking in cardinal-containing xNPs is then a result of semantic concord—a phenomenon where part of an xNP agrees with the entire xNP, also observed with features other than [$\pm$ plural] (see Corbett 1983; Wechsler & Zlatic 2003). Support for this theory comes from the fact that, both cross-linguistically and within the same language, number marking in cardinal-containing xNPs is not uniform. Thus in English, cardinals in cardinal-containing xNPs must be morphologically singular; in Dutch this is true for cardinals and measure nouns (the Dutch examples in (27) are due to Eddy Ruys, personal communication), whereas in Russian, semantic concord is all-pervasive (except for cardinals between 21 and 99 in oblique cases, as noted by an anonymous reviewer). Finally, in Turkish, Welsh, Hungarian and Finnish, the lexical xNP must be singular (see (21) above). This cross-linguistic variation would be difficult if not impossible to explain if plural marking reflected genuine semantic plurality.

(27) a. drie liter water  
    three liter water  
    ‘three liters of water’  
    b. driehonderd meisjes  
    three.hundred girls  
    ‘three hundred girls’

(28) dvadcat millionov knig Russian  
    twenty-NOM million-GEN.PL book-GEN.PL  
    ‘twenty million books’

However, as noted by an anonymous reviewer, it is not clear how this approach fits into the Agreement Hierarchy (Corbett 1983),
governing the application of semantic concord. We discuss the problem and possible solutions in Ionin & Matushansky (in preparation).

To conclude, for languages where number morphology is present in cardinal-containing xNPs, the specifier structure in (22b) appears to be preferable to both (4) and (22a). However, this structure comes with the additional assumption that a null classifier is available in such languages. Importantly, the presence of number marking on simplex cardinals inside complex ones, as in Russian, strongly suggests that complex cardinals are transparent to syntax and therefore cannot be constructed entirely in the lexicon, as required by extra-linguistic theories of cardinals (e.g. Wiese 2003) and by the hypotheses that cardinals are syntactic heads (e.g. Ritter 1991, Giusti 1991, 1997; Zamparelli 1995, 2002). An additional argument against these hypotheses comes from Case-marking in cardinal-containing xNPs discussed below. We will show that the Case-marking facts provide support for the structure in (4) over both structures in (22).

3.3 Case assignment with cardinals

Contrary to most standard views, we assume that simplex cardinals belong to one or another open lexical class available in a language, and that it is not necessarily the case that all simplex cardinals belong to the same lexical class. Specifically, we agree with Hurford (1975, 1987, 2001, 2003) that the vast majority of cardinals are singular nouns,\(^{16}\) with lower cardinals being sometimes adjectival (but see Moser & Marlett 1994 for a discussion of Seri, where cardinals start out as verbs). This hypothesis explains why simplex cardinals do not have their own declensional paradigm, but decline like adjectives or nouns,\(^{17}\) but even more relevantly, it is also compatible with the behaviour of Case-assignment within a complex cardinal.

Besides the fact that in many languages cardinals are Case-marked, simplex cardinals are also able to assign Case (Genitive in Russian, Partitive in Finnish). For an unclear reason, for most cardinals in both languages, Case-assignment is only visible in direct Cases (Accusative/Nominative), while in an oblique Case the entire xNP is marked with

\(^{16}\)The hypothesis that the vast majority of cardinals are singular nouns is supported for instance by the fact that they can require a singular article or take overt plural marking. Another possible candidate for nouns with the semantic type of modifiers (and with similar behaviour with respect to Case) is measure nouns, such as pound or liter (see Chierchia 2004 for an \(\langle e, t \rangle\) analysis of measure nouns) and vague measure nouns (a bunch of roses, a can of olives, etc.), discussed by Dodge & Wright (2002) and Doetjes et al. (1998).

\(^{17}\)Note also that if simplex cardinals belong to standard lexical classes it makes the hypothesis that they are constructed extra-linguistically even less likely.
that oblique Case (see Mel’čuk 1985; Franks 1994; Hurford 2003, among others):

(29) dvumja-stami pjat’ju-desjat’ju šagami Russian
two-INSTR hundreds-INSTR five-INSTR ten-INSTR steps-INSTR
‘(with) two hundred and fifty steps’

Taken in itself, this pattern of Case-marking is compatible with all the three structures under consideration, on the condition that simplex cardinals don’t require Case (which would entail that they are not nominal, against overwhelming cross-linguistic evidence (see Hurford 1975, 1987, 2001, 2003)). However, as discussed below, the behaviour of complex cardinals in direct Cases is only compatible with (4); we rely on Franks (1994) to explain the pattern in the oblique Cases.\(^{18}\)

3.3.1 Case assignment by simplex cardinals  In Russian and Inari Sami, cardinals assign Case to their sister nouns, and the Case depends on the cardinal. In Russian, the lower (adjectival) cardinals \(\frac{1}{2}, 1\frac{1}{2}, 2, 3\) and 4 (but not one) assign Paucal,\(^{19}\) while the higher cardinals assign Genitive, as shown in (30). In Inari Sami, cardinals 2 through 6 assign Accusative while the higher cardinals assign Partitive, as shown in (31) (Nelson & Toivonen 2000). Similar facts from other languages can be found in Hurford (2003).

(30) a. četyre šagá
    four step-PAUC
    ‘four steps’ (Russian)

b. šest’ šagov
    six step-GEN.PL
    ‘six steps’ (Russian)


\(^{19}\) The syntactic configuration between an adjectival cardinal and its sister is far from obvious. On the one hand, adjectives are generally assumed to be maximal projections (adjoined to the xNP or merged as specifiers, depending on the framework). On the other hand, Russian adjectival cardinals assign (Paucal) Case to their sisters, which can only be a property of heads. Finally, if the xNP sister of an adjectival cardinal is its complement, we expect the entire maximal projection to be an xAP, which is clearly not the case. In Ionin & Matushansky (in preparation) we propose that adjectival cardinals can have both sets of properties since they are simultaneously heads and maximal projections.
In languages where cardinals are responsible for Case assignment, the structure in (4) receives support over the structures in (22). In (4), the lowest cardinal assigns Case to its sister, the lexical xNP. In (22a) a classifier projection intervenes between the lowest cardinal and the lexical xNP, and so additional stipulations are required. Finally, in (22b) the complex cardinal is in the specifier and cannot assign Case due to the fairly standard assumption of both GB and Minimalist frameworks that maximal projections cannot assign Case.

Importantly, Case assignment within Russian complex cardinals behaves just like Case assignment from cardinals to lexical xNPs. In a complex cardinal like four thousand or five thousand in (32), the case on thousand depends on the preceding cardinal. While four assigns Paucal Case to thousand, five assigns Genitive. The fact that the syntactic process of Case assignment takes place within a complex cardinal shows that Russian complex cardinals are constructed in syntax.

(32) a. četyre tysjači šagov
    four thousand-PAUC step-GEN.PL
    ‘four thousand steps’ (Russian)

b. pjat’ tysjač šagov
    five thousand-GEN.PL step-GEN.PL
    ‘five thousand steps’ (Russian)

The same argument can be constructed outside Slavic. In Finnish, cardinals 2 and above assign Partitive to the lexical xNP, as in (33a), from Hurford (2003), and in (33b). Partitive Case assignment in Finnish also takes place within complex cardinals, as shown in (34).

(33) a. kolme saapasta
    three-NOM boot-PART
    ‘three boots’

b. viisi kirjaa
    five-NOM book-PART
    ‘five books’
We conclude that Case-assignment within complex cardinals provides evidence against the hypotheses that they are syntactic heads (Ritter 1991; Giusti 1991, 1997; Zamparelli 1995, 2002) and therefore against extra-linguistic analyses of complex cardinals.

3.3.2 The role of overt morphology Given the facts of Case assignment by and within complex cardinals discussed above, we conclude that languages with morphological Case assignment, such as Russian (and Slavic in general), Inari Sami, and Finnish must have the syntax in (4). On the other hand, languages like English, where Case assignment is not morphologically overt, could in principle have the syntax in (22a) or (22b), and still be compatible with the semantics that we have proposed for complex cardinals (with (22b) preferable to (22a) in languages like English, which have number marking on the lexical NP—see above).

English, however, presents an additional complication. It is generally assumed that only prepositions and verbs can assign Case in English, which correctly predicts that nouns should be unable to take nominal complements, as illustrated in (35). Since we agree with Hurford (1975, 1987, 2001, 2003) that English cardinals are nouns, the structure in (4) should be ruled out in English for the same reason examples (35) are.

(35) a. *son Mary
   b. *liter water

We propose that cardinals are exceptional nouns in the same way the adjectives *worth, like and near are exceptional (see Maling 1983) in that they assign (Accusative) Case to their arguments, as illustrated in (36). As a result, the structure in (4) becomes possible.

(36) a. It’s worth five dollars.
   b. It’s near the house.

None of the possible alternatives is preferable. If we continue to assume that English cardinals are nouns and do not assign Case, then the standard assumption that nouns need Case will be violated in cardinal-containing xNPs whatever structure is adopted: a cardinal-containing xNP would contain at least two nouns with only one Case (the one assigned to the entire xNP from the outside). If cardinals are
adjectives, the status of semi-lexical cardinals becomes unclear. Finally, if they are neither adjectives nor nouns, we need to assume a new lexical category for them, which is not preferable to the theory that cardinals are nouns that exceptionally assign Case. We conclude that lack of overt Case-marking does not tell us which syntactic structure is appropriate for English. As a result, it is preferable to postulate the same syntactic structure across languages, and thus we must fall back on Russian data.

This outcome seems to lead to an impasse, since Russian yields contradictory evidence for the syntactic structure of cardinal-containing xNPs: Case-marking is only compatible with the structures in (4) and in (22a), while number-marking is most easily accounted for in the specifier structure in (22b).

We propose to make use of the independently motivated generalization that classifiers are not available in the presence of number morphology (Sanches & Slobin 1973; Chierchia 1998; Borer 2005, among others). If correct, this means that even for languages where no overt Case is visible, the structure in (4) should be preferred.

We suggest therefore that morphological number marking in cardinal-containing xNPs is due to semantic concord. Even though the structure in (22b) allows for a simpler analysis of number-marking, it is incompatible with the Case-assignment facts, while the structure in (4) permits us to account for both Case and number-marking (see Ionin & Matushansky, in preparation).

### 3.4 The role of convention in complementation

Leaving aside the issues of the constituent structure of an xNP containing a complex cardinal, the analysis of complex cardinals as iterative complementation overgenerates, predicting the possibility of the complex cardinals in (37). However, cross-linguistically, only the higher simplex cardinals can function as complements to other cardinals (henceforth, multiples). (On the ordering of multiples see section 4.3.)

(37) a. *two twenty = 40
    
    b. *two [seventy five] = 150

The class of multiples usually contains most powers of the base (usually 10, but 20, 15 and 5 are also attested), and a small set of

---

20 Note that not all powers of 10 may serve as multiples. For instance, though 10 is a mathematical base, three-ten is not a possible complex cardinal of English, but is perfectly fine in Finnish (see (34)).
others (12, 20, 60...) , which may be subject to further constraints. For example, 20 serves as a multiple productively in Mixtec, Yoruba and Celtic languages, where 20 is a base, and sporadically in Danish and French, where it is not (Hurford 1975, 2003).

This is exactly a situation where our arguments against the extra-linguistic nature of cardinals do not apply: whereas syntactic phenomena inside complex cardinals (such as Case assignment or number marking) show that they are combined in syntax, the distinction between multiples and non-multiples does not correlate with any syntactical or morphological property relevant to the computational system.\(^{21}\)

The cross-linguistic and intra-linguistic variation as to which cardinals are simplex or complex is equally large. For instance, in Russian, sorok ‘forty’ is linguistically simplex, and is derived from the Old Nordic sekr ‘furs’ (see Wiese 2003, chapter 3), while dvenadcat’ ‘twelve’ is complex (lit. ‘two on ten’). In English, sixty is arguably derived from six-ten, but given the phonological changes, it may be preferable to treat it as a simplex cardinal (the same holds for forty, eighty, etc.). We consider the issue of which cardinals are simplex and which are complex to be extra-linguistic as well.

To summarize, any syntactic analysis of complex cardinals needs to be supplemented by extra-linguistic constraints (see Ionin & Matushansky, in preparation, for a discussion of extra-linguistic constraints in such non-numeric areas as measure phrases, names and titles). The alternative of treating the composition of complex cardinals as entirely extra-linguistic fails to account for Case assignment and number-marking, as well as the similarities between cardinals and measure nouns.

3.5 Summary

In this section, we have argued that the lexical xNP sister of a cardinal must be semantically singular, and have discussed two possible accounts

\(^{21}\)This said, in some languages being a multiple does appear to correlate with certain such properties, but not in any way that promises a straightforward solution. As mentioned above, simplex cardinals do not behave the same: the higher a cardinal, the more it behaves like a noun with respect to concord, the presence of an article, Case assignment and Case-marking (see Hurford 2003; Ionin & Matushansky, in preparation, for details). This makes it possible to say that higher cardinals behave more like nouns also in being able to appear in the complement of another cardinal. However, (a) it is not at all clear in what way being a noun is a gradable property and (b) being a multiple does not map directly into any of these properties. Finally, even if we just postulate a [± multiple] feature, it would not be able to account for such complex cardinals as the French quatre-vingt ‘eighty’, where vingt ‘twenty’ can be a multiple with quatre ‘four’ only. Importantly, the issue of multiples arises irrespectively of which linguistic analysis is adopted for complex cardinals.
for the presence of plural morphology in *two books* in English, Russian, etc. The first one depends on the presence of an atomizer, which is either part of the lexical entry of cardinals, or syntactically projected as a classifier. The second option is that plural marking with cardinals reflects number agreement in some form. We then discussed some of the syntactic phenomena inside complex cardinals, since it is the syntactic transparency of complex cardinals that lends weight to our claim that they are constructed in syntax. The behaviour of morphological Case and of number marking in cardinal-containing xNPs lead to opposite conclusions about their internal structure, even within the same language, which is why we sketched some arguments in favour of the iterative complementation structure in (4) cross-linguistically (except maybe in classifier languages).  

Finally, we showed that although certain phenomena in the syntax of complex cardinals must be attributed to extra-linguistic constraints, there is evidence that their semantic and syntactic composition is nonetheless done by standard linguistic means.

### 4 COMPLEX CARDINALS AND COORDINATION

We have so far been concerned with complex cardinals involving multiplication (*two hundred*), which we analyzed as complementation (cf. (4)). We now turn our attention to complex cardinals involving addition, like *twenty-two, two hundred and two*, etc.

Examples like (38) show that simplex cardinals can be combined into complex ones via coordination, which is then interpreted as addition.

(38)  

a. one hundred and two

b. *laba iyo toban*  
   \[two \text{ CONJ } ten\]  
   ‘twelve’ (Somali)

  Saeed (1999)

c. seven and two thirds

---

22 An anonymous reviewer draws our attention to the interpretation of NP-ellipsis with cardinals:

(i) Mary bought two hundred books, and Peter – three.

   a. three books
   b. three hundred books

Although the preferred interpretation of the elided xNP in (i) is (ia), the majority of the speakers we have asked also accept (ib), in particular if *two* in the first conjunct is stressed, although we have also found minor speaker variation in function of the multiple (e.g., *million* is preferred to *hundred*). The fact that ‘intermediate’ NP-ellipsis is possible lends further support to the iterative complementation structure and is incompatible with the specifier approach, but further work is required to determine why (ib) is dispreferred or unavailable for some speakers (including the reviewer in question).
d. zweiundzwanzig
   two and twenty
   ‘twenty-two’ (German)

e. Her husband was a grave looking young man of five or six and twenty
   (Jane Austen, *Sense and Sensibility*, chapter 19)

We propose that this analysis can be extended to complex cardinals not involving overt conjunction (e.g. *twenty-two*) by appealing to the notion of *asyndetic coordination*—the phenomenon where the semantics of coordination is obtained in the absence of an overt conjunction. (See Hurford 2003 for a discussion of other means of expressing non-multiplication arithmetic operations, also exemplified in (2c)).

Asyndetic coordination is also attested in the domain of measurements, as illustrated by (39a) (from Gawron 2002), which is truth-conditionally equivalent to (39b). In addition to showing that asyndetic coordination is not specific to complex cardinals, (39) points to an extra similarity between measure nouns such as *foot/inch* and cardinals (see also footnote 16), and (40) illustrates the effect for monetary units.

(39) a. six feet six inches of finest silk
    b. six feet and six inches of finest silk

(40) a. two dollars (and) seventy-five cents
    b. two dollars (*and) seventy-five

In addition, coordination without an overt conjunction is attested cross-linguistically, as noted by Payne (1985) (via Winter 1995), Stassen (2000) and Drellishak (2004):

(41) ñe niyôj nipita ni’e
    be.PST her.brother her.aunt her.sister
    ‘It was her aunt, her brother, and her sister.’
    (Andoke (Macro-Carib, Witotoan))

As noted by Drellishak (2004), asyndetic coordination means that an overt conjunction is optional. In addition, in some languages and under some circumstances, an overt conjunction is impossible, as is the case for VP-coordination in the West-Papuan language Abun (Berry & Berry 1999 via Drellishak 2004). This is consistent with the fact that in some languages (e.g. Russian), numerical expressions never contain an overt conjunction, while in others (e.g. Arabic), an overt conjunction is obligatory for addition (Zabbal 2005).
However, treating addition as coordination (whether asyndetic or overt) is not enough. We also need to establish where in the xNP containing a complex cardinal the coordination takes place and to derive compositionally the correct interpretation of such an xNP.

4.1 The syntax of coordination

Given the syntax and semantics that we have proposed, in *two hundred twenty books*, each coordinated cardinal must contain an instance of the lexical xNP *books*: *two hundred books and twenty books*. There are two ways in which *two hundred books and twenty books* could be converted into *two hundred and twenty books*: (1) right-node raising of the lexical xNP; and (2) PF-deletion of the lexical xNP in the first conjunct. We discuss both possibilities below, and note that both strategies are in principle available: some languages use right-node raising, others use PF-deletion, and still others utilize both strategies.

4.1.1 Right-node raising or NP-deletion? The first possibility is that coordination in complex cardinals involves right-node raising of the lexical xNP, as shown in (42) (for analyses of right-node raising as rightward movement, see Ross 1967, Postal 1974, Abbott 1976, Grosu 1976, Sabbagh 2003, among others).

\[
\text{(42)}
\]

Right-node raising can also account for the example in (39), which could be derived from right-node raising of the common PP, as shown in (43).

\[
\text{(43)}
\]

An alternative possibility is that coordinated cardinals are derived via PF-deletion of the NP in the first conjunct, as shown in (49a). The
measurement cases in (49b) can be dealt with in the same way (PF-deletion of the of-PP):

\[
\text{(44)}
\]

Different options may be adopted in different languages. In Ionin & Matushansky (in preparation), we argue that cross-linguistically both processes are attested: we show that the behaviour of Russian and Biblical Welsh cardinals are best explained under the xNP-deletion view, while the right-node raising view can account for the behaviour of cardinals in English and Inari Sami. Furthermore, German appears to utilize both mechanisms, and Hurford (1975) discusses Biblical Welsh data with the lexical xNP appearing in the middle of the complex cardinal (cf. also three score years and ten): 23

\[
\text{(45) } \text{onid un mlwydd cant} \quad \text{Hurford (1975: 198)}
\]

but one years hundred
‘ninety years (old) and nine...’ (Genesis 17.1, 24)

For the purposes of the present paper the choice of xNP-deletion vs. right-node raising is irrelevant: either mechanism can derive complex cardinals involving addition. However, it should also be observed that if the specifier structure in (22b) is adopted, the question need not arise when the classifier is null. This would have been an argument in favour of (22b) were it not the case that in at least some languages, more than one instance of the lexical xNP is present with a coordinated cardinal, as shown below.

4.1.2 Evidence in favour of multiple xNPs Our overall proposal that coordinated cardinals are derived from coordinated xNPs (whether via

23 (45) exemplifies the use of a regular preposition to express subtraction (also known as ‘overcounting’; see Menninger 1969 and Hurford 2003). The remaining arithmetical operation, division, is also used albeit rarely, and once again regular linguistic means are employed. We leave overcounting and division aside here but see Hurford (2003).
right-node raising or NP-deletion) finds support in Luvale (Zweig, to appear) and Biblical Hebrew, where the lexical xNP may appear in both conjuncts of a coordinated cardinal, as in (46)–(49) (see also Hurford 1975 for the same effect in Biblical Welsh).

(46) mikoko makumi atanu na-mikoko vatanu
sheep ten five and-sheep five
‘fifty-five sheep’ (Luvale)

(47) tusahaan meqot shana u-shlosim shana
nine hundred-PL year and-thirty year
‘(And all the days that Adam lived were) nine hundred and thirty years’ (Genesis, 5.5)

(48) shalos shanim wo-barbaq meqot shana
three year-PL and four hundred-PL year
‘(And Salah lived after he begat Eber) four hundred and thirty years...’ (Genesis, 11.15)

(49) tusahaan shanim u-matayim shana...
nine year-PL and hundred-DU year
‘(And Peleg lived after he begat Reu) two hundred and nine years’ (Genesis, 11.19)

The multiple lexical xNP facts provide evidence in favor of treating twenty-two books in English as having the underlying form twenty books and two books, and also against the specifier structure in (22b), which does not lead us to expect multiple instances of the lexical xNP.

4.2 The semantics of coordination

On our analysis, an xNP like twenty-two books involves the coordination of twenty books and two books, where both are predicates over semantically plural individuals. It is easy to show that the standard Boolean semantics of and (Partee & Rooth 1983), type-lifted to apply to predicates, does not yield the expected result for this xNP:

(50) \[ \text{and} = \lambda f \in D_{(e, t)} \cdot \lambda g \in D_{(e, t)} \cdot \lambda x \in D_e . f(x) \land g(x) \]

(51) \[ \text{and} \left[ \left[ \text{two books} \right] \right] \left[ \left[ \text{twenty books} \right] \right] \approx \lambda x . \left[ \left[ \text{twenty books} \right] \right](x) \land \left[ \left[ \text{two books} \right] \right](x) \]

The reading in (51) is not available for the xNP twenty-two books. This absence is fully expected for pragmatic reasons, since nothing can be
simultaneously 20 books and 2 books (cf. section 2.2.2). Instead, we
obtain a considerably more complex meaning:

\[(52) \quad \exists x. \exists y, z [x = y \lor z \land (\text{twenty-two books}) (y) \land (\text{two books}) (z)]\]

The first observation that can be made in this respect is that this
effect is not restricted to cardinals. As shown by Krifka (1990a),
of Boolean coordination leads to problems with plural predicates, and
Heycock & Zamparelli (2000, 2003) show that the same issue arises for
plural predicates inside DPs:

\[(53)\]
\[
a. \text{The books are old and new.} \\
b. \text{These men and women met in the park.}
\]

\[(54)\]
\[
a. \text{His friends and colleagues came to the party.} \\
b. \text{A set of people each of whom is his friend or his colleague came to the party.}
\]

What is relevant for us here is the fact that coordination of two plural
predicates may result in a split reading (54b) (term due to Heycock &
derive this reading by assuming that and returns a set-product, as
defined in (55).\[26\]

\[(55) \quad \text{Set-product (SP)} \\
SP(A_1, \ldots, A_n) = \{X : X = a_1 \cup \ldots \cup a_n, a_1 \in A_1, \ldots, a_n \in A_n\}\]

The split reading in (52), however it is achieved (see Krifka 1990a;
Heycock & Zamparelli 2000), is what we need for coordination inside
an xNP like twenty-two books: a set of plural individuals that are each a
sum of two plural individuals such that one of them is in the denotation
of two books and the other is in the denotation of twenty books.

4.2.1 The role of pragmatics in coordination An important observation
made by Heycock & Zamparelli (2000, 2003) concerns the possibility

\[24\] The absence of the reading in (51) means that a cardinal-containing xNP such as two books
has an exactly-reading only—if the at least reading were available, then a single plural individual
could have been at least two books and at least twenty books simultaneously. See also fn. 6.

\[25\] That the ambiguity of the conjunction occurs at the level of NP predicates is even clearer when
the DP is placed in the predicate position, as in They became his friends and colleagues.

\[26\] The definition in (55) can be adapted to plural individuals.
of overlap in the split reading of (54b). The situation where no individual may be both a friend and a colleague is called a *full split reading*. This reading is most salient in (53b): no single individual can be simultaneously a man and a woman. Strikingly, with coordination of xNPs containing cardinals, only the full split reading seems available:

(56) Twenty-two people came to the party.
   a. *A plural individual that is simultaneously 20 people and 2 people...*
   b. A plural individual that contains 20 people and 2 people...

The absence of the joint reading (56a), on which a plural individual is 20 people and two people at once, is fully expected (see above). More puzzlingly, the reading in (56b) must be a *full* split reading: no overlap is possible. This cannot be ruled out pragmatically: in principle, a plural individual containing 20 people and two people may contain the totality of 20 people, or of 21 people. However, *twenty-two people* clearly cannot denote a predicate over a plural individual with fewer than 22 subparts. So why is overlap impossible for cardinal-containing predicates, while it is allowed elsewhere?

By our hypothesis, (56) is derived from (57) by right-node raising or NP-deletion. We note that to the extent that (57) is acceptable, overlap is equally impossible in it. Furthermore, the same lack of overlap occurs even when the lexical xNPs in the two conjuncts are different: only a mathematician set on devising a puzzle would treat (57b) as being about twenty people. Finally, the situation is not specific to xNPs containing cardinals: we see the same lack of overlap in (57c).

(57) a. Twenty people and two people came to the party.
   b. Twenty professors and seven deans came to the party.
   c. All professors and several deans came to the party.

The lack of overlap is also clearly seen when measurements or money are considered: (58a) cannot be about a mere six feet of silk (with the six inches included inside the six feet), and (58b) cannot be about only five dollars (with the seventy-five cents included in the five dollars).

(58) a. I bought six feet (and) six inches of finest silk.
   b. This cost five dollars (and) seventy-five cents.

Other examples where *and* means ‘in addition to’ and that do not involve cardinals come from Hofweber (2005) and Carlson (1987):

(59) She only had an apple and dessert.  

Hofweber (2005)
As Hofweber (2005) observes, ‘A usual utterance of this wouldn’t be true if she just had an apple, even though fruit is perfectly fine dessert.’ In other words, the lack of overlap is not limited to coordinated cardinals containing a single overt lexical xNP. Furthermore, it is not even limited to coordination of xNPs, as shown by (60). Carlson (1987) observed that while in isolation John did something amazing can be true by virtue of there being a token event of John pulling a rabbit out of a hat, in (60), this phrase must denote something different from the rabbit-pulling event.

(60) John did something amazing and he pulled a rabbit out of a hat.

Carlson (1987)

What is the reason behind this lack of overlap across constructions? We suggest a pragmatic explanation of these facts, along the following lines. When the denotation of one of the coordinated xNPs (two people, six inches, etc.) is totally or partially included in the denotation of the other (twenty people, six feet, etc.), a pragmatic principle prevents the use of a coordinated structure. A good candidate is the Gricean maxim of Manner (Grice 1975), which basically requires that all professors and several deans should not be used when all professors is a simpler alternative; the same principle rules out twenty-two people when twenty people is an accurate (and simpler) description. Basically, a conversational maxim prevents the speaker from using two coordinated xNPs unless she knows that the denotation of neither of them is contained inside the denotation of the other.

4.2.2 Supporting evidence: availability of overlap If the lack of overlap is pragmatic in nature, then it should be possible to override it. Indeed, we note that overlap does become possible in certain environments. For instance, in (61a), the question is open as to whether professors who have joint appointments in linguistics and psychology may be counted twice. Similarly, in (61b), the quorum requirement may be satisfied if a total of ten professors, five of whom are deans, are present.

(61) a. Each applicant is required to meet with three professors from linguistics and two professors from psychology.

b. We need ten professors and five deans for a quorum.

Likewise, Carlson’s example (60) allows overlap under certain circumstances: for instance, if to marry the princess John needs to do something amazing and to pull a rabbit out of a hat, he may be able to persuade the princess that one action fulfills both conditions.
Eddy Ruys, personal communication, suggests that the pragmatic constraint against overlap is overruled in these examples because the same individuals or events are presented in different guises: e.g. on the overlap reading of (61b), the same person is presented under the guise of a professor as well as the guise of a dean. On the other hand, the guise strategy is unavailable when we are talking about twenty people and two people in (57a) (or twenty-two people in (56)), since the lexical nouns in the two conjuncts are identical. The guise strategy is similarly unavailable for six feet and six inches in (58a), since no measure unit can be in the guise of a foot and the guise of an inch at the same time.

Further evidence in favour of a pragmatic approach based on guises can be drawn from the fact that a plural cardinal-containing xNP can denote a single individual:

(62) a. It is as if she is really three different people.
   b. These two very different people (Jekyll and Hyde) are really one person.

Both examples in (62) are concerned with different guises of the same individual, which permits the same entity to be counted more than once. We believe that this is also what takes place in (61), where the same entity can be counted under different guises.

Likewise, overlap does seem (marginally) possible for an xNP containing a coordinated cardinal if the two conjuncts have different implicit restrictions: in (63), it is possible that we need the twenty books that our French friends recommended, plus the two books that our British friends recommended, and there is overlap between the two sets.

(63) We need twenty books and two books.

We propose that in this case, the relevant entity also appears in two guises: the guise of a book that our French friends recommended, and the guise of a book that our British friends recommended. This reading appears to be facilitated by stress on the conjunction (which then has to be overt).

4.2.3 The role of an overt conjunction There is a subtle difference between the readings of (56) and (57a): (57a), to the extent that it is

The possibility that things more abstract than entities can be counted suggests a reanalysis of the well-known ambiguity of examples like (i), from Krifka (1990b):

(i) Four thousand ships passed through the lock last year.

The so-called event-related reading of (i), purported to count the events of ships passing through the lock, can instead be surmised to count stages of ships passing through the lock. An immediate advantage of such a hypothesis would be that it would permit the standard uniform semantics to be maintained for cardinals. Due to the lack of space, we leave a detailed discussion of this matter for future research.
acceptable, suggests that a group of 20 people and a separate group of two people arrived at the party separately, while (56) suggests nothing of the kind. We believe that the difference is once again due to pragmatics: while (56) and (57a) have the same truth-conditions, (56) is the conventional way to talk about twenty-two people. If one chooses to use (57a) (the unconventional way) instead, one should have a good pragmatic reason for doing so: the most natural reason is to separate the groups of twenty people and two people in space and/or time. (Another reason is to assign different guises, as in (62) and (63).)

Without such a reading, (57a) is pragmatically odd. This relationship between (56) and (57a) can probably also be accounted for under the maxim of Manner: (57a) is a lengthier, more cumbersome way of saying what can be more succinctly expressed by (56). Like any conversational maxim, it can be overridden by pragmatic considerations—e.g. the need to separate the two groups.

4.2 Summary We have shown that the lack of an overlap reading with an xNP containing a coordinated cardinal (twenty-two books) in fact extends well beyond this phenomenon to coordination of two separate xNPs and even coordination of entire events. We have argued that the lack of overlap is pragmatic in nature, and shown that, like any pragmatic constraint, it is overruled under certain conditions.

The issue still remains of why overlap is easily available in the absence of a cardinal, as in (54). Since the question of how genuine semantic plurals are affected by the pragmatic considerations discussed above extends beyond the scope of the paper, we leave it to future research.

4.3 The role of convention in coordination

Two new questions arise now that we have a semantics for coordinated cardinals. The first one is what determines whether addition or multiplication is at work. For instance, why is (a) hundred fifteen never interpreted, through multiplication, to mean 1500? Conversely, why is fifteen hundred never interpreted, through addition, to mean 115? The second question is that of the ungrammaticality of coordinated cardinals such as twenty-seventeen (with the meaning ‘37”), which are over-generated by our system (Philippe Schlenker, personal communication). We follow Hurford (2003) for our data and the conclusion that, just as for the constraints on complementation discussed in section 3.4, the answers lie in extra-linguistic conventions.

4.3.1 Addition v. multiplication As shown in Hurford (2003), there are cross-linguistic mathematical constraints on the order of cardinals
with addition v. multiplication. In the case of multiplication, in most languages, the higher cardinal follows the lower cardinal: thus, 200 is read *two hundred, not *hundred two. In the case of addition, in the absence of an overt conjunction, the high cardinal nearly always precedes the low cardinal: thus, 22 is read twenty-two, not *two-twenty; 102 is read a hundred two, not *two hundred, though the reverse order, two and twenty, where the overt conjunction unambiguously signals addition, is also attested, e.g. in German (38d).

(38) d. zweiundzwanzig
two and twenty
‘twenty-two’  (German)

Exceptions can be found for both generalizations. Thus in Scottish Gaelic cardinals 11 through 19 exhibit the order low-high. In many other languages (e.g. English), these cardinals form a single word, with the internal order low-high (nine-teen). On the other hand, the first generalization is violated by a number of languages, among which are Sinhala and Maori, where the ordering of cardinals in multiplication is high-low.28

4.3.2 Non-standard coordination We propose that cardinal coordination is not constrained semantically but rather that extra-linguistic, arithmetical, constraints are involved: for instance, languages which use a base-ten system typically disallow the coordination of two simplex cardinals whose value is at least 10 but below 100 (hence the impossibility of twenty-seventeen, forty-thirty, ninety-ten), though there are exceptions (e.g. the French soixante-dix, lit. ‘sixty-ten’, for 70).

Interestingly, the presence of an overt coordination alleviates the constraints on which cardinals can be coordinated:

(64) a. *twenty seventeen books asyndetic coordination
b. #twenty and seventeen books overt conjunction

As indicated by the grammaticality judgments, in the presence of an overt conjunction, convention can sometimes be overridden. This is used in the following line from a children’s poem by the Russian poet Taffy:

(65) tridcat’- tri i dva kota i četyre koški Russian
thirty three and two cat.m–pl and four cat.f–pl
‘Thirty-three and two tomcats and four tabbycats.’

28 Unlike in Shona and Yoruba, where cardinals follow nouns and it can therefore be argued that the NP is head-final, in Sinhala and Maori cardinals precede nouns and this argument is not available (see Hurford 2003; Ionin & Matushansky, in preparation, for details and discussion).
The possibility of (65) argues that the constraints on coordination can be overridden when there is some reason to separate two pluralities, and this requires an overt coordination. Again, we draw a parallel with measure phrases (cf. (39)). Convention dictates that the larger measurement unit (feet) appears before the smaller one (inches), as in (66a); with asyndetic coordination, the reverse order is impossible (66b). However, when an overt conjunction is used, convention can be overridden: both orders are possible, as indicated by (66c–d). For (66d) to be acceptable, it is helpful to clearly separate the six inches of silk from the six feet of silk in space and/or time. This is parallel to what happens with cardinals in (65).

(66) a. I bought six feet six inches of blue silk.
   b. ??/∗I bought six inches six feet of blue silk.
   c. I bought six feet and six inches of blue silk.
   d. I bought six inches and (then) six feet of blue silk.

We speculate that an overt conjunction changes the prosodic properties of the cardinal-containing xNP and makes it compatible with the new interpretation.

4.3.3 Cross-linguistic and intra-linguistic variation Complex cardinals differ as to whether they use asyndetic or overt coordination, within the same language, as well as between languages. For instance, in English, twenty-two disallows an overt and, while one hundred and one requires it, and in three hundred (and) fifty, it is optional. We note that similar differences exist in other coordinated structures as well. For instance, consider times: when minutes and seconds are coordinated, overt conjunction is optional (68a), while when years and days are coordinated, overt conjunction is obligatory, or at least strongly preferred (68b). Thus, coordinated cardinals in which an overt conjunction is obligatory vs. optional have a parallel in other types of coordinated structures.

(68) a. She ran the race in five minutes (and) ten seconds.
   b. She lived there for five years *(and) ten days.

We note that the arithmetic constraints on coordination of cardinals have to be part of any theory of complex cardinals, not just ours. For instance, take a theory that treats complex cardinals as morphological compounds, constructed entirely in the lexicon: it would have to assume the role of arithmetic constraints on compound formation. We do not see why assuming arithmetic constraints in morphology is any more explanatory than assuming them in syntax. Furthermore, given the facts in (66), the morphological compound view would have to treat complex measure phrases such as six feet six inches as compounds
as well, which does not seem particularly desirable. The advantage of our syntactic view is that, once the extra-linguistic factors are taken into account, arithmetic operations can be done via entirely linguistic means, consistently with cross-linguistic data.

5 CONCLUSIONS AND NEW QUESTIONS

We have proposed that simplex cardinals are semantic modifiers and complex cardinals are constructed by regular syntactic means (complementation and coordination, for the languages discussed here). This analysis allows us to satisfactorily account both for the semantics of complex cardinals and for their cross-linguistic syntax. In Ionin & Matushansky (2004), we show that this analysis also allows us to account for the (seemingly) special properties of the English modified cardinal construction in (18). For non-classifier languages, we propose that complex cardinals do not form a constituent to the exclusion of the lexical xNP (until right-node raising or NP-deletion).

The data presented indicate that multiplication and addition in language use entirely linguistic means: standard syntax as well as independently attested principles of semantic composition. We show that properties of cardinals that do not follow from standard syntax and semantics can be accounted for by extra-linguistic constraints, which are necessary under any theory of complex cardinals and play a role in non-numerical areas. On the other hand, a theory that treats the composition of complex cardinals as entirely extra-linguistic fails to account for number marking and Case assignment within complex cardinals.

Some questions remain open for further study. We have suggested that a reconciliation between the conflicting structures suggested by the patterns of number marking and Case assignment can be derived via semantic concord. This means that the interaction of semantic concord and the Agreement Hierarchy must be examined in detail. We also need to address the question of number agreement and concord with cardinal-containing xNPs, since different patterns are attested cross-linguistically (singular or plural marking on the lexical xNP, on the verb and on the determiners and modifiers in the cardinal-containing xNP). Ordinals and non-nominal cardinals also need to be explored more deeply, as do non-multiplication arithmetic operations, exemplified in (2c) and (45).

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APPENDICES: OTHER ISSUES IN THE SYNTAX OF NUMERALS

We now consider some other issues in the syntax of numerals in order to show that they can be successfully treated under our analysis: modified numerals and cardinals inside arithmetic expressions.

APPENDIX 1: MODIFIED NUMERALS

Constructions such as (69a–c) (Kadmon 1987; Krifka 1992, 1999; Corblin, to appear) and (69d) (Corver & Zwarts 2004) are usually assumed to involve modification of the numeral, before it has combined with the lexical xNP:

\[
\begin{align*}
(69) & \quad \text{standard bracketing} \\
& \quad \begin{array}{l}
\text{a. } \left[\text{more than ten}\right] \text{ books} \\
\text{b. } \left[\text{at least ten}\right] \text{ books} \\
\text{c. } \left[\text{exactly ten}\right] \text{ books} \\
\text{d. } \left[\text{just about ten}\right] \text{ books}
\end{array}
\end{align*}
\]

However, the syntax that we have proposed for cardinals requires them to have the seemingly counter-intuitive bracketing structure in (69): the cardinal must combine with the lexical xNP before combining with the quantifier:

\[
\begin{align*}
(70) & \quad \begin{array}{l}
\text{a. } \left[\begin{array}{c}
\text{more than } \\
\text{ten books}
\end{array}\right] \\
\text{b. } \left[\begin{array}{c}
\text{at least } \\
\text{ten books}
\end{array}\right]
\end{array}
\end{align*}
\]

\[29\] Corblin (to appear) argues that while at least combines with an xNP and denotes a relation between two sets, more than is cardinal-internal and denotes a relation between two numbers. If correct, his proposal can be used as an argument in favour of the specifier structure in (22b).
There is in fact independent evidence against the bracketing in (69). First of all, for this bracketing to work, the PP must be linearized before the constituent to which it is attached (i.e. the cardinal), as in (69d), an order unattested elsewhere in English. Secondly, two books can be replaced by an xNP such as the predicted number of books—supporting our proposal that two books in the constructions in (70) is a constituent:

(71)  a. [more than [the predicted number of books]]
     b. [at least [the predicted number of books]]
     c. [exactly [the predicted number of books]]
     d. [just about [the predicted number of books]]

Thus, at least in English, an analysis of modified numerals and prepositional numerals does not necessitate that more than ten, exactly ten, etc. form a maximal projection to the exclusion of the lexical xNP. The same result can be obtained for other languages.

APPENDIX 2: COUNTING

We have been arguing against the standard assumption that a complex cardinal is a unit to the exclusion of the lexical xNP complement. The question arises how we can deal with mathematical examples like (72), where cardinals (simplex as well as complex) function as arguments.

(72)  a. Two and two is four.
     b. Seven times seven is 49.
     c. Twenty-five plus seventy-four is ninety-nine.

(73)  a. I added two/this number to two.
     b. I multiplied seven/this number by seven.

In examples like (73), cardinals behave like xNPs of the semantic type e in that they appear in normal argument positions. The question is which of the two forms is the basic one. One answer is that the counting form, as in (72)–(73), is the basic one, and the xNP-internal form is derived from it. On this view, an item such as two hundred and four refers to a particular number (an abstract entity).

30 If our approach is correct, then we predict that argument positions, usually considered to be reserved for xNPs, can be filled by PPs ([PP Between 20 and 30 people] arrived). This is not necessarily a problem, given that PPs can appear in the subject position of copular predicates (Under the bed is a weird place to sleep).
On our approach to the semantics of cardinals exactly the opposite has to be true. Two questions therefore have to be answered: (a) what is the mechanism allowing the transition from the use of cardinals as singular terms (74a) to their xNP-internal use (74b),^31 and (b) is there any independent motivation for assuming that the xNP-internal use of cardinals is basic?

(74)  
\begin{align*}
\text{a. } & \text{The number of moons of Jupiter is four.} \\
\text{b. } & \text{Jupiter has four moons.}
\end{align*}

The answer to the first question is provided by Hofweber (2005). Hofweber compares NP-ellipsis with numerals (75a) to generic statements involving ‘bare determiners’ (75b). He observes that in both cases, cardinals (except for one) trigger plural agreement, and suggests that generic statements involving ‘bare determiners’ involve ellipsis as well, although of a slightly different kind. In particular, Hofweber argues that generic statements like (75b) can be unified with the arithmetic statements like (75c):

(75)  
\begin{align*}
\text{a. } & \text{Two are for you, and two for me.} \\
\text{b. } & \text{Two are more than none.} \\
\text{c. } & \text{Two and two are four.}
\end{align*}

His concrete proposal is that semantically bare determiners involve implicit quantification:

(75)  
\begin{align*}
\text{b'. } & \text{For whatever } X, \text{ two } X \text{ are more than no } X. \\
\text{c'. } & \text{For whatever } X, \text{ two } X \text{ and two (more) } X \text{ are four } X.
\end{align*}

We concur with this proposal and assume that NP-internal cardinals are the basic form, and mathematical cardinals are nominalizations. Independent evidence for this view comes from the fact that while mathematical cardinals always behave as nouns, xNP-internal cardinals don’t: as Hurford (2001) observes, cross-linguistically, lower cardinals are often adjectives. It depends on the language whether a particular xNP-internal cardinal behaves like a noun or like an adjective, so if mathematical cardinals are treated as the basic form and xNP-internal cardinals are derived from them, an additional stipulation is required to account for the categorial diversity of the latter. Our account is therefore more economical, and also relies on the transition from a less abstract meaning (the property of a set) to a more abstract one (a theoretical entity).

^31 Hofweber (2005) calls this question Frege’s other puzzle (Frege 1884).
Our account differs from Hofweber’s concerning cases where singular agreement is used, as in (75d). In such cases, Hofweber claims, we’re not dealing with bare determiners but with singular terms, and the copula is one of identity.

(75) d. Two and two is four.

Hofweber suggests that the transition from (75c) to (75d) involves ‘type lowering’, from operations on determiners (type \((e, t, e, t)\)) to operations on entities. This ‘cognitive type coercion’ is required for cognitive reasons, unlike the more familiar type shifting.

We suggest that this last step is unnecessary, and the cases in (74c) and (74d) should be treated the same. Our motivation for this proposal comes from the following variation:

(76) a. Two hours in the alien ship was/were clearly not enough.

b. 10 miles separate(s) the castle from the dragon lair.

Either plural or singular agreement can be used with bona fide cardinal-containing xNPs when the subject is an abstract measure xNP. This informal description certainly applies both to the ‘bare determiners’ in (75c) and ‘mathematical cardinals’ in (75d). The slight preference for the singular agreement can be assimilated to the same preference with more abstract predicates:

(77) a. Two hours is/?? are such a short time, really.

b. 10 miles is/?? are not much of a distance.

We conclude that Hofweber’s implicit quantification view, in combination with the known variation in agreement, can account for mathematical cardinals on the basis of regular syntax.

The behaviour of plus and minus, which would seem to be purely mathematical expressions, lends further support to the intuition that simple arithmetic operations are expressed via standard linguistic means. Besides their mathematical sense, they also have a regular linguistic meaning of (xNP-connecting) as well as and excepting, as shown by their use in informal contexts (all my friends plus my colleagues came to the party). Nonetheless, historically, plus and minus started out in many languages as purely mathematical terms and their informal use is a back-formation. This back-formation is inexplicable without a most natural assumption that mathematical (and other scientific) usage always rests on standard linguistic means—back-formation is then driven by parallelism with other terms. However, this natural assumption also forces us to conclude that the mathematical use of numerals is derived from the xNP-internal one.
REFERENCES


University of Amsterdam, Amsterdam. 483–512.


