No More Shall We Part: Quantifiers in English Comparatives*

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July 16, 2013

Abstract: It is well-known that the interpretation of quantificational expressions in the comparative clause poses a serious challenge for semantic analyses of the English comparative. In this paper, we develop a new analysis of the comparative clause designed to meet this challenge, in which a silent occurrence of the negative degree quantifier NO (observed overtly in collocations like *no more, no greater*, etc.) interacts with other quantificational expressions to derive the observed range of interpretations. Although our analysis incorporates ideas from previous analyses (in particular, negation and maximality based analyses of the comparative clause), we show that it is able to account for a broader range of facts than other approaches, with a minimum of construction-specific stipulations. We demonstrate that the analysis can be embedded within a general syntactic and semantic architecture for the comparative, and although our analysis focuses on the English data, we demonstrate its potential for cross-linguistic application. We conclude with a discussion of the implications of our approach for the analysis of "split-scope" phenomena in nominal quantification.
Keywords Comparatives, quantifiers, modals, negation, scope

1 Universal quantifiers in the comparative clause

The interpretation of quantificational elements appearing within the comparative clause presents numerous puzzles for a satisfactory semantic analysis of the comparative. These puzzles can best be illustrated by examining the readings that arise upon the inclusion of different universal quantifiers. Consider, for instance, the necessity modals *supposed to* and *need to*, which are traditionally analyzed as universal quantifiers over (deontically accessible) possible worlds. These modals display an intriguing split when they occur within the comparative clause.

*To appear in *Natural Language Semantics.* We are grateful to Jason Merchant, Anastasia Giannakidou, Roger Schwarzschild, and audiences at the University of California, Santa Cruz, the University of Chicago, and the University of Tübingen for valuable feedback on various parts of this work, as well as to two anonymous reviewers for *Natural Language Semantics* for their helpful comments. This paper is based upon work supported by the National Science Foundation under Grant No. 0620247.

Scenario: Chuck wants to be a fighter pilot. Air Force regulations require all pilots to be between 5'4" and 6'5" tall.

(1) a. Chuck is taller than he is supposed to be.
b. Chuck is taller than he needs to be.

According to (1a), Chuck's height exceeds the maximum permissible height, i.e., he is too tall to comply with Air Force regulations. Under its most natural reading, (1b) conveys something rather different. According to (1b), Chuck's height exceeds the minimum permissible height, i.e., he could be less tall and still comply with Air Force regulations. In what follows, we will refer to the readings respectively manifested by (1a) and (1b) as the "$>_{max}$" and the "$>_{min}$" readings.

Other deontic necessity modals display this same split. Some of them, such as *should* and *ought to*, resemble *supposed to*.

(2) a. Chuck is taller than he is supposed to be.
b. Chuck is taller than he should be.
c. Chuck is taller than he ought to be.
'Chuck's height exceeds the maximum permissible height.'

Others, including *have to* and *required to*, instead pattern with *need to*.

(3) a. Chuck is taller than he needs to be.
b. Chuck is taller than he has to be.
c. Chuck is taller than he is required to be.
'Chuck's height exceeds the minimum permissible height.'

However, although the modals in (3) typically yield $>_{min}$ readings, this is not always the case. Several authors (Heim 2006b; Krasikova 2008; Beck 2010) have observed that the $>_{max}$ reading, although generally less salient, is nonetheless possible for these modals, given the proper discourse context. For example, in all of the sentences in (4), the $>_{max}$ reading is the intended one.

(4) a. Frank's income was higher last year than it needed to be for him to receive the tax exemption.
'Frank's income exceeded the maximum allowable income.'
b. The faux-leather case is a little too bulky and makes the computer bigger than it needs to be to fit into a small carry bag.
(http://astore.amazon.com/samsung.omnia.mobile-20/detail/B0019M7R9K)
'Its size exceeds the maximum possible size for something to fit into a small carry bag.'
c. Frank was older than he had to be to qualify for the youth discount.
'Frank's age exceeded the maximum permitted age.'

These data raise questions that must be answered by any semantic analysis of the comparative. First, how should the $>_{max}$ and $>_{min}$ readings observed in (2)-(4) be derived? What exactly are the differences between their logical representations? Second, what determines the reading(s) that a particular necessity modal will give rise to? Why is the $>_{min}$ reading possible in the presence of *need to*, but not *supposed to*? Obviously, it would be desirable to relate this difference to other, independently observable differences between the two classes of modals.

Alongside the lexical verb *need to*, English also possesses the modal auxiliary *need*, which is a negative polarity item. The latter governs a bare VP complement rather than an infinitive, and does not require *do*-support.

(5) You need *(not) answer the question.

Interestingly, NPI *need* has only a $>_{min}$ reading when it occurs in the comparative clause. The examples in (6) are quite natural, because they evoke such a reading, but the examples in (7) sound strange: they evoke a $>_{max}$ interpretation (i.e., that Frank's income is too high and that the computer in its case is too big), but such a reading is unavailable.

(6) a. Frank requested more letters of recommendation than he need have requested for this position.

b. Due to his ignorance of the income tax code, Hank paid more than he need have paid.

(7) a. ?? Frank's income was higher last year than it need have been for him to receive the tax exemption.

b. ?? The leather case is too bulky and makes the computer bigger than it need be to fit into a small bag.

The behavior of NPI *need* in the comparative clause raises further questions: what licenses the appearance of *need* in (6)? And why is the $>_{max}$ reading possible (if dispreferred) with the lexical verb *need to*, but not with *need*?

The previous examples involve modals; when we turn to other quantificational expressions, things are different. For example, universal quantifiers over individuals invariably give rise to $>_{max}$ readings when they appear in the comparative clause: (8a-d) can only be understood to mean that Max's height exceeds the height of his tallest student(s), not that his height exceeds the height of his shortest student(s).

(8) a. Max is taller than each of his students is.

b. Max is taller than every one of his students is.

c. Max is taller than all of his students are.

d. Max is taller than both of his students are.

The same is true for conjoined DPs, when these are interpreted distributively.

(9) Max is taller than Lee and Hank are.

(9) means that Max's height exceeds Lee's height, and also Hank's height, not that Max's height exceeds the height of the shorter of Lee and Hank. Another question that arises, then, is why universal and conjoined DPs do not display the $>_{max}$ vs. $>_{min}$ split that was observed for necessity modals.

Although universal DPs do not participate in $>_{min}$ readings, another sort of reading sometimes emerges when they appear in the comparative clause. The following example is in fact ambiguous:

(10) More students have read *Lord of the Rings* than have read every other novel by Tolkien. (Bhatt and Takahashi 2011, fn. 18)

Under one of its readings, (10) conveys that for each Tolkien novel *x* other than *Lord of the Rings*, the number of students who have read *Lord of the Rings* exceeds the number of students who have read *x*. This is a kind of $>_{max}$ reading for (10), since it follows that *Lord of the Rings* has more readers than the most-read other Tolkien novel. Under another reading, (10) instead conveys that the number of students who have read *Lord of the Rings* exceeds the number of students who have read all of the other Tolkien novels. A semantic analysis of the comparative should also show how this latter reading comes about.

Taken together, the above facts provide a fertile testing ground against which to evaluate competing semantic analyses of the comparative construction. It therefore comes as no surprise that these (and similar) puzzles figure prominently in almost every comprehensive semantic analysis of the comparative, with several recent proposals wholly devoted to them (see e.g. von Stechow 1984a; Larson 1988; Meier 2002; Schwarzschild and Wilkinson 2002; Heim 2006b; Büring 2007; Gajewski 2008; van Rooij 2008; Schwarzschild 2008; Beck 2010). Despite all of this attention, an analysis that derives the full range of facts with a minimum of construction-specific stipulations has remained elusive.

In this paper, we present a new analysis of the comparative that incorporates and extend several ideas present in the recent literature on this topic, and, we believe, provides the most general account of the facts to date. Our core account of the patterns described above follows the proposals of Heim (2006b) and Schwarzschild (2008) in attributing the different readings to the presence of a silent, scopally active operator within the comparative clause. As Beck (2010) has observed, the basic challenge that such an approach faces is to somehow restrict the range of possible

scope relations that this operator may enter into, in order to prevent the overgeneration of unattested readings, e.g., the $>_{min}$ reading with universal DPs. Furthermore, whatever restrictions operate within the comparative clause to prevent this sort of overgeneration should as well be observable outside of the realm of comparatives, lest the analysis become entirely *ad hoc*. Beck's conclusion, with which we agree, is that none of the existing scope-based analyses manages to answer this challenge successfully, and this conclusion partly motivates her own analysis, which does not rely on scope alternations within the comparative clause.

Our task, then, is to develop and motivate a scope-based solution to these puzzles that meets Beck's challenge. Specifically, we suggest that the readings attested for various quantifiers point to the presence of a silent negative element within the comparative clause. In this respect, our proposal parallels the accounts of Marques (2003), Schwarzschild (2008), and Gajewski (2008), who also posit a negative operator within the comparative clause. Although our proposal draws from and builds upon these earlier proposals, we disagree over the precise identity of this operator: whereas earlier accounts place a silent counterpart to sentential negation within the comparative clause, we instead take the operator to be a silent counterpart to the negative differential degree quantifier *no* which appears in comparatives like *Sarah is* no *taller than Frank is*. This difference may appear inconsequential, but we demonstrate that it both resolves some empirical problems that arise from treating the negative element in comparatives as sentential negation (in particular involving the interpretation of differential measure phrases), and also imposes constraints on scopal interactions in the comparative clause that are just the ones we need to respond to Beck's challenge.

The paper is structured as follows. In section 2, we introduce three prominent negation-based analyses of comparatives and assess their capacity to explain the pattern of data presented above, ultimately concluding that none of the current proposals succeed in accounting for the full range of facts. In section 3, we present our own semantic analysis of comparatives, which is centered around the hypothesis that the negative element in comparatives is a variant of the overt degree quantifier *no* (in e.g. *no taller than*), and then in section 4 we show that our analysis successfully explains both the facts discussed above as well as the truth conditions and acceptability of examples involving other kinds of quantificational expressions in the comparative clause. Section 5 provides a comparison to alternative analyses, section 6 extends the proposals we develop for comparative deletion structures to additional comparative construction types, and section 7 shows how our proposals may be extended to account for similar patterns of interpretation with negative DPs (so-called "split-scope" phenomena). Section 8 concludes.

2 Negation and maximality in comparatives

The idea that the meaning of the comparative involves negation has a long history in both theoretical and historical studies of the comparative, and is motivated generally by a number of facts, most central of which are the acceptability of negative polarity items in the comparative clause and the unacceptability of positive polarity items (Ross 1968; Seuren 1973; Mittwoch 1974; von Stechow 1984a; though see Schwarzschild and Wilkinson 2002; Heim 2006a; Giannakidou and Yoon 2010 for more nuanced views), and the overt occurrence of negative morphology in some languages (see e.g. Joly 1967; Napoli and Nespor 1976; Stassen 1985; Price 1990). Our interest in these approaches is that they provide a means of accounting for the $>_{max}$ and $>_{min}$ readings observed in Section 1 in terms of scopal interactions between quantifiers and negation. However, as we will see, the existing negation-based analyses of the comparative are not completely successful in accounting for the facts.

2.1 The A/¬A Analysis

The most direct way to introduce negation into the meaning of the comparative is by taking advantage of the inherent ordering in the lexical semantics of gradable predicates. The intuition underlying this approach is that a simple example like (11a) has the truth conditions paraphrased in (11b).

(11) a. Sarah is taller than Frank is.
b. There is a way to make 'tall' true of Sarah and false of Frank.

There are different ways to implement this analysis, which differ most fundamentally in their assumptions about the denotation of gradable predicates like *tall*. Here we begin from the assumption that the semantic function of a gradable predicate (e.g., *tall*) is to locate an individual along some scalar dimension of measurement (e.g., height). A scalar dimension, or just a "scale", is formalized as a linearly ordered set of values, or "degrees" (type d), which themselves correspond to particular measures along the scale (e.g., particular heights). In this paper, we will adopt the most common analysis of gradable predicates that follows this assumption, which treats them as relations between individuals and degrees (see Cresswell 1976; von Stechow 1984a; Heim 1985 and many subsequent works). A further requirement of the A/¬A analysis, which we also adopt throughout this paper, is that these relations be (downward) monotone, as defined in (12).

(12) A relation R is (*downward*) *monotone* iff
$\forall x \forall d \forall d' [[R(x, d) \wedge d > d'] \rightarrow R(x, d')]$ (Heim 2000: (3))

The adjective *tall* thus has the meaning in (13) (type $\langle d, \langle e, t\rangle\rangle$), where **height** is a function that maps an individual to its height, i.e., to a particular degree along the adjective's associated scale.

(13) $[\![\text{tall}]\!] = \lambda d \lambda x.\textbf{height}(x) \succeq d$

For perspicuity, we will write this relation as $\textbf{tall}(x, d)$.

The *locus classicus* for the A/¬A analysis is Seuren 1973, in which both the existential quantification over degrees and the negation in (11b) are contributed by the meaning of the comparative morpheme (which we gloss throughout the paper as MORE), as shown in (14). (See also Ross 1968; McConnell-Ginet 1973, Klein 1980, and Larson 1988 are variants that involve different initial assumptions about the semantics of gradable predicates).

(14) $[\![\text{MORE}]\!] = \lambda g_{\langle d,et\rangle} \lambda C_{\langle d,t\rangle} \lambda x.\exists d[g(x, d) \wedge \neg C(d)]$

Syntactically, the comparative clause instantiates a *wh*-movement structure in which a silent relative pronoun binds the degree argument position of the gradable predicate, which is subsequently deleted under identity with the matrix predicate (see Bresnan 1973; Chomsky 1977; Izvorski 1995), as shown in (15a); its denotation is a degree property, as shown in (15b).

(15) a. Sarah is taller than [$_{\text{CP}}$ wh Frank is ~~*t* tall~~]
b. $[\![[_{\text{CP}}\ wh \text{ Frank is } t \text{ tall}]]\!] = \lambda d.\textbf{tall}(\textbf{f}, d)$
c. $[\![\text{MORE}]\!]([\![\text{tall}]\!])([\![[_{\text{CP}}\ wh \text{ Frank is } t \text{ tall}]]\!])([\![\text{Sarah}]\!])$
d. $\exists d[\textbf{tall}(\textbf{s}, d) \wedge \neg \textbf{tall}(\textbf{f}, d)]$

The comparative morpheme combines with the gradable predicate and the comparative clause to yield a one-place predicate over individuals, as shown in (15c), and the resulting truth conditions in (15d) accurately reflect the initial intuition about the meaning of this example: *Sarah is taller than Frank is* is true just in case there is a degree of height d such that Sarah's height is at least as great as d (making $[\![\text{tall}]\!]$ true of Sarah) and Frank's height is not at least as great as d (making $[\![\text{tall}]\!]$ false of Frank).

A consequence of this standard formulation is that the negation contributed by *more*/*-er* necessarily scopes over any quantificational elements occurring within the comparative clause. As many authors have observed (see e.g. Larson 1988; van Rooij 2008), this makes the wrong predictions for comparative clauses that contain universal quantifiers which give rise to $>_{max}$ readings. For example, the interpretation assigned to (16a) is (16b), where w_0 designates the world of evaluation.

(16) a. Chuck is taller than [$_{\text{CP}}$ wh he is supposed to be t tall]

b. $\exists d[\textbf{tall}(\textbf{c}, d, w_0) \wedge \neg\forall w[ACC_{w_0}(w) \rightarrow \textbf{tall}(\textbf{c}, d, w)]]$

According to (16b), (16a) is true just in case there is a degree of height such that Chuck is at least that tall in the actual world, and it is not the case that he is at least that tall in every deontically accessible world. These truth conditions allow for deontically accessible worlds in which he is taller than he actually is, which is a good result if we want to derive a $>_{min}$ reading (this is one way of characterizing the truth conditions of such a reading), but is a bad result for (16a), which has only a $>_{max}$ reading: this sentence entails that there are no deontically accessible worlds in which Chuck is taller than he actually is. Similarly, the interpretation assigned to (17a) is (17b), which is true so long as Lee has a degree of height that not all of his students have, and so fails to entail that there is no student taller than him.

(17) a. Lee is taller than [$_{\text{CP}}$ wh each of his students is t tall]
b. $\exists d[\textbf{tall}(\textbf{l}, d) \wedge \neg\forall x[\textbf{student-of}(x, \textbf{l}) \rightarrow \textbf{tall}(x, d)]]$

2.2 Free sentential negation

Schwarzschild (2008) develops an alternate formulation of the A/¬A analysis which provides a way of deriving the correct interpretations for these examples, by divorcing negation from the comparative morpheme and giving it independent status within the comparative clause.[1] (See also Marques 2003 and Gajewski 2008, which we discuss in more detail below.)

(18) Sarah is taller than [$_{\text{CP}}$ wh Frank is NOT ~~t tall~~]

The comparative clause now contains the silent element NOT, which is the covert counterpart to the overt sentential negation *not*. Correspondingly, the sole contribution of the comparative morpheme is existential quantification over degrees:

(19) $[\![\text{MORE}]\!] = \lambda g_{\langle d,et\rangle}\lambda C_{\langle d,t\rangle}\lambda x.\exists d[g(x, d) \wedge C(d)]$

[1] Actually, Schwarzschild ultimately proposes to replace NOT in the comparative clause with a different silent operator, FALL-SHORT, in order to derive the correct meaning for comparatives with differentials. This modification ensures that the comparative clause in a sentence like *Sarah is (at least) two inches taller than Frank is* denotes the set of heights that Frank's height falls short of by (at least) two inches (see Schwarzschild's paper for the semantic details). Like NOT, the operator FALL-SHORT may take variable scope in the comparative clause with respect to other quantificational items. However, since there appears to be no overt counterpart to this operator, there is no way to independently verify its scope-taking characteristics. For this reason, we focus here on the predictions of Schwarzschild's initial analysis in terms of sentential negation.

The compositional analysis of (18) is shown in (20); for a simple example like this, Schwarzschild's analysis derives exactly the same truth conditions as the classic version of the A/¬A analysis.

(20) a. $[\![[_{\text{CP}}\ wh$ Frank is NOT t tall$]\!]$ = $\lambda d.\neg$**tall**$(\mathbf{f}, d)$
b. $[\![$MORE$]\!]([\![$tall$]\!])([\![[_{\text{CP}}\ wh$ Frank is NOT t tall$]\!])([\![$Sarah$]\!])$
c. $\exists d[$**tall**$(\mathbf{s}, d) \wedge \neg$**tall**$(\mathbf{f}, d)]$

However, the introduction of an independent NOT within the comparative clause creates the possibility this negation will enter into scope interactions with other quantificational elements. In particular, the $>_{max}$ and $>_{min}$ readings observed with different universal quantifiers in the comparative clause can now be derived by varying their scope relative to NOT (see also Larson 1988; Büring 2007; van Rooij 2008). The $>_{max}$ reading results when a universal scopes over NOT, as shown in (21b), a logical representation for (21a) that correctly characterizes its truth conditions.

(21) a. Chuck is taller than [$_{\text{CP}}\ wh$ he is supposed NOT to be t tall]
b. $\exists d[$**tall**$(\mathbf{c}, d, w_0) \wedge \forall w[ACC_{w_0}(w) \rightarrow \neg$**tall**$(\mathbf{c}, d, w)]]$

Here the comparative clause introduces a degree property that is true of those heights that Chuck does not reach in any deontically admissible world. If Chuck's height in the admissible worlds ranges from, say, 5 feet to 6 feet, the comparative clause is true only of heights above 6 feet, and (21a) is correctly predicted to mean that Chuck's height in the world of evaluation is greater than 6 feet.

An analogous result is derived for universal quantifiers over individuals: scoping negation below the universal quantifier in subject position derives (22b) as the interpretation of (22a).

(22) a. Lee is taller than [$_{\text{CP}}\ wh$ each of his students is NOT t tall]
b. $\exists d[$**tall**$(\mathbf{l}, d) \wedge \forall x[$**student-of**$(x, \mathbf{l}) \rightarrow \neg$**tall**$(x, d)]]$

This says that there is a height that Lee reaches and all of his students fail to reach, which is the right meaning for (22a).

By introducing clausal negation as an independent constituent in the comparative clause, separated from the denotation of the (matrix) comparative morphology, Schwarzschild's analysis allows for scopal interactions in places that the classic A/¬A analysis does not, and so succeeds where the classic analysis does not in deriving $>_{max}$ readings. This analysis also makes the following prediction: since the readings associated with a universal quantifier depend upon its scope relative to NOT, the revised A/¬A analysis predicts that for any particular quantifier, its

observed readings should reflect independently observable restrictions on its scope relative to sentential negation.

Let us consider modals first. It is in fact well-known that modals differ in their scopal possibilities relative to sentential negation, (see e.g. Cormack and Smith 2002; von Fintel and Iatridou 2007). In particular, the modals that always give rise to $>_{max}$ readings, such as *supposed to*, *should*, and *ought to* (cf. (2)), also necessarily have wide scope relative to negation. This is illustrated by the examples in (23), which unambiguously convey the requirement that one not perform that many push-ups.

(23) a. You are not supposed to do that many push-ups.
b. You should not do that many push-ups.
c. You ought not to do that many push-ups.
'It is necessary that you not do that many push-ups.'

In contrast, the modals that allow $>_{min}$ readings, such as *need to*, *have to*, and *required to* (cf. (3) and (4)), always scope under sentential negation. This is illustrated by the examples in (24), which unambiguously convey the absence of any requirement that one perform that many push-ups.

(24) a. You do not need to do that many push-ups.
b. You do not have to do that many push-ups.
c. You are not required to do that many push-ups.
'It is not necessary for you to do that many push-ups.'

As we saw in section 2.1, scoping the modal under negation derives the $>_{min}$ readings in comparatives, so the pattern in (24) is consistent with the facts.

However, we also saw in the introduction that modals such as these actually give rise to both $>_{min}$ and $>_{max}$ readings. Consider for example (25), uttered in a context in which we are interested in whether Frank qualifies for a tax deduction that applies only to people who make between $75,000 and $100,000 per year.

(25) Frank's income is higher than it needs to be to qualify for this deduction.

In this context, (25) can be understood either as saying that Frank makes between $75,000 and $100,000, and so qualifies for the deduction ($>_{min}$), or that he makes over $100,000, and so fails to qualify ($>_{max}$). If the modal necessarily takes scope under sentential negation, then the revised A/¬A analysis is not able to capture the second reading.

Turning now to universal quantifiers, the situation is even more problematic. A universal DP's scope relative to sentential negation is not generally fixed: all

of the examples in (26) are ambiguous between a reading in which the universal outscopes negation, and another in which negation outscopes the universal.[2]

(26) a. Each nurse didn't visit that patient.
b. Every nurse didn't visit that patient.
c. All of the nurses didn't visit that patient.
d. Both nurses didn't visit that patient.
'For every nurse x, x didn't visit that patient.' OR 'Not every nurse visited that patient (though perhaps some did).'

Without any additional, construction-specific scope restrictions, Schwarzschild's revised A/¬A analysis predicts the same scopal possibilities in comparatives. However, as we have already seen, scoping NOT over a universal produces the incorrect $>_{min}$ reading:

(27) $\exists d[\textbf{tall}(\textbf{l}, d) \wedge \neg\forall x[\textbf{student-of}(x, \textbf{l}) \rightarrow \textbf{tall}(x, d)]]$

(27) says that there is a height that Lee reaches and not all of Lee's students do, which is too weak.

Here, then, is a place where the default prediction of the revised A/¬A analysis, namely that the $>_{min}$/$>_{max}$ distinction should track independent constraints on scopal interactions with negation, is not met. In order to exclude the $>_{min}$ reading, and maintain Schwarzschild's analysis, some other principle would have to be invoked to prevent universal DPs from scoping below NOT. One possibility, suggested by van Rooij (2008), is that the $>_{min}$ reading is blocked by principles of informativity. Noting that the $>_{max}$ reading is logically stronger than the $>_{min}$ reading, van Rooij proposes to exclude the $>_{min}$ reading for universal DPs by extending Dalrymple et al.'s (1998) Strongest Meaning Hypothesis to comparatives. The Strongest Meaning Hypothesis, which was originally formulated for reciprocals, requires that a sentence be assigned its strongest possible interpretation that is consistent with the non-linguistic information supplied by its context of use.

We see a couple of problems for this proposal. First, as Beck (2010) observes, there appears to be no way for the weaker $>_{min}$ reading to surface, even

[2] A reviewer asks whether negation can really take scope over the universals in (26). Such readings have been disussed for many years in the theoretical and descriptive literature (see Horn 1989 for a comprehensive summary); they have been reproduced experimentally (see e.g. Musolino, Crain, and Thornton 2000); and they can be found in corpora:

(i) Every chef wasn't a madman. Most weren't, in fact. But many were and are, and the very best chefs, I knew, as I wrote my book at what my chef, Chef Pardus, would call production speed, were a little twisted in the dark spaces of their brain. (Michael Rulman, *Soul of a Chef*, p. 133)

when the stronger $>_{max}$ reading is ruled out: (28) is only understood as false (28a), and never as true (28b).

(28) The next-to-last sprinter ran faster than every other sprinter did.

a. The next-to-last sprinter's speed exceeded the fastest sprinter's speed.

b. * The next-to-last sprinter's speed exceeded the slowest sprinter's speed.

Second, the Strongest Meaning Hypothesis makes the wrong predictions in examples that include matrix-level clausal negation:

(29) Kim didn't run faster than everyone else did.

a. It is false that Kim's speed exceeded the speed of the fastest of the others.

b. It is false that Kim's speed exceeded the speed of the slowest of the others.

Here the strength relations are reversed: the $>_{min}$ reading in (29b) asymmetrically entails the $>_{max}$ reading in (29a), yet (29) can only be understood as conveying (29a). Finally, as we saw above, the *need to*-class of necessity modals gives rise to both $>_{min}$ and $>_{max}$ readings. If the Strongest Meaning Hypothesis rules out weaker meanings, then it ought to apply uniformly and block the $>_{min}$ readings in these examples as well.

2.3 The maximality analysis

The chief competitor to the A/¬A analysis in the literature on comparatives is the maximality-based analysis of the comparative clause developed by von Stechow (1984a). Von Stechow proposes that the comparative clause be analyzed as a definite degree description: the comparative clause in (30) denotes the maximal degree of height d such that Frank's height is at least as great as d, or more simply, Frank's height.

(30) Sarah is taller than Frank is.

This analysis is typically implemented by translating the wh-element in comparatives as the maximality operator in (31a) (assuming a Heim and Kratzer-style analysis of movement, where the sister of the moved expression is a function over objects of the semantic type associated with its base position), where MAX is defined as in (31b).[3]

[3]Various other options are possible, including building maximality into the denotation of the comparative morphology and treating the comparative clause as a property of degrees (Cresswell 1976), or assuming a generalized mapping from degree properties to unique degrees of the sort hypothesized by Jacobson (1995) for free relatives. For our purposes in this paper, these distinctions are not important.

(31) a. $[\![wh_{comp}]\!] = \lambda C_{\langle d,t\rangle}.\textsc{max}(C)$
b. For any degree property C, $\textsc{max}(C) = \iota d[C(d) \wedge \neg\exists d'[C(d') \wedge d' > d]]$

The comparative morpheme can then be analyzed either as term that turns regular degree relations into comparative degree relations (von Stechow 1984a), or as a degree quantifier in its own right (Heim 2000, 2001). Here we will adopt von Stechow's version of the former type of analysis, which is given in (32).[4]

(32) $[\![\textsc{more}]\!] = \lambda g_{\langle d,\langle e,t\rangle\rangle}\lambda d_1\lambda d_2\lambda x.g(d_1 + d_2)(x)$

Here $d_1 + d_2$ represents the concatenation of d_1 and d_2; the truth conditional contribution of (32) is to require that the adjective's individual argument have a degree of the measured property that is at least as great as the standard degree (d_1) plus some "differential" degree (d_2).

As an illustration, consider the composition of the comparative predicate in (30), where (33a) spells out the full syntactic structure typically assumed for the comparative clause.

(33) a. $[\![[_{\text{CP}}\ wh$ Frank is ~~t tall~~$]]\!] = \textsc{max}(\lambda d.\textbf{tall}(\textbf{f}, d))$
b. $[\![\textsc{more}]\!]([\![\text{tall}]\!]) = \lambda d_1\lambda d_2\lambda x.\textbf{tall}(x, d_1 + d_2)$
c. $[\![\text{taller than } wh \text{ Frank is } t \text{ tall}]\!] = \lambda d_2\lambda x.\textbf{tall}(x, \textsc{max}(\lambda d.\textbf{tall}(\textbf{f}, d)) + d_2)$

The comparative clause has the denotation in (33a): it picks out the maximal degree d such that Frank's height is at least as great as d, which is just Frank's height. The comparative adjective has the denotation in (33b); putting these two things together gives us (33c) which is true of a degree d_2 and an individual x if x's height is at least as great as Frank's height plus d_2.

We now need to say something about how the differential degree argument d_2 is saturated. In some cases, a modifier or measure phrase does the job, as in the following examples:

(34) a. Sarah is 2 cm taller than Frank.
b. Sarah is much taller than Frank.
c. Sarah is slightly taller than Frank.

[4]We make this choice primarily for reasons of simplicity, but as will become clear in the next section, in our overall approach to the semantics of comparatives, none of the traditional arguments for a quantificational analysis of comparative morphology — scope ambiguities, identity conditions in comparative ellipsis, etc. — apply, because we are able to explain such facts strictly in terms of the semantic contribution and syntactic position of the *than*-phrase. This gives us the freedom to assign MORE the kind of meaning that we would most naturally expect it to have given its morphosyntactic distribution: one that takes a word meaning and gives back a new word meaning.

But in many cases there is no overt expression in the syntax to provide the value for this argument. For the moment, let us assume that in such cases the differential argument is saturated by a default existential quantifier which ranges over positive degrees on the scale (i.e., degrees greater than zero), so that the full comparative predicate in an example like (30) denotes the property in (35).[5]

(35) $\lambda x.\exists d'[\textbf{tall}(x, \textsc{max}(\lambda d.\textbf{tall}(\textbf{f}, d)) + d')]$

(35) is true of an individual x if and only if x's height is at least as great as Frank's height plus some positive degree, which entails that there is some positive difference between x's height and Frank's height, which is exactly what we want.

With this background in hand, let us now return to the analysis of quantifiers in comparative clauses. For our purposes in this paper, what is important about the maximality-based approach to comparatives is that it introduces negation into the meaning of the comparative clause, through the definition of the MAX operator. (35), for example, is equivalent to (36), where the entire underlined part is the denotation of the comparative clause, and the double-underlined part is the negative component.

(36) $\lambda x.\exists d'[\textbf{tall}(x, \underline{\iota d[\textbf{tall}(\textbf{f}, d) \wedge \underline{\underline{\neg \exists d''[\textbf{tall}(\textbf{f}, d'') \wedge d'' > d]}}]} + d')]$

Like the A/¬A analysis, then, the maximality-based analysis predicts a truth-conditional interaction between negation and quantifiers in the comparative clause. The problem is that the interaction is not the one that we want, a point that has been observed by many authors (see Schwarzschild and Wilkinson 2002 for a comprehensive discussion). In particular, just as with the classic A/¬A analysis, negation is predicted to always take wide scope relative to comparative clause-internal quantifiers, because it is introduced externally (either by the wh-element, or, in some analyses, by the comparative morpheme). And again, just as with the classic A/¬A analysis, this scope relation invariably yields a $>_{min}$ reading for universals in the comparative clause, leaving the maximality-based analysis incapable of deriving the $>_{max}$ reading for universals.

[5]Note that the situation for comparative predicates without differential expressions is no different from the situation with unmodified positive form adjectives, given the assumptions we have made in this paper: both *tall* and *taller than Frank is* are semantically type $\langle d, \langle e, t\rangle\rangle$, and so need to saturate a degree argument. The usual analysis of the former case is that a silent "positive" morpheme does this job, and indeed nothing stops us from adopting such an analysis here, provided we give the right semantics to the positive morpheme. In particular, it should be one that treats the comparative as a special kind of "minimum standard absolute adjective" (Kennedy 2007b), imposing an existential rather than norm-related requirement on the degree argument. See Svenonius and Kennedy 2006; Kennedy and Levin 2008; Grano and Kennedy 2012 for a way of working this out that makes slightly different assumptions about the semantics of adjectives and the comparative morphology than we have made here.

As a simple illustration, consider the analysis of (37), with a universal DP in the comparative clause. The denotation of the comparative clause is (37a), which is equivalent to (37b).

(37) Lee is taller than [$_{\text{CP}}$ wh each of his students is ~~*t* tall~~]

a. $\text{MAX}(\lambda d.\forall x[\textbf{student-of}(x, \textbf{l}) \rightarrow \textbf{tall}(x, d)])$

b. $\iota d[\forall x[\textbf{student-of}(x, \textbf{l}) \rightarrow \textbf{tall}(x, d)] \wedge$
$\neg\exists d'[[\forall x[\textbf{student-of}(x, \textbf{l}) \rightarrow \textbf{tall}(x, d')] \rightarrow d' > d]$

The unique degree that satisfies the ι-description in (37b) is the one that every student is at least as tall as, such that there is no other degree greater than it that each student is at least as tall as. There is such a unique degree, namely the one that corresponds to the shortest student's height, so the result is the incorrect $>_{min}$ truth conditions for the comparative. Exactly the same problem will arise in examples containing universal modals like *supposed to*, with the result that the classic maximality-based analysis cannot account for the observed interpretations of a wide range of sentences, for the same reasons we saw with the classic A/¬A analysis: because the negative component of the meaning obligatorily takes wide scope relative to quantifiers in the comparative clause.

3 The NO MORE analysis

As stated in the introduction, our analysis of the semantics of the comparative clause takes the maximality-based analysis as its starting point (though as we will see in the end, our overall semantics of comparative constructions is probably best thought of as a hybrid of the maximality-based and A/¬A analyses). Focusing first on the comparative clause, the interpretation that we want to derive differs from the traditional MAX-based representation in two respects, which we illustrate using our simple example in (30), repeated below.

(30) Sarah is taller than Frank is.

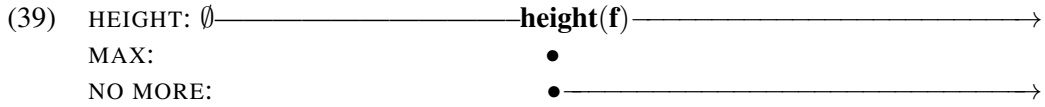
First, we analyze the comparative clause as a degree property (type $\langle d, t\rangle$), rather than a definite degree description (type d). Second, we want to eliminate the first conjunct of the maximality-based analysis: the requirement that a degree that satisfies the comparative clause be one that (in this example) Frank's height is at least as great as. This leaves only the negative portion of the meaning behind, as shown in (38b), which (for reasons that will become clear shortly) we refer to as the NO MORE analysis.

(38) a. $\iota d[\textbf{tall}(\textbf{f}, d) \wedge \neg\exists d'[\textbf{tall}(\textbf{f}, d') \wedge d' > d]$ (MAX Analysis)

b. $\lambda d.\neg\exists d'[\textbf{tall}(\textbf{f}, d') \wedge d' > d]$ (NO MORE Analysis)

According to the NO MORE analysis, the comparative clause denotes the set of heights that Frank does not exceed, i.e., those heights *h* such that Frank is no taller than *h*, which is the set of heights that extend onwards from Frank's height, inclusive. The difference between the degrees that the comparative clause picks out in the MAX analysis and in the NO MORE analysis is illustrated schematically in (39).

(39)



Before showing how we compositionally derive denotations like (38b) for the comparative clause, and how our analysis explains the observed pattern of interactions with quantifiers, let us first demonstrate that our proposal is adequate as a baseline semantics for comparatives, since it differs from almost all existing analyses in the kind of the denotation it assigns to the comparative clause.[6] In fact, the only modification we need to make to the proposals we have already adopted — von Stechow's denotation for the comparative morpheme in (32) and default existential quantification in examples without an overt differential phrase — is one that is arguably welcome: we need to give the standard morpheme *than* a role to play in composition.

Specifically, we analyze the standard morpheme as an existential quantifier over degrees, with the denotation in (40).[7]

(40) $[\![than]\!] = \lambda P_{\langle d,t\rangle}\lambda Q_{\langle d,t\rangle}.\exists d[Q(d) \wedge P(d)]$

The first argument of *than* (its restriction) is provided by its complement, the comparative clause, and the second argument of *than* (its scope) is provided by quantifying in, i.e. by abstracting over the first argument of the comparative adjective

[6]The one exception we know of is Marques 2003, who also adopts syntactic and semantic assumptions about the comparative clause that are comparable to the ones will make below, in which the "missing" predicate in the comparative clause has comparative morphology and semantics. The main difference between our account and Marques' is that he, like Schwarzschild (2008), takes the negative element in the comparative clause to be contributed by a sentential negation operator, but we do not. As a result, his analysis, like Schwarzschild's, predicts interactions between e.g. negation and quantificational subjects that are not actually observed. On the other hand, his analysis shares with ours the ability to straightforwardly handle differential expressions.

[7]Our denotation for *than* is identical to the denotation given to the comparative morpheme by Schwarzschild (2008), and also by Gajewski (2008) on his "E-Theory" analysis of comparatives. A major advantage of treating the standard morpheme as the expression that introduces existential quantification is that we can then assign a meaning to the comparative morpheme that straightforwardly handles differential arguments, as we will demonstrate below, namely the von Stechow denotation in (32).

(the base position of the *than*-phrase).[8] The denotation of the former under the NO MORE analysis is (38b), and the denotation of the latter given our assumptions about the composition of the comparative predicate is (41).

(41) $\lambda d.\exists d''[\textbf{tall}(\textbf{s}, d + d'')]$

To make it easier to illustrate our analysis, though, let us instead use the representations in (42a-b), which are equivalent to (38b) and (41), respectively, given our assumptions about the denotations of gradable adjectives (see (13) in section 2.1).

(42) a. $\lambda d.\textbf{height}(\textbf{f}) \leq d$ *restriction of* than *in (30)*
b. $\lambda d.\textbf{height}(\textbf{s}) > d$ *scope of* than *in (30)*

Supplying (42a-b) as the restriction and scope arguments to the denotation for *than* in (40) gives us (43) as the truth conditions of *Sarah is taller than Frank is*.

(43) $\exists d[\textbf{height}(\textbf{s}) > d \wedge \textbf{height}(\textbf{f}) \leq d]$

According to this analysis, (30) is true just in case there is a degree of height that is exceeded by Sarah's height, but which Frank's height does not exceed; i.e., whenever there is an overlap between the regions of the scale that are picked out by the restriction and the scope. As illustrated in (44a-b), this will be the case when Sarah's height is strictly ordered above Frank's, but not when Sarah's height is equal to or below Frank's.

(44) a. HEIGHT: $\emptyset$——————-**height**(**f**)—-**height**(**s**)——————————→
(42a): •——————————→
(42b): $\emptyset$——————————○
⇒ (43) satisfied; (30) TRUE

b. HEIGHT: $\emptyset$——————-**height**(**s**)—-**height**(**f**)——————————→
(42a): •——————————→
(42b): $\emptyset$——————————-○
⇒ (43) not satisfied; (30) FALSE

[8] As explained in more detail in Alrenga, Kennedy, and Merchant 2012, the hypothesis that the *than*-phrase takes scope in comparatives has a number of significant empirical advantages over standard approaches, in which it is the comparative morpheme which takes scope. At a superficial level, it better matches the surface syntactic fact that it is the *than*-phrase, not the comparative morpheme, which can adjoin at different levels in the clause. And at a deeper level, this aspect of our analysis straightforwardly derives the fact that the surface position of the *than*-phrase corresponds to the scope of comparison (Williams 1977; Gawron 1995; Bhatt and Pancheva 2004), and helps explain a number of typological generalizations.

In examples with an overt differential phrase, the region picked out by the scope expression will be shifted by an appropriate amount, and in a direction that is appropriate for the polarity of the adjective (down for positive adjectives, up for negative adjectives). For example, the scope argument of *than* in (45) is (45a), which is equivalent to (45b), and the predicted truth conditions are as shown in (46).

(45) Sarah is 2 centimeters taller than Frank is.

a. $\lambda d.\textbf{tall}(\textbf{s}, d + \textbf{2cm})$

b. $\lambda d.\textbf{height}(\textbf{s}) - \textbf{2cm} \geq d$

(46) $\exists d[\textbf{height}(\textbf{s}) - \textbf{2cm} \geq d \wedge \textbf{height}(\textbf{f}) \leq d]$

If, say, Sarah's height is 150 centimeters, then (46) is satisfied as long as Frank's height is 148 centimeters or less; we thus derive an "at least" semantics for differentials as a baseline. This meaning can be strengthened to an "exactly" meaning pragmatically in the usual way, through a quantity implicature, or semantically by adopting the degree quantifier semantics for numerals and measure phrases advocated by Kennedy (2013) and discussed in section 4.4 below.

Now that we have shown that the NO MORE meaning for the comparative clause is both consistent with our initial assumptions about the meaning of the comparative and derives the correct truth conditions in basic cases, we will present our proposal for how it is derived compositionally. Like Schwarzschild (2008), we propose that the comparative clause includes an independent negative expression, and that it is this element which interacts with quantifiers and modals to derive the various readings discussed in section 1. Unlike Schwarzschild, we do not analyze this expression as sentential negation, for the reasons outlined in section 2.2: it derives too many readings for universal DPs in comparatives and too few for the *need to*-class of modals.

Instead, we propose that the negative element in comparatives is a negative degree quantifier which we gloss as NO, and to which we assign the denotation in (47).

(47) $[\![\text{NO}]\!] = \lambda P_{\langle d,t\rangle}.\{d \mid P(d)\} = \emptyset$

More specifically, we claim that the occurrence of NO in the comparative clause is just a silent allomorph of the very same negative degree quantifier that we see overtly in examples like (48) (and possibly also of the negative "determiner" in examples like *Frank read no books*, a point to which we return in section 7).

(48) Sarah is no taller (than ...).

At a general level, our analysis of the negative element in comparatives as a degree quantifier will allow us to explain the observed pattern of interpretations in comparatives in terms of independent constraints on the interaction of degree quantifiers and other quantificational expressions, as we will document in detail in the next section. At a more specific level, our claim that NO in the comparative clause is the same expression that appears overtly in (48) provides us with an extra evaluation metric for our analysis, independently of how it handles the facts we have been concerned with so far: as a default, our analysis predicts full parallelism between the interpretive properties of the comparative clause and the intepretive properties of matrix comparatives with differential *no*. As we will see, this prediction is borne out.[9]

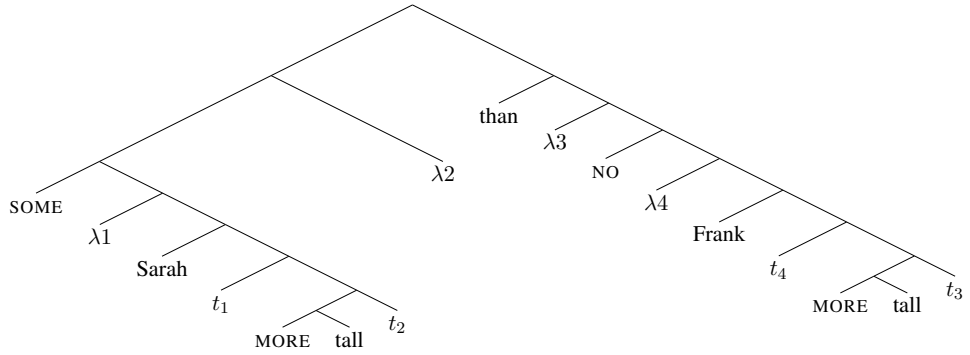
Turning now to the composition of the comparative clause, in order to derive the NO MORE truth conditions, and to provide argument positions for both NO and the relative pronoun which turns the whole comparative clause into a degree property, we make the non-standard but crucial assumption that the degree predicate inside the comparative clause is not an occurrence of a regular type $\langle d, \langle e, t\rangle\rangle$ gradable predicate, but is rather an occurrence of a type $\langle d, \langle d, \langle e, t\rangle\rangle\rangle$ *comparative* predicate, deleted under identity with the matrix comparative predicate (cf. Marques 2003). Moreover, since NO is a degree quantifier, it must take scope. To achieve parallelism in the matrix clause for purposes of licensing deletion and to capture the truth conditions of comparatives without overt differentials, we modify our earlier assumption about default existential quantification over the differential argument in the matrix and posit a corresponding positive degree quantifier SOME with the denotation in (49).

(49) $[\![\text{SOME}]\!] = \lambda P_{\langle d,t\rangle}.\{d \mid P(d)\} \neq \emptyset$

[9] Gajewski (2008) also treats the negative element in comparatives as a kind of degree quantifier (cf. Heim 2006a), but one that essentially reproduces the semantic effect of sentential negation. In this way, he is able to appeal to the same constraints on the interaction of degree and individual quantifiers that we do to solve the over-/undergeneration problems that face Schwarzschild's analysis; however, as he points out, his analysis cannot accommodate comparatives with differential arguments. Furthermore, it is not clear what kinds of predictions Gajewski's analysis makes outside of comparatives. Gajewski assigns the same denotation to his silent operator as is proposed by Heim (2006a) for the overt measure phrase *little* (see the appendix to his paper), and indeed, overt *little* seems to show the requisite scope behavior. However, Beck (2012) points out certain facts concerning the interpretation of sentences containing overt *little*, e.g. *Ida weighs that little*, that Heim's analysis fails to capture. This suggests that the correct denotation for overt *little* is in fact different from the denotation that Gajewski assigns to his silent operator, and consequently, the former's behavior cannot be used to independently confirm the latter's scope-taking properties.

Together, these assumptions derive the (simplified) Logical Form in (50) for (30).[10]

(50)



The denotation of the matrix degree predicate (the scope of the *than*-phrase) is shown in (51), which is equivalent to the denotation we assumed above: it picks out the set of degrees such that there is a positive difference between those degrees and Sarah's height.

(51) $\lambda d.\{d' \mid \textbf{tall}(\mathbf{s}, d + d')\} \neq \emptyset$ $(\equiv \lambda d.\textbf{height}(\mathbf{s}) > d)$

[10]Our simplifications are as follows: we assume a QR-treatment of scope-taking for convenience; we assume that the relative wh-word in the comparative clause introduces λ-binding but otherwise has no semantic content; we treat the individual-denoting subject as though it has lowered into a predicate-internal position; we ignore the contributions of tense, etc.; and we omit category labels. We do not believe that any of these simplifications bear on our core proposals.

For the syntax of NO, we assume that it is licensed by (or recoverable from) the selectional requirements of the standard marker *than*. Specifically, we assume that *than* selects for the uninterpretable feature [uNO] on the embedded C that heads its clausal complement. This uninterpretable feature must be checked against its interpretable counterpart [iNO], which is borne by the operator NO. (Compare to Landau's (2002) treatment of (one class of) negative verbs in Hebrew.) In this way, the standard marker *than*, via its selection for [uNO], manages to encode the presence of NO in the comparative clause.

We further assume that as a silent operator, NO is licensed only if its presence can be deduced from the surrounding structure, specifically from the overt standard marker *than*. Zeijlstra (2008, 2009) suggests that covert operators are subject to an economy condition that prevents their insertion unless they rescue a structure that would be otherwise ungrammatical. Since NO carries the feature [iNO], its presence rescues structures in which an instance of its uninterpretable counterpart [uNO] appears. Put another way, the appearance of [uNO] on the embedded C guarantees the presence of NO inside the comparative clause, while economy prohibits the appearance of NO in a structure in which [uNO] does not appear.

The more significant issue is the compositional analysis of the comparative clause, which is spelled out in the following denotation: (52a) shows the denotation of the constituent containing the comparative and all its arguments; (52b) shows the denotation of the scope of NO (the result of λ-binding the differential degree argument); (52c) shows the denotation of the constituent containing NO and its scope; and (52d) shows the denotation of the entire clause (the result of λ-binding the standard argument).

(52) a. $[\![\text{MORE}]\!]^g([\![\text{tall}]\!]^g)([\![t_3]\!]^g)([\![t_4]\!]^g)([\![\text{Frank}]\!]^g) = \textbf{tall}(\mathbf{f}, g(3) + g(4))$

b. $\lambda d.\textbf{tall}(\mathbf{f}, g(3) + d)$

c. $[\![\text{NO}]\!](\lambda d.\textbf{tall}(\mathbf{f}, g(3) + d)) = \{d' \mid \textbf{tall}(\mathbf{f}, g(3) + d')\} = \emptyset$

d. $\lambda d.\{d' \mid \textbf{tall}(\mathbf{f}, d + d')\} = \emptyset$ $\quad (\equiv \quad \lambda d.\textbf{height}(\mathbf{f}) \leq d)$

(52d) is true of those degrees d such that the the set of degrees d' that can be added to d to derive a degree that Frank's height is at least as great as is empty. Any degree ordered below Frank's height will not satisfy this description, but Frank's height and all the degrees that exceed it will; (52d) is thus equivalent to the NO MORE denotation we introduced in (38b).

(53) shows the result of providing (51) and (52d) as the scope and restriction arguments to *than*, and the truth conditions for the comparative in (30):

(53) $\exists d[\{d' \mid \textbf{tall}(\mathbf{s}, d + d')\} \neq \emptyset \wedge \{d' \mid \textbf{tall}(\mathbf{f}, d + d')\} = \emptyset]$

According to our analysis, *Sarah is taller than Frank is* is true just in case there is a degree d such that the set of degrees that can be added to d to get to Sarah's height is non-empty, but the set of degrees that can be added to d to get to Frank's height is empty. Or, to put it more perspicuously: there is a positive difference between Sarah's height and d, and no positive difference between Frank's height and d. This will be the case only if d is at least as great as Frank's height itself, in which case it follows that Sarah is taller than Frank.

It should now be clear why our analysis really represents a hybrid of the A/¬A and maximality-based analyses: our semantics for the comparative clause is based on the "no more than" part of the definition of the maximality operator, but satisfying the truth conditions of the comparative as a whole involves finding a degree that satisfies the matrix degree predicate and the comparative clause degree predicate, with the latter being a (kind of) negative variant of the former. The crucial differences from traditional versions of these analyses are that: 1) the negative component of the comparative clause comes from the differential degree operator NO; 2) existential quantification is over the standard argument of a comparative adjective, not over the (single) degree argument of a non-comparative gradable adjective; and 3) the standard morpheme *than* plays a crucial role in semantic composition as

a scope-taking degree determiner.[11] In the next section, we show how our analysis accounts for the observed interactions — and non-interactions — with quantifiers in the comparative clause.[12]

4 Scopal (non-)interactions in the comparative clause

In a nutshell, our proposal is that the observed pattern of $>_{max}$ and $>_{min}$ interpretations in the comparative clause reflects the interaction of the differential degree quantifier NO with other quantificational expressions, and in particular, the interaction of the negative component of NO's meaning with other quantifiers. In fact, although the NO MORE analysis introduces negation differently from Schwarzschild's version of the A/¬A analysis, the basic scope relations that deliver the two readings are the same: the $>_{max}$ reading arises whenever a universal quantifier has scope over NO, and the $>_{min}$ reading arises when NO has scope over a universal. The crucial difference between the two analyses is that there are independent constraints governing the relation between degree quantifiers and other quantificational expressions which, as we will see, filter out the unattested readings that are problematic for the revised A/¬A analysis. More generally, we will demonstrate that there is full parallelism between the scope-taking ability of overt NO and the covert NO that is the crucial element of the NO MORE analysis. We go through the analysis of the core cases of necessity modals and universal DPs in sections 4.1 and 4.2, and then take a look at other kinds of interactions in section 4.3: possibility modals (4.3.1), existential DPs (4.3.2), overt negation and negative quantifiers (4.3.3) and negative polarity items (4.3.4). In section 4.4, we turn to comparatives involving (overt) matrix-level differentials.

4.1 Necessity modals

[11] At the same time, *than* is not necessary to derive the semantics of comparison: an "intransitive" comparative like *Sarah is taller* can be interpreted on its own, without positing an implicit *than*-phrase, given a suitable analysis of the missing standard argument as a kind of definite anaphor. See Alrenga et al. 2012 for discussion of the various combinatoric and typological options that become available when the standard morpheme is given a proper role to play in the semantic analysis of comparative constructions.

[12] We have focused entirely on "comparative deletion" constructions so far, but there are several additional types of comparatives in English that our analysis needs to address, including "subcomparatives" like *Sarah is taller than the bed is long*, phrasal comparatives like *Sarah is taller than Frank* (where *Frank* is syntactically just a DP, not the remnant of ellipsis), and comparatives with measure phrase standards such as *Sarah is taller than five feet.* We believe that our proposals can be augmented to account for all of these constructions in a principled way, but to keep the main line of exposition as clear as possible, we show how this can be done in section 6.

Let us begin with the *need to*-class of necessity modals. Recall that an example like (54) is ambiguous: in a situation in which Frank qualifies for a certain class of tax benefits just in case his income is between $50,000 and $74,999, (54) can be used to say either that Frank fails to qualify for the benefits because his income is too high (the $>_{max}$ reading), or that he qualifies for the benefits with some wiggle room (the $>_{min}$ reading).

(54) Frank's income is higher than it needs to be.

As in Schwarzschild's version of the A/¬A analysis, we derive the two readings from the different scope relations that may obtain between the modal and the negation, in our case the negation contributed by the differential operator NO.

Consider first the result of interpreting NO below the modal, as shown in (55).

(55) a. Frank's income is higher than [$_{\text{CP}}$ wh_1 it needs NO$_2$ to be t_2 higher t_1]

b. $\exists d[\{d' \mid \textbf{high}(\textbf{inc}, d+d', w_0)\} \neq \emptyset \wedge$
$\forall w[ACC_{w_0}(w) \rightarrow \{d' \mid \textbf{high}(\textbf{inc}, d+d', w)\} = \emptyset]]$

On this parse, an utterance of (54) is true just in case there is a degree d such that there is a positive difference between Frank's income and d in w_0, and in every accessible world w, there is no positive difference between Frank's income and d in w. In the scenario in question, Frank's income in the accessible worlds ranges from $50,000 to $74,999. The only way for it to be true of all of these worlds that there is no positive difference between d and his income is for d to be at least $74,999; given (55b), it follows that his income in w_0 is at least $75,000. This is the $>_{max}$ reading, and Frank fails to get the tax benefit.

Now consider the result of interpreting NO above the modal:

(56) a. Frank's income is higher than [$_{\text{CP}}$ wh_1 NO$_2$ it needs to be t_2 higher t_1]

b. $\exists d[\{d' \mid \textbf{high}(\textbf{inc}, d+d', w_0)\} \neq \emptyset \wedge$
$\{d' \mid \forall w[ACC_{w_0}(w) \rightarrow \textbf{high}(\textbf{inc}, d+d', w)]\} = \emptyset]$

On the parse in (56a), (54) is true just in case there is a degree d such that there is a positive difference between Frank's income and d in w_0, and in at least one accessible world, there is no positive difference between Frank's income and d. This will be false if d is $49,999 or lower, because in such a case there is a positive difference between Frank's income and d in the world in which he has the lowest income ($50,000), and so there is a positive difference between his income and d in every accessible world. But if d is $50,000 (or more), then there will be no positive difference between d and his income in (at least) the world in which he makes $50,000, and so the sentence will be true so long as Frank makes at least $50,001 in the actual world. This is the $>_{min}$ reading, and Frank gets the benefit.

Recall that one of the problems for Schwarzschild's revised A/¬A analysis was that it could not easily account for the ambiguity of comparative clauses containing members of the *need to*-class of modals. In particular, it is not able to account for the $>_{max}$ reading, because sentential negation obligatorily takes wide scope with respect to *need to, have to*, etc. This is illustrated by the examples in (57), which cannot be used to make the strong assertion that income above $74,999 is not allowed (for, e.g., the relevant tax benefit), only the weaker (and not particularly helpful) assertion that incomes below $75,000 are allowed.

(57) a. Frank's income does not need to be higher than $74,999.
b. Frank's income does not have to be higher than $74,999.
c. Frank's income is not required to be higher than $74,999.

Our analysis does not suffer from this problem, because differential NO differs crucially from sentential negation in being able to take scope above or below the *need to*-class of modals. This is illustrated by the examples in (58) (cf. Nouwen's (2008) discussion of modified numerals of the form *no more than n*, which give rise to the same pattern of interpretation).

(58) a. Frank's income needs to be no higher than $74,999.
b. Frank's income has to be no higher than $74,999.
c. Frank's income is required to be no higher than $74,999.

Under one reading, these sentences are equivalent to those in (57). However, they can also be used to make the stronger claim that an income above $74,999 is not allowed (for the purpose in question).

These two readings are derived from logical forms in which overt NO scopes above and below the modal, which derive the truth conditions in (59a-b), respectively. (For the compositional interpretation of measure phrase standards like *than $74,999*, see section 6. We discuss clausal comparatives with overt differential NO in section 4.4.)

(59) a. $\{d' \mid \forall w[ACC_{w0}(w) \rightarrow \exists d[\textbf{high}(\textbf{inc}, d + d', w) \wedge d = \textbf{74,999}]]\} = \emptyset$
b. $\forall w[ACC_{w0}(w) \rightarrow \{d' \mid \exists d[\textbf{high}(\textbf{inc}, d + d', w) \wedge d = \textbf{74,999}]\} = \emptyset]$

(59a) says that there are no degrees d' such that in every accessible world, Frank's income is at least as high as d' plus $74,999. This entails that there are some worlds in which his income is $74,999 or less but allows for worlds in which his income is $75,000 or more, which is the reading that is equivalent to the one reading that we get with sentential negation in (57a-c). (59b) says that in every accessible world, there are no degrees d' such that Frank's income is at least as high as d' plus $74,999. This entails that there are no worlds in which his income is $75,000 or higher (while

still allowing for it to be lower), which is the stronger reading that is missing from the examples involving sentential negation.

Further evidence that both scopal interpretations are possible for overt differential NO is provided by the attested examples in (60) and (61), which are clearly intended to convey the NO $>$ $\forall w$ and $\forall w$ $>$ NO readings, respectively.

(60) NO $>$ $\forall w$

a. Foyt, a racing legend by virtue of winning more Auto Club championship races than any other American, has never won an IROC championship but needs to finish no better than sixth here today to take this year's title.
'It is not necessary for Foyt to finish any better than sixth.'

b. Many families once had to travel up to 25km to reach health facilities; today, most have to travel no further than 1km to reach their village clinics at the home of local health workers.
'It is not necessary for a family to travel any further than 1km.'

c. Following the start of meetings, which will be within fifteen (15) minutes of the end of the students' day, teachers shall be required to stay no longer than sixty (60) minutes.
'It shall not be necessary for a teacher to stay any longer than 60 min.'

(61) $\forall w$ $>$ NO

a. In order [for Sabbatini] to win, Sabbatini needs to finish first here and Woods needs to finish no better than 14th at the East Lake Golf club.
'It is necessary that Woods not finish better than 14th.'

b. I made it short because the town covenants say structures have to be no higher than the brick wall at the end of the property.
'It is necessary that structures not be higher than the brick wall.'

c. Men are required to be clean shaven except in certain situations. Even in those situations, though, beards are required to be no longer than a quarter of an inch.
'It is necessary that one's beard not be longer than a quarter of an inch.'

Crucially, these examples differ clearly from corresponding examples with sentential negation. The sentences in (60) have equivalent paraphrases with negation, as expected, but the ones in (61) do not:

(62) a. Foyt does not need to finish (any) better than sixth here today to take this year's title. (= (60a))

b. Most families do not have to travel (any) further than 1km to reach their village clinics at the home of local health workers. (= (60b))

c. Teachers shall not be required to stay (any) longer than sixty (60) minutes. (= (60c))

(63) a. In order for Sabbatini to win, he needs to finish first here and Woods does not need to finish (any) better than 14th. ($\neq$ (61a))

b. I made it short because the town covenants say structures do not have to be (any) higher than the brick wall at the end of the property. ($\neq$ (61b))

c. Men are required to be clean shaven except in certain situations. Even in those situations, though, beards are not required to be (any) longer than a quarter of an inch. ($\neq$ (61c))

This pattern is exactly what we expect given our hypothesis that the negative element in the comparative clause is a silent version of the differential degree quantifier NO, rather than an occurrence of sentential negation.

Before leaving the *need to*-class of modals, let us briefly note that our analysis also accounts for the fact that comparatives containing NPI *need*, unlike the regular modal *need to*, are unambiguous. (We will have more to say about NPIs in general in section 4.3.4.) This is illustrated by (64), where the continuation in (64a) is odd because the context is one that promotes a $>_{max}$ reading, but only the $>_{min}$ interpretation is available. In contrast, the continuation in (64b) is fine, because the $>_{max}$ reading is possible.

(64) Frank does not qualify for these tax benefits, because...

a. ?? ...his income is higher than it need be. (only $>_{min}$ available)

b. ...his income is higher than it needs to be. ($>_{max}$ possible)

This contrast is expected on our analysis. In (64b), NO may take scope either above or below the modal; the latter parse derives the $>_{max}$ interpretation. In contrast, NO must take scope over *need* in (64a) in order to satisfy its licensing properties as a polarity item. This rules out the $\forall w >$ NO parse and the corresponding $>_{max}$ reading. And as above, we see exactly the same behavior with overt *no* when we look at an example that promotes a narrow scope interpretation for NO, such as (65):

(65) ?? Men are required to be clean shaven except in certain situations. Even in those situations, though, beards need be no longer than a quarter of an inch.

Let us now turn to the *supposed to* class of modals, which differ from the *need to* class in only allowing $>_{max}$ interpretations: all of the sentences in (66), when uttered in the tax benefit context we introduced earlier, entail that Frank's income is $75,000 or higher.

(66) a. Frank's income is higher than it is supposed to be.

b. Frank's income is higher than it should be.

c. Frank's income is higher than it ought to be.

And indeed, as is expected on our analysis, these modals also must take wide scope relative to overt differential NO:

(67) a. Frank's income is supposed to be no higher than $74,999.
b. Frank's income should be no higher than $74,999.
c. Frank's income ought to be no higher than $74,999.

These sentences can only be understood as saying that it is necessarily the case that incomes not exceed $74,999, not that it is not necessary for incomes to exceed $74,999. Similarly, the example in (68a) only has the somewhat strange reading (given the way that scoring works in automobile racing) that Foyt must finish at sixth place or lower in order to win the title, while (68b) has the more natural reading that he will win with a finish of sixth place or higher.

(68) a. Foyt should finish no better than sixth in today's race in order to win the title.
b. Foyt need finish no better than sixth in today's race in order to win the title.

Again, this parallelism between overt and silent NO are exactly what we expect to find if our analysis is correct.

More significantly, the specific pattern that we observe — NO can take scope below both the *need to*-class of modals and the *supposed to*-class, but can only take scope above the former — is also something that we expect to see given our analysis of NO as a degree quantifier. It is already known that these two groups of modals pattern differently regarding scopal interactions with degree quantifiers, with the former class behaving as though they allow either wide or narrow scope for degree quantifiers, and the latter class behaving as though they block wide scope for degree quantifiers (Heim 2000, 2001; Bhatt and Pancheva 2004; Büring 2007; Krasikova 2008; van Rooij 2008; Lassiter 2012). It remains an open question why this effect is observed, and indeed whether it is a true blocking effect, or whether it instead reflects a difference in the semantics of the two classes of modals such that it is only for the *need to*-class that the relative position of a degree quantifier makes a difference to the truth conditions. Since answering this question goes well beyond the scope of this paper, we will continue to talk as though this is a kind of blocking effect, and simply point out that the pattern of interpretation that we see in comparatives is exactly what we predict given our proposal that the negative part of the meaning of the comparative clause is introduced by the differential degree quantifier NO.[13]

[13]That said, it is easy to show that our proposals are consistent with an analysis like the one in Lassiter 2012, in which the *should* class allows wide scope for a degree quantifier, but the resulting truth conditions are not distinct from the narrow scope truth conditions. Lassiter's denotation for *should* is given in (i), where μ_{good} is a measure of the "goodness" of a proposition; *should p* basically

Finally, although our discussion so far has centered entirely around deontic necessity modals, the same scope correlations may be observed for other necessity modals as well, which behave in general like the *supposed to* class in ruling out wide scope for degree quantifiers (see Büring 2007). Consider, for example an utterance of (69) in a context in which tomorrow's weather forecast predicts that the high temperature will be between 65° and 75°.

(69) It is already warmer today than it will/shall be tomorrow.
  a. *Current temperature is 83°*: TRUE
  b. *Current temperature is 73°*: FALSE

The pattern of truth judgments shown in (69a-b) demonstrates that the modals *will* and *shall* only allow $>_{max}$ readings in the comparative clause, and correspondingly, *will* and *shall* also take scope over overt NO: (70) can only be understood to mean that my paper is guaranteed to not be longer than ten pages; it cannot have the weaker meaning in which my paper is not guaranteed to be longer than ten pages.

(70) My paper will/shall be no longer than ten pages.

To the best of our knowledge, the above correlations between a modal's scope relative to NO and the reading(s) that it will yield in the comparative clause hold throughout the inventory of English necessity modals.

4.2 Universal DPs

We now turn to the case of universal quantifiers over individuals. Recall that examples like the following have $>_{max}$ readings, but lack $>_{min}$ readings.

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means "it is better that p than that not p."

(i) $[\![should]\!] = \lambda p_{\langle s,t\rangle}.\mu_{good}(p) >_s \mu_{good}(\neg p)$

In a sentence like *Your abstract should be no longer than two pages*, scoping NO below *should* gives the truth conditions in (iia), and scoping NO above *should* gives the truth conditions in (iib).

(ii) a. $\mu_{good}(\{d \mid \textbf{length}(\textbf{abs}) \geq \textbf{2pp} + d\} = \emptyset) >_s \mu_{good}(\{d \mid \textbf{length}(\textbf{abs}) \geq \textbf{2pp} + d\} \neq \emptyset)$
  b. $\{d \mid \mu_{good}(\textbf{length}(\textbf{abs}) \geq \textbf{2pp} + d) >_s \mu_{good}(\textbf{length}(\textbf{abs}) < \textbf{2pp} + d)\} = \emptyset$

(iia) says that it is better that there is no positive difference between the length of the abstract and two pages than that there is a positive difference between the length of the abstract and two pages. (iib) says that there is no degree such that it is better that the length of the abstract is at least as great as that degree plus two pages than that its length is less than that degree plus two pages. Both sets of truth conditions rule out lengths greater than two pages, and are noncommittal about lengths less than two pages.

(71) a. Max is taller than each of his students is.
b. Max is taller than every one of his students is.
c. Max is taller than all of his students are.
d. Max is taller than both of his students are.

Just as in the A/¬A analysis, we derive the $>_{max}$ readings when NO scopes below a universally quantified subject, as shown in (72a).

(72) a. Max is taller than [$_{\text{CP}}$ wh_1 each of his students$_2$ NO$_3$ [t_2 is t_3 taller t]]
b. $\exists d[\{d' \mid \textbf{tall}(\textbf{m}, d + d')\} \neq \emptyset \wedge$
$\forall x[\textbf{student-of}(x, \textbf{m}) \rightarrow \{d' \mid \textbf{tall}(x, d + d')\} = \emptyset]]$

The truth conditions in (72b) say that there is a degree d such that there is a positive difference between Max's height and d, and for every one of Max's students x, there is no positive difference between x's height and d. This will be the case only if Max's height exceeds that of his tallest student, which is the $>_{max}$ reading.

If it were also possible for NO to take scope over a universally quantified subject, then we would incorrectly predict all of the examples in (71) to be ambiguous. However, as was the case with the *supposed to*-class of modals, it has already been observed in the literature that there are constraints on the possible scopal relations that may obtain between degree quantifiers and quantificational DPs (Szabolcsi and Zwarts 1993; Kennedy 1999; Heim 2000, 2001; Bhatt and Pancheva 2004; Takahashi 2006). Heim (2000, p. 27) states the constraint as follows:

(73) If the scope of a quantificational DP contains the trace of a DegP, it also contains that DegP itself.

As was the case with the modals, there are differences of opinions in the literature as to what underlies (73), and as to whether it represents a real grammatical constraint or whether it follows from some other principle (see e.g. Gajewski 2008; van Rooij 2008; Lassiter 2012), and it is not our intention to resolve these debates here. Instead, for the purposes of this paper we will simply assume that (73) holds as a descriptive constraint, and will focus as we did above on the specific prediction of the NO MORE analysis that comparatives should pattern identically to examples with the overt version of NO with respect to quantifier interactions. As shown by the examples in (74), this prediction is correct for universally quantified subjects: these sentences have only "*every ... not*" interpretations; they lack "*not ... every*" interpretations.

(74) a. Each teacher is no taller than his students.
b. Every teacher is no taller than his students.
c. All of the teachers are no taller than their students.

d. Both teachers are no taller than their students.

Crucially, these examples contrast with those in (75), which involve sentential negation and allow "*not ... every*" interpretations.

(75) a. Each teacher is not taller than his students.
b. Every teacher is not taller than his students.
c. All of the teachers are not taller than their students.
d. Both teachers are not taller than their students.

Recall from section 2.2 that it was precisely because of examples like these that we were led to reject Schwarzschild's revised A/¬A analysis, because there the (incorrect) prediction is that we see parallelism between comparatives and overt sentential negation. In contrast, on the NO MORE analysis, the lack of an ambiguity in comparative clauses with universal quantifiers fits into a larger pattern of observed interactions between individual and degree quantifiers.

Note, however, that although (73) rules out structures in which NO is raised out of the syntactic scope of the universal, correctly predicting the lack of $>_{min}$ readings in the examples discussed so far, it says nothing about a case in which NO takes scope over the universal in virtue of its surface position, i.e., in which the universal does not c-command the base position of NO. It is this possibility that accounts for the ambiguity that we observed earlier in (10), repeated here with the paraphrases of the two readings in (76a-b).

(76) More students have read Lord of the Rings than have read every other Tolkien novel.
a. The number of students who have read LOTR exceeds the number of students who have read all of the other Tolkien novels.
b. The number of students who have read LOTR exceeds the number of students who have read the most-read Tolkien novel other than LOTR.

We will assume here that amount *more* is the comparative form of an underlying adjective **many** which denotes a relation between plural objects and their cardinalities (see e.g., Landman 2004; Ionin and Matushansky 2006; Rothstein 2011; see Cresswell 1976; Krifka 1989 and Hackl 2000 for variants of this analysis), and that the individual argument is bound by a default existential quantifier. This gives us (77a) and (78a) as two potential logical forms for (76), with the truth conditions given in (77b) and (78b), respectively.

(77) a. ...than [wh_1 NO$_2$ [$_{\text{DP}}$ [t_2 MORE MANY t_1] students]$_3$ [$_{\text{DP}}$ every other Tolkien novel]$_4$ have [$_{\text{VP}}$ t_3 read t_4]]

b. $\exists n[\{n' \mid \exists x[\textbf{many}(x, n+n') \wedge \textbf{students}(x) \wedge \textbf{read}(x, \textbf{lotr})\} \neq \emptyset \wedge$
$\{n' \mid \exists x[\textbf{many}(x, n+n') \wedge \textbf{students}(x) \wedge \forall y[\textbf{tolnov}(y) \rightarrow \textbf{read}(x, y)]]\} = \emptyset]$

(78) a. ...than $[wh_1$ [$_{\text{DP}}$ every other Tolkien novel $]_4$ NO$_2$ [$_{\text{DP}}$ [t_2 MORE MANY t_1] students $]_3$ have [$_{\text{VP}}$ t_3 read t_4]]

b. $\exists n[\{n' \mid \exists x[\textbf{many}(x, n+n') \wedge \textbf{students}(x) \wedge \textbf{read}(x, \textbf{lotr})\} \neq \emptyset \wedge$
$\forall y[\textbf{tolnov}(y) \rightarrow \{n' \mid \exists x[\textbf{many}(x, n+n') \wedge \textbf{students}(x) \wedge \textbf{read}(x, y)]\} = \emptyset]]$

According to (77b), (76) is true with the logical form in (77a) if and only if there is a number n such that there is a positive difference between the number of students who read *The Lord of the Rings* and n, and there is no positive difference between n and the number of students who read all of the other Tolkien novels, which is the reading paraphrased in (76a). In contrast, on the parse in (78a), it must be the case that there is a number n such that there is a positive difference between the number of students who read *The Lord of the Rings* and n, and for every other Tolkien novel, there is no positive difference between n and whatever number of students read it. It follows that the number of students who read *The Lord of the Rings* is greater than the number of students who read the most-read of Tolkien's other novels. This is the reading in (76b).

Since the ambiguity in (76) is derived by varying the relative scope of the DPs *every book* and *more students* (= [MORE **many** *students*]), the NO MORE analysis further predicts that when this scope relation is fixed, the ambiguity should disappear. This prediction is borne out by the following example:

(79) There were more women at the LSA than there were at every other conference during this past year.

a. The number of women who attended the LSA exceeded the number of women who attended the conference with the most female attendees other than the LSA.

b. * The number of women who attended the LSA exceeded the number of women who attended all of the other conferences during this past year.

In (79), the grammar of the English existential construction demands that the universal DP *every other conference* outscope the pivot DP *more women* within the comparative clause (Francez 2007): compare *No more than three women attended every conference* (ambiguous) to *There were no more than three women at every conference* (unambiguous). And in fact, (79) is unambiguous, and only possesses the reading corresponding to (76b), in which the universal takes wide scope relative to the DP that provides the base position for NO, as shown in (78a).[14]

[14]How might the A/¬A analysis derive the ambiguity in (76)? Under that analysis, the reading in (76b) is derived when the universal DP takes highest scope within the comparative clause, above

4.3 Interactions with other operators

Universal quantifiers over possible worlds and individuals are, obviously, not the only kinds of operators that can occur in the comparative clause. In this section, we go through several other crucial cases that have been discussed in the previous literature: possibility modals, existential DPs, negation and negative quantifiers, and NPIs.[15]

4.3.1 Possibility modals

Like their necessity counterparts, possibility modals also give rise to both $>_{max}$ and $>_{min}$ readings in the comparative clause, and may similarly be divided into two groups. The first group consists of possibility modals that give rise to both $>_{max}$ and $>_{min}$ readings. As the following examples illustrate, this group includes deontic *could*, ability *can*, and dispositional *willing to* (Heim 2000, 2001; Meier 2002; Büring 2007).

(80) $>_{max}$ *readings*

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both NOT and the DP **many** *women*:

(i) $\exists n[\exists x[\textbf{many}(x,n) \wedge \textbf{students}(x) \wedge \textbf{read}(x,\textbf{lotr})] \wedge \forall y[\textbf{tolkeinnovel}(y) \rightarrow \neg\exists x[\textbf{many}(x,n) \wedge \textbf{students}(x) \wedge \textbf{read}(x,y)]]]$

The reading in (76a) would arise when the universal DP takes lowest scope within the comparative clause:

(ii) $\exists n[\exists x[\textbf{many}(x,n) \wedge \textbf{students}(x) \wedge \textbf{read}(x,\textbf{lotr})] \wedge \neg\exists x[\textbf{many}(x,n) \wedge \textbf{students}(x) \wedge \forall y[\textbf{tolkeinnovel}(y) \rightarrow \textbf{read}(x,y)]]]$

Just as before, the scope relations that derive the two possible readings for (76) are essentially the same for both analyses. Note, however, that the universal DP scopes under NOT in (ii). Earlier, we observed that this scope relation must generally be prevented, in order to account for the absence of the $>_{min}$ reading in examples such as *Max is taller than each of his students is*. This means that under the A/¬A analysis, whatever additional restrictions are assumed in order to rule out ¬∀ scope relations in the comparative clause more generally must allow for this order in (ii).

[15]There are, of course, still other types of quantificational expressions that may appear in the comparative clause, such as proportional quantifiers such as *most* and *usually* and propositional attitude verbs (e.g., *Bill did better on the exam than Max predicted that most of his students would do*, on which see Schwarzschild and Wilkinson 2002). While we do not consider them in any detail here, we do not see any serious obstacles that would prevent the straightforward extension of our proposals to these quantifiers. At any rate, the basic predictions of our analysis are clear enough: in each case, we expect to find that the reading(s) that these quantifiers give rise to in the comparative clause parallel the one(s) observed in the presence of the overt differential *no* (e.g., the readings associated with *Max predicted that most his students would get no better than 90% on the exam*).

a. Chuck is taller than he could be under the previous regulations (but he may be eligible under the new ones).
'Chuck's height exceeds the previous maximum permissible height.'

b. Derrick jumped higher than I can jump.
'Derrick's vertical leap exceeded my maximum possible vertical leap.'

c. Lee traveled further into the jungle than I was willing to go.
'The distance that Lee traveled exceeded the maximum distance that I was willing to go.'

(81) $>_{min}$ *readings*

a. I became so inspired by the topic that I ended up writing more than I could have (while still meeting the page requirement).
'My paper's length ended up exceeding the minimum permissible length.'

b. His prices are a bit higher than I can find elsewhere (but I'm happy to pay a little extra for his attention to detail).
'His prices exceed the minimum prices that can be found elsewhere.'

c. Our initial asking price was about 10 percent higher than we were willing to accept.
'Our initial asking price exceeded the minimum amount that we were willing to accept (by about 10 percent).'

The second group consists of possibility modals that invariably yield $>_{min}$ readings, such as epistemic *might* and *may*.

(82) It is warmer today than it might/may be tomorrow.
'Today's temperature exceeds tomorrow's minimum possible temperature.' (I.e., it may be less warm tomorrow.)

As with the necessity modals discussed earlier, it has already been pointed out in the literature that these two groups of modals differ in their interactions with degree quantifiers, with the first group allowing scope permutations, and the second group giving rise to the same kind of "intervention effect" as the *supposed to*-class of necessity modals. The fact that the more restrictive modals like *might* and *may* only allow $>_{min}$ readings, while their universal counterparts only allow $>_{max}$ readings, follows automatically from the hypothesis that the semantics of comparatives crucially involves negation. In particular, this follows from the equivalences in (83a-b).

(83) a. $\neg\forall \Leftrightarrow \exists\neg$

b. $\forall\neg \Leftrightarrow \neg\exists$

In other words, the scope relations underlying the two readings for possibility modals are the opposite from those that we previously identified for necessity modals

(cf. Heim 2000, 2001): the $>_{min}$ reading arises when a possibility modal scopes over NO, and the $>_{max}$ reading arises when NO scopes over the modal. Since the *might*-class of modals behaves as though it is subject to the constraint in (73), the latter scope relation is ruled out, and only the $>_{min}$ reading is available.

Moreover, just as we saw with necessity modals, when we look at scope interactions between possibility modals and the overt version of NO, we find a pattern of interpretations that conforms exactly to the predictions of the NO MORE analysis. Sentences constructed out of modals in the *could*-class are ambiguous, as shown by the attested examples in (84) and (85). (See Nouwen (2008, ex. (27)) on analogous ambiguities involving *allowed to* and modified numerals of the form *no more than n*.)

(84) NO $> \exists w$

a. Ten years later in 1869, another rule was added that stated the baseball bat could be no longer than 42 inches in length — the same maximum length allowed today.
'It was not allowed for baseball bats to be any longer than 42 inches.'

b. If you want to make a dollar in profit off of an ice cream sundae that you can sell for $2, you can pay no more than $1 to produce and sell each ice cream sundae.
'It is not possible for you to pay any more than $1.'

c. Republican Ryan Frazier claims to be "100%" pro-life" but he is not! [...] He supports certain limits on abortion, such as de-funding Planned Parenthood, but is willing to go no further.
'It is not compatible with his disposition to go any further.'

(85) $\exists w >$ NO

a. Having so found that, the Court could have written no more. Instead, it spend multiple paragraphs discussing...
'It was allowable that the Court write no more.'

b. With wholesale pricing, you can pay no more than $6.00 [per] bottle.
'It is possible that you pay no more than $6.00 per bottle.'

c. (From a website entitled "What do I do if the party who sued me actually owes me money?") You can file what is called a Counterclaim, which is, essentially, a Complaint filed by the Defendant against the Plaintiff. [...] You may keep your counterclaim in Justice Court by stating that even though the amount listed in your counterclaim exceeds $12,000, you are willing to accept no more than the $12,000 if you prevail.
'It is compatible with your disposition that you receive no more than $12,000 upon prevailing.'

In contrast, the epistemic modals *might* and *may* do not give rise to ambiguity with overt NO: (86) unambiguously conveys the possibility that John's paper does not exceed ten pages in length.

(86) John's paper might/may be no longer than ten pages.
'It is possible that John's paper is no longer than ten pages.'
'*It is not possible that John's paper is longer than ten pages.'

It is worthwhile to compare the behavior of *willing to* to that of the modal auxiliary *dare*. Although these two modals possess similar meanings, auxiliary *dare* (like *need*) is a negative polarity item:

(87) I dare *(not) ask what you think of me.

As a result, only the $\neg\exists$ reading is possible when *dare* combines with NO:

(88) a. I will support certain limits on abortion, but I dare go no further than that.
'It is not compatible with my disposition to go any further.'
b. ?? Even though my counterclaim exceeds $12,000, I dare accept no more than the $12,000 if I prevail.

And likewise, *dare* only gives rise to $>_{max}$ readings in the comparative clause:

(89) a. Lee wants to travel further into the jungle than I dare go.
'The distance that Lee wants to travel exceeds the maximum distance that I am willing to go.'
b. ?? Our initial asking price is higher than we dare accept.

This difference between *dare* and *willing to* follows straightforwardly, once it is assumed that NO must take scope over *dare* in the comparative clause in order to satisfy the latter's licensing requirements.

4.3.2 Existential DPs

As shown by the following examples, monotone-increasing existential quantifiers have only $>_{min}$ readings:

(90) a. Sarah is taller than some of my students are.
b. Sarah is taller than (at least) three of my students are.

These sentences are true as long as it is possible to find a relevantly-sized group of students such that Sarah's height exceeds theirs; they cannot be understood to require Sarah's height to be greater than the maximal height such that some/at least three students are that tall. (In the case of (90a), this would be equivalent to saying

that Sarah is taller than any of my students, a point to which we return when we discuss NPIs in section 4.3.4.)

The lack of $>_{max}$ interpretations for these examples is exactly what our analysis predicts, given the assumption that the existential DPs in (90a-b) are in fact quantificational, and so are subject to the intervention constraint in (73). It follows that NO must take scope below the subject, and as we have already seen, scoping NO below an existential quantifier derives the $>_{min}$ interpretation. This is shown for (90a) in (91).

(91) $\exists d[\{d' \mid \textbf{tall}(\textbf{s}, d + d')\} \neq \emptyset \wedge \exists x[\textbf{student-of}(x, \textbf{me}) \wedge \{d' \mid \textbf{tall}(x, d + d')\} = \emptyset]]$

(91) is true if there is a degree d such that there is a positive difference between Sarah's height and d, and there is a student of mine such that there is no positive difference between that student's height and d. This will be true as long as Sarah is at least taller than the shortest of all my students.[16]

Unfortunately, this line of analysis does not quite give us an account of the interpretation of non-monotonic quantifiers such as *exactly three of my students* in (92a), which have presented problems for many other analyses of comparatives (see Schwarzschild and Wilkinson 2002 and Gajewski 2008 for discussion). (92b) shows the truth conditions that we get for (92a) when NO scopes under *exactly three of my students*. (For perspicuity, we adopt Nouwen's (2010) convention of using $\exists!x$ to indicate a uniqueness requirement on values of x for the "exactly" meaning; see Kennedy 2013 for a different approach to numerals which achieves the same results, and is more in line with the overall program advanced in this paper, but would make the representations less easy to read.)

(92) a. Sarah is taller than exactly three of my students are.

b. $\exists d[\{d' \mid \textbf{tall}(\textbf{s}, d + d')\} \neq \emptyset \wedge \exists x![\textbf{student-of}(x, \textbf{me}) \wedge \#(x) = 3 \wedge \{d' \mid \textbf{tall}(x, d + d')\} = \emptyset]]$

These truth conditions are satisfied just in case there is a degree d such that there is a positive difference between Sarah's height and d, and there is a unique plurality

[16] Indefinite DPs can give rise to $>_{max}$ interpretations in certain contexts. In particular, they appear to have such interpretations when they are understood generically:

(i) a. A BMW is more expensive than a Honda is.

b. Volvos are safer than Fiats are.

Figuring out exactly what is going on with generic comparatives would take us well beyond the scope of this paper (see Nickel 2010 for a comprehensive study), but one possibility worth considering is that the indefinite subjects in examples like these are not quantificational (as in e.g. Diesing 1992), and so do not interact with (73) in the same way as truly quantificational existentials.

of students of cardinality three such that (for each of them) there is no positive difference between their heights and d. This certainly requires there to be three students whose heights are less than Sarah's, but it can be true even when there are more than three such students, which is not the case for the actual English sentence. (E.g., if there are four students a, b, c, and d who are successively shorter than Sarah, then we can satisfy the truth conditions in (92b) by picking a degree between a's height and b's height.)

As pointed out by Gajewski (2008), this problem arises quite generally for analyses in which the "top-level" quantificational force in a comparative is existential, as in our approach and as in the formulations of the A/¬A analysis that we discussed earlier.[17] For the purpose of this paper, we will adopt Gajewski's solution to the problem, which is to assume that in examples like these, an exhaustivity operator can be inserted at the matrix level, above the scope of *than* and the existential quantification over degrees that it introduces, which ensures in e.g. the case of (92) that there are no stronger true alternatives to this sentence in which *three* is replaced by a different numeral.[18]

4.3.3 Sentential negation and negative quantifiers

Sentential negation and negative quantifiers are in general unacceptable in the comparative clause:

(93) a. * Sarah is taller than Frank isn't.
b. * Sarah is older than none of her colleagues are.
c. * Sarah ran faster than few of her students did.

In the maximality-based analysis, the ungrammaticality of e.g. (93a) follows from the fact that the comparative clause lacks a denotation: there is no maximal degree d such that it is not the case that Frank is at least as tall as d (von Stechow 1984a; Rullmann 1995). This type of explanation is obviously unavailable to us, and indeed, facts like these have proven difficult for accounts which provide mechanisms for effectively scoping quantifiers out of the comparative clause in order

[17] In contrast, analyses which find ways of effectively scoping *exactly three students* out of the comparative clause (such as Larson 1988) can account for these kinds of cases; the problem for such analyses, as Gajewski and others have observed, is that they overgenerate in many other cases (e.g., for negative quantifiers).

[18] A promising alternative approach would be to adopt Brasoveanu's (2013) analysis of modified numerals as introducing "post-suppositions," which constrain the size of the set individuals introduced by a summation operator over the individual argument of the relevant NP. If this operator can also be inserted above the scope of *than*, we will achieve the same results that Gajewski's approach achieves. Fully working out the details of this alternative and comparing it to Gajewski's analysis goes well beyond the scope of this paper, however.

to explain the interactions with universals that we have already observed (Larson 1988; Schwarzschild and Wilkinson 2002; Heim 2006b).

However, working within the A/¬A tradition, Gajewski (2008) and van Rooij (2008) suggest an alternative explanation for these facts. Both authors observe that the inclusion of an overt negative element in the comparative clause yields systematically trivial truth conditions, and argue that it is this grammatically determined uninformativity which underlies the unacceptability of the examples. We would like to argue that the same type of explanation can be extended to our analysis: even if we allow NO and sentential negation to interact scopally, the resulting truth conditions on either scope relation are uninformative, and the sentence is unacceptable.

(94a-b) show the truth conditions that we get for (93a) when NO has narrow and wide scope relative to negation, respectively.

(94) a. $\exists d[\{d' \mid \textbf{tall}(\mathbf{s}, d+d')\} \neq \emptyset \wedge \{d' \mid \textbf{tall}(\mathbf{f}, d+d')\} \neq \emptyset]$ $(\neg > \text{NO})$

b. $\exists d[\{d' \mid \textbf{tall}(\mathbf{s}, d+d')\} \neq \emptyset \wedge \{d' \mid \neg\textbf{tall}(\mathbf{f}, d+d')\} = \emptyset]$ $(\text{NO} > \neg)$

(94a) is true just in case there is a degree d such that there is a positive difference between Sarah's height and d, and there is a positive difference between Frank's height and d, which is true no matter what Sarah and Frank's heights actually are. (94b) on the other hand is true just in case there is a degree d such that there is a positive difference between Sarah's height and d, and there is no d' that can be added to d such that Frank's height is not as great as the result. But given that there is no maximal degree on the height scale, this requirement can never be met, and the result is a contradiction.

For examples with negative quantifiers, we assume that NO must take scope below the quantifier in accord with (73). This means that the truth conditions of (93b) are as shown in (95). (Here we characterize negative quantifiers as universals because it makes it a bit easier to see where the contradiction arises; of course the same results obtain if such quantifiers are treated as existentials.)

(95) $\exists d[\{d' \mid \textbf{old}(\mathbf{s}, d+d')\} \neq \emptyset \wedge \forall x[\textbf{colleague}(x, \mathbf{s}) \rightarrow \neg\{d' \mid \textbf{old}(x, d+d')\} = \emptyset]]$

According to (95), *Sarah is older than none of her colleagues are* is true if there is a degree d such that there is a positive difference between Sarah's age and d, and for each of her colleagues, it is false that there is no positive difference between the colleague's age and d — which is to say that for each of them there *is some* positive difference between their age and d. But this, too, is a tautology. If scales are dense (Fox and Hackl 2007), the first conjunct can always be satisfied, because the set of degrees that are strictly ordered below Sarah's age is non-empty. But for exactly the same reason, the second conjunct can also always be satisfied, because for each colleague, the set of degrees that are strictly ordered below that colleague's

age is also non-empty. The result is that the sentence is a tautology, and tells us nothing about the relation between Sarah's height and the heights of her colleagues. A similar line of reasoning can be applied to the example in (93c).

4.3.4 NPIs

We conclude this section by looking briefly at what our analysis has to say about the distribution and interpretation of NPIs in comparatives. We have already seen examples involving NPI *need* and *dare*; the following examples (some adapted from Seuren 1973; some pulled from the web), illustrate other kinds of NPIs that are acceptable in comparatives:

(96) a. The emperor was more inclined to amuse himself than to do *a damn thing* for his country.
b. * The emperor liked to amuse himself by doing a damn thing for his country.

(97) a. The amount of spaghetti was more than I was *all that keen* to eat.
b. * The amount of spaghetti was significant, and I was all that keen to eat it.

(98) a. The fifth glass of whiskey was more than I *cared to* drink.
b. * The fifth glass of whiskey was on the house because I cared to drink it.

(99) a. My urge to steal was stronger than I *could help*.
b. * My urge to steal was strong, but I could help it being that strong.

(100) a. For several reasons, including lifestyle, failing eyes, and the invention of the iPod, I listen to many more books than I read *anymore.*
b. * I listen to a lot of books nowadays, and I read a few anymore.

(101) a. I have also been seeing lots more turtles and snakes than I have seen *in years.*
b. * I continue to see the same turtles and snakes that I've seen in years.

(102) a. Right now it is leafing out and looks happier than I have seen *yet*, with lots of healthy new growth.
b. * The tree is leafing out and looks happy yet.

There is a continuing debate in the literature about what the licensing conditions for NPIs are (see e.g. Ladusaw 1979; Linebarger 1980; Kadmon and Landman 1993; Krifka 1995; Giannakidou 1998, 1999, 2006; Chierchia 2004), and whether NPIs in comparatives specifically are licensed or "rescued" (Giannakidou and Yoon 2010). Furthermore, most of the literature on NPIs has focused on their interaction with propositional negation, so it is certainly appropriate to ask whether expressions that are licensed via their relation to propositional negation can be equally well licensed via a relation to degree negation. But it is clearly the case that from a logical perspective, at least, NO is downward entailing relative to degrees:

(103) For any individual x, degrees d_1, d_2 and degree relation g:
  a. If x *is more* g *than* $d_1 \Rightarrow x$ *is more* g *than* d_2,
  b. then x *is no more* g *than* $d_2 \Rightarrow x$ *is no more* g *than* d_1.

If this logical property of NO enables it to license NPIs, then certainly the pattern that we see in the preceding examples is exactly what we would expect.

Sentences like the following have also sometimes been presented as evidence that comparatives license NPIs:

(104) a. Max is taller than any of his students are.
  b. Dr. Jones treated more patients than any of his colleagues did.

If the *any* phrases in these examples are in fact NPIs, then it must be the case that NO takes scope over the subject, otherwise the NPIs would fail to be licensed. This would derive the truth conditions in (105) for (104a), which (assuming NPIs are indefinites) is the $>_{max}$ interpretation.

(105) $\exists d[\{d' \mid \textbf{tall}(\mathbf{m}, d + d')\} \neq \emptyset \wedge \{d' \mid \exists x[\textbf{student-of}(x, \mathbf{m}) \wedge \textbf{tall}(x, d + d')]\} = \emptyset]$

Unfortunately, there is a problem for this analysis. Overt NO does not license an *any*-phrase in subject position:

(106) a. * Any of these students are no taller than Max.
  b. * Any of the doctors treated no more than five patients.

Fortunately, there is an alternative account of the facts, which is that the *any*-phrases in examples like (104a-b) are actually free choice items (FCIs) with universal force, and not NPIs at all (Giannakidou and Yoon 2010).[19] Assuming that universal FCIs are like other universal quantifiers in blocking wide scope for NO, this hypothesis also ensures a $>_{max}$ interpretation, since (104a) will have the truth conditions in (107).

(107) $\exists d[\{d' \mid \textbf{tall}(\mathbf{m}, d + d')\} \neq \emptyset \wedge \forall x[\textbf{student-of}(x, \mathbf{m}) \rightarrow$
$\{d' \mid \textbf{tall}(x, d + d')\} = \emptyset]]$

There are several pieces of evidence that argue in favor of an analysis of these *any*-phrases as FCIs. First, as Heim (2006a) and Giannakidou and Yoon (2010) point out, these *any*-phrases can be modified by *almost*, but NPI-*any* cannot be so modified:

[19] For similar reasons, we will probably want to adopt a free-choice analysis of the "taller than both" reading of disjunctions in examples like (i).

(i) Max is taller than Lee or Hank is.

(108) a. Max is taller than almost any of his students are.
b. If (*almost) any of his students are as tall as he is, Max will surprised.

Second, although the examples in (106) are unacceptable, they become fully acceptable (and accept modification by *almost*) when we add an expression that licenses a free choice interpretation, such as an epistemic modal:[20]

(109) a. (Almost) any of these students could be no taller than Max.
b. (Almost) any of these doctors could have treated no more than five patients.

Finally, as Heim (2006a) points out (see also Schwarzschild and Wilkinson 2002), the subject position in a comparative clause is an upward-entailing context, and so is not the type of environment that licenses NPIs in the first place:

(110) a. Frank is taller than some professional athletes are. $\not\rightarrow \leftarrow$
b. Frank is taller than some professional basketball players are.

All of these properties are expected in our analysis: given that downward monotonicity is introduced by NO, environments that are outside its scope — such as subject position when filled by a quantificational expression — are not expected to be downward monotonic. Instead, given the existential semantics for *than*, such contexts should (all other things being equal) be upward monotonic. In this respect, our analysis is similar to Heim's own account, in which downward monotonicity is also introduced by an operator that can take scope at different positions in the comparative clause. (Though as we point out in section 5, Heim's "Π" operator is not as successful as our NO in capturing the pattern of readings that we see in comparatives.)

4.4 Remarks on differentials

We conclude this section by taking a closer look at what our analysis has to say about examples involve different kinds of overt differential expressions. We begin with the overt version of NO:

(111) Sarah is no taller than Frank is.

In principle, overt NO can take scope in two places: either below or above *than*, deriving (112a) and (112b), respectively.

(112) a. $\exists d[\{d' \mid \textbf{tall}(\textbf{s}, d + d')\} = \emptyset \wedge \{d'' \mid \textbf{tall}(\textbf{f}, d + d'')\} = \emptyset]$

[20]Note in particular that NO must take scope under the epistemic modals in these examples, as expected given what we saw in section 4.3.1, so it cannot be the case that NO is licensing *any* here.

b. $\{d' \mid \exists d[\textbf{tall}(\textbf{s}, d + d') \wedge \{d'' \mid \textbf{tall}(\textbf{f}, d + d'')\} = \emptyset]\} = \emptyset$

(112b) represents the correct truth conditions for (111): the set of degrees that correspond to positive differences between Sarah's height and Frank's height is empty. (112a), on the other hand, is tautologous, because the existential is satisfied by any degree that exceeds both of Sarah's and Frank's heights, regardless of the actual ordering between them. This Logical Form can therefore be ruled out by the same principles that forbid the co-occurrence of sentential negation with the silent occurrence of NO in the comparative clause. (See the discussion of this point in section 4.3.3.) The result is that overt *no* must take scope over *than*.

We now turn to an interesting fact discussed by Beck (2010). Beck observes that while (113a) does not imply anything about the heights of Max's students (other than that the tallest of them is shorter than Max), (113b) entails not only that there is a 2cm difference between Max's height and the height of the tallest student, but also that every student has the same height, an inference which Beck refers to as the "assumption of equality."

(113) a. Max is taller than every one of his students is.
b. Max is exactly 2 cm taller than every one of his students is.

At first this looks like a problem for us. If differentials are singular terms which saturate the differential argument in the matrix (as we assumed in section 3), we get the truth conditions in (114) (cf. (46)), which merely requires the tallest student to have a height at least two centimeters below Max's height.

(114) $\exists d[\textbf{tall}(\textbf{m}, d + \textbf{2cm}) \wedge \forall x[\textbf{student-of}(x, \textbf{m}) \rightarrow \{d' \mid \textbf{tall}(x, d + d')\} = \emptyset]]$

However, as Beck points out, it is also possible to treat the measure phrase as a generalized quantifier over degrees, with the denotation in (115) (cf. von Stechow 1984a; Kennedy 2013).

(115) $[\![\text{exactly 2 cm}]\!] = \lambda P_{\langle d,t\rangle}.max(P) = \textbf{2cm}$

At first, this doesn't appear to help very much. If *exactly 2 cm* has the quantificational meaning in (115), it should be able to take scope either below or above *than*, just like overt *no*, giving rise to the two interpretations shown in (116a-b),

(116) a. $\exists d[max\{d' \mid \textbf{tall}(\textbf{m}, d+d')\} = \textbf{2cm} \wedge \forall x[\textbf{student-of}(x, \textbf{m}) \rightarrow \{d' \mid \textbf{tall}(x, d+d')\} = \emptyset]]$
b. $max\{d' \mid \exists d[\textbf{tall}(\textbf{m}, d+d') \wedge \forall x[\textbf{student-of}(x, \textbf{m}) \rightarrow \{d' \mid \textbf{tall}(x, d+d')\} = \emptyset]]\} = \textbf{2cm}$

But careful scrutiny shows that these two denotations are logically equivalent, and differ from (114) only in requiring the height of the tallest student to be exactly (and not merely at least) two centimeters below Max's height; neither gives rise to the desired inference that all of the students have equal height.

However, we believe that there is a way to derive the result we want. First, assume that whenever a differential degree quantifier in the matrix introduces a maximality requirement over its scope, NO must introduce a corresponding maximality requirement over its scope in the comparative clause. Second, assume (quite naturally) that max introduces an existence presupposition over its domain. Such a presupposition would appear to be incompatible with the basic meaning of NO, but we would like to suggest that both the existence presupposition of max and the core meaning of NO as a negative degree quantifier can be combined by coercing the usual denotation of NO into the one in (117).

(117) $[\![\text{NO}_{max}]\!] = \lambda P_{\langle d,t\rangle}.max(P) = \mathbf{0}$

Here **0** is the zero degree, which represents no amount of the property in question. Such degrees are needed independently to handle "minimum standard absolute" adjectives like *bent, open, exposed, impure*, and so forth (Kennedy 2007b), and are in fact formally equivalent to the empty set if degrees are modeled as intervals (i.e., as convex sets of points; see Seuren 1978; von Stechow 1984b; Kennedy 2001).

With these assumptions in hand, the interpretation of (113b) is as shown in (118).

(118) $\exists d[max\{d' \mid \textbf{tall}(\mathbf{m}, d + d')\} = \textbf{2cm} \wedge \forall x[\textbf{student-of}(x, \mathbf{m}) \rightarrow max\{d' \mid \textbf{tall}(x, d + d')\} = \mathbf{0}]]$

This is true just in case there is a degree d such that the difference between d and Max's height is exactly two centimeters, and for every student x, the difference between d and x's height is zero. These truth conditions are satisfied only if all of the students have the same height, deriving the assumption of equality.

Of course, a crucial question for us is *why* the introduction of maximization in the matrix by the differential phrase entails a corresponding introduction of maximization in the comparative clause, and the viability of our account of the assumption of equality effect will ultimately depend on how well we answer it. Providing a full answer to this question would take us too far away from the main focus of this paper, so we will not attempt to do so, but we will instead point to two additional facts which suggest that the core ideas we have articulated here are on the right track.[21]

[21] We are very grateful to an anonymous reviewer whose comments on differentials led us to a full appreciation of the importance of the following facts.

The first is the simple observation that (119) has only an "exactly" interpretation: it is false if Sarah is more than two centimeters taller than Frank.

(119) Sarah is exactly 2 cm taller than Frank is.

This fact is important because, given everything else we have said, merely assuming the quantificational denotation for *exactly 2 cm* in (115) is not sufficient to derive it, thanks to the scopal interaction between the differential and the *than*-clause. When the differential scopes over *than*, as in (120a), we get the correct truth conditions for (119), but when the differential scopes below *than*, as in (120b), we get incorrect truth conditions: the existential is satisfied here as long as Frank is shorter than Kim's height minus two centimeters.

(120) a. $max\{d' \mid \exists d[\textbf{tall}(\textbf{s}, d + d') \wedge \{d' \mid \textbf{tall}(\textbf{f}, d + d')\} = \emptyset]]\} = \textbf{2cm}$
b. $\exists d[max\{d' \mid \textbf{tall}(\textbf{s}, d + d')\} = \textbf{2cm} \wedge \{d' \mid \textbf{tall}(\textbf{f}, d + d')\} = \emptyset]]$

Here we cannot appeal to the kinds of blocking principles that rule out the narrow scope interpretation of *no* in (112b): the truth conditions in (120b) are fully informative; they are just wrong as a characterization of the meaning of (119). We could stipulate that quantificational differentials must take wide scope relative to *than*, but this would obviously not be a particularly explanatory move.

However, if the analysis of assumption-of-equality effects that we gave above is correct, then we can account for the actual meaning of (119) without stipulating anything about the relative scope of the differential and *than*. If the use of a differential with maximality semantics in the matrix clause requires corresponding use of NO_{max} in the comparative clause, then the truth conditions that we derive for the two scopal relations between *exactly two cm* and *than* in (119) are not the ones shown in (120a-b), but rather the ones shown in (121a-b).

(121) a. $max\{d' \mid \exists d[\textbf{tall}(\textbf{s}, d + d') \wedge max\{d' \mid \textbf{tall}(\textbf{f}, d + d')\} = \textbf{0}]]\} = \textbf{2cm}$
b. $\exists d[max\{d' \mid \textbf{tall}(\textbf{s}, d + d')\} = \textbf{2cm} \wedge max\{d' \mid \textbf{tall}(\textbf{f}, d + d')\} = \textbf{0}]]$

(121a-b) are logically equivalent, just as (116a-b) are, and require that the difference in height between Sarah and Frank be exactly two centimeters. This is exactly what we want.

The second fact which suggests that our approach is on the right track is the following: indefinite or "vague" differentials such as those in (122) do not introduce an assumption of equality inference: the *much* version of (122), for example, entails that the difference between Max's height and the height of any of his students is large, but it perfectly consistent with the students having different heights.[22]

[22]Differential *no* also fails to show an assumption of equality effect:

(122) Max is *a bit/much/a lot* taller than every one of his students is.

We are not sure what the best analysis of these differentials is, but there is good reason to think that they do not have a maximality semantics. Unlike *exactly*-differentials, these expressions are consistent with the difference between target and standard of comparison being greater than the amount named by the differential:

(123) a. # Max is exactly 2cm taller than Lee, if not more than 2 cm taller.
b. Max is a bit taller than Lee, if not a lot taller.
c. Max is much taller than Lee, if not very much taller.
d. Max is a lot taller than Lee, if not a truly tremendous amount taller.

(123a) is odd because the maximality part of the meaning of *exactly 2cm* entails that the difference between Max's height and Lee's height is no more (and no less) than two centimeters, but the *if not* continuation indicates that the difference might be greater. The fact that (123b-d) are fine therefore indicates that they do not introduce maximality semantics in the matrix.

Turning back to (122), if the assumption of equality inference arises when NO is coerced into NO_{max} in the comparative clause, as we claimed above, and if this happens if *and only if* the matrix differential introduces a corresponding maximality semantics, then the fact that this example does not involve an assumption of equality follows. Indefinite differentials like *a bit*, etc. do not introduce maximality, so the comparative clause in in (122) contains NO rather than NO_{max}, and we end up with a "regular" $>_{max}$ interpretation for the universal quantifier.

There is, for sure, more to say — both about "maximality parallelism" and about the difference between NO and NO_{max} (in particular, about the potential consequences of introducing a zero degree) — but this will have to be done in a different paper.

5 Comparison to previous approaches

Having completed the exposition of our own approach to the problems outlined at the outset of this paper, we would like to briefly situate our analysis amongst others that have been proposed. Early approaches to these problems systematically undergenerated, as they failed to account for entire classes of readings. We have already observed in section 2 that the classical formulations of both the A/¬A analysis and the maximality-based analysis suffer from the same empirical limitations: each is

(i) Max is no taller than any one of his students is.

But this is arguably because the *any* in this example is (existential) NPI *any*, rather than (universal) free-choice *any*, as shown by the fact that it cannot felicitously be modified by *almost*.

only able to derive the $>_{min}$ reading for universal quantifiers in the comparative clause, and the $>_{max}$ reading for existentials. This is because in both analyses, the negation scopes over any quantifier in the comparative clause. (Recall that under the A/¬A analysis, negation is contributed by the matrix-level comparative morpheme, while under the maximality-based analysis, negation is contributed by the maximality operator.) Larson (1988) recasts the A/¬A analysis in a way that uniformly gives such quantifiers scope over the matrix-level comparative AP, and thus, over the negation contributed by MORE. This is achieved not by scoping the quantifier itself, a move which Larson shows to be problematic for a variety of reasons, but rather by assigning scope over the comparative AP to the entire comparative clause. As a result, Larson's version of the A/¬A analysis exhibits exactly the opposite limitation: only $>_{max}$ readings for universals and $>_{min}$ readings for existentials are predicted to be possible. Working closer to the maximality-based tradition, Schwarzschild and Wilkinson (2002) observe further reasons for why scoping the quantifier out of the comparative clause is undesirable. These authors pursue a strategy that at least initially appears to be quite different from Larson's (depending instead on an interval semantics for gradable predicates), but turns out to yield identical predictions regarding possible readings for universal and existential quantifiers.

Heim (2006b), who observes that Schwarzschild and Wilkinson's analysis may be more similar to Larson's than initial appearances suggest, develops a hybrid solution to these problems. For a given quantifier, her analysis is able to derive both the logical representation delivered by the classical accounts, as well as the one delivered by the Larsonian/Schwarzschild and Wilkinson accounts. This is achieved by postulating a silent, variable-scope operator within the comparative clause; the precise scope of this operator in turn determines how much of the material in the comparative clause will come to scope over the matrix-level comparative AP. Although Heim's paper is firmly rooted in the maximality-based tradition, Büring (2007) shows that her assumptions are equally compatible with an A/¬A-style approach. The resulting analysis, however, now suffers from overgeneration, as it predicts that both the $>_{max}$ and the $>_{min}$ reading should be observable with any universal or existential quantifier. Furthermore, because the analysis allows negation and negative quantifiers in the comparative clause to scope over the matrix-level comparative AP, the unacceptability of such examples goes unexplained: a sentence like **Sarah is taller than Frank isn't* is incorrectly predicted to be grammatical, with the reading 'Sarah's height does not exceed Frank's height.' The solution, of course, is to restrict the scope options available to the operator within the comparative clause. However, it is not clear that this can be done in a non-*ad hoc* manner, given that the proposed operator is specific to comparatives (and perhaps, other degree constructions).

Most recently, the problem of quantifiers in the comparative clause has been

addressed by Schwarzschild (2008), Gajewski (2008), van Rooij (2008), Krasikova (2008), Beck (2010) and Matushansky (2011). We discussed problems for van Rooij's approach in section 2.2, and also provided a thorough review there of Schwarzschild's revised A/¬A analysis, pointing out that the analysis undergenerates for certain modals and overgenerates for certain quantificational DPs and adverbs. As we have noted in several places, Gajewski's analysis is arguably the most similar to our own, but its predictions about the scope parallism between overt differential *no* and scopal interactions in the comparative clause are unclear (see note 9) and it does not support a ready account of differential phrases (as Gajewski acknowledges). In unpublished work, Matushansky (2011) develops a variant of Gajewski's analysis that can accommodate differentials, but the result is a theory in which comparatives with and without differentials receive quite distinct compositional analyses. In contrast, our approach provides a uniform treatment of comparatives with and without differentials.

Building on ideas in Krasikova 2008, Beck (2010) shows how a fairly simple semantics for the comparative construction, in which the comparative morpheme introduces an ordering and the *than*-clause introduces an interval, can be augmented with a selection principle for choosing the "right" interval from the *than*-clause denotation (in a principled way) to account for the core facts involving universal quantifiers and the *supposed to*-class of modals. Beck then shows how additional principles governing exhaustification, the interpretation of indefinites, and the resolution of imprecision and granularity can be brought to bear on the rest of the facts (*need to*-modals, indefinites and non-monotonic quantifiers, the effects of differentials, etc.). The result is a rich and complex analysis, and so providing a full comparison of our approach and hers would dramatically extend the length of a paper that is probably already longer than it should be (though not longer than it needs to be). We believe that it is at least a theoretical (and potentially an empirical) advantage of our analysis that we are able to account for the full range of data entirely in terms of our basic semantics of the English comparative construction, though it is indeed possible that we, too, will need to invoke some extra machinery as well. (Indeed, we already saw this to a certain extent in our account of the "assumption of equality" inference with differentials and embedded universal quantifiers.) Moreover, our account straightforwardly explains why we see exactly the same pattern of interpretation in the matrix clause interpretation of the overt differential *no* that we see inside the comparative clause: this just reflects the interpretive and distributional properties of NO. This is a clear advantage of our analysis, and one that we will capitalize on again below in section 7, where we show how our account provides a new way of understanding "split scope" interpretations of negative DPs.

6 Other kinds of comparative constructions

The analysis of comparatives presented in section 3 provides a fully compositional semantics for comparative deletion and comparative ellipsis constructions: comparatives in which the complement of *than* is a (possibly reduced) clause, which contains an underlying (and deleted) occurrence of a predicate which is identical to the matrix comparative predicate. But there are at least three other types of comparative constructions in English that need a different kind of syntactic analysis, which raises the question of whether they also need a different kind of semantic analysis: subcomparatives, phrasal comparatives, and measure phrase comparatives. We think that the answer is no, and will demonstrate in this section how our analysis can be extended to deal with them.

6.1 Subcomparatives

Let us begin with subcomparatives, which are illustrated in (124).

(124) a. Sarah is taller than the hole is deep.
b. The shapes are longer than they are wide.

In most analyses, examples like these are unsurprising: they are simply overt versions of the underlying structure posited for all clausal comparatives, in which the degree argument of the embedded adjective is bound by a wh-operator, and the entire comparative clause then denotes a property of degrees that is interpreted in whatever way is dictated by the rest of the compositional analysis (e.g., as described for the A/¬A and maximality-based analyses in section 2 or for more recent approaches in section 5.)

The central problem presented by subcomparatives for our analysis is that the form of the embedded predicate is not what we expect: since NO targets the differential degree argument of a comparative adjective, we would expect to see examples like (125a-b), which are quite clearly ungrammatical.

(125) a. * Sarah is taller than the hole is deeper.
b. * The shapes are longer than they are wider.

Alrenga et al. (2012) argue that examples like these are ungrammatical because comparative morphology bears a PF-uninterpretable feature. The matrix occurrence of the comparative morpheme is able to delete this feature in agreement with *than*, but there is no similar licenser of the feature in the embedded predicate. In the case of comparative deletion structures, such uninterpretability is not a problem, because the comparative morphology is part of the constituent that is ultimately deleted,

and so cannot trigger a PF violation (cf. Kennedy and Merchant 2000; Merchant 2001). Among other things, this proposal has the empirical advantage of deriving the fact that deletion in comparative deletion constructions is obligatory, a fact that is mysterious on most accounts.

Alrenga et al. then propose that well-formed subcomparatives like the ones in (124) must be formed using a construction-specific, phonologically null degree morpheme. While this solution appears on the surface to be *ad hoc*, it also has an important empirical advantage over the standard account, in which all clausal comparatives are derived from the same underlying structure. The fact is that true subcomparatives — examples in which non-norm related (or non-evaluative, in the sense of Rett 2008) comparisons of two objects along different dimensions are achieved via structures that are parallel to comparative deletion except that they include an overt occurrence of a gradable predicate in the comparative clause — are exceedingly rare in the world's languages (Beck, Krasikova, Fleischer, Gergel, Hofstetter, Savelsberg, Vanderelst, and Villalta 2009).[23] On the standard view, in which all clausal comparatives have the same underlying structure (in particular, structures that are isomorphic to the examples in (124)), this fact is a mystery. In contrast, if subcomparatives require the presence of a special functional morpheme, we can explain (or at least model) the variation as an instance of language-specific differences in functional morphology inventories. Moreover, as long as this morpheme introduces the same semantic content that we find in comparative deletion, the compositional analysis of the examples in (124) will be completely parallel to the other clausal comparatives we have analyzed in this paper.

Specifically, we will propose a denotation for the subcomparative-forming degree expression that takes an observation made by Snyder, Wexler, and Das (1994) and Beck et al. (2009) about the cross-linguistic distribution of subcomparatives as its starting point: if a language has subcomparatives, then it also allows noncomparative adjectives to combine directly with measure phrases (in collocations like *two meters tall, ten years old*). This generalization suggests that the denotation for the subcomparative-forming morpheme should be that of a measure phrase; a minor modification to the denotation that we have already posited for NO

[23] Note, in particular, that amount comparatives like (ia), which are attested in languages that lack adjectival subcomparatives (such as Japanese), are not true subcomparatives, despite the fact that they are usually described as such.

(i) a. Sarah has published more articles than she has published books.
b. * Sarah has published more articles than she has published many/few books.

This is because the gradable predicate — the cardinality predicate **many** — is, in fact, (obligatorily) deleted. Leaving it overt, as in (ib), which is the true mark of a subcomparative, results in ungrammaticality.

gives us exactly what we need:

(126) $[\![\text{NO}_{sub}]\!] = \lambda d \lambda P_{\langle d,t\rangle}.\{d' \mid P(d + d')\} = \emptyset$

NO_{sub} is, in effect, a lexicalization of the *no-more-than* part of a modified numeral or measure phrase like *no more than two (meters)*: it introduces the same semantic content into a subcomparative that is provided separately by the comparative morpheme and NO in comparative deletion structures. In particular, it combines with a "standard" argument that provides the target for the property-generating wh-operator in comparatives, and gives back a negative degree quantifier that should be subject to the same distributional constraints and give rise to the same interpretive possibilities that we have already seen with (regular) NO.[24]

Finally, the fact that NO_{sub} is semantically a type of (quantificational) measure phrase provides us with a basis for explaining the relation between measure phrase constructions and subcomparatives observed by Snyder et al. and Beck et al. Working through the details of this account would go well beyond the scope of this paper, but the basic idea is that noncomparative adjectives project a position for a measure phrase only on a language specific and idiosyncratic basis. (See Murphy 1997; Schwarzschild 2005; Svenonius and Kennedy 2006; Grano and Kennedy 2012 for specific proposals along these lines.) If this is not an option in a particular language, or is not an option for a particular adjective, then subcomparatives should be ruled out. For this reason, too, we want to reject the (otherwise attractive) hypothesis that NO_{sub} is active in all clausal comparatives — both subcomparatives and comparative deletion structures. As the reader can verify, the resulting meanings and truth conditions would be the same as what we derived earlier for comparative deletion by assuming that (regular) NO combines with a deleted occurrence of the comparative predicate. The problem with this hypothesis is that we would then be forced to say that NO_{sub} can combine with any gradable predicate, not just those that project measure phrases, and so would lose an account of the limited distribution of subcomparatives and the Snyder et al./Beck et al. observations.[25]

6.2 Phrasal comparatives and measure phrase comparatives

[24]Type-wise, NO_{sub} is similar to Heim's (2006b) Π operator, and semantically it is quite similar to Gajewski's (2008) "degree negation" operator. The crucial difference with the latter is that it effectively introduces negation just over differences, and so interacts with the rest of our system in the way that we need it to.

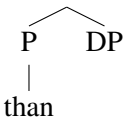
[25]For similar reasons, we want to resist another possible account of the ungrammaticality of examples like (125a-b), which says that the embedded adjective in a subcomparative is as our theory predicts, except that the comparative morpheme must be silent (perhaps to avoid the uninterpretability problems that Alrenga et al. 2012 hypothesize). This would certainly account for the surface form of subcomparatives, but it would give us no traction on the problem of their limited distribution.

Phrasal comparatives are comparatives in which the complement of *than* is syntactically a DP and semantically denotes an individual:

(127) a. Sarah is taller than me.
b. Who is Sarah taller than?
c. No one is taller than himself.

It has sometimes been argued that all English comparatives are underlyingly clausal (Lechner 2001; Bhatt and Takahashi 2011), but such an analysis has a difficult time explaining why the standard marker in (127a) is in accusative case, why extraction of the standard is possible in (127b), and why the standard can be an accusative reflexive pronoun bound by the subject in (127c). On the other hand, these facts are exactly what we would expect to find if the syntactic structure of the *than*-phrase in these examples is just that of a PP, as in (128) (Hankamer 1973; Merchant 2009) [26]

(128)



But if the structure of the *than*-constituents in (127) is as shown in (128), then clearly the semantic analysis we developed in section 3 is not applicable to these examples. How, then, can we derive their truth conditions?

The answer is that we need two *than*s in English, as originally argued by Hankamer (1973). The first, which we will write as $than_C$, is the one that appears in clausal comparatives, combines with two degree properties, and has the meaning we gave it in section 3:

(129) $[\![than_C]\!] = \lambda P_{\langle d,t\rangle}\lambda Q_{\langle d,t\rangle}.\exists d[Q(d) \wedge P(d)]$

The second, which we will call $than_P$, appears in phrasal comparatives and combines with two individuals and a comparative degree relation (i.e., a degree relation

[26] Of course, Lechner (2001), Bhatt and Takahashi (2011) and other proponents of a uniform clausal analysis of comparatives such as Pancheva (2006) are well aware of facts like those in (127), and have proposed mechanisms for dealing with them. Our intent in this section is not to justify the claim that these kinds of examples must have a phrasal analysis, but simply to show that, if they do, we can fit them in within our overall approach to the semantics of English comparatives. (If comparatives are uniformly clausal, then there is nothing for us to show.) We leave it to interested readers to decide for themselves whether the full range of facts is better explained by a syntactic analysis of English that allows for both phrasal and clausal comparatives, or by one that allows only clausal comparatives plus other mechanisms to accommodate data like (127a-c).

with an extra, differential degree argument). The denotation that we need for *than*$_P$, which can be derived from the denotation of *than*$_C$, is shown in (130).

(130) $[\![than_P]\!]$ =
$\lambda x \lambda g_{\langle d,\langle d,\langle e,t\rangle\rangle\rangle} \lambda d \lambda y. \exists d'[g(y,d',d) \wedge \{d'' \mid g(x,d',d'')\} = \emptyset]$ =
$\lambda x \lambda g_{\langle d,\langle d,\langle e,t\rangle\rangle\rangle} \lambda d \lambda y. [\![than_C]\!](\lambda d'.\{d'' \mid g(x,d',d'')\} = \emptyset)(\lambda d'.g(y,d',d))$

In essence, *than*$_P$ incorporates the negation over the differential argument that is normally supplied by NO. Note that, as a result, a matrix differential in a phrasal comparative must take scope over the *than*-phrase, which ensures that a differential is not incorrectly "used twice" in the composition of a phrasal comparative. (We are grateful to an anonymous reviewer for pointing this out as a problem for an earlier formulation of *than*$_P$.)

The correct truth conditions for examples like those in (127) are derived by interpreting the *than*-phrase below the matrix differential and below the target of comparison. In the case of a simple example like (127a), this basically just means interpreting the *than*-phrase in situ, as in (131a); binding the differential position with SOME and adding in the external argument gives us (131b), which correctly captures the truth conditions for (127a).

(131) a. $[\![than_P]\!]([\![me]\!])([\![taller]\!])$ =
$\lambda d \lambda x. \exists d'[\textbf{tall}(x, d' + d) \wedge \{d'' \mid \textbf{tall}(\textbf{me}, d' + d'')\} = \emptyset]$

b. $\{d \mid \exists d'[\textbf{tall}(\textbf{sarah}, d' + d) \wedge \{d'' \mid \textbf{tall}(\textbf{me}, d' + d'')\} = \emptyset]\} \neq \emptyset$

In more complex examples (e.g., in phrasal amount comparatives, or when the target is a non-subject), this will require making use of the "parasitic scope" relation (Sauerland 1998; Barker 2007); see Kennedy 2007a; Kennedy and Stanley 2009; Bhatt and Takahashi 2011 for discussion.

Measure phrase comparatives are syntactically similar to phrasal comparatives in that the complement of *than* is a DP, in particular a DP measure phrase:

(132) a. Sarah is taller than five feet.

b. Lee drove faster than 65 mph.

The difference is a semantic one: measure phrases denote degrees (or generalized quantifiers over degrees) rather than individuals. However, our approach to these is essentially the same as our approach to regular phrasal comparatives: we just need to add a third *than* — *than*$_M$ — which selects for a standard of type d. The denotation we want is the one in (133), which again can be derived from the denotation of *than*$_C$.

(133) $[\![than_M]\!] = \lambda m \lambda g_{\langle d,\langle d,\langle e,t\rangle\rangle\rangle} \lambda d \lambda x. \exists d'[g(x,d',d) \wedge d' = m]$
$= \lambda m \lambda g_{\langle d,\langle d,\langle e,t\rangle\rangle\rangle} \lambda d \lambda x. [\![than_C]\!](\lambda d'.d' = m)(\lambda d'.g(x,d',d))$

A general advantage of this type of approach to phrasal and measure phrase comparatives is that we locate the semantic differences between the various constructions in the semantics of the standard morphology. Unlike approaches to the phrasal/clausal distinction which posit multiple denotations for the comparative morphology (Heim 1985; Bhatt and Takahashi 2011), the kind of analysis we have outlined here provides a much more natural way of explaining the robust cross-linguistic generalization that whenever a language marks a distinction between phrasal and clausal comparatives, it does so in the standard morphology, never in the comparative morphology (Pancheva 2006; Merchant 2009).

7 Beyond comparatives: "Split-scope" readings of negative DPs

Finally, we would like to show how our analysis can provide a new way of thinking about "split-scope" readings of negative DPs, which have been extensively studied in Dutch and German (see e.g. Jacobs 1980; Kratzer 1995; Geurts 1996; de Swart 2000; Zeijlstra and Penka 2005; Abels and Martí 2010; Penka 2011), and to a somewhat lesser extent in English (see Potts 2000; von Fintel and Iatridou 2007; Iatridou and Sichel 2011). Examples are shown in (134) and (135), and the readings we are interested in are the ones given in the paraphrases.

(134) The company is required to fire no employees.
  a. It is necessary that there are no employees that the company fires.
  b. It is not necessary that there are some employees that the company fires.

(135) The company is allowed to fire no employees.
  a. It is permitted that there are no employees that the company fires.
  b. It is not permitted that there are some employees that the company fires.

The (a) paraphrases are standard *de dicto* readings which can be derived by treating *no employees* as a generalized quantifier and assigning it scope below the modal. The "split-scope" readings are the ones in the (b) paraphrases, and are so called because the descriptive content and existential force of *no employees* appears to be in the scope of the modal, but the negative force is clearly scoping over the modal.

What is particularly interesting for us is that the class of modals that allow for split-scope interpretations is exactly the class of modals that generate ambiguities in comparatives: replacing *required to* with *supposed to* and *allowed to* with *might* derives sentences that do not have the split-scope interpretations:

(136) The company is supposed to fire no employees.
  a. It is necessary that there are no employees that the company fires.
  b. * It is not necessary that there are some employees that the company fires.

(137) The company might fire no employees.
  a. It is possible that there are no employees that the company fires.
  b. * It is not possible that there are some employees that the company fires.

Furthermore, Potts 2000 and von Fintel and Iatridou (2007) observe that universal DPs also do not participate in split-scope interpretations:[27]

(138) Every company hired no employees.
  a. Every company is such that there are no employees that it hired.
  b. * Not every company is such that there are employees that it hired.

There are numerous accounts of split-scope in the literature, and we do not have the space here to review them all. Instead, we simply want to point out that our analysis of the interpretation of modals and quantifiers in comparatives provides the basis for implementing a conjecture first articulated (as far as we know) by Irene Heim: scope splitting is DegP movement (Heim 2001, p. 225). This view is also considered by Penka (2012, p. 529), who rejects it on the basis that the negative indefinite *no* in *no employees* does not cooccur with degree morphology, in contrast to e.g. a comparative numeral like *more than three*. But our hypothesis here is that the *no* in *no employees* is itself the degree morpheme, and not an indefinite determiner at all. In particular, it is (or at least can be) the very same degree morpheme NO that we find in English comparatives, which, in collocations like *no employees*, binds the very same quantity position inside the nominal projection that is bound by numerals and amount comparatives.

On this view, a simple example like (139) has the logical form in (139a), and the truth conditions in (139b). (Here we assume as earlier that nominals contain a cardinality predicate **many** and that their individual argument is bound by existential closure.)

(139) The company hired no employees.

[27] Universal DPs in German do appear to participate in split-scope readings; (i) is a canonical example:

(i) Jeder Arzt hat kein Auto.
every doctor has no car
'Not every doctor has a car.'

However, Abels and Martí (2010) argue that this constitutes a distinct phenomenon from split-scope readings arising with modals and other intensional verbs. Whereas the latter do not depend on any particular intonational contour, split-scope readings with universal DPs emerge only under the so-called "hat contour". Furthermore, even with the appropriate intonation, the split-scope reading for (i) is not available for some German speakers, while it appears that all German speakers allow for split-scope readings with modals and intensional verbs.

a. [NO$_1$ the company hired [t_1 **many** employees]]
b. $\{n \mid \exists x[\textbf{hire}(\textbf{c}, x) \wedge \textbf{employee}(x) \wedge \#(x) = n]\} = \emptyset$

The ambiguity of (134) can then be accounted for in exactly the same way that we saw with comparatives: in terms of the relative scope of NO and the modal. (140) is the regular *de dicto* reading, and (141) shows the logical form and truth conditions for the split-scope reading.

(140) a. [needs [NO$_1$ the company to fire [t_1 **many** employees]]]
b. $\forall w[ACC_{w0}(w) \rightarrow \{n \mid \exists x[\textbf{hire}(\textbf{c}, x, w) \wedge \textbf{employee}(x, w) \wedge \#(x, w) = n]\} = \emptyset]$

(141) a. [NO$_1$ needs [the company to fire [t_1 **many** employees]]]
b. $\{n \mid \forall w[ACC_{w0}(w) \rightarrow \exists x[\textbf{hire}(\textbf{c}, x, w) \wedge \textbf{employee}(x, w) \wedge \#(x, w) = n]]\} = \emptyset$

Obviously there is much more to say here, but we think that this is a promising avenue for future exploration. In particular, if the approach outlined here is correct, then we will succeed in explaining what was previously thought to be two distinct phenomena (the scopal properties of comparatives and split-scope readings for negative DPs) in terms of a single, uniform semantics for negative degree quantifiers.

8 Conclusion

We hope to have achieved the following two goals with this paper. First, we have presented and motivated our own solution to the vexing problem posed by quantifiers occurring within the comparative clause. Our proposed solution follows one line of recent approaches to this problem (exemplified by Heim 2006b, Schwarzschild 2008 and Gajewski 2008), which traces the readings triggered by different quantifiers to their scopal interactions with a silent, comparative clause-internal operator. Crucially, our proposal departs from this previous work over the exact nature of this operator, which we have identified as the negative degree quantifier NO. Because we further claim that this covert operator is simply a silent allomorph of the overt degree quantifier that appears in comparatives like *no taller than...*, and perhaps also in DPs like *no employees*, we predict that the attested interpretation(s) for a given quantifier in the comparative clause should match its interpretive properties relative to overt *no*. For all of the quantifiers that we have investigated, this prediction is indeed borne out. We thus believe that our solution constitutes an advancement over previous scope-based approaches, which are afflicted by systematic over-/undergeneration problems.

Our second goal has been to show that our operator/scope-based analysis of the interpretation of quantifiers inside the comparative clause can be embedded in a general and independently motivated syntax and semantics for comparative constructions. Here we depart from most previous work in two ways. First, we posit a semantically contentful occurrence of the comparative morpheme inside the comparative clause. (This assumption is also made by Marques (2003), though in the context of an analysis that involves sentential negation, and so suffers from the problems we identified for such accounts in section 2.2.) Second, we analyze the standard marker *than* as a degree determiner, which combines with the comparative clause to form a generalized quantifier over degrees. Our treatment of the *than*-phrase thus resembles Heim's (2006b) proposal that the comparative clause consitutes a degree quantifier, although Heim maintains the common assumption that *than* is semantically vacuous. In contrast, our proposal provides both semantic and syntactic content to *than*, a move that has important and positive consequences for our understanding of both intra- and cross-linguistic variation in the expression of comparison, which are explored in more detail in Alrenga et al. 2012.

Stepping back a bit further, our treatment of NO as a negative degree quantifier effectively turns the problem posed by quantifiers in the comparative clause into one particular instantiation of a larger problem, namely how to account for the scopal interactions of degree quantifiers with modals and quantificational DPs. Most of the previous work that addresses this question has been concerned primarily with how these interactions play out at the matrix level, e.g. in sentences like *The paper is required to be exactly 5 pages long / longer than that* (see e.g. Heim 2000, 2001; Takahashi 2006; Bhatt and Pancheva 2004). A consequence of our proposed syntax and compositional semantics for the comparative is that the comparative clause becomes another grammatical environment in which such interactions can be observed. And if our (admittedly preliminary) suggestion in section 7 regarding a degree-quantifier treatment of the *no* that appears in negative DPs is on the right track, then the "split scope" reading that is sometimes observed with such DPs can be viewed as still another instantiation of the same problem. Our overall approach thus leads to the attractive possibility that these seemingly disparate sets of facts will ultimately submit to a single explanation. At the same time, of course, it makes the need for an answer to the following question that much more pressing: what exactly *does* account for the restrictions governing a degree quantifier's scope?

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