Ordering Combination for Modal Comparison

Background. On Kratzer's well-known analysis (Kratzer 1981, forthcoming), comparative modal predicates such as *more likely* and *more permissible* are treated in terms of the relation **better possibility** which itself is defined in terms of an ordering on possible worlds induced by contextual factors. As is familiar, a contextually-determined *modal base* $f(w)$ picks out the accessible worlds ($\cap f(w)$), and these are ordered by a set of relevant propositions—the *ordering source* $g(w)$—such that for any two worlds v, z in $\cap f(w)$, $v \leq_{\mathrm{g(w)}} z$ ("v comes at least as close to the ideal $g(w)$ as z does") iff every ordering source proposition which holds in z also holds in v. One way of defining comparative possibility is given in (1).

(1) p is **at least as good a possibility** as q in w with respect to f and g iff there is no accessible world in $q - p$ that is more ideal (higher ranked by $\leq_{\mathrm{g(w)}}$) than every accessible world in $p - q$; p is a **better possibility** than q iff p is at least as good a possibility as q but not vice versa. (Kratzer, forthcoming)

A number of challenges have called into question the viability of the ordering-based approach in general (e.g., Portner 2009, Yalcin 2010, Lassiter 2011). In this talk, we address two puzzles which exemplify important features of these challenges, and present solutions to these puzzles which point toward a deeper understanding of comparative modality. Our main idea is to introduce mechanisms for constructing derived ordering sources used to interpret modal expressions. The resulting ordering sources model (i) how expectations or requirements "add up", and (ii) how ranked sets of expectations or priorities are combined.

Challenge 1: Expectations adding up. Lassiter (2011) argues that Kratzer's approach to comparative likelihood yields unintuitive results in cases such as (2), in which the number of expectations satisfied seems central to determining what is likely.

(2) [Context: Bill is extremely predictable. He almost always drives to and from work, arrives home by 6 p.m., and has macaroni for dinner.] It is more likely that Bill will have something other than macaroni for dinner than it is that he will both fail to be home by 6 p.m. *and* fail to drive his car.

(2) would typically be taken to be true here. But the **better-possibility** relation induced by $\leq_{\mathrm{g(w)}}$ (where the ordering source $g(w)$ contains the three expectations that Bill drives, that he is home by 6, that he has macaroni), does not predict the truth of (2), because certain worlds in which Bill fails to have macaroni are not related by $\leq_{\mathrm{g(w)}}$ to certain worlds in which he fails on the other two expectations. The judgement that (2) is true appears to be based on a different, derived, ordering source: one that models the intuition that the more expectations in $g(w)$ are satisfied, the better.

(3) **Expectations/priorities adding up**
For any ordering source A, $OS_{add\text{-}up}(A) =_{def.} \bigcup_i p_i$,
where $p_i =_{def.} \{w :$ at least i propositions in A are true in $w\}$.

The **better-possibility** relation induced by the derived ordering source $OS_{add\text{-}up}(g(w))$ correctly models truth judgements about (2), since worlds in which only one expectation fails are more highly ranked according to $\leq_{OS_{add\text{-}up}(g(w))}$ than those in which two fail. We claim such derived ordering sources are often the basis for statements of comparative modality.

Challenge 2: Multiple orderings. Comparative modality is also often sensitive to multiple orderings. The truth of (4) in the context given, for example, is sensitive to the likelihood of outcomes as well as their desirability (Goble 1996, Lassiter 2011):

(4) [Context: A doctor must choose one of two medicines—A or B—to administer to a critically ill patient. A has a small chance of producing a total cure and a large chance of killing the patient. B is sure to save the patient's life, but will leave him slightly debilitated.] It is better to administer medicine B than to administer medicine A.

We propose that cases such as this are to be analyzed in terms of a **better-possibility** relation based on an ordering source derived from a prioritized sequence of ordering sources (cf. Kratzer 1981, von Fintel and Iatridou 2008). For (4), there is a stereotypical ordering source that models the likelihood of outcomes (OS_1) which takes priority over an ordering source capturing desirability of outcomes (OS_2). In (5) we define a general merging operation for ordering sources which gives priority to the considerations encoded in the first.

(5) **Ordered merging of expectations/priorities**

$g_1 * g_2 =_{def.} g_1 \cup \{\bigvee\{\bigwedge x | x \in c_{n+1}\} \vee ((\bigvee\{\bigwedge x | x \in c_n\}) \wedge y),$
where for $n \geq 0$, c_n is the set of all subsets of g_1 of cardinality n, and $y \in g_2\}$

This operation of *-merging is analogous to the lexicographical *-combination of posets: the secondary ordering g_2 only plays a role in ordering a pair of worlds when the primary g_1 doesn't determine a linear ordering between them

For concreteness, we characterize the context of (4) in (6). L1 and L2 are expectations about biological processes (e.g., "the patient's endocrine system produces the normal variant of gutsophine"; "the patient's immune system reacts to medicine A"), and knowledge about the interaction of L1 and L2 with medicines A and B is encoded in the modal base:

(6) Modal base: taking B leads to survival of the patient but not full recovery,
taking A when L2 occurs leads to death,
taking A when ¬L2 and ¬L1 occur leads to death,
taking A when ¬L2 and L1 occur leads to complete recovery, ...

OS_1: $OS_{add\text{-}up}(\{L1, L2\}) = \{ L1 \wedge L2, L1 \vee L2 \}$
(The most likely worlds are those in which both L1 and L2 happen;
The least likely are those in which neither happen.)

OS_2: { The patient lives, The patient is perfectly healthy }
(The most desirable worlds are those in which patient lives perfectly healthy;
the least desirable are those in which patient dies.)

Given (6), the merged ordering source $OS_1 * OS_2$ ranks highest those worlds in which L1 and L2 both occur and yields a **better-possibility** ordering according to which (4) is true.

Summary. We show how two problems for ordering semantics can be solved through the use of ordering sources derived by adding up and merging of simpler ordering sources. These proposals have application to other puzzles which arise with expressions of comparative modality and weak necessity, and future work should extend them to the compositional treatment of gradable modals, and to quantitative expressions of probability and possibility.

References. von Fintel, K., and S. Iatridou. 2008. How to say *ought* in foreign. In *Time and modality*, Guéron and Lecarme (eds.), 115–141. Goble, L. 1996. Utilitarian deontic logic. *Philosophical Studies* 82:317–357. Kratzer, A. 1981. The notional category of modality. In *Words, worlds, and contexts*, Eikmeyer and Rieser (eds.), 38–74. Kratzer, A. forthcoming. *Collected papers on modals and conditionals*. Oxford University Press. Lassiter, D. 2011. Measurement and modality: The scalar basis of modal semantics. PhD Dissertation, NYU. Portner, P. 2009. *Modality*. Oxford University Press