The objective of this chapter is to demonstrate that degrees should be formalized as intervals on a scale rather than as points, as traditionally assumed. Using cross-polar anomaly as the empirical basis for my claims, I will argue that gradable adjectives denote functions from objects to intervals on a scale, or extents, and assume an ontology which distinguishes between two sorts of extents: positive extents and negative extents (as in Seuren 1978, von Stechow 1984b, and Löbner 1990). I characterize the difference between positive and negative adjectives as a sortal one: positive adjectives denote functions from objects to positive extents, and negative adjectives denote functions from objects to negative extents. After setting this analysis into the semantic framework developed in chapter 2, I show that cross-polar anomaly can be explained in the same way as incommensurability: a consequence of the sortal characterization of adjectival polarity is that the ranges of positive and negative adjectives are disjoint; as a result, the comparison relation in examples of cross-polar anomaly is undefined. Section 3.2 continues with an examination of comparison of deviation constructions, which at first glance appear to be counterexamples to the proposals made in section 3.1, but upon closer examination turn out to fit in naturally with the analysis of degree constructions developed in chapter 2. Finally, section 3.3 demonstrates that the algebra of extents, in conjunction with the semantic analysis of gradable adjectives as measure phrases, has the additional positive result of providing the basis for an insightful explanation of the monotonicity properties of gradable adjectives.

3.1.1 Degrees and Polar Opposition

As observed in chapter 1 (see section 1.1.4), a characteristic of many (though not all) gradable adjectives is that they come in pairs: tall - short, safe - dangerous, sharp - dull, etc. In a basic sense, the meanings of the members of such pairs are fundamentally the same: they apply to the same objects, and they provide the same kind of information about the degree to which an object possesses some gradable property. For example, the antonymous adjectives tall and short both provide information about an object's height. At the same time, positive and negative adjectives represent different—and in some sense complementary—perspectives on the degree to which an object possesses a gradable property.

3.1.2 Degrees and Polar Opposition

The semantic analysis of gradable adjectives and degree constructions developed in chapter 2, in conjunction with the semantic analysis of gradable adjectives as measure phrases, has the additional property of providing the basis for an insightful explanation of the monotonicity properties of gradable adjectives.
property.  Intuitively, tall measures the height an object has; short measures the height an object does not have. The empirical manifestation of this distinction in perspective can be observed in differences in the interpretations of absolute and comparative constructions with positive and negative adjectives. The basic analysis of the semantics of positive adjectives can be illustrated by considering interpretation of (1). The logical form of (1) is (2), where \text{slong} denotes a standard of longness relative to some comparison property appropriate for The Brothers Karamazov (e.g., the property of being a Russian novel).1

(1) The Brothers Karamazov is long.

(2) \text{abs} (\text{long}(\text{BK})) (\text{slong})

According to the truth conditions for the absolute construction proposed in chapter 2, (1) is true just in case the degree to which The Brothers Karamazov is long is at least as great as the standard value; i.e., if and only if the relation illustrated in (3) holds, where \text{dBK} is the degree of The Brothers K.'s length.

(3) length: 0 \quad \text{slong} \quad \text{dBK} \quad \infty

Absolute constructions with negative adjectives may be analyzed in basically the same way, with one important modification: the relation between the reference value and the standard value must be reversed. This point is illustrated by the analysis of (4).

(4) The Dream of a Ridiculous Man is short.

1See the discussion of the derivation of implicit standards in chapter 2, section 2.3.2; I use the variable \text{slong} rather than the complex expression \text{stnd} (\text{long} (\text{p}(\text{The Brothers Karamazov}))) for perspicuity.

The logical representation of (4) is (5), where \text{sshort} denotes the standard of shortness appropriate for The Dream of a Ridiculous Man, which may or may not be the same as the standard of longness in a given context. The crucial difference between (4) and (1) is that in the former the partial ordering relation associated with the absolute construction must be reversed, whereas in the latter it is left intact since the function \text{abs} is only applied to the reference value and the standard value is left unchanged. Accordingly to the truth conditions for the absolute construction proposed in chapter 2, (4) holds when \text{sshort} is the standard value (e.g., if only if the relation illustrated in (6) holds, where \text{dD} is the degree of The Dream of a Ridiculous Man's length).

(6) length: 0 \quad \text{dD} \quad \text{sshort} \quad \infty

Comparatives with positive and negative adjectives show exactly the same difference in ordering relations as absolute constructions. Consider, for example, (7), which has the logical representation in (8).

(7) The Brothers Karamazov is longer than The Idiot.

(8) \text{more} (\text{long}(\text{BK})) (\text{long}(\text{Idiot}))

2In this context, a novel whose length is between 400 and 600 pages is neither long nor short; in Klein's (1980) terminology, it falls in the "extension gap" on the scale of length. This example points out the fact that antonymous adjectives are, in effect, contraries: although an object may be neither long nor short, no object is both long and short. This relation is enforced by a general constraint on the relation between positive and negative standards; see Bierwisch 1989 for discussion.

The Brothers Karamazov is long.
The relational differences between positive and negative adjectives illustrated by these examples suggest a means of formally representing adjectival polarity within the algebra of degrees outlined in section 3.1.1. As noted above, the members of a positive and negative pair of gradable adjectives not only apply to the same objects (i.e., share the same domain), they also provide the same basic information about the objects to which they apply: they measure the degree to which an object possesses some gradable property. Another way of stating this observation is that positive and negative adjectives measure objects according to the same dimension. The crucial difference between positive and negative adjectives, illustrated by the examples discussed above, is that the measurements they return permit complementary orderings on their domains. This was most clearly shown in the comparatives in (7) and (10): when \( \text{The Brothers Karamazov} \) "exceeds" \( \text{The Idiot} \) with respect to longness, \( \text{The Idiot} \) "exceeds" \( \text{The Brothers Karamazov} \) with respect to shortness.

The analysis of gradable adjectives as measure functions supports a formal representation of adjectival polarity which captures the intuitions about polar adjectives outlined here. The claim that gradable adjectives denote functions from objects to degrees supports a formal representation of gradable adjectives as measure functions that captures the intuition that positive and negative pairs of adjectives measure objects according to the same dimension. To make this idea more precise, I will assume the general principle in (13) (cf. Rullmann 1993).

\[
(x,y) = (y,x)
\]

and

\[
\forall x \in \mathbb{D}, \quad (x, \phi x) = (\phi x, x)
\]

for any scale \( \phi \), domain \( \mathbb{D} \), and morpheme \( \phi \). This principle states that the orderings induced by the positive and negative versions of any morpheme are complementary. This is illustrated by the comparatives in (7) and (10): when \( \text{The Brothers Karamazov} \) "exceeds" \( \text{The Idiot} \) with respect to longness, \( \text{The Idiot} \) "exceeds" \( \text{The Brothers Karamazov} \) with respect to shortness.

Negative comparatives can be analyzed in the same way as their positive counterparts, with the exception that the ordering relation introduced by the degree morpheme must be reversed, as in negative absolute constructions. This is illustrated by (11).

\[
\text{The Idiot is shorter than The Brothers Karamazov.}
\]

In the context illustrated by (11), where \( \phi x \) indicates the degree to which \( x \) possesses some gradable property, the degree which \( \text{The Idiot} \) possesses the property of shortness is long, which contradicts the claim that \( \text{The Brothers Karamazov} \) "exceeds" \( \text{The Idiot} \) with respect to longness.
Specifically, I will assume that the range of the positive is the set of degrees on the scale under the basic ordering provided by the scale ("<"), while the range of the negative the set of degrees on the scale under the reverse ordering (">"). On this view, a negative adjective is the dual of its positive counterpart (cf. Cresswell 1976, Gawron 1995, Sánchez-Valencia 1995, Rullmann 1995).

The claim underlying this representation of adjectival polarity is that positive and negative pairs of adjectives map the objects in their domains to degrees on the same basic scale, but their ranges are differentiated by virtue of being associated with opposite ordering relations. Since there is a one-to-one mapping between the range of \( \phi \) and the range of \( \phi \) (the identity function), however, the two sets are isomorphic. In other words, for any scale \( S \), the set of positive degrees on \( S \) and the set of negative degrees on \( S \) contain the same objects. This has a very important empirical result: it explains the validity of statements like (14), a minimal requirement of any theory of the semantics of polar adjectives and comparatives.

(14) The Brothers Karamazov is longer than The Idiot if and only if The Idiot is shorter than The Brothers Karamazov.

If positive and negative adjectives define identical mappings from objects in their domains to degrees on a scale, but differ the ordering they induce, then assuming that the comparative (and absolute) morphemes "inherit" the ordering associated with the adjective that heads the degree construction, it follows that the coordinated propositions in (14) (i.e., (7) and (10)), are true in the same situations: whenever the degree of The Brothers' length exceeds the degree of The Idiot's length.3

3If a negative adjective is the dual of its positive counterpart, then the validity of (14) follows from the Duality Principle: if a statement \( \phi \) is true in all orders of type T (partial order, total order, etc.), then its dual is also true in all orders of type T (see Landman 1991:88).

3.1.3 Cross-polar Anomaly

Sentences such as (15)-(18), which exemplify the phenomenon that I referred to in chapter 1 as "cross-polar anomaly", present an interesting puzzle for the algebra of degrees discussed in the previous two sections, in which degrees are represented as points on a scale.

(15) # The Brothers Karamazov is longer than The Idiot is short.
(16) # The Idiot is shorter than The Brothers Karamazov is long.
(17) # Mars is closer than Pluto is distant.
(18) # A brown dwarf is dimmer than a blue giant is bright.

These examples indicate that comparatives constructed out of a positive and negative pair of adjectives are semantically anomalous. Crucially, (15)-(18) contrast with examples of comparative subdeletion involving adjectives of the same polarity, such as (19)-(20).

(19) The table is longer than it is wide.
(20) # The table is longer than it is short.

See also Bierwisch 1989 for a discussion of similar facts in German.

(193193) 4The anomaly of sentences like (15)-(18) was first observed by Hale (1970), who notes the

contrary prediction (i) and (ii):

(i) The table is longer than it is short.
(ii) The table is shorter than it is short.

The anomaly of sentences like (15)-(18) was first observed by Hale (1970), who notes the

contrary prediction (i) and (ii):

(i) The table is longer than it is short.
(ii) The table is shorter than it is short.
Carmen's Cadillac is wider than Mike's Fiat is long.

Fortunately, the ficus was shorter than the ceiling was low, so we were able to get it into the room.

The puzzle of cross-polar anomaly is that the very same assumptions which provided an intuitive characterization of adjectival polarity, as well as an explanation of the validity of (14), make the wrong predictions in the case of cross-polar anomaly.

Examples of cross-polar anomaly are structurally instances of comparative subdeletion, therefore, assuming the analysis of subdeletion structures presented in section 2.4.1.2 of chapter 2, the logical representation of (15) is (21).

\[
\text{(21)} \quad \text{more (long (The Brothers K)}) = \text{max}(\lambda d [\text{abs} (\text{short (The Idiot)}) (d)])
\]

The standard value in (21) is the degree introduced by the comparative clause: the maximal degree \(d\) such that \(\text{The Idiot}\) is at least as short as \(d\). According to the analysis of adjectival polarity outlined in section 3.1.2, the adjectives \(\text{long}\) and \(\text{short}\) define the same mapping from the objects in their domains to points on a scale of length. It follows that degrees of longness and shortness are the same objects, therefore, the comparative clause in (21) should pick out not only the maximal degree in the range of \(\text{short}\) that represents \(\text{The Idiot}\)'s length, but also the maximal degree in the range of \(\text{long}\) that represents \(\text{The Idiot}\)'s length, since these two degrees are in fact the same objects. That is, even though the orderings on the ranges of \(\text{long}\) and \(\text{short}\) are distinct, since degrees of longness and degrees of shortness are the same objects are in fact the same objects, this result is not dependent on the choice of the specific adjectives \(\text{long}\) and \(\text{short}\), it holds for any positive-negative pair. Since positive and negative degrees denote the same objects, any equivalence relation of the form in (22) holds, with the result (4) that the degrees of longness and shortness share the same objects of comparison.

\[
\text{(22)} \quad \text{max}(\lambda d [\text{abs} (\text{long (The Idiot)}) (d)]) = \text{max}(\lambda d [\text{abs} (\text{short (The Idiot)}) (d)])
\]

If this is the case, however, then (15) should not only be interpretable, it should be logically equivalent to (7) and (10). This is clearly the wrong result.

What is important to observe is that the result in (15) does not depend on the choice of the adjectival polarities, it should be logically equivalent to (7) and (10) when the same adjectives are used.

The standard value in (15) is the degree introduced by the comparative clause: the maximal degree \(d\) such that \(\text{The Brothers K}\) is at least as long as \(d\). According to the analysis of adjectival polarity outlined in section 4.4, the degree of longness and shortness share the same objects of comparison. Since the orderings on the ranges of \(\text{long}\) and \(\text{short}\) are distinct, since degrees of longness and degrees of shortness are the same objects, any equivalence relation of the form in (22) holds, with the result (4) that the degrees of longness and shortness share the same objects of comparison.

\[
\text{(23)} \quad \text{deg} (\text{dpos}) (\text{dneg})
\]

\[
\text{(24)} \quad \text{deg} (\text{dneg}) (\text{dpos})
\]

Simply put, there is no property of an algebra of degrees which explains the anomaly of comparing positive and negative adjectives.

The standard value in (15) is the degree introduced by the comparative clause: the maximal degree \(d\) such that \(\text{The Brothers K}\) is at least as long as \(d\). According to the analysis of adjectival polarity outlined in section 4.4, the degree of longness and shortness share the same objects of comparison. Since the orderings on the ranges of \(\text{long}\) and \(\text{short}\) are distinct, since degrees of longness and degrees of shortness are the same objects, any equivalence relation of the form in (22) holds, with the result (4) that the degrees of longness and shortness share the same objects of comparison. Since the orderings on the ranges of \(\text{long}\) and \(\text{short}\) are distinct, since degrees of longness and degrees of shortness are the same objects, any equivalence relation of the form in (22) holds, with the result (4) that the degrees of longness and shortness share the same objects of comparison.

\[
\text{(25)} \quad \text{max}(\lambda d [\text{abs} (\text{long (The Brothers K)}) (d)])
\]

If this is the case, however, then (15) should not only be interpretable, it should be logically equivalent to (7) and (10). This is clearly the wrong result.

What is important to observe is that the result in (15) does not depend on the choice of the adjectival polarities, it should be logically equivalent to (7) and (10) when the same adjectives are used.

The standard value in (15) is the degree introduced by the comparative clause: the maximal degree \(d\) such that \(\text{The Brothers K}\) is at least as long as \(d\). According to the analysis of adjectival polarity outlined in section 4.4, the degree of longness and shortness share the same objects of comparison. Since the orderings on the ranges of \(\text{long}\) and \(\text{short}\) are distinct, since degrees of longness and degrees of shortness are the same objects, any equivalence relation of the form in (22) holds, with the result (4) that the degrees of longness and shortness share the same objects of comparison. Since the orderings on the ranges of \(\text{long}\) and \(\text{short}\) are distinct, since degrees of longness and degrees of shortness are the same objects, any equivalence relation of the form in (22) holds, with the result (4) that the degrees of longness and shortness share the same objects of comparison.

\[
\text{(26)} \quad \text{max}(\lambda d [\text{abs} (\text{short (The Idiot)}) (d)])
\]

If this is the case, however, then (15) should not only be interpretable, it should be logically equivalent to (7) and (10). This is clearly the wrong result.

What is important to observe is that the result in (15) does not depend on the choice of the adjectival polarities, it should be logically equivalent to (7) and (10) when the same adjectives are used.
3.1.4 On the Unavailability of an Incommensurability Explanation

Before I present an alternative characterization of degrees, I should address what at first glance appears to be a possible explanation of cross-polar anomaly within the algebra of degrees outlined here. This explanation rejects the claim that positive and negative pairs of adjectives share the same scale, and explains the anomaly of examples like (15)-(18) in terms of incommensurability.

(25) #Mike is taller than Carmen is clever.

(26) # The Idiot is more tragic than my copy of The Brothers Karamazov is heavy.

The explanation of the anomaly of examples like (25) and (26) developed in chapter 2 builds on the semantic analysis of subdeletion outlined above. According to these proposals, the logical representation of an example like (25) is (27).

(ii) $\exists d [d > \zeta d'.\text{short}(\text{Mike},d')\&(tall(\text{Carmen},d))]$

According to (ii), (i) is true just in case there is a degree which exceeds the maximum degree of Mike's shortness and Carmen is that tall. This means that (i) should be true in the situation represented in (iii), since $dM$ denotes both the degree to which Mike is short and the degree to which Carmen is tall.

(iii) \begin{align*}
\text{height} : & 0 \\
\hdashline
\text{dM} & \text{----------}
\text{dC} & \text{----------}
\end{align*} \infty

See Kennedy 1997b for more detailed discussion of the problem of cross-polar anomaly for within the context of a relational analysis of gradable adjectives.

The standard value introduced by the comparative clause in (27) is the maximal degree $d$ such that Carmen is at least as clever as $d$, therefore, in order to evaluate the truth of (27), an ordering must be established between this degree and the degree to which Mike is tall. Although the incommensurability explanation of cross-polar anomaly in these

\begin{align*}
\text{(ii) & height} & \text{----------} \\
\text{dM} & \text{----------} \\
\text{dC} & \text{----------} \\
\end{align*} \infty

The standard value introduced by the comparative clause in (27) is the maximal degree $d$ such that Carmen is at least as clever as $d$, therefore, in order to evaluate the truth of (27), an ordering must be established between this degree and the degree to which Mike is tall. Although the incommensurability explanation of cross-polar anomaly in these

\begin{align*}
\text{(ii) & height} & \text{----------} \\
\text{dM} & \text{----------} \\
\text{dC} & \text{----------} \\
\end{align*} \infty

The explanation of the anomaly of examples like (15) and (20) developed in chapter 2 builds on the semantic analysis of subdeletion outlined above. According to these proposals, the logical representation of an example like (15) is (28).

(28) #The idiot is more tragic than my copy of The Brothers Karamazov is heavy.

The adjectives "tall" and "clever" map their arguments onto different scales, however, so the ordering relation introduced by the comparative is undefined for these two degrees, resulting in semantic anomaly. Although the incommensurability explanation of cross-polar anomaly in these situations may appear to be a solution, it is not possible to establish an ordering between the degrees of tallness and cleverness which will make (28) true. The problem is that it must be the case that a mapping between e.g. degrees of tallness and degrees of shortness can be defined, otherwise the validity of (i) would remain unexplained (see Rullmann 1995 for additional discussion of this point). The incommensurability explanation fails because any algebra of degrees which explains the validity of (i) must also be able to explain the validity of (24) which is a minimal requirement of descriptive adequacy. Hence, no algebra of degrees which explains the validity of (24) can be used to explain the validity of (i).
Although it fails to explain cross-polar anomaly, the proposal that positive and negative adjectives are incommensurable contains an interesting hypothesis: that degrees of tallness and degrees of shortness are different sorts of objects, with the consequence that the range of a positive adjective is disjoint from the range of its negative counterpart. If this were the case, then cross-polar anomaly could be explained in the same terms as incommensurability: it would be impossible to evaluate the relation introduced by the comparative morpheme. Without an ad hoc stipulation to this effect, however, there is no way to build the necessary sortal difference between positive and negative degrees into a degree algebra: if corresponding positive and negative degrees on the same scale denote the same objects, then they are sortally indistinguishable from one another. In the following section, I will develop an alternative algebra of degrees in which positive and negative degrees are analyzed as distinct objects, and I will show that this distinction not only provides an insightful representation of adjectival polarity and an account of the validity of statements like (14), it also provides the basis for an explanation of cross-polar anomaly.

3.1.5 The Algebra of Extents

In order to develop a solution to the problem of cross-polar anomaly, I will adopt an ontology of comparison in which the projection of an object on a scale is represented not as a discrete point, but rather as an interval, which I will refer to as an extent (cf. Seuren 1978, 1984, von Stechow 1984b, Bierwisch 1989, Löbner 1990). In terms of the analysis of gradable adjectives developed here, the basic claim remains the same: every gradable adjective denotes a function from objects to scalar values, and scales are differentiated through association with a dimension. The difference is in the structure of the scalar values: whereas the algebra of degrees outlined in section 3.1.2 represents scalar values as discrete points, the extent algebra that I will formalize below represents scalar values as intervals. As I will demonstrate, this modification provides the basis for an analysis of adjectival polarity which both predicts the validity of statements like (14) and supports an explanation of cross-polar anomaly.
To make things more precise, assume that for any object $a$ which can be ordered according to some dimension $\delta$, there is a function $d$ from $a$ to a unique point on the scale $S^\delta$. Let the positive extent of $a$ on $S^\delta$, $\text{pos}^\delta(a)$, be as defined in (30), and let the negative extent of $a$ on $S^\delta$, $\text{neg}^\delta(a)$, be as defined in (31).

\begin{align}
\text{pos}^\delta(a) &= \{ p \in S^\delta | p \leq d(a) \} \\
\text{neg}^\delta(a) &= \{ p \in S^\delta | d(a) \leq p \}
\end{align}

According to the definitions in (30) and (31), the positive and negative projections of an object on a scale are (join) complementary. This is illustrated by (32), which shows the positive and negative extents of an object $a$ on the scale $S^\delta$.

\begin{align}
S^\delta: 0 &\quad \cdots \quad \text{pos}^\delta(a) \quad \cdots \quad \text{neg}^\delta(a) \quad \cdots \quad \infty
\end{align}

Given these background assumptions, we can now construct a formal analysis of adjectival polarity in terms of an algebra of extents. In section 3.1.2, I noted the following distinction between antonymous pairs of adjectives: although the members of a positive and negative pair of adjectives measure objects according to the same dimension, they represent complementary perspectives on the projection of an object onto scale. For example, the sentences "Carmen is tall" and "Mike is short" both provide information about the height of Carmen and Mike, respectively, but the information is qualitatively different in each case: the positive adjective "tall" conveys information about the height an object has, while the negative adjective "short" conveys information about the height an object does not have (cf. von Stechow 1984b:196). This point corresponds to a degree in the traditional algebra.

The distinction between positive and negative extents provided by the algebra of extents—specifically, the complementarity illustrated in (32)—provides a means of representing adjectival polarity in a way that captures these intuitions. Specifically, I will claim that positive adjectives denote functions from objects to positive extents, and negative adjectives denote functions from objects to negative extents. Put another way, I am claiming that adjectival polarity is a sortal distinction between positive and negative adjectives. All gradable adjectives denote functions from objects to extents, but the set of such functions can be sorted according to their range: positive adjectives denote gradable properties whose range is the set of positive extents on a scale; negative adjectives denote gradable properties whose range is the set of negative extents on a scale. This distinction, I argue, is the key to resolving the puzzles posed by the notion of positive and negative degree.

Section 1.9.4 (page 149)

According to the definitions in (30) and (31), the positive and negative projections of an object on a scale are (join) complementary. This is illustrated by (32), which shows the positive and negative extents of an object $a$ on the scale $S^\delta$. Let $\text{pos}^\delta(a)$ and $\text{neg}^\delta(a)$ denote the same object. In contrast, the extent algebra provides a means of representing adjectival polarity in a way that captures these intuitions. Specifically, I will claim that positive adjectives denote functions from objects to positive extents, and negative adjectives denote functions from objects to negative extents. Put another way, I am claiming that adjectival polarity is a sortal distinction between positive and negative adjectives. All gradable adjectives denote functions from objects to extents, but the set of such functions can be sorted according to their range: positive adjectives denote gradable properties whose range is the set of positive extents on a scale; negative adjectives denote gradable properties whose range is the set of negative extents on a scale. This distinction represents the fundamental difference between this approach to adjectival polarity and the one provided by the degree algebra outlined in section 3.1.2: in the former, but not the latter, the ranges of a positive and negative pair of adjectives are disjoint.

As observed in section 3.1.2, because degrees are defined as points on a scale, there is no way to differentiate positive and negative degrees. If gradable adjectives denote functions from objects to points on a scale, then for any object $a$ and any pair of positive and negative adjectives $\phi^\text{pos}$ and $\phi^\text{neg}$, $\phi^\text{pos}(a)$ and $\phi^\text{neg}(a)$ denote the same object. In contrast, extents have additional structure, namely the intervals extending to the relevant end of the scale. As a result, positive and negative extents are distinct objects, and for any object $a$ and any pair of positive and negative adjectives $\phi^\text{pos}$ and $\phi^\text{neg}$, $\phi^\text{pos}(a)$ and $\phi^\text{neg}(a)$ denote different objects. A consequence of this distinction is that, for any scale $S^\delta$, the sets of positive extents on $S^\delta$ and the sets of negative extents on $S^\delta$ are disjoint subsets of the total set of extents on $S^\delta$. This aspect of the extent algebra provides the foundation for the explanation of cross-polar
The ontological assumptions that I have outlined here have their roots in the work of Seuren (1978) (see also Seuren 1984), who analyzes gradable adjectives as relations between individuals and extents and makes a distinction between positive and negative extents along the lines of the one proposed in (28) and (29); this approach is also adopted in von Stechow 1984b and Löbner 1990.

The important contribution that the work reported here makes to this line of research is that it introduces a strong empirical argument—the phenomenon of cross-polar anomaly—for formalizing scalar values in terms of intervals and, in particular, for using the sortal distinction between positive and negative extents as the basis for a theory of adjectival polarity (see also the discussion of the distribution of measure phrases in section 3.1.8). Before I go through the explanation of cross-polar anomaly, however, I will show how the algebra of extents provides an explanation for the different relational characteristics of positive and negative adjectives observed in section 3.1.2, and how it accounts for the validity of statements like (14).

3.1.6 Extents and Polar Opposition

The formal representation of adjectival polarity proposed in the previous section has an important consequence: it derives the relational difference between positive and negative pairs of gradable adjectives, eliminating the need to assume that the comparative and absolute degree morphemes "inherit" their ordering relations from the adjectives they combine with. Instead, we can adopt a uniform set of truth conditions for degree morphemes as in (33)-(36), where the ordering relations on extents \( \{>,<,\geq\} \) are defined in a standard Boolean fashion, as in (37)-(39).

Consider, for example, the analysis of positive and negative absolute constructions, such as (40) and (41), in the context illustrated by (42), where \( \text{long} \) denotes the positive extent of \( \text{The Brothers Karamazov} \) on the scale of length, \( \text{short} \) its negative extent, and \( \text{slong} \) and \( \text{sshort} \) denote standards of longness and shortness (for e.g. Russian novels). The logical representations of (40) and (41) are (43) and (44) respectively.

As above, I assume that \( \text{slong} \) and \( \text{sshort} \) are derived as a function of the meaning of the adjective and a contextually determined comparison property, as described in chapter 2, section 2.3.
English

According to the truth conditions for the absolute in (33), (40) is true just in case the extent to which *The Brothers Karamazov* is long is at least as great as the standard value. In the context in (42), (40) is true, because long(BK) \(\geq\) slong holds.

The negative absolute in (41) can be analyzed in exactly the same way: it is true just in case the extent to which *The Brothers Karamazov* is short is at least as great as the standard of shortness. (43) is false in the context illustrated by (42), because short(BK) \(\geq\) sshort does not hold. What is important to observe is that the same ordering relation is used to calculate the truth of both the positive and negative absolute constructions. This contrasts with the analysis of this sentence in the algebra of degrees discussed in section 3.1.2, which required the ordering relation associated with the absolute to be reversed for negative adjectives.

The analysis of positive and negative comparatives is similar. Consider, for example, (45) and (46) in the context represented by (47), where long(BK) and short(BK) are as defined above, and long(Idiot) and short(Idiot) represent the positive and negative extents of *The Idiot*’s length, respectively.

(45) *The Brothers Karamazov* is longer than *The Idiot*.

(46) *The Idiot* is shorter than *The Brothers Karamazov*.

(47) length: 0 \[\ldots\] \(\infty\) 0 \[\ldots\] long(BK) \[\ldots\] short(BK) \[\ldots\] long(Idiot) \[\ldots\] short(Idiot) \[\ldots\] \(\infty\)

The logical representation of (45) is (48).

(48) more(long(BK)) \(\cdot\) more(long(Idiot))

According to the truth conditions for the comparative in (34), (45) is true just in case long(BK) > long(Idiot), i.e., just in case the positive projection of *The Brothers Karamazov* on the scale of length exceeds the positive projection of *The Idiot* on the scale of length. These conditions are met in the context represented by (47), so (45) is true. Again, there is no need to assume a change in the ordering relation associated with the comparative: the structure of extents is such that the same relation can be used to characterize the truth conditions of both positive and negative comparatives.

The negative comparative in (46) is treated in the same way. The logical form of (46) is (49).

(49) more(short(Idiot)) \(\cdot\) more(short(BK))

(46) is true just in case short(Idiot) > short(BK) holds, i.e., just in case the negative projection of *The Idiot* on the scale of length exceeds the negative projection of *The Brothers Karamazov* on the scale of length. These conditions are met in the context represented by (47) and (48), so (46) is true. Again, there is no need to assume a change in the ordering relation associated with the absolute: the structure of extents is such that the same relation can be used to characterize the truth conditions of both absolute and comparative constructions, which contrasts with the analysis of this sentence in the algebra of degrees discussed in section 3.1.2, where the ordering relation associated with *The Brothers Karamazov* is short is at least as great as the ordering relation associated with *The Idiot*. What is important to observe is that the same ordering relation is used in both cases.

(46) is true just in case the extent to which *The Idiot* is short is at least as great as the extent to which *The Idiot* is long. This discussion of (45) and (46) illustrate another important aspect of the analysis, namely that it explains the validity of constructions like (50).

(50) *The Idiot* is shorter than *The Brothers Karamazov* iff *The Brothers Karamazov* is longer than *The Idiot*.

(50) can be paraphrased in the following way: *The Idiot* is at least as long as it is short, and *The Brothers Karamazov* is at least as short as it is long. This is true because long(BK) > short(BK) holds. According to the truth conditions for the absolute in (33), (40) is true just in case the extent to which *The Brothers Karamazov* is long is at least as great as the extent to which *The Brothers Karamazov* is short.
statements like (50) can be viewed as substitution instances of (51), where \( \phi_{pos} \) and \( \phi_{neg} \) are antonymous gradable adjectives.

(51)
\[
\phi_{pos}(a) > \phi_{pos}(b) \text{ iff } \phi_{neg}(b) > \phi_{neg}(a)
\]

The validity of (51) follows from the claim that the positive and negative projections of an object on a scale are join complementary. In section 3.1.4, I claimed that the positive and negative projections of an object onto a scale along dimension \( \delta \), \( \text{pos}_{\delta}(a) \) and \( \text{neg}_{\delta}(a) \), correspond to (30) and (31), repeated below, where \( d \) is a function from an object to a point on the scale.

(30)
\[
\text{pos}_{\delta}(a) = \{ p \in S_{\delta} | p \leq d(a) \}
\]

(31)
\[
\text{neg}_{\delta}(a) = \{ p \in S_{\delta} | d(a) \leq p \}
\]

From these definitions, we can define the complements of positive and negative extents as follows:

(52)
\[
\text{neg}_{\delta}(x) = \text{pos}_{\delta}(x) - \{ d(x) \}
\]

(53)
\[
\text{pos}_{\delta}(x) = \text{neg}_{\delta}(x) - \{ d(x) \}
\]

Returning to (51), since the result of applying \( \phi_{pos} \) and \( \phi_{neg} \) to an object is a positive or negative extent on the scale shared by \( \phi_{pos} \) and \( \phi_{neg} \), we can rewrite (51) as the more general statement in (54), and use the equivalences in (52) and (53) to prove its validity.

(54)
\[
\text{pos}_{\delta}(a) > \text{pos}_{\delta}(b) \text{ iff } \text{neg}_{\delta}(b) > \text{neg}_{\delta}(a)
\]

If \( \text{pos}_{\delta}(a) > \text{pos}_{\delta}(b) \), then \( \text{pos}_{\delta}(a) - \{ d(a) \} > \text{pos}_{\delta}(b) - \{ d(b) \} \), since \( d(a) \) and \( d(b) \) are the maximal elements of \( \text{pos}_{\delta}(a) \) and \( \text{pos}_{\delta}(b) \), respectively. It follows that \( \text{neg}_{\delta}(a) > \text{neg}_{\delta}(b) \), by substitution, and finally that \( \text{neg}_{\delta}(b) > \text{neg}_{\delta}(a) \), by contraposition. The other direction of the biconditional can be proved in exactly the same way.

3.1.7 Cross-Polar Anomaly Revisited

We are now in a position to explain cross-polar anomaly. Recall that in section 3.1.3 I investigated the possibility of an explanation of this anomaly within a degree algebra in terms of incommensurability. The basic idea behind this type of explanation is that degrees of e.g. tallness and shortness, like degrees of e.g. tallness and cleverness, should be analyzed as distinct objects in different ordered sets. If this were the case, then the relation introduced by the comparative morpheme would be undefined, and cross-polar anomaly could be explained in the same way as incommensurability. I observed that this explanation is unavailable if degrees are defined as points on a scale, because the algebra of degree does not make a structural distinction between positive and negative degrees: for any scale, the set of positive degrees and the set of negative degrees are isomorphic. As a result, the set of positive degrees defined on a scale by the algebra of degree would be indistinguishable from the set of negative degrees defined on the same scale by the algebra of degree. Therefore, if we adopt the interpretation of examples of cross-polar anomaly, the substitution instance of (23) or (24) (repeated below), which schematically represent the interpretation of examples of cross-polar anomaly, should be perfectly interpretable.

(23)
\[
\text{deg}(d_{pos}(d_{neg}(d_{pos}(\cdot)))
\]

(24)
\[
\text{deg}(d_{neg}(d_{pos}(\cdot)))
\]

In contrast, the algebra of extents makes a structural distinction between positive and negative extents: positive and negative extents are different sorts of objects, which represent complementary perspectives on the projection of an object on a scale. This distinction was used to characterize adjectival polarity as a sortal distinction: positive and negative extents, and not negative and positive extents, are different sorts of objects, which can be viewed as substitution instances of (51), where \( \text{pos}_{\delta} \) and \( \text{neg}_{\delta} \) are the maximal elements of \( \text{pos}(\cdot) \) and \( \text{neg}(\cdot) \), respectively. It follows that if \( \text{pos} < \text{neg} \), then \( \text{pos}_{\delta} < \text{neg}_{\delta} \).

\[
\text{deg}(d_{pos}(d_{neg}(\cdot)))
\]

The validity of (55) follows from the claim that positive and negative projections of an object on a scale along dimension \( \delta \) are join complementary. In section 3.1.4, I claimed that the positive and negative projections of an object onto a scale along dimension \( \delta \), \( \text{pos}_{\delta}(a) \) and \( \text{neg}_{\delta}(a) \), correspond to (30) and (31), repeated below, where \( d \) is a function from an object to a point on the scale.

(30)
\[
\text{pos}_{\delta}(a) = \{ p \in S_{\delta} | p \leq d(a) \}
\]

(31)
\[
\text{neg}_{\delta}(a) = \{ p \in S_{\delta} | d(a) \leq p \}
\]

From these definitions, we can define the complements of positive and negative extents as follows:

(52)
\[
\text{neg}_{\delta}(x) = \text{pos}_{\delta}(x) - \{ d(x) \}
\]

(53)
\[
\text{pos}_{\delta}(x) = \text{neg}_{\delta}(x) - \{ d(x) \}
\]

If \( \text{pos}_{\delta}(a) > \text{pos}_{\delta}(b) \), then \( \text{pos}_{\delta}(a) - \{ d(a) \} > \text{pos}_{\delta}(b) - \{ d(b) \} \), since \( d(a) \) and \( d(b) \) are the maximal elements of \( \text{pos}_{\delta}(a) \) and \( \text{pos}_{\delta}(b) \), respectively. It follows that \( \text{neg}_{\delta}(a) > \text{neg}_{\delta}(b) \), by substitution, and finally that \( \text{neg}_{\delta}(b) > \text{neg}_{\delta}(a) \), by contraposition. The other direction of the biconditional can be proved in exactly the same way.
functions from objects to negative extents. The crucial consequence of these assumptions is that the range of a positive adjective is disjoint from the range of its negative counterpart. If the sets of positive and negative extents are disjoint, then any expression that is a substitution instance of (55) or (56), where $\deg$ is an ordering on extents and $e_{pos}$ and $e_{neg}$ are positive and negative extents, will fail to return a truth value, since the ordering relation introduced by $\deg$ will be undefined.

\begin{align*}
(55) & \deg(e_{pos})(e_{neg}) \\
(56) & \deg(e_{pos})(e_{neg})
\end{align*}

The end result is that within the algebra of extents, cross-polar anomaly can be explained in the same terms as incommensurability: the degree relation introduced by the comparative morpheme is undefined for the compared extents.

For illustration, consider an example like (57), which has the logical representation in (58).

\begin{align*}
(57) & \text{The Brothers Karamazov is longer than The Dream of a Ridiculous Man is short.} \\
(58) & \text{\emph{more}}(\text{long}(\text{The Brothers K}) \text{\emph{than}} \text{short}(\text{The Dream of a Ridiculous Man}))
\end{align*}

The denotation of the comparative clause in (58) is the maximal extent $e$ such that $\text{The Dream of a Ridiculous Man}$ is at least as short as $e$, which is a negative extent. The reference value, however, is derived by applying the adjective $\text{long}$ to the denotation of $\text{The Brothers K}$, returning the positive extent of $\text{The Brothers K}$'s length. Since positive and negative extents come from disjoint sets, the relation introduced by $\text{more}$ is undefined for its arguments, and the structure is anomalous.

The same explanation applies to sentences in which the polar adjectives are reversed, such as (59).

\begin{align*}
(59) & \text{The Dream of a Ridiculous Man is shorter than The Brothers Karamazov is long.} \\
(60) & \text{\emph{more}}(\text{short}(\text{The Dream of a Ridiculous Man}) \text{\emph{than}} \text{long}(\text{The Brothers K}))
\end{align*}

In this example, the standard value is a positive extent—the maximal extent $e$ such that $\text{The Brothers K}$ is at least as long as $e$, which is a positive extent. The reference value, however, is derived by applying the adjective $\text{short}$ to the denotation of $\text{The Dream of a Ridiculous Man}$, returning the negative extent of $\text{The Dream of a Ridiculous Man}$'s length. Again, the relations introduced by $\text{more}$ are undefined for its arguments, and the structure is anomalous.

It should be observed that the analysis of cross-polar anomaly that I have outlined here makes a more general prediction: if subdeletion structures are interpreted by directly supplying the denotation of the comparative clause as the standard value, then any instance of comparative subdeletion in which the adjective in the main clause is of different polarity than the $\text{than}$-clause should be anomalous, not just examples involving antonymous pairs such as (15)-(18). The following minimal pairs verify this prediction.

\begin{align*}
(61) & \text{Unfortunately, the ficus turned out to be taller than the ceiling was high.} \\
(62) & \text{Unfortunately, the ficus turned out to be taller than the ceiling was low.} \\
(63) & \text{Luckily, the ficus turned out to be shorter than the doorway was short.} \\
(64) & \text{Luckily, the ficus turned out to be shorter than the doorway was high.}
\end{align*}

The logical representations of the comparative clauses in (62) and (64) are:

\begin{align*}
(62) & \text{\emph{more}}(\text{tall}(\text{the ficus}) \text{\emph{than}} \text{low}(\text{the ceiling})) \\
(64) & \text{\emph{more}}(\text{short}(\text{the ficus}) \text{\emph{than}} \text{high}(\text{the doorway}))
\end{align*}

In (62), the standard value is a positive extent—the maximal extent $e$ such that $\text{The ceiling}$ is at least as high as $e$, which is a positive extent. The reference value, however, is derived by applying the adjective $\text{low}$ to the denotation of $\text{The ficus}$, returning the negative extent of $\text{The ficus}$'s height. Since degrees are not members of the same order, the comparison relation is undefined, and the structure is anomalous. The same explanation applies to (64), where the adjective $\text{short}$ is applied to the denotation of $\text{The doorway}$, returning the positive extent of $\text{The doorway}$'s height. Since positive and negative extents are disjoint, the relation introduced by $\text{more}$ is undefined, and the structure is anomalous.

The same analysis applies to the following sentence.

\begin{align*}
(65) & \text{The Dream of a Ridiculous Man is longer than the ceiling was high.} \\
(66) & \text{The Dream of a Ridiculous Man is longer than the ceiling was low.} \\
(67) & \text{The Dream of a Ridiculous Man is shorter than the doorway was short.} \\
(68) & \text{The Dream of a Ridiculous Man is shorter than the doorway was high.}
\end{align*}
the comparative relation is undefined. Cross-polar anomaly is thus a more general phenomenon than the original facts suggest: it does not require that the two adjectives in the comparative are members of a specific positive-negative pair, only that they are of different polarity. Indeed, the analysis proposed here predicts that this should be the case.

To summarize, the explanation of cross-polar anomaly outlined here is available because the algebra of extents permits a sortal distinction between positive and negative extents to be made at a very basic, structural level. More importantly, the fact that this anomaly is observed in the first place provides support for the hypothesis that “degrees” should be characterized as intervals on a scale, rather than points on a scale, and for adopting the description of cross-polar anomaly as a compelling empirical argument for this characterization. If adjectival polarity is represented as a sortal distinction between positive and negative adjectives, and if positive and negative extents are distinct objects, then we predict that comparatives constructed out of polar opposites should be anomalous. That is, any formula which is a substitution instance of (55) and (56) should fail to return a truth value, because the relation introduced by \( \text{deg} \) will be undefined. Consequently, the logical representation of (55) is interpretable, but the logical representation of (56) is not.

3.1.8 The Distribution of Measure Phrases

As observed by von Stechow (1978), the logical representation (69) is not interpretable. The order relation introduced by the measure phrase is undefined in (69) but not in (66).

The explanation runs as follows. If the scale of height has a minimal element, then the scale of height is bounded but the scale of length is not. If the scale of length is bounded but the scale of height is not, then the logical representation of (66) is interpretable, but the logical representation of (67) is not.

If we assume that measure phrases denote bounded extents (see the discussion of comparison of deviation in section 3.2), and that dimensional adjectives such as long, tall, and wide are associated with scales with a minimal element, then we predict that measure phrases should be interpretable, because the extent of my Cadillac’s length is bounded but the extent of my Fiat’s shortness is not.

If we assume that measure phrases denote bounded extents (see the discussion of comparison of deviation in section 3.2), the logical representation of (66) is interpretable, but the logical representation of (67) is not.

As observed by von Stechow (1978), the logical representation (69) is not interpretable. The order relation introduced by the measure phrase is undefined in (69) but not in (66).

The explanation runs as follows. If the scale of height has a minimal element, then the scale of height is bounded but the scale of length is not. If the scale of length is bounded but the scale of height is not, then the logical representation of (66) is interpretable, but the logical representation of (67) is not.

If we assume that measure phrases denote bounded extents (see the discussion of comparison of deviation in section 3.2), and that dimensional adjectives such as long, tall, and wide are associated with scales with a minimal element, then we predict that measure phrases should be interpretable, because the extent of my Cadillac’s length is bounded but the extent of my Fiat’s shortness is not.

If we assume that measure phrases denote bounded extents (see the discussion of comparison of deviation in section 3.2), the logical representation of (66) is interpretable, but the logical representation of (67) is not.

To summarize, the explanation of cross-polar anomaly outlined here is available. If the scale of height has a minimal element, then the logical representation of (66) is interpretable. The extent of my Cadillac’s length is bounded but the extent of my Fiat’s shortness is not.

The explanation runs as follows. If the scale of height has a minimal element, then the scale of height is bounded but the scale of length is not. If the scale of length is bounded but the scale of height is not, then the logical representation of (66) is interpretable, but the logical representation of (67) is not.

If we assume that measure phrases denote bounded extents (see the discussion of comparison of deviation in section 3.2), the logical representation of (66) is interpretable, but the logical representation of (67) is not.

To summarize, the explanation of cross-polar anomaly outlined here is available. If the scale of height has a minimal element, then the logical representation of (66) is interpretable. The extent of my Cadillac’s length is bounded but the extent of my Fiat’s shortness is not.
Because the ordering relation in (69) is undefined, (67) is correctly predicted to be anomalous. If this analysis is correct, then it suggests an explanation for the fact that while (67) is anomalous, (70) is not.

(70) My fiat is shorter than 8 feet. Structurally, (70) is a phrasal comparative. Given the analysis of phrasal comparatives presented in chapter 2, the logical representation of (70) should be (71).

(71) \text{more (short (my Fiat)) (short (5 feet))}

The crucial difference between (71) and (69) is that the standard value in the latter is provided directly by the measure phrase, while the standard value in the former is derived by applying the adjective (short) to the measure phrase. If it can be shown that the result of this operation is the negative extent that ranges from the point on the scale corresponding to 5 feet to the upper end of the scale, then we will have an explanation for this contrast. I will leave this as a point for future investigation.

3.1.9 Positives that Look Like Negatives

Chris Barker (personal communication) observes that sentences like (72)-(74) appear to be counterexamples to the claim that comparatives formed out of positive and negative pairs of adjectives are anomalous.

(72) Your C is sharper than your D is flat.
(73) My watch is faster than your watch is slow.
(74) She was earlier than I was late.

If the adjectives in (72)-(74) are actual positive-negative pairs, then the analysis outlined in the previous sections incorrectly predicts that these sentences should be anomalous. There are at least three reasons to believe that the adjectives in (72)-(74) are not positive-negative pairs. First, the adjectives are interpreted in a way that measures divergence from some common point of reference. In (72), the extent to which your C is sharper than your D is flat can only mean that the extent to which my C is sharper is greater than the extent to which your D is flat. Second, the adjectives are interpreted in a way that measures divergence from some common point of reference. In (73), the extent to which my watch is faster than your watch is slow can only mean that the extent to which my watch is faster is greater than the extent to which your watch is slow.

The crucial problem involves interpretation. The claims made in this section are counterexamples to the claim that comparatives formed out of positive and negative pairs are anomalous. Chris Barker (personal communication) observes that comparatives formed out of positive and negative pairs of adjectives are anomalous.

(75) #My car is faster than your car is slow.

The second argument comes from the distribution of overt measure phrases. A characteristic of negative adjectives is that they cannot be modified by overt measure phrases.

(76) #Mr. Reich is 5 feet short.

(77) #Maureen was driving 14 mph slow.

213213

214214
Both of the adjectives in each of (72)-(74) permit measure phrases, however, when they are interpreted in the way described above, so that they measure divergence from a reference point. (78)-(80) illustrate this point.

(78) Your C is 30 Hz flat/sharp.
(79) My watch is 10 minutes fast/slow.
(80) She was an hour early/late.

The third argument makes use of the observation that statements whose logical forms are substitution instances of (51), repeated below, are valid for any positive-negative pair of adjectives.

\[
\phi_{\text{pos}}(a) > \phi_{\text{pos}}(b) \text{ iff } \phi_{\text{neg}}(b) > \phi_{\text{neg}}(a)
\]

If flat-sharp, fast-slow, and early-late, are actual positive-negative pairs, then (81)-(83) should be valid.

(81) Your A is sharper than your D iff your D is flatter than your A.
(82) My watch is faster than yours iff yours is slower than mine.
(83) She was earlier than I was iff I was later than she was.

These statements are not valid, however, on the relevant interpretation. Although (83), for example, is valid if late and early describe the relative temporal ordering of two individuals (with respect to some event), it is not valid if the adjectives describe deviation from some reference point indicating "on time". On the latter interpretation, the first conjunct would be true in a context in which we were both early, but she was earlier than I was, while the second conjunct would be false, because neither of us was late. This latter interpretation is the one the adjectives must have in (74), however: if the adjectives are interpreted so that (84) is anomalous.

The data discussed here support the conclusion that both members of the adjective pairs in (72)-(74) are positive: adjectives like sharp and flat (as well as fast-slow and early-late, on the relevant interpretations), which measure divergence from some reference point (e.g., the point at which a tone is neither sharp nor flat), not only project their arguments onto the same scale, they define the same sorts of projections onto a scale. If the adjectives in (72)-(74) are sortally the same, as the facts indicate, then these sentences do not represent counterexamples to the analysis developed in this section.

3.1.10 Summary

The extent approach to the semantics of gradable adjectives consists of two principal claims. First, gradable adjectives denote relations between individuals and extents. An extent is defined as an interval on a scale, and a structural distinction is made between two sorts of extents: positive extents and negative extents. Second, positive and negative adjectives are distinguished by the sort of their extent arguments: positive adjectives denote relations between individuals and positive extents; negative adjectives denote relations between individuals and negative extents. The extent approach to the semantics of gradable adjectives consists of two principal claims.

The discussion so far has shown this analysis to have several interesting results. If individuals and negative extents are distinguished from individuals and positive extents, negative adjectives do not have the same reference to a scale as positive adjectives do. If the adjectives in (72)-(74) are sortally the same, the facts indicate, then these sentences do not represent counterexamples to the analysis developed in this section.

The data discussed here support the conclusion that both members of the adjective pairs in (72)-(74) are positive: adjectives like sharp and flat (as well as fast-slow and early-late, on the relevant interpretations), which measure divergence from some reference point (e.g., the point at which a tone is neither sharp nor flat), not only project their arguments onto the same scale, they define the same sorts of projections onto a scale. If the adjectives in (72)-(74) are sortally the same, as the facts indicate, then these sentences do not represent counterexamples to the analysis developed in this section.

Although (83), for example, is valid if late and early describe the relative temporal ordering of two individuals (with respect to some event), it is not valid if the adjectives describe deviation from some reference point indicating "on time". On the latter interpretation, the first conjunct would be true in a context in which we were both early, but she was earlier than I was, while the second conjunct would be false, because neither of us was late. This latter interpretation is the one the adjectives must have in (74), however: if the adjectives are interpreted so that (84) is anomalous.

The data discussed here support the conclusion that both members of the adjective pairs in (72)-(74) are positive: adjectives like sharp and flat (as well as fast-slow and early-late, on the relevant interpretations), which measure divergence from some reference point (e.g., the point at which a tone is neither sharp nor flat), not only project their arguments onto the same scale, they define the same sorts of projections onto a scale. If the adjectives in (72)-(74) are sortally the same, as the facts indicate, then these sentences do not represent counterexamples to the analysis developed in this section.

Although (83), for example, is valid if late and early describe the relative temporal ordering of two individuals (with respect to some event), it is not valid if the adjectives describe deviation from some reference point indicating "on time". On the latter interpretation, the first conjunct would be true in a context in which we were both early, but she was earlier than I was, while the second conjunct would be false, because neither of us was late. This latter interpretation is the one the adjectives must have in (74), however: if the adjectives are interpreted so that (84) is anomalous.

The data discussed here support the conclusion that both members of the adjective pairs in (72)-(74) are positive: adjectives like sharp and flat (as well as fast-slow and early-late, on the relevant interpretations), which measure divergence from some reference point (e.g., the point at which a tone is neither sharp nor flat), not only project their arguments onto the same scale, they define the same sorts of projections onto a scale. If the adjectives in (72)-(74) are sortally the same, as the facts indicate, then these sentences do not represent counterexamples to the analysis developed in this section.
explains the validity of statements like (14), satisfying the minimal requirement of
descriptive adequacy, and in addition, it provides a uniform account of the semantics of
positive and negative adjectives in both the absolute and comparative forms. Most
importantly, it provides a principled explanation of cross-polar anomaly within a general
analysis of incommensurability.

3.2 Comparison of deviation

3.2.1 The Semantic Characteristics of Comparison of Deviation

As noted in chapter 1, any analysis of cross-polar anomaly must also account for the existence
of comparatives involving positive and negative pairs of adjectives which are superficially
similar to examples of cross-polar anomaly but are not anomalous. (84)-(87) exemplify
sentences of this type, which I referred to as "comparison of deviation" constructions.

(84) William is as tall as Robert is short.
(85) Francis is as reticent as Hilary is long-winded.
(86) San Francisco Bay is more shallow than Monterey Bay is deep.
(87) The Tenderloin is more dirty than Pacific Heights is clean.

Comparison of deviation and cross-polar anomaly are linked in the following way: an
explanation of cross-polar anomaly must not have the consequence that comparison of
deviation sentences are predicted to be ungrammatical; similarly, it should not be the case
that an account of comparison of deviation permits non-anomalous interpretations of cross-
polar anomaly sentences. This was a problem for the vague predicate analysis.

In chapter 1 (section 1.1.4.3), I noted three important characteristics distinguish the
former from the latter. The first difference involves basic interpretation. Unlike standard
comparative constructions, which compare the projections of two objects onto a scale,
comparison of deviation constructions compare the extents to which two objects differ from
a relevant standard value. This is most clearly illustrated by the equative construction (84).

(84) can only mean that the extent to which William exceeds some standard value of tallness
is (relatively) the same as the extent to which Robert exceeds some standard value of
shortness; (84) cannot mean that William and Robert are equal in height. This interpretation
should be contrasted with that of a more typical example of equative subdeletion, such as (88),
in which it is asserted that the height and width of the doorway are the same.

(88) The doorway is as tall as it is wide.

William is as tall as Robert is short.

The comparatives in (86) and (87) have similar interpretations. (86), for example, means
that the extent to which the Tenderloin exceeds a standard of dirtiness is greater than the
extent to which Pacific Heights exceeds a standard of cleanliness.

That these sentences have only interpretations of this sort is made clear by the
second difference between them and standard comparative constructions. Whereas
standard comparatives do not entail that the property predicated of the compared objects
is true in the absolute sense, comparison of deviation constructions do carry this
entailment. For example, (84) entails that William is tall and that Robert is short, but (88)
entails neither that the doorway is tall nor that it is wide: this sentence could be truthfully
used to describe a one foot by one foot opening.

Similarly, while (86) entails that San Francisco Bay is shallow and Monterey Bay is deep,
(89), though this information is conveyed, does not, though this information is conveyed, in
a way that clearly shows that the standard value used to compare San Francisco Bay
and Monterey Bay is not the same.

(89) San Francisco Bay is shallower than Monterey Bay.

(90) San Francisco Bay is shallower than Monterey Bay, though they're both fairly deep.

The comparatives in (98) and (99) cannot have such interpretations. (89), for example, means
that San Francisco Bay is shallower than Monterey Bay. However, (89) can only mean that
San Francisco Bay is shallower than Monterey Bay, though these sentences have only
interpretations of this sort is made clear by the
second difference between them and standard comparative constructions. Whereas
standard comparatives do not entail that the property predicated of the compared objects istrue in the absolute sense, comparison of deviation constructions carry this entailment.

For example, (84) entails that William is tall and that Robert is short, but (88) entails
neither that the doorway is tall nor that it is wide: this sentence could be truthfully used to
describe a one foot by one foot opening. Similarly, while (86) entails that San Francisco Bay
is shallow and Monterey Bay is deep, (89) does not, though this information is conveyed, in
a way that clearly shows that the standard value used to compare San Francisco Bay
and Monterey Bay is not the same.

(89) San Francisco Bay is shallower than Monterey Bay.

(90) San Francisco Bay is shallower than Monterey Bay, though they're both fairly deep.

The comparatives in (98) and (99) cannot have such interpretations. (89), for example, means
that San Francisco Bay is shallower than Monterey Bay. However, (89) can only mean that
San Francisco Bay is shallower than Monterey Bay, though these sentences have only
interpretations of this sort is made clear by the
second difference between them and standard comparative constructions. Whereas
standard comparatives do not entail that the property predicated of the compared objects istrue in the absolute sense, comparison of deviation constructions carry this entailment.

For example, (84) entails that William is tall and that Robert is short, but (88) entails
neither that the doorway is tall nor that it is wide: this sentence could be truthfully used to
describe a one foot by one foot opening. Similarly, while (86) entails that San Francisco Bay
is shallow and Monterey Bay is deep, (89) does not, though this information is conveyed, in
a way that clearly shows that the standard value used to compare San Francisco Bay
and Monterey Bay is not the same.

(89) San Francisco Bay is shallower than Monterey Bay.

(90) San Francisco Bay is shallower than Monterey Bay, though they're both fairly deep.

The comparatives in (98) and (99) cannot have such interpretations. (89), for example, means
that San Francisco Bay is shallower than Monterey Bay. However, (89) can only mean that
San Francisco Bay is shallower than Monterey Bay, though these sentences have only
interpretations of this sort is made clear by the
second difference between them and standard comparative constructions. Whereas
standard comparatives do not entail that the property predicated of the compared objects istrue in the absolute sense, comparison of deviation constructions carry this entailment.

For example, (84) entails that William is tall and that Robert is short, but (88) entails
neither that the doorway is tall nor that it is wide: this sentence could be truthfully used to
describe a one foot by one foot opening. Similarly, while (86) entails that San Francisco Bay
is shallow and Monterey Bay is deep, (89) does not, though this information is conveyed, in
a way that clearly shows that the standard value used to compare San Francisco Bay
and Monterey Bay is not the same.

(89) San Francisco Bay is shallower than Monterey Bay.

(90) San Francisco Bay is shallower than Monterey Bay, though they're both fairly deep.
In contrast, denying that San Francisco Bay is shallow after an utterance of (86) is contradictory:

(91) San Francisco Bay is more shallow than Monterey Bay is deep. #San Francisco Bay isn't shallow, though.

Finally, comparison of deviation interpretations do not license morphological incorporation of the adjective and the comparative morpheme. (86) and (87) contrast with (92) and (93), respectively, which are instances of cross-polar anomaly.

(92) #San Francisco Bay is shallower than Monterey Bay is deep.
(93) #The Tenderloin is dirtier than Pacific Heights is clean.

On the surface, comparison of deviation constructions appear to be problematic for the analysis of cross-polar anomaly developed in section 3.1, which predicted that any comparative constructed out of a positive and negative pair of adjectives should trigger incommensurability. On closer inspection, however, comparison of deviation constructions like (84)-(87) actually provide interesting support for the analysis of cross-polar anomaly.

Consider, for example, the interpretation of (84). What is crucial to observe is that this sentence does not simply permit the type of interpretation discussed above, in which the compared objects are the extents to which the compared individuals exceed appropriate standards of tallness and shortness, (84) has only this type of interpretation. In particular, (84) cannot be interpreted as the extent to which the compared individuals exceed appropriate standards of tallness and shortness.

The puzzle that remains to be solved is how the comparison of deviation interpretation arises in the first place. I will address this question in the next section.

### Differential Extents and Comparison of Deviation

To see why, consider the logical representation of (94) on this interpretation:

(94) \[ \forall x (\text{tall}(x) \leftrightarrow \exists y \forall z (\text{short}(y) \land \text{short}(z) \rightarrow \text{short}(y))) \]

Assuming the truth conditions for the equative given in section 3.1.6, the interpretation of (84) represented by (94) is ruled out as an instance of cross-polar anomaly: the arguments of \( \forall x \) are not elements of the same ordered set, so the partial ordering relation introduced by the degree morpheme is undefined. A similar case can be made for (85). The puzzle that remains to be solved is how the comparison of deviation interpretation arises in the first place. I will address this question in the next section.

### 3.2.2 Differential Extents and Comparison of Deviation

The puzzle that remains to be solved is how the comparison of deviation interpretation arises in the first place. I will address this question in the next section.

### 3.2.2 Differential Extents and Comparison of Deviation

The puzzle that remains to be solved is how the comparison of deviation interpretation arises in the first place. I will address this question in the next section.
defined in terms of set difference as in (96). If absolute extents can be partitioned as in (95), then $e_1$ denotes the difference between $tall(w)$ and $stall$, and $e_2$ denotes the difference between $short(r)$ and $sshort$. That is, $e_1$ and $e_2$ represent the extents to which Bradley and Reich exceed standards of tallness and shortness, respectively.

I would like to suggest that comparison of deviation involves comparison of such "differential extents". Specifically, the comparison of deviation interpretation of (84) involves a relation between $e_1$ and $e_2$ in (95). Building on the standard interpretation of the equative, the comparison of deviation interpretation of (84) can be characterized as in (97):

$$||\text{(84)}|| = 1 \text{ iff } e_1 \geq e_2,$$

where $e_1$ denotes the extent to which $W$ is tall compared to $R$.

If the hypothesis that measure phrases denote bounded extents is correct (see section 3.1.8), we make the following prediction: only in these cases will the corresponding positive phrases allow measure phrases. If this is the case, then $e_1$ and $e_2$ in (95) are equivalent to $e'1$ and $e'2$ respectively, and it follows that if $e'1 \geq e'2$, then $e_1 \geq e_2$.

The importance of (i) is its strictness in favor of an interpretation on extents that is independent of intervals. Moreover, it allows us to make a stronger assumption on the intervals $e'1$ and $e'2$ that are used to interpret the comparison of deviation indicating that they must be the case that there is an interval $[a, b]$ such that $e'1 \geq e'2$ if and only if $e_1 \geq e_2$. If the hypothesis that measure phrases denote bounded extents is correct (see section 3.1.8), we make the following prediction: only in these cases will the corresponding positive phrases allow measure phrases. If this is the case, then $e_1$ and $e_2$ in (95) are equivalent to $e'1$ and $e'2$ respectively, and it follows that if $e'1 \geq e'2$, then $e_1 \geq e_2$.
In order for the general approach to comparison of deviation outlined here to go through, however, it must be shown that comparison of differential extents in examples involving positive and negative adjectives (such as (84)-(87)) does not result in incommensurability. That is, it must be shown that differential extents are sortally the same, regardless of whether they are subintervals of positive extents or subintervals of negative extents, so that the comparison relation is defined in examples like (84)-(87). In fact, this result follows from the definition of extent concatenation in (96). Two initial conditions are important. First, since we are only interested in cases in which $e_d$ is the argument of a gradable predicate, it must be the case that $e'$ is either positive or negative, therefore $e'$ must be a proper extent. Second, $e$ and $e'$ must be extents of the same sort: both positive or both negative. This follows from the definition of an extent as a convex subset of a totally ordered set of points: if e.g. $e$ and $e'$ were of opposite polarity, then the value of $e' - e$, which equals $-(e' \cap e)$, would not be convex, and therefore not an extent.

Assuming these initial conditions, two questions must be answered: what is the sort of $e_d$ when $e$ is positive, and what is the sort of $e_d$ when $e$ is negative? In the case in which $e$ is positive, $e_d$ must be a bounded extent (i.e., a proper subset of $S_e$), otherwise $e'$ would not satisfy the condition that it be a proper extent. When $e$ is negative, $e_d$ again must be bounded in order to ensure that $e'$ is a proper extent. The conclusion is that a differential extent is bounded when concatenated with either a positive or a negative extent, a fact which I interpret as indicating that differential extents are (structurally) of the same sort. This is consistent with the hypothesis mentioned above that all differential extents are elements of the set of bounded extents on a scale. If all differential extents are members of the same set of bounded extents, and assuming that mappings of the sort defined above can be determined in composition of extent concatenation, it should be possible to establish an ordering relation between any two differential extents. Therefore, the comparison relation associated with comparison of differential extents is defined in terms of the comparison relation between two bounded extents, and assuming the mappings of the sort defined above can be determined, it follows that all differential extents have members of the same sort.

In order for the general approach to comparison of deviation outlined here to go through, however, it must be shown that comparison of differential extents in examples involving positive and negative adjectives (such as (84)-(87)) does not result in incommensurability. That is, it must be shown that differential extents are sortally the same, regardless of whether they are subintervals of positive extents or subintervals of negative extents. This follows from the definition of extent concatenation in (96). Two initial conditions are important. First, since we are only interested in cases in which $e_d$ is the argument of a gradable predicate, it must be the case that $e'$ is either positive or negative, therefore $e'$ must be a proper extent. Second, $e$ and $e'$ must be extents of the same sort: both positive or both negative. This follows from the definition of an extent as a convex subset of a totally ordered set of points: if e.g. $e$ and $e'$ were of opposite polarity, then the value of $e' - e$, which equals $-(e' \cap e)$, would not be convex, and therefore not an extent.

Assuming these initial conditions, two questions must be answered: what is the sort of $e_d$ when $e$ is positive, and what is the sort of $e_d$ when $e$ is negative? In the case in which $e$ is positive, $e_d$ must be a bounded extent (i.e., a proper subset of $S_e$), otherwise $e'$ would not satisfy the condition that it be a proper extent. When $e$ is negative, $e_d$ again must be bounded in order to ensure that $e'$ is a proper extent. The conclusion is that a differential extent is bounded when concatenated with either a positive or a negative extent, a fact which I interpret as indicating that differential extents are (structurally) of the same sort. This is consistent with the hypothesis mentioned above that all differential extents are elements of the set of bounded extents on a scale. If all differential extents are members of the same set of bounded extents, and assuming that mappings of the sort defined above can be determined in composition of extent concatenation, it should be possible to establish an ordering relation between any two differential extents. Therefore, the comparison relation associated with comparison of differential extents is defined in terms of the comparison relation between two bounded extents, and assuming the mappings of the sort defined above can be determined, it follows that all differential extents have members of the same sort.
Before concluding this section, it should be observed that comparison of deviation is not unique to comparatives involving polar opposites; it represents a possible interpretation of subdeletion structures in general, as shown by (99).

(99) The Bay Bridge is as long as the Empire State Building is tall.

(99) is ambiguous between a standard equative interpretation, in which it is asserted that the length of the Bay Bridge equals the height of the Empire State Building (which is false), and a comparison of deviation interpretation, in which the two objects are asserted to deviate to a (relatively) equal extent from the appropriate standards of longness and tallness (which is true). This ambiguity is predicted by the analysis of comparison of deviation that I have outlined here. Nothing about the analysis restricts it to examples involving positive and negative pairs of adjectives. As a result, in examples which do not involve adjectives of opposite polarity, such as (99), it should not only be possible to construct a comparison of deviation construction, but it should also be possible to construct an interpretation in terms of the standard meaning of the equative morpheme.

Before concluding this section, it should be observed that comparison of deviation is a more general construction, as compared to subdeletion constructions, in that it allows for comparison of more than two objects. For example, (100) is a valid comparison of deviation construction, but it is not a valid subdeletion construction.

(100) The Bay Bridge is more long than the Empire State Building is tall.

In chapter 1 (section 1.1.4.1), I observed that negative polarity item licensing and entailment patterns indicate that positive adjectives generate monotone decreasing contexts, while negative adjectives generate monotone increasing contexts. This is supported by empirical evidence (see Seuren 1978, Ladusaw 1979, Linebarger 1980, Sánchez-Valencia 1996). (100)-(107) review the crucial data.

18Note that a comparative version of (99) also permits a comparison of deviation interpretation if incorporation does not occur:

(i) The Bay Bridge is more long than the Empire State Building is tall.

(ii) William is not very tall in front of a huge building.

The standard meaning of the equative morpheme is that the Bay Bridge is as long as the Empire State Building is tall. The Bay Bridge is longer than the Empire State Building is tall. It is also possible to construct a comparison of deviation construction in terms of the standard meaning of the equative morpheme, such as (99), it should not only be possible to construct a comparison of deviation construction, but it should also be possible to construct an interpretation in terms of the standard meaning of the equative morpheme. However, this is not possible with subdeletion constructions, as they do not involve cross-polar oppositions. For example, (100) is a valid comparison of deviation construction, but it is not a valid subdeletion construction. Therefore, the analysis of comparison of deviation construction is more general than the analysis of subdeletion construction.
It is difficult/*easy for him to admit that he has ever been wrong.

It would be foolish/*clever of her to even bother to lift a finger to help.

It is strange/*typical that any of those papers were accepted.

It's lame/*cool that you even have to talk to any of these people at all.

It's dangerous to drive in Rome.

⇒ It's dangerous to drive fast in Rome.

It's safe to drive in Des Moines.

⇐ It's safe to drive fast in Des Moines.

It's strange to see Frances playing electric guitar.

⇒ It's strange to see Frances playing electric guitar poorly.

It's common to see Frances playing electric guitar.

⇐ It's common to see Frances playing electric guitar poorly.

Monotonicity properties represent one of several factors which have traditionally been used to classify gradable adjectives according to their “logical polarity” (in the sense of H. Klein 1996): adjectives which license negative polarity items and downward entailments in clausal complements, such as difficult and strange, are classified as negative, while adjectives which do not license negative polarity items but do permit upward inferences, such as easy and safe, are classified as positive.

The goal of this section is to show that the monotonicity properties of polar adjectives follow directly from the theory of polarity developed in section 3.1, in which polar adjectives are treated as functions from individuals to positive or negative extents.

3.3.2 Degrees and Monotonicity

Recall from the discussion in section 3.1 that logical polarity is represented in a degree algebra as a distinction in the range of gradable adjectives. As defined above, a scale $S_\delta$ is a linearly ordered set of degrees along some dimension. If we assume that a positive adjective preserves the order on $S_\delta$, while a negative one reverses it, then for any scale $S_\delta$ and any pair of positive and negative adjectives associated with $S_\delta$, the set of degrees associated with the positive $D_{pos}$ and the set of degrees associated with the negative $D_{neg}$ stand in the dual relation.

Moreover, since there is a one-to-one mapping between the set of degrees associated with the positive $D_{pos}$ and the set of degrees associated with the negative $D_{neg}$, their membership is the same. Moreover, since the set of degrees associated with the positive $D_{pos}$ is the dual of the set of degrees associated with the negative $D_{neg}$, whenever (109) holds, (108) holds.

This representation has an additional positive result: it entails that negative adjectives are monotone decreasing. To see why, consider an arbitrary case of ordering along a dimension, for example, safety. If $a$ is safer than $b$, then the relation $a <_{safety} b$ holds. If the ordering relation associated with a negative adjective is the dual of the relation associated with its positive counterpart, then given the analysis of gradable adjectives as functions that map individuals to degrees, whenever (110) holds, (111) also holds.

\[
\begin{align*}
\text{if } & a <_{\text{safety}} b \text{ then } \text{dangerous}(a) <_{\text{dangerous}} \text{dangerous}(b) \\
\text{if } & \text{safe}(a) <_{\text{safe}} \text{safe}(b) \text{ then } \text{dangerous}(b) <_{\text{dangerous}} \text{dangerous}(a)
\end{align*}
\]

Philosophers consider ethical judgments to be judgments about the content of actions. If we assume that a positive adjective preserves the order on $S_\delta$, while a negative one reverses it, then for any scale $S_\delta$ and any pair of positive and negative adjectives associated with $S_\delta$, the set of degrees associated with the positive $D_{pos}$ and the set of degrees associated with the negative $D_{neg}$ stand in the dual relation.

Moreover, since there is a one-to-one mapping between the set of degrees associated with the positive $D_{pos}$ and the set of degrees associated with the negative $D_{neg}$, their membership is the same. Moreover, since the set of degrees associated with the positive $D_{pos}$ is the dual of the set of degrees associated with the negative $D_{neg}$, whenever (109) holds, (108) holds.

This representation has an additional positive result: it entails that negative adjectives are monotone decreasing. To see why, consider an arbitrary case of ordering along a dimension, for example, safety. If $a$ is safer than $b$, then the relation $a <_{safety} b$ holds. If the ordering relation associated with a negative adjective is the dual of the relation associated with its positive counterpart, then given the analysis of gradable adjectives as functions that map individuals to degrees, whenever (110) holds, (111) also holds.

\[
\begin{align*}
\text{if } & a <_{\text{safety}} b \text{ then } \text{dangerous}(a) <_{\text{dangerous}} \text{dangerous}(b) \\
\text{if } & \text{safe}(a) <_{\text{safe}} \text{safe}(b) \text{ then } \text{dangerous}(b) <_{\text{dangerous}} \text{dangerous}(a)
\end{align*}
\]
Given the definitions in (112), it follows that positive adjectives denote monotone increasing functions and negative adjectives denote monotone decreasing functions.

$$f(a) < f(b) \implies a < b$$  \hspace{1cm} (112) \\
$$f(a) > f(b) \implies a > b$$  \hspace{1cm} (111)

Although this is a positive result, it must be acknowledged that the monotonicity of negative adjectives does not follow from an independent aspect of the algebra of degrees. Rather, it is a definitional property of negative adjectives: that negative adjectives are scale reversing is assumed in order to construct a semantics which correctly accounts for the interpretations of positive and negative adjectives in the absolute and comparative forms. Nothing about the system itself requires negatives to be monotone decreasing; rather, it is the data which force this assumption. An alternative situation would be one in which independently motivated characteristics of the ontology have the additional consequence that negative adjectives are scale reversing. The discussion of cross-polar anomaly in section 3.1 demonstrated that the distinction between positive and negative extents provided by an extent algebra is independently motivated, as it supports an explanation of cross-polar anomaly (something the algebra of degrees fails to do). In the following section, I will show that this distinction has the additional positive result of deriving the monotonicity properties of gradable adjectives.

### 3.3.3 Extents and Monotonicity

The conclusion of section 3.1 was that scalar values ("degrees") should be formalized in terms of intervals of a scale, as originally proposed by Seuren (1978) (see also von Stechow 1984b and Löbner 1990), and polar opposition should be represented as a sortal distinction.

Assuming a standard Boolean ordering on extents in (112), whenever (112) holds, the

$$a < b \implies \neg a > b$$  \hspace{1cm} (111) 

$$a > b \implies \neg a < b$$  \hspace{1cm} (111)

When the relation between \(a\) and \(b\) shown in (112) holds, then the values of \(f(a)\) and \(f(b)\) (i.e., the positive and negative extents of \(a\) and \(b\) on the scale of safety) are as shown in (113).

$$(113)$$

Consider again the case of an ordering along a dimension of safety in which an object

$$a$$

is ordered below an object \(b\) in which an object

$$b$$

Consider again the case of an ordering along a dimension of safety in which an object

$$a$$

is ordered below an object \(b\) in which an object

$$b$$

are monotonic increasing functions. Entailing that positive adjectives are monotone increasing functions and negative adjectives are monotone decreasing functions is a definition of the positive and negative extents of a sortal distinction.

In addition to supporting a principled explanation of cross-polar anomaly (something the algebra of degrees fails to do), this neat definition of positive and negative adjectives allows us to formulate a neat definition of the degree of a concept. This neat definition of a function is monotone increasing.

$$f(a) < f(b) \implies a < b$$  \hspace{1cm} (111) 

This neat definition of a function is monotone decreasing.

$$f(a) > f(b) \implies a > b$$  \hspace{1cm} (111) 

Given the definition in (112), it follows that positive adjectives denote monotone increasing functions and negative adjectives denote monotone decreasing functions.
The positive adjective safe preserves the ordering on a and b, but the negative adjective dangerous reverses it. Therefore, safe is monotone increasing, and dangerous is monotone decreasing. This result does not follow from some property of safe and dangerous; rather it is a general consequence of the hypothesis that logical polarity is represented as a sortal distinction between gradable adjectives. What distinguishes this result from the one obtained in the degree approach is that it follows directly from the algebra of extents, not from prior assumptions about implicit ordering relations associated with the adjectives safe and dangerous.

3.3.4 Summary

Taking differences in monotonicity properties of gradable adjectives as a starting point, this section considered two alternative representations of logical polarity. The degree analysis makes the assumption that negative adjectives are scale reversing at a basic level; rather than explaining why negative adjectives are monotone decreasing, it defines them as such. The extent approach, on the other hand, derives the monotonicity properties of polar adjectives from the sortal distinction between positive and negative adjectives provided by the algebra of extents. This approach is independently motivated because, unlike the degree algebra, it supports an explanation of cross-polar anomaly.

It is worth noting that the proposals I have made here do not predict whether an adjective is going to be positive or negative—nor are they intended to. The goal is rather to present a formal representation of adjectival polarity which accounts for the relational properties of positive and negative adjectives in as robust and explanatorily adequate a way as possible.

If the nominal determiners many and few are analyzed as positive and negative gradable predicates, respectively (Klein 1980, see also Sapir 1944), then it should be possible to explain their properties of gradable adjectives in terms of the extent approach, which correctly predicts that the adjective safe is going to be positive or negative, depending on the context. This alternative approach provides a principled analysis of the cross-polar anomaly, which follows directly from the algebra of extents.

The facts I have considered in this paper show that a formal system in which polarity is represented as a sortal distinction between gradable adjectives achieves this goal.

3.4 Conclusion

The puzzle which formed the starting point for this chapter was the anomaly of comparatives formed of positive and negative pairs of adjectives. Observing that this anomaly is unexplained if degrees are analyzed as points on a scale, I claimed that degrees should instead be formalized as intervals on a scale, or extents, and I made a structural distinction between two sorts of extents: positive extents, which range from the lower end of the scale to some positive point, and negative extents, which range from some point to the upper end of the scale. Adjectival polarity was then characterized as a sortal distinction between positive and negative adjectives: positive adjectives denote functions from objects to positive extents; negative adjectives denote functions from objects to negative extents. This approach not only supports a principled analysis of the cross-polar anomaly, it also explains why negative adjectives are monotone decreasing and positive adjectives are monotone increasing, in a way that is consistent with the assumption that negative adjectives are scale reversing. The extent approach provides a principled representation of logical polarity, which explains the different relational characteristics of positive and negative adjectives, and provides the basis for an explanation of the interpretation of comparatives of degree adjectives. Finally, the extent approach provides a principled representation of logical polarity which accounts for the relational properties of positive and negative adjectives in as robust and explanatorily adequate a way as possible.

The extent approach shows that a formal system in which polarity is represented as a sortal distinction between gradable adjectives achieves this goal.