Conjunction is Parallel Computation

I propose a new, game theoretical, analysis of conjunction which provides a single logical translation of *and* in its sentential, predicate, and NP uses, including both Boolean and non-Boolean cases. In essence it analyzes conjunction as parallel composition, based on game-theoretic semantics and logical syntax by Abramsky (2007).

I aim to account i.a. for conjunctions of quantified NPs in the context of group predicates:

(1) *Every man and every woman kissed (each other)* in the reading 'For every man x and for every woman y, x and y kissed (each other)', or 'every man-woman pair kissed.' In a model with n men and m women, this requires $n \times m$ kissings.

Proposals on the interpretation of conjoined NPs include algebraic Boolean operations (Keenan and Faltz, 1985), mereological sums (Link, 1983), and combinations of sums with type shifting (Hoeksema, 1988). Examples like (1) are challenging for all of these proposals. The straightforward order-theoretic approach to Boolean compounding of quantifiers assigns *every man and every woman* the type of an ordinary generalized quantifier, and predicts (1) to be equivalent to **every man kissed (each other) and every woman kissed (each other).* Translation of *and* as mereological sums is not directly applicable in (1) because of a type mismatch: mereology is defined on entities but not on quantifiers. Hoeksema's solution comes closer to adequacy; *and* is interpreted as Linkian sums, but the two NPs scope out of the conjoined structure, predicting correct truth conditions for (1).

Hoeksema's solution, however, fails for two reasons. First, it runs contrary to independent evidence that quantifiers normally don't scope out of a conjoined structure. Second, Hoeksema's technique falsely predicts availability of scope dependency between the two quantifiers. In fact, conjoined quantifiers are generally scope-independent, compare:

(2) a. Three boys kissed three girls. **no conjunction**

b. Three boys and three girls kissed (each other). **conjoined quantifiers**

(2a) but not (2b) has the scope dependent reading 'there are three boys such that each of them kissed three girls' (triples of girls kissed may vary with the boy). (Both (2a) and (2b) have a scope-independent group reading 'a group of three boys was engaged in kissing with a group of three girls' whereby each of the boys may have kissed fewer than three girls).

(3) a. Every man kissed almost every woman
Scopal dependency: each man kissed a vast majority of women; the set of women kissed may vary arbitrarily with the man, to the degree that e.g. there might be few or no women that all men kissed.

b. Every man and almost every woman kissed each other
No scopal dependency: there's a fixed majority of women that all men kissed.

Desiderata for an adequate analysis of conjunction include compositionality, capturing scope independence of conjoined quantified NPs, and semantic generalization of the meaning of *and* across its various uses (branching (Sher, 1990) satisfies all but the last desideratum). My proposal relies on game-theoretic semantics (GTS), a theory designed to treat scope independence, where different types of meanings (quantifiers, sentences) can be represented uniformly as games. I propose to analyze sentences like (1) through paraphrases like

(4) Take an arbitrary man x and take an arbitrary woman y; they kissed each other.

'Take an arbitrary x' is an informal description of the game theoretic semantics for the universal quantifier $\forall x$. Note that the paraphrase translates NP conjunction by sentential *and*, and is similar to paraphrases for non-Boolean conjunction in Schein (1993). As the paraphrase suggests, the quantifier meaning is taken to be an instruction ('take an x') rather than a function onto truth values (as in generalized quantifier theory).

So my analysis is dynamic, more specifically game-theoretic (Hintikka, 1979). In game-theoretic semantics (GTS) sentences are interpreted as instructions for evaluating truth of a statement, formalized as games with two players, Verifier and Falsifier. An atomic formula ϕ is a trivial game in which the Verifier wins iff ϕ is true in the classical sense. Quantifiers denote moves in the game. $\exists x$ instructs the Verifier to pick a value for the variable x, and $\forall x$ is an instruction for the Falsifier to assign a value to x. (Note that $\forall x$ is a minimal game, too.) Truth in GTS is a notion secondary to verification procedure but is equivalent to classical truth in first order logic. A formula is true iff it denotes a game in which the Verifier has a winning strategy, i.e. Verifier can win no matter how Falsifier plays.

I propose to treat conjunction uniformly as parallel composition, an operation on games that Abramsky (2007) symbolizes as $||$. In the game $\phi \mid\mid \psi$, both ϕ and ψ are played in parallel, without temporal or causal relation between ϕ and ψ, and the Verifier wins iff she wins in both subgames. In addition to parallel composition, Abramsky proposes a sequential composition operator $\cdot$. Note that $\phi \mid\mid \psi \equiv \psi \mid\mid \phi$, but $\phi \cdot \psi$ does not equal $\psi \cdot \phi$. I assume that quantifiers combine with predicates via sequential combination (provably $\forall x \cdot \phi \equiv \forall x \phi$, $\exists x \cdot \phi \equiv \exists x \phi$), and interpret coordination of both sentences (*it rains and it is cold*) and quantifiers (*every man and almost every woman*) by parallel composition. For sentential coordination, parallel composition ($\phi \mid\mid \psi$) is truth-conditionally equivalent to standard conjunction: both ψ and ϕ must be true to make $\phi \mid\mid \psi$ true. Boolean predicate conjunction is analogous to the sentential case, given that predicates are interpreted as sentential formulas with an open variable: $[\![\text{everyone dances and sings}]\!]^{M,g} = \forall x.(\texttt{sing}(x) \mid\mid \texttt{dance}(x))$.

Formalizing the paraphrase in (4), the compositional logical translation of (1) is

(5) $[\![\text{Every man and every woman kissed each other}]\!]^{M,g} = [\forall^{[MAN]}x \mid\mid \forall^{[WOMAN]}y] \cdot \mathbf{kissed}(x, y)$
(notation for quantifier restriction $Q^{[A]}$ from Peters and Westerståhl (2006, p. 87).

The complex NP *every man and every woman* is translated as $\forall^{[MAN]}x \mid\mid \forall^{[WOMAN]}y$ which is a combination of semantic values for *every man* ($\forall^{[MAN]}x$) and *every woman* ($\forall^{[WOMAN]}x$).

Parallel composition is designed to be a representation of scope independence, so the proposal immediately covers examples like (2b) and (3b) which crucially involve quantifier independence. In (3b), for instance, $[\![\text{almost every woman}]\!]^{M,g}$ can be formalized as a game where the Verifier picks a sufficiently big subset WOMAN$'$ $\subset$WOMAN, and the Falsifier picks an arbitrary $x \in$WOMAN$'$; since parallel games are independent, the set of women involved in kissing doesn't vary with men.

Parallel combination is a compositional, unified translation of *and* in sentential and NP conjunction. Originally proposed in the game-theoretic framework, the idea of parallel composition is in principle compatible with other dynamic theories such as DPL (Groenendijk and Stokhoff, 1991). However, combining quantified NPs with $||$ is more natural in GTS, where both universal and existential quantifiers are interpreted dynamically, than in DPL, where universal quantification is static.