A Scalar Semantics for Scalar Readings of Number Words*

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1 Introduction

Number words have played a central role in debates about the relation between context and meaning for decades. While current analyses of these terms and the sentences in which they appear differ in their details, they typically agree that interpretations of sentences containing number words crucially involve pragmatic enrichment of a more basic meaning. In the first part of this paper, I present a set of challenges for existing semantic and pragmatic accounts of number word meaning. In the second part, I develop and motivate a fully semantic and compositional analysis of scalar readings of sentences containing numerals, in which number words denote generalized quantifiers over degrees, and scalar readings arise through scopal interactions between number words and other constituents, rather than through pragmatic enrichment.

1.1 The Classic Analysis

The dominant view of number word meaning — or more accurately, the contribution of number words to sentence and utterance meaning — comes from Horn 1972:

> Numbers, then, or rather sentences containing them, assert lower boundedness — *at least n* — and given tokens of utterances containing cardi-

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> nal numbers may, depending on the context, implicate upper boundedness — *at most n* — so that the number may be interpreted as denoting an exact quantity. (Horn 1972, p. 33)

Horn's analysis is based on the observation that number words appear to pattern with other scalar terms in leaving open whether propositions involving higher quantities are true, while generating entailments that propositions involving lower quantities are false, as illustrated by the examples in (1).

(1) a. Kim read three of the articles on the syllabus, if not more/#fewer.
    b. Kim read most of the articles on the syllabus, if not all of them/#many of them.

Although an assertion of the first part of (1a) (*Kim read three of the articles on the syllabus*) would usually be taken to communicate the information that Kim didn't read more than three books, just as the first part of (1b) would be taken to mean that Kim didn't read more than merely most of the books (i.e., that Kim didn't read all of them), the fact that (1a-b) can be followed by *if not more/all* is taken as evidence by Horn that the UPPER BOUND inference associated with utterances of these sentences (*not four, five six...; not all*) is a conversational implicature. In contrast, the infelicity of the *if not fewer/many* continuations is taken as evidence that the LOWER BOUND inference (*at least three; more than half*) is an entailment.

The details of Horn's account of the "exact quantity" interpretation of sentences with number words — what Horn (1992) calls the TWO-SIDED meaning — runs as follows. Assume that sentences containing number words entail a lower bound, so that the first part of (1a) is true just in case the number of articles read by Kim is equal to or greater than three. If this is correct, then any sentence just like (1a) in which *three* is replaced by a number word that introduces a lower bound greater than 3 (*four, five, six*, etc.) entails (1a). On the assumption that a speaker intends to be as informative as possible without saying something that she knows to be false, in accord with Grice's Maxims of Quality and Quantity, an utterance of (1a) instead of a stronger alternative generates the implicature that the speaker believes that none of the alternatives are true. The resulting combination of sentence meaning (*the number of books Kim read is equal to or greater than three*) and implicature (*for all n greater than three, the speaker doesn't believe that the number of books Kim read is equal to or greater than n*) corresponds to the two-sided meaning.

Additional evidence for the Classic Analysis comes from the interpretation of sentences in which number words occur in a downward-entailing context. Unlike (1a), an utterance of (2a) is most naturally understood as conveying the information that everyone who missed three *or more* of the questions failed the exam, not as a claim that everyone who missed exactly three of the questions failed; (2b-c) are understood in a similar way.

(2) a. Everyone who missed three of the questions failed the exam.
b. Everyone who missed some of the questions failed the exam.
c. Everyone who missed most of the questions failed the exam.

The difference between these examples and the ones in (1) is that the kind of entailment pattern that motivates an analysis of the upper bounding interpretations of (1a-c) in terms of scalar implicature does not obtain, because the numeral and scalar quantifiers appear in a downward entailing context. Replacing *three* in (2a) with a numeral higher in the counting list does not generate an entailment to (2a) (given lower bounded truth conditions); instead the entailment relations are flipped: if it is true that everyone who missed at least three of the questions failed the exam, then it is true that everyone who missed at least four, five, six, etc. failed. The Classic Analysis therefore predicts that examples like (2) should not have two-sided interpretations, which indeed appears to be the case.

Horn's version of the Classic Analysis is a neo-Gricean one, but its general features are shared by "grammatical" theories of scalar implicature as well (e.g. Chierchia 2004, 2006; Chierchia, Fox, and Spector 2012; Fox 2007; Magri 2011, etc.). In these approaches, the upper bounding inference is introduced compositionally by an exhaustivity operator EXH which composes with a sentence, computes a set of alternative meanings for the sentence in a way that is fully parallel to the reasoning outlined above, and returns a meaning that consists of the original sentence meaning plus a denial of its stronger alternatives. The central difference between the neo-Gricean version of the Classic Analysis and the grammatical one is that on the former view, the two-sided understanding of the utterances in (1) is part sentence meaning, part speaker meaning; while on the latter view, it is entirely a matter of sentence meaning, and so can interact compositionally with other expressions. However, both versions of the Classic Analysis share two core features. First, upper bounding inferences arise through consideration and subsequent exclusion of stronger scalar alternatives. Second, upper bounding inferences of numerals and other scalar terms (both quantifiers like *some, many, most* as well as modals, aspectual verbs, and so forth) are derived in exactly the same way. As we will see below, these two features lead to problems for both versions of the Classic Analysis.

1.2 Number word meaning

Before moving to a discussion of these problems, I want to point out that although the Classic Analysis is often thought of as an analysis of the meanings of number words themselves, this is incorrect. As the quote from Horn 1972 makes clear, it is instead an analysis of the information conveyed by an utterance of a sentence containing a cardinal numeral. As such, it is compatible with several distinct hypotheses about the meanings of number words themselves and the way in which the "basic" lower bounded semantic content is introduced. There are three main approaches that have been taken in the literature.

The first is to analyze number words as quantificational determiners, as in Barwise and Cooper 1981. On this view, the hypothesis that numerals introduce lower bounded content reflects one of two potential analyses of the denotation of the determiner, specifically the one in (3a) (using the numeral 'three' as an example), in which it introduces a lower bound on the cardinality of the intersection of the determiner's restriction and scope.

(3) a. $[\![three]\!] = \lambda P \lambda Q. \mid P \cap Q \mid \geq 3$
    b. $[\![three]\!] = \lambda P \lambda Q. \mid P \cap Q \mid = 3$

This denotation contrasts with the one in (3b), advocated by Breheny (2008), which introduces a two-sided meaning as a matter of semantic content, and so represents a significantly different hypothesis about how two-sided inferences are generated.

The second option is to treat number words as cardinality predicates, or properties of pluralities, as in Krifka 1998; Landman 2003, 2004; Chierchia 2010; Rothstein 2011 (or, in a more elaborate way, Ionin and Matushansky 2006). This approach typically assumes a denotation along the lines of (4), where # returns the number of atoms that comprises a plural individual.

(4) $[\![three]\!] = \lambda x.\#(x) = 3$

Even though the number word introduces a relation to an exact quantity (which accounts for obligatory two-sided interpretations of predicate numerals; see Geurts 2006), the resulting truth conditions for a sentence in which a numeral modifies a noun are lower bounded, thanks to existential closure over the individual argument. The truth conditional content of the first part of (1a), for example, is as shown in (5) (ignoring partitivity).

(5) $\exists x[\textbf{read}(x)(\textbf{kim}) \wedge \textbf{articles}(x) \wedge \#(x) = 3]$

(5) entails the existence of a plurality of books read by Kim of size three, but does not rule out the existence of pluralities of greater size, and so has lower bounded truth conditions. However, this analysis can be turned into one that introduces two-sided truth conditions either by adding an extra condition to the effect that the plurality which satisfies the cardinality predicate uniquely satisfies it (Nouwen 2010), or by type-shifting the predicate denotation into a determiner meaning (Geurts

2006).[1]

A third option is to treat numerals as singular terms, as advocated by Frege (1980 [1884]), i.e. as names of numbers. Recasting Frege's view in the context of contemporary semantic theory, we may assume a model-theoretic interpretation in which numbers are of the same semantic type as the objects introduced by measure phrases like *two meters* and quantified over by comparatives and other kinds of quantity morphology, i.e. type d (for DEGREES; see e.g. Cresswell 1976; von Stechow 1984; Klein 1991; Kennedy 1999; and and many others). Like these other expressions, number words must compose with an expression of type $\langle d, \alpha \rangle$; such an expression can be derived from the quantificational determiner or adjectival meanings of number words by abstracting over the position of the numeral in the metalanguage representation of the meaning, deriving either the "parameterized" quantificational determiner in (6a) (Hackl 2000) or the gradable adjective in (6b) (Cresswell 1976; Krifka 1989).[2]

(6) a. $[\![\text{MANY}_{Det}]\!] = \lambda n \lambda P \lambda Q. \mid P \cap Q \mid \geq n$
b. $[\![\text{MANY}_{Adj}]\!] = \lambda n \lambda x. \#(x) = n$

Composing (6a) or (6b) with the singular term denotation for e.g. *three*, namely the number 3, then gives back the determiner meaning in (3a) or the cardinality predicate meaning (4), respectively.[3] This approach is similar to previous two in making

[1] In fact, Geurts shows that both the two-sided and lower bounded determiner meanings in can be derived from the cardinality predicate meaning in using standard type-shifting principles, and uses this to build a "polysemy" analysis of number word meaning which can accommodate most of the facts we will discuss in the next section, with the exception of those involving existential modals.

[2] In fact, Cresswell (1976) and Krifka (1989) treat nouns themselves as type $\langle d, \langle e, t \rangle \rangle$, basically incorporating the meaning of MANY_{Adj} into the meaning of the noun. I actually prefer this way of doing things for English, but since working it out requires dealing with issues of composition that are orthogonal to the purposes of this paper (having to do with how the degree argument gets passed up, and how it gets saturated in the absence of an overt numeral), I will stick with the compositionally simpler MANY_{Adj} version of the analysis.

[3] An apparent disadvantage of this kind of analysis is that it must stipulate that MANY is deleted (or unpronounced) when it composes with a numeral, though it may turn out that there is a principled explanation for this. Note in particular that other putative degree-denoting expressions either allow deletion of their corresponding scalar predicates in prenominal position (ia) or require it (ib):

(i) a. a 2 meter (long) board
b. a 10 kilogram (*heavy) stone

(Murphy 1997) proposes that reason that *heavy* must be omitted in (ib) is because the measure phrase unambiguously determines the dimension of measurement. (This is not the case in (ia), where *2 meters* could be a measure of length, width, depth, etc.) It is tempting to apply the same reasoning to the case of numerals and MANY in English, since MANY is (by hypothesis) the only scalar predicate that numerals combine with. This would lead us to expect that if a language had the number word part of a determiner or adjective meaning (and in this way, it can accommodate whatever syntactic evidence there might be for one analysis or the other), but differs crucially in allowing for more independence between the numeral and the rest of the nominal projection: since the numeral itself is a syntactic and semantic argument, rather than a determiner or a modifier, it may have an interpretive independence from the rest of the nominal. Specifically, it can in principle take scope independently from the rest of the nominal projection. This will turn out to be crucial in the alternative to the Classic Analysis that I will propose in Section 3.

2 Challenges to the Classic Analysis

In this section, I will discuss a number of problems for the Classic Analysis, which challenge both the view that number words introduce lower bounded sentence meaning, and the view that the the upper bounding inference and two-sided speaker meaning is derived in the same way for number words as it is for other scalar expressions.

2.1 Two-sided meaning and semantic composition

In a neo-Gricean model of speaker meaning, conversational implicatures are based partly on contextual information, partly on reasoning based on the Cooperative Principle, and partly on sentence meaning. Because sentence meaning is part of the input to implicature calculation, a prediction of the neo-Grician model is that implicatures are invisible to semantic composition. This is a central tenet of the neo-Gricean program, and is the basis for its resulting "simplification" of semantics. In the case of number words, if they introduce only lower bounded semantic content, with two-sided meanings arising as implicatures, the prediction is that only lower bounded content should interact compositionally with other expressions in a sentence. As it turns out, there is reason to believe that that this prediction is wrong, and that instead number words interact with other expressions in a way that indicates that two-sided inferences are a matter of semantic content. I will examine three such cases here: interactions with negation, quantifiers and modals.[4]

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more than one type of MANY-term, it would need to be expressed, a scenario that sounds very much like a classifier system.

[4]Space considerations preclude a discussion of the full range of data that has been presented in recent years as evidence for two-sided content in number words; so I focus here on a few cases that I think make the point in a particularly clear and relevant way. For additional discussion, though, see Sadock 1984; Koenig 1991; Scharten 1997; Carston 1998; Krifka 1998; Bultinck 2005; Geurts 2006 and Breheny 2008.

2.1.1 *Interactions with negation*

Consider first the question-answer dialogue in (7). If the semantic content of B's response to A involves a lower bounded meaning for the number word, then her denial amounts to the claim that it is false that she has at least three children. This is consistent with her follow-up in (7B-i), but it should be contradicted by her follow-up in (7B-ii).

(7) A: Do you have three children?
B: No, I don't have three children.
(i) I have two.
(ii) I have four.

In fact, either (7B-i) or (7B-ii) is a perfectly acceptable way for B to elaborate on her initial denial that she has three children. Horn (1972) was well aware of this, and argued that only B's responses in (7B-i) involves a negation of the semantic content of the number word. (7B-ii), on the other hand, is an instance of METALINGUISTIC NEGATION, in which B's use of negative morphology is designed to signal that the upper bounding implicature should be suppressed.

However, this analysis has been challenged in recent years, including by Horn himself. Horn (1992) observes that that in question/answer contexts, putative denials of an upper bound inference in answers with number words have a different status from those those involving clear scalar quantifiers. He points to pairs like the following, observing that B's response in (8) is perfectly acceptable even with a neutral intonation, but B's response in (9) is acceptable only with the strong intonational prominence on *all* that is indicative of a metalinguistic interpretation.

(8) A: Do you read three of the articles on the syllabus?
B: No, I read four of them.

(9) A: Did you read many of the articles on the syllabus?
B: ?No, I read all of them.

Horn (1996) elaborates on this difference with a discussion of examples like those in (10).

(10) a. ??Neither of them read many of the articles on the syllabus. Kim read one and Lee read them all.
b. ??Neither of them started the assignment. Kim didn't even look at it, and Lee finished it.
c. ??Neither of them liked the movie. Kim hated it, and Lee absolutely loved it.

The negative quantifier in the first sentence of these examples can in principle be understood either semantically or metalinguistically. If it is understood semantically, it interacts with the lower bounded semantic content of the scalar quantifier *many*, the aspectual verb *start* and the scalar verb *like*, generating the meaning that it is false of the two individuals picked out by *them* (Kim and Lee) that an appropriate lower bound was reached. (The number of articles read was below the threshold for counting as "many"; the assignment was not started; the evaluative attitude towards the movie is insufficient to count as liking it.) If it is used metalinguistically, with appropriate intonational prominence on the scalar term, it can deny the corresponding scalar inference: *"Neither of us LIKED the movie. We both LOVED it!"* The examples in (10) are strange because they require us to simultaneously understand the negative quantifier semantically and metalinguistically, something that is evidently impossible, or at the very least triggers a zeugma effect. If number words are semantically lower bounded and only pragmatically upper bounded, then we should see exactly the same infelicity in (11), but this is not the case:

(11) Neither of them read three of the articles on the syllabus. Kim read two and Lee read four.

This example is perfect, even without special intonational prominence on the number word. This is expected only if the number word introduces two-sided semantic content, which can then be targeted by the negative quantifier, deriving the intuitively correct truth conditions for (11): it is false of both Kim and Lee that they each read exactly three articles on the syllabus.

The interaction of negation and number words is a problem for both the neo-Gricean and grammatical variants of the Classic Analysis, though in different ways. As noted above, the neo-Gricean analysis must explain apparent denials of the upper bound as instances of metalinguistic negation, a move that does not appear to square very well with the facts. The grammatical approach can in fact account for the facts by hypothesizing that EXH, the exhaustivity operator involved in generating upper bounding inferences, can be inserted below negation. In the case of (11), for example, the relevant reading can be derived by assuming the representation in (12a), which derives the truth conditions paraphrased in (12b) (keeping things informal for now).

(12)
a. Neither of them EXH [read three of the articles on the syllabus]
b. For each $x \in \{$Kim, Lee$\}$, it is not the case that:
   (i) x read at least three articles and
   (ii) it is not the case that x read more than three articles

To satisfy the negation, it is necessary for each of Kim and Lee to make one of (12b-i) or (12b-ii) false; this will be the case if they read fewer than three articles ((12b-i) false) or more than three articles ((12b-ii) false), i.e. if the two-sided truth conditions fail to hold.

However, note that in the examples under consideration here, the upper bounding inference appears in a downward entailing context. As I mentioned in Section 1.1, it is generally the case that scalar implicatures disappear in such contexts. This is expected in a neo-Gricean framework, because such contexts invert entailments, and explains why (9) and (10) are odd. At the same time, this is precisely why examples like (8) and (11) are such problems for the neo-Gricean version of the Classic Analysis: if the two-sided interpretation of number words derives from a scalar implicature, then it should disappear in downward entailing contexts, contrary to fact.

In contrast, the fact that scalar implicatures generally disappear in downward entailing contexts does not follow automatically in the grammatical theory of scalar implicatures, and this difference is precisely what gives the grammatical version of the Classic Analysis the tools it needs to derive two-sided meanings for number words in the examples above. But instead of solving the problem, this just creates a new one: if examples like (8) and (11) show that EXH can be inserted in a downward entailing context, generating two-sided inferences for number words, then why can it evidently not be inserted in the corresponding examples in (9) and (10)?

In fact, according to Chierchia et al. (2012), insertion of EXH in such examples is possible, but dispreferred. Chierchia et al. point out that whenever a weak scalar term is part of a propositional constituent S that occurs in a downward entailing context, then adjunction of EXH to S derives a meaning that is asymmetrically entailed by the corresponding structure without EXH. This allows them to appeal to a "strongest meaning" principle as a way of limiting upper bounding inferences downward entailing contexts: insertion of EXH is dispreferred if it results in a weaker meaning. This is a default constraint, but it is not a hard one, as it can be violated in the right context. On this view the unacceptability of (9) and the examples in (10) is indicative of the constraint in action, while the fact that strong phonological emphasis on the scalar terms can improve the examples indicates that the constraint can be bypassed.

Regardless of whether this line of thought is a correct or fruitful way of understanding the distribution of upper bounding inferences with scalar terms in general (a debate that I do not wish to engage here), it does not provide a way of salvaging the Classic Analysis of number words. The problem, quite simply, is that there is no evidence that two-sided interpretations of sentences with number words are dispreferred in downward entailing contexts, or that it is necessary to add a special prominence or emphasis on the number word to derive this meaning. Quite the opposite: in the examples considered above, the two-sided interpretations appear to be the default, and need not be signalled by special intonational prominence. In short, the central challenge for the grammatical version of the Classic Analysis is the same as the one faced by the neo-Gricean version: sentences with number words systematically have two-sided meanings in contexts in which other scalar terms do not.

2.1.2 Interactions with quantifiers

Interactions with universal and negative quantifiers make the same point. Earlier I presented (2), repeated below, as evidence that upper bounding inferences disappear in the scope of a universal quantifier.

(2) a. Everyone who missed three of the questions failed the exam.
b. Everyone who missed some of the questions failed the exam.
c. Everyone who missed most of the questions failed the exam.

While it is true that (2a) is most naturally understood as conveying the information that everyone who missed three or more of the questions failed, we actually cannot be sure that this is because *three* introduces lower-bounded content. Even if *three* introduced two-sided content, (2a) would be still be true in a situation in which everyone who missed three or more of the questions failed the exam, because such a situation is also one in which everyone who missed exactly three questions failed.[5] We could then explain our understanding of the message conveyed by an utterance of (2a) in terms of a particularized scalar implicature: generally, missing more questions on an exam is worse than missing fewer, so if it's true that everyone who missed exactly three questions failed, we can reason that everyone who missed more than three failed too (cf. Breheny 2008).

But more importantly, it is easy to construct examples in which sentences containing numerals in downward-entailing quantifier restrictions clearly have two-sided truth conditions. Consider, for example, a test-taking situation in which there are five answers, and students are rewarded with different color stars depending on how well they do: students who get exactly three correct answers receive a red star; students who get exactly four correct answers get a silver star, and students

[5]The crucial difference between lower bounded content and two-sided content is that the former would be true and the latter false if e.g. some people missed four or more questions and all of them failed, but nobody missed exactly three questions. But this situation is not really a problem, given the existence presupposition of the relative clause. Indeed, the fact that (2a) appears to presuppose only that there are some people who missed exactly three questions on the exam, and not that there are some people who missed more than three questions, could potentially be interpreted as another argument that number words introduce two-sided semantic content.

who get five correct answers get a gold star. In this scenario, the sentences in (13) can be understood in two different ways. They both can be understood in a way parallel to (2a), with the number word introducing a lower bound, in which case they falsely describe the situation under consideration. This reading might indicate the existence of lower bounded truth conditions, but it could also be the result of pragmatic reasoning, as described above.

(13) a. Everyone who correctly answered three of the questions got a red star.
b. No one who correctly answered three of the questions got a gold star.

However, both of the sentences in (13) also can be understood as saying something true about the situation in (12), a result that is possible only if the number words are understood as providing both a lower and upper bound on the number of missed examples, i.e., only if the sentences have two-sided interpretations.

The same cannot be said of the examples in (14), in which the number words are replaced by scalar determiners. These sentences are unambiguously false as descriptions of the situation in (12), which means that they have only their "basic," lower bounded truth conditions.

(14) a. Everyone who correctly answered most of the questions got a red star.
b. No one who correctly answered most of the questions got a gold star.

The absence of two-sided readings in (14a-b) is expected on the neo-Gricean version of the Classic Analysis, but the presence of two-sided readings in (13a-b) is not. In the absence of strong intonational prominence on the scalar words (something not required for the numerals), these readings can be derived in the grammatical version of the Classic Analysis by inserting EXH in the relative clause (despite the fact that this is a downward entailing context), but then we incorrectly predict two-sided readings to be equally accessible in (14a-b).

2.1.3 Interactions with modals

Let us finally consider sentences in which number words appear in the scope of a modal verb.[6] At first glance, the interaction of modals and number words appears to provide strong evidence that the lower bound inference is a constituent of the semantic content of sentences containing number words. In the examples in (15),

[6] I restrict my attention here to interactions between number words and deontic modals, mainly because focusing on one case is sufficient to make the main point that number words are able to introduce two-sided truth conditional content, and because deontic modals support the construction of fairly clear examples. However, I will say a bit more about the relation between number words and other kinds of modals in section 3.

the number word is part of the semantic scope of the modal, and is understood as though it introduces lower bounded semantic content (Scharten 1997; Carston 1998).

(15) a. In Britain, you have to be 18 to drive a car.
b. Mary needs to get three As if she wants to graduate.
c. Recipients of the education benefit must have two children enrolled in primary school.

An utterance of (15a), for example, is most naturally understood as an assertion that in all worlds consistent with the relevant regulations, one must be at least 18 to drive, i.e. that 18 is the minimum legal driving age. Likewise, (15b) places a lower bound on the number of *As* Mary needs to graduate, and (15c) implies that anyone with two or more students in primary school will get the benefit. These readings are exactly what we expect if number words have lower bounded semantic content, and if only the semantic content of the expression that provides the scope of the modal (the infinitival clause) plays a role in determining the truth conditions of the sentence.[7]

(15a-c) can also have two-sided understandings, in which an upper bound is placed on the obligation: (15b), understood in this way, says that Mary needs to get three As to graduate, but she does not need to get more than three As. This reading can be derived in the usual way, either as a quantity implicature or by inserting EXH at the root.

The problem is that examples like (15a-c) can, in addition, have a two-sided understanding which is not derivable as a scalar implicature, at least not in the normal way. Such interpretations are stronger than the ones I just described: instead of merely placing an upper bound on what is required, they involve requirements which are themselves upper bounded. World knowledge makes such readings implausible for (15a-b), but they emerge quite clearly in (16a-b), for example.

(16) a. In "Go Fish", each player must start with seven cards.
b. Assignments have to be five pages long.

The point of (16a) is to say that the rules of "Go Fish" stipulate that players begin with exactly seven cards. Likewise, (16b) can be used to convey the information that acceptable assignments are no more and no less than five pages long. Here again, the upper bounding inference appears to be part of the semantic content of

[7]As pointed out by Geurts (2006), these examples also appear to challenge a semantic analysis of number words as two-sided quantificational determiners (as in (3b)), a position that is advocated by Breheny (2008). Breheny acknowledges this challenge, and attempts to derive the appearance of one-sided content in examples like (15) pragmatically.

an expression that composes with another element of the sentence, something that is problematic for the neo-Gricean version of the Classic Analysis.

The facts can be accommodated by the grammatical version of the Classic Analysis by inserting EXH below the modal (cf. Chierchia et al. 2012). As with the other phenomena we looked at, however, this would again predict a parallelism between number words and other scalars that is not actually observed. Consider the following scenario. A major midwestern research university seeks to build up enrollments in its for-profit MA program by requiring PhD-granting departments to identify applicants to their graduate programs who are promising and likely to accept an offer of admission, but not fully competitive for the PhD program, and then to pass them on for consideration by the MA program admissions staff. The administration wants to maximize the applicant pool, but not overwhelm the limited MA program staff. In such a context, (17a) is most naturally understood as imposing a two-sided obligation: the Linguistics Department must send over ten applicants, and may not send over more than ten applicants.

(17) a. The Linguistics Department is required to select ten of its PhD program applicants for consideration by the MA program.
b. The Linguistics Department is required to select some of its PhD program applicants for consideration by the MA program.

(17b), on the other hand, does not have an understanding in which the requirements prohibit selection of Q of the Department's PhD applicants, where Q is a scalar alternative to *some*. That is, although norms of behavior and expectation typically ensure that a department given the instructions in (17b) will not send all — or even most or many — of its PhD applicants to the MA program (don't make the MA people do too much work; save some applicants for the PhD program), (17b) cannot be understood as an actual rule prohibiting such a move.

The preceding examples all involve interactions between number words and universal modals. But number words also interact in an interesting and very important way with existential modals. In particular, existential modals naturally give rise to readings in which number words are associated only with an upper bound (Scharten 1997; Carston 1998). (18a) is understood to say that as long as Lee has 2000 or fewer calories, there will be no problem of weight gain; (18b) gives prisoners permission to make up to three phone calls, and (18c) licenses attendance in six courses or fewer.

(18) a. Lee can have 2000 calories without putting on weight.
b. Prisoners are allowed to make three phone calls.
c. You may take six courses next year.

Carston (1998) claims that the salient, upper bounded interpretation of such examples cannot be accounted for on the Classic Analysis, but this is not correct: as I will show below, it can be derived in the usual way, as a scalar implicature. More significantly, given the traditional approaches to the semantic analysis of number words discussed in section 1.2, the upper bounded reading can *only* be derived as an implicature, regardless of whether we assume that number words introduce lower bounded or two-sided content. To see why, let us take a close look at the analysis of (18c).

First, observe that the upper bounded reading is consistent with a *de dicto* interpretation of the nominal, so we need to be able to derive it when the nominal takes scope under the modal. The assume that the numeral either introduces lower bounded content, or two-sided content, in one of the ways discussed in section 1.2. If the numeral introduces lower-bounded content, we get the truth conditions informally stated in (19a); if it introduces two-sided content, we get the truth conditions in (19b).

(19) a. $\exists w \in Acc_{w0}[\textit{the number of courses you take in w} \geq 6]$
b. $\exists w \in Acc_{w0}[\textit{the number of courses you take in w} = 6]$

On either of these options, the truth-conditional content of (18c) is exceedingly weak: there is a deontically accessible world in which you take at least/exactly six courses. Neither of these meanings forbids enrollment in more than six courses, and neither expressly allows enrollment in fewer than six. And indeed, (18c) can be understood in this weak way. However, it is far more natural to understand it in a stronger way, which forbids enrollment in more than six courses, but allows enrollment in six or fewer. This is the upper bounded reading which Carston claims to be unavailable to the Classic Analysis.

In fact, this meaning is straightforwardly derivable as an implicature, given either the one-sided semantics in (19a) or the two-sided semantics in (19b). In particular, we can derive the inference that registration in more than six classes is prohibited as a scalar implicature from (19a), since it is asymmetrically entailed by alternatives in which the number word is replaced by a higher value. The resulting implicature — *for all n greater than 6, there is no deontically accessible world in which the number of courses you take is at least n* — gives the upper bound and the strong meaning typically associated with an utterance of this sentence. Somewhat suprisingly, if the basic meaning of the number word involves two-sided content, as in (19b), the strong reading *cannot* be derived as a scalar implicature, because (19b) is not entailed by alternatives in which 6 is replaced by higher values. But both the upper bound inference and the inference that lower quantities are allowed can be derived as a particularized conversational implicature from general reasoning about permission (Breheny 2008).[8].

The significance of the upper bounded reading of sentences like (18a-c), then, is not that it presents a particular problem for the Classic Analysis, as Carston claims, but is rather that this is a reading which is only derivable via the implicature system, no matter which of the traditional hypotheses about the semantic content of number words we adopt. This means that if we can find evidence that these readings are derived semantically, rather than pragmatically, we have a strong reason to reconsider the traditional semantic assumptions about number word meaning. And indeed, there is evidence that the one-sided, upper-bounded readings we are interested in here are not (or at least do not have to be) derived via implicature, but are instead a matter of semantic content. One piece of evidence will be discussed in the next section, where we will see that there is populations of speakers who systematically fail to calculate scalar implicatures, yet not only assign two-sided meanings to sentences with number words, but also appear to understand sentences like those in (18) in the usual, strong way. Both facts are problematic for accounts of scalar readings of number words that crucially invoke the scalar implicature system.

The second piece of evidence comes from examples similar to the ones we have already been considering, in which the upper bounding inference is retained in a context in which implicatures normally disappear. To set up the example, imagine a situation in which there are three different groups of people who can be distinguished according to how many of four possible exemptions they are allowed to claim on their tax returns: zero for individuals in Group A, two for individuals in Group B, and four for individuals in Group C. These individuals are members of an exemption-maximizing but law-abiding society, so everyone in Group A claims zero exemptions, everyone in Group B claims exactly two, and everyone in group C claims exactly four. Now consider the following utterances as descriptions of this situation:

(20) a. No individual who was allowed to claim two exemptions claimed four.
b. No individual who was allowed to claim some exemptions claimed four.

(20a) has a reading in which it is true in this scenario, because the quantifier restriction is understood to pick out individuals who were allowed to to claim two exemptions, and not allowed to claim more than two exemptions, i.e. the ones in

[8]The reasoning goes roughly as follows. Knowing that it is permissible to take exactly six courses, does not allow one to conclude that it is permissible to take more than six courses, because doing so involve a greater use of existing resources. On the other hand, if it is permissible to take six courses, then surely it is not a problem to take five or fewer, because that would involve a smaller use of existing resources.

Group B. This is an upper bounded reading, but it occurs in a downward entailing context (and in the argument of a logical operator), which we have already seen is a context that is resistant to implicature calculation. And indeed, (20b), in which the number word is replaced by the scalar quantifier *some*, is clearly false in this situation, because the quantifier is understood to range over all individuals who were allowed to claim exceptions, which includes the ones in Group C. The sentence does not have a reading in which the restriction ranges over individuals who were allowed to claim some exemptions but not allowed to claim all exemptions. This would exclude the individuals in Group C, and would make the sentence true.

These facts provide further evidence, in addition to what we have seen so far, that upper bounding inferences in sentences with number words are a matter of semantic content, and as such, they provide yet another argument for rejecting the Classic Analysis. But the real significance of these facts is that they give us new insight on what an alternative analysis of number word meaning must look like: it should be one that generates an upper bounding inference for sentences with existential modals as a matter of semantic content. We have just seen that none of the semantic analyses discussed in Section 1.2, has this feature, including ones which posit two-sided semantic content for number words. The right analysis, then, is going to look different from what we have seen so far. In Section 3, I will say what I think it should look like; before doing that, though, I want to take a look at some experimental data which provides additional support for the conclusions that we have reached here.

2.2 Experimental studies of number word meaning

In addition to the kinds of semantic facts we have been discussing, there is a a large set of experimental evidence based on different methodologies and studies of both child and adult behavior which indicates that two-sided interpretations of sentences with number words are retained in contexts in which upper bounded interpretations of other scalar terms disappear. For example, in separate studies involving the Truth Value Judgment Task, Noveck (2001) and Papafragou and Musolino (2003) found that children (unlike adults) fail to assign upper bounded readings to sentences with weak modals, scalar quantifiers, and aspectual verbs (e.g., *might, some, start*), but consistently assign two-sided interpretations to corresponding sentences with number words. Hurewitz, Papafragou, Gleitman, and Gelman (2007) and Huang, Snedeker, and Spelke (2009) achieved similar results in picture-matching and act-out tasks, and Huang and Snedeker (2009) found an asymmetry between number words and scalar terms in a set of eye-tracking experiments that suggested the same conclusion. Huang and Snedeker (2009) moreover showed that in a task in which adults' calculation of upper bounding implicatures were suppressed for the scalar quantifier *some*, they were retained for numerals.

Taken as a whole, these studies provide evidence that two-sided interpretations of sentences with number words are available to speakers and in contexts in which two-sided interpretations of other scalar terms are not. The developmental work is particularly important, because it shows that during a stage in which children generally tend not to compute scalar implicatures (though they can do so when other cognitive demands are decreased, and the communicative task is highlighted; see Papafragou and Tantalou 2004), they nevertheless consistently assign two-sided interpretations to sentences with number words. The behavioral data thus supports the same conclusion that we drew in the previous section from observations about the linguistic data: two-sided interpretations of number words are not derived in the same way as two-sided interpretations of other scalar terms. The most straightforward explanation for the full pattern of data is that two-sided readings are not derived via the implicature system, whatever that may be (neo-Grecian or grammatical; generalized or particularized), but instead arise in a fully compositional way, in virtue of the meanings of the number words themselves — meanigns which are evidently different from the options we considered in Section 1.2, given what we saw with existential modals in the previous section.[9]

The experimental literature also includes (at least) one apparent challenge to this conclusion, however. Musolino (2004) reports a set of experiments which show that in contexts in which adults most naturally understand sentences containing number words in a one-sided way — either as introducing only a lower bound on a particular quantity, or as introducing only an upper bound — children do the same.[10] For example, children heard sentences like the following in a scenario in which an "owner" character has more objects than another character needs. (21) was presented in a context in which Troll needs just two cookies for a party, and Goofy presents him with a tray containing four cookies.

(21) Let's see if Goofy can help the Troll. The Troll needs two cookies. Does Goofy have two cookies?

[9]Though see Barner and Bachrach (2009) for an alternative explanation of the difference between numerals and other scalar terms, which is based on the idea that the alternatives involved in computing implicatures for numerals are easier for children to access, given their knowledge of the counting list. I do not have space here to adequately assess this interpretation of the data, other than to point out that in addition to its plausibility as an account of the developmental facts, it should be assessed in light of facts like those discussed in section 2.1 (and additional facts discussed in the papers mentioned in footnote 4), which give us strong reasons to believe that two-sided inferences in sentences containing number words are a matter of semantic content, not implicature.

[10]The following is a simplification and condensation of Musolino's study; please see the original article for the full details of design and statistical analysis of results, as well as a discussion of controls and adult responses, which were exactly as one would expect.

Children systematically respond “yes” to the question, which means that they are understanding it in a one-sided, lower bounded way. Similarly, when (22) is used to describe a situation in which Troll has missed only one hoop, children report that he does in fact win the coin, which indicates that the number word imposes at most an upper bound on number of misses, but not a lower one.

(22) Goofy said the Troll could miss two hoops and still win the coin. Does the Troll win the coin?

Unfortunately, we do not know from Musolino’s experiments whether children understand this sentence to actually impose an upper bound, because he did not consider scenarios in which more than two hoops were missed. My own experience with five-year olds tells me that the typical child knows exactly what would make the Troll lose here — missing more than two hoops — and I’m sure that other parents, aunts, uncles and older siblings will agree with me on this. This hunch should be confirmed experimentally, but if it turns out to be correct, we have a significant puzzle not only for the Classic Analysis of number word meaning, but also for alternatives which posit two-sided semantic content. The problem, as we saw in the previous section, is that given the current options for number word meaning, an upper bounding inference for examples with existential modals, like (22), can only be generated by implicature: because the truth conditional content alone does not preclude the existence of deontically accessible worlds in which more than two hoops are missed. This result is in conflict with all the other evidence suggesting that children do not calculate upper bounding implicatures for scalar terms, including for modals.

2.3 Summary

Summarizing, we have now seen compelling evidence that the upper bounding inference which gives rise to two-sided interpretations of sentences with number words is a matter of semantic content. The first kind of evidence comes from interactions between sentences containing number words and other logical operators that show that two-sided inferences are a matter of semantic content. This is a serious problem for Horn’s original neo-Gricean version of the Classic Analysis, a fact that has already been pointed out in the literature by many researchers, including by Horn himself (Horn 1992; see also Sadock 1984; Carston 1988, 1998; Koenig 1991; Scharten 1997; Geurts 2006 and Breheny 2008). The grammatical version of the Classic Analysis can accomodate the compositional facts by inserting an exhaustivity operator in the minimal clause containing the number word, but then predicts that other scalar terms should just as easily give rise to two-sided interpretations in the same contexts, contrary to fact. In short, on either version of the Classic Analysis, it remains a mystery why number words easily introduce two-sided content into the truth conditions, while other scalar terms do not.

A second type of evidence for the same conclusion comes from experimental results which show that sentences with number words strongly prefer two-sided interpretations, even in contexts and developmental stages in which sentences with other scalar terms are assigned one-sided interpretations. These results complement the linguistic data and indicate that two-sided interpretations of sentences with number words are a matter of semantic content, and not derived by whatever reasoning or mechanisms are involved in assigning upper bounding inferences to sentences containing other kinds of scalar terms. At the same time, children in the relevant developmental stage are also able to correctly assign one-sided interpretations to sentences containing number words in contexts in which adults to the same. The crucial question, then, is: what is the crucial difference between the contexts in which only two-sided meanings appear to be available and those which also allow one-sided readings? Let us now answer that question, and see how it leads to a new, fully semantic analysis of scalar readings of number words.

3 Numerals as generalized quantifiers over degrees

3.1 Scalar readings of comparative numerals

The generalization that emerges from both the linguistic and experimental literature is that two-sided readings are the norm, but that the two classes of one-sided readings (lower- and upper bounded) appear most naturally in sentences that contain other logical operators, in particular modals. Moreover, as originally observed by Scharten (1997), there is a pattern of interaction: lower bounded readings are associated with sentences in which number words occur in the scope of a universal (typically deontic) modal, and upper bounded readings are associated with sentences in which number words occur in the scope of an existential (deontic) modal. The following examples illustrate the pattern:

(23) *Lower bounded interpretations*
- a. In Britain, you have to be 17 to drive a motorbike and 18 to drive a car.
- b. Mary needs three As to get into Oxford.
- c. Goofy said that the Troll needs to put two hoops on the pole in order to win the coin.
- d. You must provide three letters of recommendation.
- e. You are required to enroll in two classes per quarter.

(24) *Upper bounded interpretations*

a. She can have 2000 calories a day without putting on weight.
b. You may have half the cake.
c. Pink panther said the horse could knock down two obstacles and still win the blue ribbon.
d. You are permitted to take three cards.
e. You are allowed to enroll in four classes per quarter.

All of these sentences also allow for two-sided interpretations, though in most of the examples in (23), real-world knowledge renders such a reading unlikely, and for the examples in (24), such a reading results in exceedingly weak truth conditions, as we saw in section 2.1.

The ambiguity of number words in modal contexts is strikingly similar to a pattern of ambiguity that has been previously discussed in the literature, involving the interaction between modals and comparatives (see e.g. Heim 2000; Hackl 2000; Bhatt and Pancheva 2004; Nouwen 2010). Consider, for example, (25a-b).

(25) a. Kim is required to take fewer than three classes.
b. Kim is allowed to take fewer than three classes.

(25a) has a reading in which the maximum number of courses that Kim is allowed to take is fewer than three; i.e., enrollment in three or more is prohibited. This is similar to the two-sided interpretation of the corresponding sentence with a bare numeral, where *fewer than three* is replaced by *three*. (25a) has a second reading, however, in which the <u>minimum</u> number of courses that Kim is required to take is less than three, e.g., Kim must take two, but is allowed to enroll in more. This reading is parallel to the one-sided, lower bounded reading of the corresponding sentence with the bare numeral. (25b) is similar. One reading allows enrollment in two or fewer courses, but says nothing about enrollment in three or more courses; this is parallel to the weak, two-sided reading of the sentence with the bare number word. The other reading forbids enrollment in more than two courses; this is parallel to the upper bounded, one-sided reading of the sentence with the bare number word.

Hackl (2000) shows that the ambiguity of sentences like (25) can be explained as a scope ambiguity arising from the interaction of the modal verbs and the comparative. Crucial to the analysis is the hypothesis that a comparative numeral like *fewer than three* is not an unanalyzed, complex quantificational determiner with the denotation in (26a) (as in e.g., Barwise and Cooper 1981; Keenan and Stavi 1986), but is rather a full-fledged generalized quantifier on its own, with the denotation in (26b).

(26) $[\![\text{fewer than 3}]\!]$

a. $\neq \lambda P_{\langle e,t\rangle}\lambda Q_{\langle e,t\rangle}.|P \cap Q| < 3$
b. $= \lambda D_{\langle d,t\rangle}.max\{n|D(n)\} < 3$

On this view, comparative numerals denote generalized quantifiers over degrees (type $\langle\langle d,t\rangle,t\rangle$). They combine with a degree property that is created by abstracting over a degree argument position inside the nominal projection, which we may assume to be introduced by the null cardinality predicate MANY discussed in Section 1.2, and to return true just in case the maximal degree (number) that satisfies the property is greater or less than (depending on the type of comparative) the degree (number) corresponding to the modified numeral.[11] Like other quantificational expressions, comparative numerals must take scope. This means that in sentences containing modal verbs, such as (25a-b), we predict an ambiguity depending on whether the comparative numeral scopes below or above the modal.

In the case of (25a), if the comparative has narrow scope relative to the modal, we derive the proposition in (27a); if it has wide scope relative to the modal, we derive the proposition (27b).

(27) a. $\lambda w.\forall w' \in Acc_w[max\{n|\exists x[\mathbf{classes}(w')(x) \wedge \#(w')(x) = n \wedge \mathbf{take}(w')(x)(\mathbf{kim})]\} < 3]$
b. $\lambda w.max\{n|\forall w' \in Acc_w[\exists x[\mathbf{classes}(w')(x) \wedge \#(w')(x) = n \wedge \mathbf{take}(w')(x)(\mathbf{kim})]\} < 3$

(27a) is fairly straightforward: it is true of a world iff in every world deontically accessible from it, the maximum number of classes taken by Kim is less than three. (27a) is therefore false of a world if there are worlds deontically accessible from it in which Kim takes three or more classes; this is the "two-sided" interpretation. (27b) is a bit more complex. It is true of a world w iff the maximum n, such that in every world deontically accessible from w there is a plurality of classes taken by Kim of size n, is less than three. In this case, the argument of the modal is a proposition with lower bounded truth conditions: the proposition that there is a plurality of classes of size n taken by Kim, for some value of n. This means that the set of values that the comparative is maximizing over represents the numbers of enrolled-in classes that all of the deontically accessible worlds agree on; maximizing over this therefore derives the *minimum* enrollment requirements, making (27b) false of

[11] As noted in Section 1.2, Hackl treats MANY as a determiner that introduces existential quantification over the individual argument of the noun. As such, MANY and its nominal argument constitute a DP which must themselves take scope, resulting in a "split-scope" analysis of DPs with comparative numerals. I adopt the "MANY$_{Adj}$" (cardinality predicate) analysis mainly to keep the logical forms simpler, but my proposals are entirely consistent with the "MANY$_{Det}$" analysis. Hackl also treats the numeral part of the comparative numeral complex as the remnant of a clausal structure, derived by ellipsis, an option that I will not discuss here.

a world if there are no worlds deontically accessible from it in which enrollment in at least two classes is required. This is the one-sided, lower bounded interpretation of (25a).

Turning to (25b), the two scope relations in (26) derive the truth conditions in (28a-b), which are a bit easier to understand.

(28) a. $\lambda w.\exists w' \in Acc_w[max\{n|\exists x[\textbf{classes}(w')(x) \wedge \#(w')(x) = n \wedge \textbf{take}(w')(x)(\textbf{kim})]\} < 3]$

b. $\lambda w.max\{n|\exists w' \in Acc_w[\exists x[\textbf{classes}(w')(x) \wedge \#(w')(x) = n \wedge \textbf{take}(w')(x)(\textbf{kim})]\} < 3$

(28a) is true of a world just in case there is a world deontically accessible from it in which the maximal number of classes that Kim takes is less than three; this is the "two-sided", weak reading of (25b), which merely allows enrollment in two or fewer classes, and doesn't forbid anything. (28b), on the other hand, is true of a world just in case the maximal n, such that there is world deontically accessible from it in which Kim takes n courses, is less than three. This is the one-sided, upper bounded reading of (25b), which forbids enrollment in greater than two classes.

Stepping back a bit, let us observe that there are two components to this analysis of the ambiguity of sentences containing comparative numerals and modals. The first is the hypothesis that comparative numerals are generalized quantifiers over degrees, which can take scope independently of the nouns that they are in construction with in the surface form, and so can interact scopally with the modals. The second is the hypothesis that part of the truth conditional content that a comparative numeral contributes involves maximization over the degree property denoted by its semantic scope, an operation (or its equivalent) which is necessary to get the comparative truth conditions right.[12] I would now like to propose that both of these features are also present in the semantics of bare numerals, in a way that derives the facts discussed in the previous section.

3.2 Unmodified numerals as degree quantifiers: Deriving scalar readings as scope ambiguities

Let us begin with scopability. That bare numerals must be able to take scope independently of the rest of the nominal has recently been argued by Kennedy and Stanley (2009). Kennedy and Stanley base their argument on the interpretation of

[12] As we saw in Section 1.2 and above, the combination of MANY plus existential quantification over the individual argument of the noun gives the scope expression a "Classic Analysis" semantics, so it picks out all the numbers/degrees such that there is a plurality of objects of at least that size which satisfy the restrictions imposed on the nominal argument.

"average" sentences such as (29a-b).

(29) a. The average American family has 2.3 children.
b. American families on average have 2.3 children.

What is important to observe about these examples is that they do not entail the existence of families with 2.3 children. This is in contrast to (30a-b), which are odd precisely because they do introduce such an entailment.

(30) a. ??A normal American family has 2.3 children.
b. ??American families in general have 2.3 children

The oddity of (30a-b) is expected if the denotation of the verb phrases in these examples is as in (31), which is what we would expect if 2.3 denotes a degree and directly saturates a degree position introduced by MANY_{Adj}.[13]

(31) $\lambda x.\exists y[\textbf{have}(y)(x) \wedge \textbf{children}(y) \wedge \#(y) = 2.3]$

(31) is a property that is true of an object if it has a quantity of children of size 2.3; assuming the truth conditions of (30a-b) entail that there are families which have such a property, we have an explanation for why the examples sound strange.

Conversely, the fact that the examples in (29) do not entail of any families that they have such an odd property suggests that (31) is not the denotation of of a constituent in the logical forms of these examples. And according to Kennedy and Stanley, the reason that (31) is not the denotation of a constituent of the logical form of these examples is because the numeral can (and must, in these cases) take scope independently of the rest of the nominal and outside of the verb phrase, just as we saw above with comparative numerals. This derives the relation between

[13]Clearly, if the numeral is a degree-denoting singular term, then MANY_{Adj} cannot actually be relation between pluralities and cardinalities, which is how I have been describing it so far. It must instead take quantities of stuff as its input, which may be composed of atoms as well as parts, and return a value that represents the measure of that stuff relative to a stuff-appropriate counting metric (possibly provided by the noun denotation; cf. Krifka 1989; Salmon 1997), with the values not limited to ones corresponding to whole numbers (cf. Fox and Hackl 2007). Quantities of children are normally not measured in this way, but many other things are, for example:

(i) It normally takes one bushel of apples to make 4 gallons of apple juice, each bushel is 42 pounds of apples, each pound is 3 apples, so it would take **3.7 apples** to make 15.2 fluid ounces of apple juice.
http://www.chacha.com/question/how-many-apples-does-it-take-to-make-15.2-fl-oz

Gennaro Chierchia (p.c.) wonders whether the use of the decimal in *3.7 apples* is natural language. Perhaps not; on the other hand, we would like to understand why such expressions so naturally slot into the same position as cardinals once they have been introduced into the language.

individuals and the (whole) number of children they have shown in (32), which then feeds into the semantics of *average* to derive correct truth conditions for the sentence as a whole.

(32) $\lambda n \lambda x.\exists y[\textbf{have}(y)(x) \wedge \textbf{children}(y) \wedge \#(y) = n]$

Let us now turn to maximization. Kennedy and Stanley assume that when a bare numeral like *three* takes scope, it either retains its status as a singular term and denotes the number 3, or it can be lifted in the standard way to a type $\langle\langle d, t\rangle, t\rangle$ generalized quantifier denotation $\lambda P_{\langle d,t\rangle}.P(3)$. If scope-taking is optional, then a simple example like (33) has the three possible logical analyses shown in (33a-c).

(33) Kim has three children.

a. $\exists x[\textbf{have}(x)(\textbf{kim}) \wedge \textbf{children}(x) \wedge \#(x) = 3]$

b. $\lambda n.\exists x[\textbf{have}(x)(\textbf{kim}) \wedge \textbf{children}(x) \wedge \#(x) = n](3)$

c. $[\lambda P.P(3)](\lambda n.\exists x[\textbf{have}(x)(\textbf{kim}) \wedge \textbf{children}(x) \wedge \#(x) = n])$

It is easy to see that these three these parses are logically equivalent, and moreover have lower-bounded truth conditions. The two scope-taking parses in (33b) and (33c) provide a way of explaining why a numeral need not be interpreted in its base position, explaining the average data, but we need to add something else in order to explain the pattern of data we have seen in this paper.

In particular, what we need to add is maximization over the degree property that constitutes the numeral's scope, as shown in (34) for the bare numeral *three*.

(34) $[\![\text{three}]\!] = \lambda D_{\langle d,t\rangle}.max\{n \mid D(n)\} = 3$

According to (34), *three* is a property of properties of degrees — a generalized quantifier over degrees — which is true of a degree property just in case the maximal number that satisfies it equals 3. The denotation of *three* in (34) is just like the denotation of *fewer than three* in (26b), except that the maximal number that satisfies the scope predicate is required to be equal to 3, rather than to be less than 3. If *three* takes sentential scope in (34), composition with the degree property that provides its scope gives the truth conditions in (35).

(35) $max\{n|\exists x[\textbf{have}(x)(\textbf{kim}) \wedge \textbf{children}(x) \wedge \#(x) = n]\} = 3$

These are two-sided truth conditions: (35) is true just in case the maximal n such that Kim has (at least) n children is equal to 3, i.e., just in case Kim has exactly three children. This analysis therefore straightforwardly account for the linguistic and experimental data discussed in Sections 2.1 and 2.2 which indicate that sentences with numerals can have two-sided semantic content.

More significantly, this analysis correctly derives lower bounded and upper bounded readings of sentences with bare numerals in modal contexts in exactly the same way that we saw for comparative numerals: as a scopal interaction between number words and modals. Consider first the case of universal modals. The sentence in (36) can be interpreted either with the number word inside the scope of the modal, deriving the proposition in (36a), or with the modal inside the scope of the number word, deriving the proposition in (36b).

(36) Kim is required to take three classes.

a. $\lambda w.\forall w' \in Acc_w[max\{n \mid \exists x[\textbf{classes}(w')(x) \wedge \#(w')(x) = n \wedge \textbf{take}(w')(x)(\textbf{kim})]\} = 3]$

b. $\lambda w.max\{n \mid \forall w' \in Acc_w[\exists x[\textbf{classes}(w')(x) \wedge \#(w')(x) = n \wedge \textbf{take}(w')(x)(\textbf{kim})]]\} = 3$

(36a) is true of a world if in every world deontically accessible from it, the maximum number of classes taken by Kim is three. This is the two-sided reading. (36b) is true of a world if the maximum number, such that in every world deontically accessible from it there is a plurality of classes of at least that size taken by Kim, is three. This means that the minimum number of deontically acceptable classes is three, which is the lower bounded meaning.

In the case of a sentence with an existential modal like (37), we get exactly the same scopal relations, but the resulting truth conditions are quite different:

(37) Kim is allowed to take three classes.

a. $\lambda w.\exists w' \in Acc_w[max\{n \mid \exists x[\textbf{classes}(w')(x) \wedge \#(w')(x) = n \wedge \textbf{take}(w')(x)(\textbf{kim})]\} = 3]$

b. $\lambda w.max\{n \mid \exists w' \in Acc_w[\exists x[\textbf{classes}(w')(x) \wedge \#(w')(x) = n \wedge \textbf{take}(w')(x)(\textbf{kim})]]\} = 3$

(37a) is the "weak" reading, which merely says that there is a deontically accessible world in which the maximum number of classes taken by Kim is three. (37b) is true of a world if the maximum number, such that *there is a deontically acceptable world in which Kim takes at least that many classes*, is three. On this reading, the proposition is false of a world if there is another world deontically accessible from it in which Kim takes more than three classes. This is the strong reading of (37), and unlike all of the traditional analyses of number word meaning, the proposal I am advocating here derives it as a matter of semantic content. This is an extremely positive result, given the evidence we saw in Sections 2.1 and 2.2 which indicated that this reading is in fact a matter of semantic content, and not derived via implicature.

3.3 Revisiting the evidence for lower-bounded content

The semantic analysis of numerals as generalized quantifiers over degrees that I presented in the previous section has a clear advantage over the alternatives that we examined in section 1.2 in that it both derives the correct scalar readings of numerals in the correct modal contexts, as a matter of semantic content, and also correctly predicts that numerals can introduce two-sided semantic content in non-modalized sentences. However, the latter result immediately raises an important question: what about the data that motivated the neo-Gricean account in the first place? For example, how do we explain the fact that the continuation *if not more* in (38a) does not give rise to a contradiction, and the fact that (38b) is most naturally understood as saying that everyone who missed three questions *or more* failed the exam?

(38) a. Kim read three of the articles on the syllabus, if not more.
b. Everyone who missed three of the questions failed the exam.

There are three ways to bring examples like these in line with my proposals. The first is to maintain the strong hypothesis that numerals always denote degree quantifiers with denotations like (34), so that (38a-b) both have semantic content, but then to attempt to explain the appearance of one-sided readings in non-modalized examples as particularized conversational implicatures (Breheny 2008), or as the result of an interaction with a covert modal (cf. Nouwen 2010). The former approach would work for (38b): if missing exactly three questions is sufficient to fail the exam, then surely missing more questions is too, while missing fewer questions may not be. For (38a), we could hypothesize the presence of an implicit relativization of the assertion to worlds compatible with my knowledge state, with the follow-up *if not more* provided as a way of indicating uncertainty about the completeness of my knowledge. Note that I am not proposing that the numeral takes scope over an implicit epistemic modal; rather, I am suggesting that the first part of (38a), two-sided content and all, is claimed to be true only in worlds compatible with my knowledge, and the second part is a way of saying that I am open to the possibility that my knowledge is partial, and so does not in fact rule out the possibility that a variant of this sentence containing a higher-valued numeral could be true of the actual world.[14]

[14] In fact, if my overall analysis is on the right track, then I predict that numerals can never take scope over epistemic modals. An empirical generalization that has emerged from research on degree quantification over the past fifteen years or so is that degree quantifiers may take scope higher than some kinds of quantificational expressions but not others (see e.g. Kennedy 1999; Heim 2000; Hackl 2000; Bhatt and Pancheva 2004; Takahashi 2006; Lassiter 2012). In particular, while degree quantifiers may scope above deontic modals of the sort we looked at in the previous sections, they may not

A second option, which also maintains the strong hypothesis that numerals always have denotations like (34), is to hypothesize that the individual variable contributed by the associated noun can be bound either below the numeral (as I have been tacitlyassuming so far) or above it. (I am grateful to Paul Marty for suggesting this approach.) On this view, the first half of (38a) is ambiguous. If the individual variable is existentially bound below the numeral, as shown in (39a), we derive two-sided truth conditions, as we have already seen.

(39) a. $max\{n \mid \exists x[\textbf{read}(x)(\textbf{kim}) \wedge \textbf{articles}(x) \wedge \#(x) = n]\} = 3$
b. $\exists x[max\{n \mid \textbf{read}(x)(\textbf{kim}) \wedge \textbf{articles}(x) \wedge \#(x) = n\} = 3]$

If, however, the individual variable is existentially bound above the numeral, we get lower-bounded truth conditions: (39b) is true just in case there is a plurality of books that Kim read whose maximal size is three, which rules out lower values but not higher ones. Whether this option is a viable alternative or not depends mainly on compositional considerations, in particular on where and how existential closure over the individual argument is introduced.

The third — and perhaps most likely — option is to hypothesize that the quantificational denotation of a number word is derived from a more basic singular term denotation. On this view, number words are in effect polysemous between the denotations in (40a) and (40b) (continuing to use *three* as our example).

(40) ⟦three⟧ =
a. 3
b. $\lambda D_{\langle d,t\rangle}.max\{n \mid D(n)\} = 3$

As we have already seen, combination of the singular term denotation of a numeral with e.g. an adjectival analysis of MANY derives lower-bounded truth conditions, so the consequence of hypothesizing (40a) as a potential denotation for the numeral is that lower-bounded truth conditions become available across the board, and in particular for non-modalized like (40a-b). This should not create any problems, since the data we considered in the first part of the paper showed only that two-sided content must be an option, not that one-sided, lower bounded content is *not* an option.[15]

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take scope above epistemic modals (Büring 2007). So the fact that (i) lacks a "strong" reading, to the effect that it is epsitemically impossible that Kim read more than three articles on the syllabus, is expected under the current proposal.

(i) Kim might have read three of the articles on the syllabus.

[15]Note also that for sentences like (40b) — and more generally for any structure in which the numeral appears in a downward entailing context — a parse in which the numeral is hypothesized

I do not have space here to conduct a full exploration of these three options, but I will briefly mention two factors that may argue in favor of the third approach. First, if the basic denotation of a numeral is a number (if numerals are singular terms), we can construct a simple mapping to the denotations of modified numerals such as *more/fewer than three, at least/most three*, and so forth: they are mappings from denotations like (40a) to denotations like (40b), with the appropriate ordering relation substituted for "=" ($>$, $<$, $\geq$, $\leq$, etc.). .[16] In Kennedy (2011) I show how this move allows us to explain a number of puzzles involving modified numerals and modals discussed by Nouwen (2010)

Second, it may be possible to say simply that the singular term meaning of the numeral is the lexical meaning, and to derive the maximal degree meaning in (40b) in a quite general way, without any assumptions specific to numerals. Specifically, we could assume that numerals in general are number-denoting singular terms, but like e.g. proper names, when they take scope they undergo a Montagovian type-shift to a generalized quantifier denotation: in the case of *three*, a move from 3 to $\lambda D_{\langle d,t\rangle}.D(3)$. As we saw in the previous section, this move alone does not derive interesting truth-conditional results. However, if we further assume that the generalized quantifier denotation composes with an exhaustivity operator of the sort originally proposed in Groenendijk and Stokhof 1984 to derive exhaustive readings of answers to questions (see also Zeevat 1994; van Rooij and Schulz 2004; Schulz and van Rooij 2006), then we should be able to derive an interpretation that is equivalent to (40b). Working out the details of this hypothesis is beyond the scope of current paper, but it seems like a promising approach. In particular, it may provide a nice account of some facts involving the interaction of questions and context in the promotion of two-sided readings discussed by Scharten (1997).

4 Conclusion

I hope to have done two things in this paper. First, I have tried to show that sentences containing numerals can have two-sided meanings as a matter of semantic content, rather than as the result of implicature calculation. Previous work has come to similar conclusions (Sadock 1984; Carston 1988, 1998; Koenig 1991; Horn 1992;

to have the denotation in (40a) will result in more informative truth conditions than one in which the numeral is hypothesized to have the denotation in (40b), and so might represent a preferred parse given general informativity principles. What is important, though, is that both readings are derived fully semantically; neither invokes the implicature system.

[16]A modifier like *exactly*, on the other hand, which also introduces an equality relation, can be analyzed as a "slack regulator" in Lasersohn's (1999): its function is not to introduce a truth-conditional mapping from a lower-bounded interpretation to a two-sided one, but just to signal increased pragmatic precision.

Scharten 1997; Geurts 2006; Breheny 2008); one of the contributions of the current paper is to show that even the "grammatical" account of scalar implicature runs into many of the same problems as the neo-Gricean one. Second, I have argued for a new semantic analysis of number words as generalized quantifiers over degrees, true of a property of degrees D just in case the maximal degree that satisfies D is equal to a specific value (a number). I showed that this account generates two-sided interpretations in the basic case, and that scalar (upper/lower-bounded) readings of sentences containing numerals arise through scopal interactions with modals.

I would like to close by briefly comparing my proposal to one with a much longer history. The idea that number words denote second order properties rather than determiners, cardinality predicates, or singular terms is not new: this idea goes back to Frege (1980 [1884]), who considers an analysis of numbers as second order properties of individuals before eventually rejecting it in favor of a treatment of numbers as singular terms, and it has been more recently proposed by Scharten (1997). The Fregean analysis of *three* as a second order property of individuals, for example, can be stated as in (41a); the "neo-Fregean" analysis of *three* as a second order property of degrees that I have proposed in this paper is repeated in (41b) for comparison.

(41) a. $[\![\text{three}]\!] = \lambda P_{\langle e,t\rangle}.\#(\{x \mid P(x)\}) = 3$
b. $[\![\text{three}]\!] = \lambda D_{\langle d,t\rangle}.max\{n \mid D(n)\} = 3$

The Fregean analysis also returns two-sided truth conditions for the basic cases we are interested in, and it has the advantage of not necessitating the introduction of degrees into the semantics. Space unfortunately prohibits a full examination of how the Fregean semantics interacts with modals; the various scope configurations do not derive exactly the same truth conditions as the degree quantifier analysis (because the Fregean semantics tracks individuals across worlds, rather than degrees), but it may be that the same range of readings eventually emerges.

One potential advantage of the degree-quantifier analysis of numerals is that it fits seamlessly into a quite general, well-motivated, and well-understood syntax and compositional semantics for quantity and degree constructions in the nominal projection. Number words stand alongside comparatives and other degree words (*too, how, so*, etc.) as expressions that provide information about the quantity of objects that satisfy the nominal description by saturating a degree argument position in an (overt or covert) occurrence of MANY. Because this position is distinct from the individual argument position that the nominal itself saturates, these words can take scope independently of the nominal, as we have seen. It may be that an equally general syntax and semantics can be developed for numerals and degree words under a Fregean analysis (see Scharten 1997 for some first steps in this direction), but this is an investigation that I will have to leave for another time.

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