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Bare Numerals and Scalar Implicatures

Benjamin Spector*
CNRS-ENS-EHESS, Institut Jean Nicod, École Normale Supérieure

Abstract

Bare numerals present an interesting challenge to formal semantics and pragmatics: they seem to be compatible between various readings ('at least', 'exactly', and 'at most' readings), and the choice of a particular reading seems to depend on complex interactions between contextual factors and linguistic structure. The goal of this article is to present and discuss some of the current approaches to the interpretation of bare numerals in formal semantics and pragmatics. It discusses four approaches to the interpretation of bare numerals, which can be summarized as follows:

1. In the neo-Gricean view, the basic, literal meaning of numerals amounts to an 'at least interpretation', and the 'exactly *n*' reading results from a *pragmatic* enrichment of the literal reading, i.e. it is accounted for in terms of *scalar implicatures*.
2. In the underspecification view, the interpretation of numerals is 'underspecified', with the result that they can freely receive the 'at least', 'exactly', or 'at most' reading, depending on which of these three construals is contextually the most relevant.
3. In the 'exactly'-only view, the numerals' basic, literal meaning corresponds to the 'exactly' reading, and the 'at least' and 'fewer than *n*' readings result from the interaction of this literal meaning with background, non-linguistic knowledge.
4. In the ambiguity view, numerals are ambiguous between two readings, the 'at least' and 'exactly' readings.The article argues that in order to account for all the relevant data, one needs to adopt a certain version of the ambiguity view. But it suggests that numerals should not necessarily be thought of as being lexically ambiguous, but rather as giving rise to ambiguities through their interactions with so-called exhaustivity operators. According to such a view, the ambiguities triggered by numerals are to be explained in terms of a non-Gricean theory of scalar implicatures.

1. Introduction

Numerical expressions ('numerals' for short) have a multiplicity of uses, some of which are illustrated in (1):

(1) a. *Determiner use*: Three men came in.
  b. *Degree use*:
    (i) Mary is six feet tall.
    (ii) Fred drove at 55 mph.
  c. *Predicative use*: We are three.

In this paper, we will be mostly concerned with the interpretation of the determiner use and, to a lesser extent, with the degree use. Furthermore, we will ignore cases where numerals are part of more complex phrases (*more than 10*, *less/fewer than 10*, *at least 10*, *at most 10*, '*exactly*' *10*, *approximately 10*, *between 5 and 10*, etc.), i.e. we will focus on 'bare' numerals (even though we will often use such complex phrases in order to paraphrase some of the readings that bare numerals give rise to).

Bare numerals present an interesting challenge to formal semantics and pragmatics: they seem to be ambiguous between various readings, and the choice of a particular reading seems to depend on complex interactions between contextual factors and linguistic structure. Thus, consider the following three sentences.

(2) Fred has three children.
(3) In order to pass, Fred must have solved three problems.
(4) If you have three children, you do not qualify for tax exemptions.

In (2), the numeral 'three' is preferably understood as equivalent to 'exactly three', i.e. (2) is perceived as true just in case Fred has 'exactly' three children (no more, no less) – we will refer to such readings as 'exactly' readings. This is not so in the case of (3): (3) does not normally convey that Fred's requirement is to solve 'exactly' three problems, which would imply that Fred would *fail* if he solved more than three problems. Rather, (3) is most naturally interpreted as stating that Fred must have solved *at least* three problems. That is, 'three' can be adequately paraphrased as 'three or more'. We will refer to such readings as 'at least' readings. Finally, given reasonable assumptions about how tax exemptions work, one can infer from (4) that you don't qualify for tax exemptions if you have three *or fewer than three* children. In this case, it seems that 'three' could be paraphrased as 'three or fewer than three' or 'at most three' ('at most' readings).

One major goal for a semantic and pragmatic theory of numerals is to account for these various interpretations on the basis of (a) a uniform semantics for numerals and (b) pragmatic principles, which account for the actual interpretation of numerals in various syntactic environments and contexts of use. In this paper, I aim at comparing and assessing various attempts at providing such a theory.

I will discuss four approaches to the interpretation of bare numerals, which can be briefly summarized as follows:

1. In the neo-Gricean view (e.g. Horn 1972; van Rooij et al. 2006), the basic, literal meaning of numerals amounts to an 'at least interpretation', and the 'exactly *n*'-reading results from a *pragmatic* enrichment of the literal reading, i.e. it is accounted for in terms of *conversational implicatures*.
2. In the underspecification view (Carston 1988; Carston & Carston 1998), the interpretation of numerals is 'underspecified', with the result that they can freely receive the 'at least', 'exactly', or 'at most' reading, depending on which of these three construals is contextually the most relevant.
3. In the 'exactly'-only view (Breheny 2008), numerals' basic, literal meaning corresponds to the 'exactly' reading, and the 'at least' and 'fewer than *n*' readings result from the interaction of this literal meaning with background, non-linguistic knowledge.
4. In the ambiguity view (e.g. Geurts 2006), numerals are ambiguous between two readings, the 'at least' and 'exactly' readings.

I will show that in order to account for all the relevant data, one needs to adopt a certain version of the ambiguity view. In particular, I will argue that numerals should not be thought of as being lexically ambiguous, but rather as giving rise to ambiguities through their interactions with so-called exhaustivity operators (Chierchia et al., forthcoming, and references cited therein). Section 2 presents the neo-Gricean approach and discusses its strengths and weaknesses. Section 3 shows that there is no compelling evidence that numerals can ever give rise to a genuinely 'at most' reading. Section 4 discusses the 'exactly'-only view. Section 5 argues that the ambiguity account is necessary and sufficient in order to predict all the relevant data, and Section 6 discusses two possible implementations of the ambiguity account – accounts based on a lexical ambiguity and accounts based on covert exhaustivity operators. Section 7 concludes the paper.

2. The Neo-Gricean Approach: Strengths and Weaknesses

2.1. A SKETCH OF THE NEO-GRICEAN APPROACH

The neo-Gricean approach has two components: a *semantic* component, which determines the contribution of numerals to the literal meaning of the sentences in which they occur, and a *pragmatic* component, which determines the way in which this literal meaning is enriched as a result of a reasoning about speakers' communicative intentions.

Consider the sentence in (5).

(5) Three girls went to the party.

According to the neo-Gricean approach, the truth conditions assigned to (5) by the grammatical compositional rules of English make it equivalent to (6).

(6) At least three girls went to the party.

Before describing the way in which this meaning is enriched so as to yield the 'exactly' interpretation, let us be more explicit about how the equivalence between (5) and (6) comes about. Following, e.g. Kadmon (1985) and Kadmon and Kadmon (2001), one can start with the following semantic rule for a numeral such as 'three':

(7) A sentence of the form *three P Q*, where *P* and *Q* are one-place predicates (noun-phrases or verb-phrases) is true if and only if there is a collection *C* of individuals, which contains 'exactly' three members, and such that *C* belongs to the extension of both *P* and *Q*.

It may seem that such a meaning for 'three' gives rise to an 'exactly' meaning for a sentence such as (5), but this is not so. Let us see why. In the case of (5), the noun 'girls' plays the role of the predicate *P* in (7), and the verb phrase 'went to the party' that of predicate *Q*. Hence, given (7), (5) is true just in case there is a collection made up of 'exactly' three girls and this collection of girls went to the party. Now, this is the case not only if three girls and no more went to the party but also if more than three girls went to the party. For suppose, for instance, that 'exactly' seven girls went to the party. Then consider any three girls among these seven girls. They constitute a collection of 'exactly' three girls that went to the party. Therefore, (5) is predicted to be true in such a situation and in any situation where more than three girls went to the party. Note that the equivalence between (5) and (6) is due to the fact that 'girls' and 'went to the party' are *distributive* predicates. Namely, whenever a certain collection *C* belongs to the extension of either predicate, any subcollection of *C* also belongs to their extensions.

According to the neo-Gricean approach, the fact that (5) is generally understood to imply that no more than three girls went to the party is due to a reasoning that the hearer performs regarding the speaker's communicative intentions. This reasoning goes as follows:

1. The author of (5) must believe that three or more than three girls went to the party. This follows from the assumption that a cooperative speaker only says things that she believes – Grice's maxim of *quality*.
2. Had she furthermore believed that more than three girls came to the party, it would have been better for her to say 'four girls came to the party'. This is due to the fact that (a) numerals are natural 'alternatives' of each other (they form a 'scale' in neo-Gricean

parlance) and (b) a cooperative speaker, when choosing between different alternative sentences, picks the one that provides as much relevant information as possible compatible with her beliefs – and the proposition that four or more girls came to the party asymmetrically entails the proposition that three or more girls came to the party, hence is strictly more informative.
3. Hence, the speaker does not have the belief that more than three girls came to the party.
4. Assuming that the speaker is knowledgeable, she must in fact believe that 'exactly' three girls came to the party.

2.2. STRENGTHS AND WEAKNESSES OF THE NEO-GRICEAN APPROACH

The neo-Gricean approach makes a number of non-trivial predictions. According to it, the strengthening from an 'at least' interpretation into an 'exactly' interpretation is a special case of a more general phenomenon, known as the phenomenon of *scalar implicature*, and it should therefore display the pattern characteristic of this phenomenon (see, e.g. Horn 1972; Gazdar 1979; Atlas & Levinson 1981; Levinson 1983 for early works on scalar implicatures within the neo-Gricean approach; Spector 2003; Sauerland 2004; van Rooij & Schulz 2004; Spector & Spector 2007; van Rooij et al. 2006 for more recent, formally explicit accounts in the Gricean tradition; and Chierchia 2004; Chierchia et al., forthcoming; Fox 2007 for a non-Gricean alternative).

For instance, a completely similar explanation is given for the fact that a sentence such as (8a) tends to be interpreted as conveying (8b).

(8) a. Fred solved the first or second problem.
b. Fred solved the first or second problem but not both.

On the assumption that the word 'or' systematically evokes the alternative word 'and' (that these two words belong to a 'scale'), one can reason that someone who utters (8a) must not believe that Fred solved both the first and second problems, so that (8a) can finally be interpreted as conveying (8b). This kind of inference is called a *scalar implicature* because (a) it is a *conversational implicature* in the sense that it is not a logical entailment, but an inference based on a reasoning about speakers' goals, and (b) it is *scalar* in the sense that it is triggered by a specific lexical item (numerals, 'or', 'some', etc.), which belongs to a *scale*, i.e. a set of expressions that can be thought of as natural alternatives to each other and are ordered in terms of logical strength.

Now, here are a number of interesting predictions of this approach for the case of 'or' and 'and', which we will systematically compare with the corresponding predictions made for numerals.

2.2.1. 'Wide-scope' Implicatures – Some Good Predictions of the Neo-Gricean Approach to Numerals

Consider the following sentence:

(9) Fred is required to (either) solve the first or second problem.[1]

According to the neo-Gricean approach, it is expected that (9) will *implicate* the negation of the sentence that results from replacing 'or' with 'all' in (9). Namely, (9) is expected to be interpreted as follows:

(10) Fred is required to solve either the first or second problem, and he is not required to solve both the first and second problems.

This is a correct result, and one that could not have been achieved by assuming that 'or' is equivalent to an exclusive disjunction (where '*P* or *Q*' entails the negation of '*P* and *Q*'). For then, we would have expected (9) to mean that Fred is required to solve either the first or second problem and not to solve both, hence that he is forbidden to solve both the problems. Rather, we want to reach a weaker conclusion, namely that Fred is *allowed* not to solve both the first and second problems, and this is 'exactly' what the neo-Gricean approach derives.

Turning now to numerals, we see that the neo-Gricean approach fares quite well as well. Consider indeed (11):

(11) Fred is required to solve three problems.

Because (11) now competes with 'Fred is required to solve four problems', which is a strictly more informative alternative, the neo-Gricean reasoning predicts the following inference for (11):

(12) Fred is not required to solve four problems.

Note that the resulting reading is not equivalent to the reading that would have arisen had the numeral 'three' been interpreted as equivalent to 'exactly three'. For in such a case, (11) would have meant that Fred's requirement was that the number of problems solved by him be equal to three, which would have entailed that he is *forbidden* to solve four. In contrast with this, the neo-Gricean approach correctly predicts (a) that the numeral 'three' can lose its 'exactly interpretation' when embedded under a necessity modal and (b) that nevertheless a negative inference is triggered (namely the negation of the stronger alternative where 'four' replaces 'three' is inferred).

2.2.2. Downward-entailing Environments – Partly Incorrect Predictions

In the case of 'or', the derivation of the relevant inferences relies on the fact that the alternative sentence with 'and' *entails* the original sentence, hence can be said to be more informative. But if 'or' occurs in a syntactic environment which reverses the direction of entailment, i.e. a so-called *downward-entailing* environment, replacing 'or' with 'and' does not give rise to a more informative sentence, but, on the contrary, to a *less* informative sentence. The most straightforward type of downward-entailing environment ('DE environment' for short) consists of *negative environments*. Other DE environments include the antecedent clause of conditional construction, the complement clause of 'negative' attitude verbs and modals ('doubt', 'refuse', 'forbid', and 'unlikely'), restrictors of universal quantifiers, restrictors and nuclear scopes of determiners such as 'no', 'few', and 'fewer than three', among others.

To illustrate, consider the two following pairs:

(13) a. Fred solved (both) the first and second problems.
b. Fred solved (either) the first or second problem.

(14) a. It's very unlikely that Fred solved (both) the first and second problems.
b. It's very unlikely that Fred solved (either) the first or second problem.

(13a) asymmetrically entails (13b). But when these sentences are embedded under the scope of a negative operator such as 'unlikely', as in (14), the direction of entailment is reversed: not only does (14a) fail to entail (14b), but now it is the sentence with 'or', i.e. (14b), which entails the one with 'and', i.e. 14(a) (if I don't think that Fred solved either the first or second problem, I necessarily also don't think that he solved both of them.)

Consider how pragmatic reasoning applies to (14b). Note that the reason why the speaker chose (14b) rather than (14a) cannot be that he did not believe (14a) to be true. For since he must believe (14b) to be true (maxim of quality) and since (14b) entails (14a), he must also believe (14a) to be true. Hence, no scalar implicature is expected in this case on the basis of the competition between 'or' and 'and'. This again seems to be correct. In particular, 'or' in (14b) cannot be interpreted as equivalent to an exclusive disjunction, with one important qualification: if 'or' is stressed and if the sentence is followed by an appropriate continuation, the exclusive reading becomes appropriate:

(15) It's very unlikely that Fred solved the first OR second problem. He must have solved BOTH of them.

This phenomenon has been explained in terms of a *metalinguistic* use of negation (Horn 1985, 1992): the idea is that while scalar implicatures cannot normally be 'embedded' (since they result from a reasoning regarding a full speech act), there is a special use of negation (called 'metalinguistic') in which negation is used to deny the appropriateness of the speech act that could normally be performed by the negated clause. In this particular case, the speaker intends to reject the scalar implicature normally associated with 'Fred solved the first or second problem', but not its truth.

Do numerals behave in a similar way as 'some' in DE contexts? If so, they should lose their 'exactly' interpretation in negative contexts (unless negation is used metalinguistically) and more generally in DE contexts and have only the 'at least reading'. This prediction is only half correct. Consider (16):

(16) Peter didn't solve 10 problems.

It seems that (16) can easily be interpreted as conveying that Peter solved fewer than 10 problems, i.e. that it is false that he solved 10 problems or more, as expected according to the neo-Gricean approach. In particular, the 'exactly' reading, which is clearly the most salient one if negation is removed (i.e. in 'Peter solved 10 problems'), loses its privilege. However, and this is inconsistent with neo-Gricean predictions, the 'exactly' reading is nevertheless accessible and does not seem to require the kind of prosodic contour which is the hallmark of metalinguistic negation, as has been argued by various authors (e.g. Horn 1992; Carston & Carston 1998; Geurts 2006; Breheny 2008).The following dialog is for instance perfectly natural.

(17) a. Fred solved 10 problems. How many problems did Peter solve?
b. I don't know, but I don't think he solved 10 problems. He may have solved fewer than 10 or more than 10 problems, but not just 10.

In contrast with this, the following dialog, where a disjunction replaces the numeral, is infelicitous unless the first occurrence of 'or' is stressed in the answer.

(18) a. Fred solved the first or second problem. What about Peter?
b. # I don't know, but I don't think he solved the first or second problem. He either solved both or neither, but not just one.

More generally, there are clear cases where a numeral is not interpreted under its 'at least' interpretation in DE contexts. Consider again (4), repeated below in (19):

(19) If you have three children, you do not qualify for tax exemptions.

According to the neo-Gricean approach, the literal meaning of this sentence (i.e. before pragmatic strengthening) makes it equivalent to 'If you have three children or more, you do not qualify for tax exemptions', which entails the following:

(20) If you have more than three children, you do not qualify for tax exemptions.

Now, the neo-Gricean approach, under all its variants, is unable to generate 'pragmatically strengthened' readings, which fail to entail the literal reading of the relevant sentence. This is so because one key assumption underlying the Gricean derivation of scalar implicatures is that the speaker believes that the literal reading of the uttered sentence is true (maxim of quality). Hence, the intended meaning of the utterance has to entail its literal meaning. In particular, (19) is predicted to be understood as entailing (20). But we saw above that (19) can be construed as true in a situation where one must have three or *fewer* than three children in order to qualify for tax exemptions. And in such a situation, (20) is false. It follows that there is a reading for (19), which is not within the reach of the traditional neo-Gricean approach.

Another complication for the neo-Gricean view is the following. Consider again the following sentence:

(21) Peter didn't solve 10 problems.

Let us focus only on the reading where the numeral takes scope below negation, i.e. the reading equivalent to 'It is not the case that 10 problems were solved by Peter', i.e. under an 'at least' meaning for numerals, 'It is not the case that there exist 10 or more than 10 problems that Peter solved'.

According to the neo-Gricean view, (21)'s scalar alternatives include the following:

(22) a. Peter didn't solve nine problems.
b. Peter didn't solve 11 problems.

Now, due to the fact that negation reverses logical entailment, (22a) asymmetrically entails (21), which itself asymmetrically entails (22b). On the one hand, as we have seen, this means that no scalar implicature will be derived for (21) on the basis of the alternative in (22b), since this alternative is less informative than (21) itself. But, on the other hand, since (22a) asymmetrically entails (21), the negation of (22a) is expected to be a scalar implicature of (21). In other words, (21) is predicted to implicate that Peter solved 'exactly' nine problems, which does not seem to be a correct prediction. This reasoning is entirely parallel to the reasoning whereby a sentence such as 'Peter did not solve both the first and second problem' is taken to implicate that Peter solved one of the problems. Because 'Peter did not solve both the first and second problem' is asymmetrically entailed by 'Peter didn't solve (either) the first or second problem', it is correctly predicted to implicate the negation of the latter sentence, i.e. that Peter solved the first or second problem (cf. Horn 1972; Fauconnier 1975). Following Chierchia (2004), we call such scalar implicatures, which are triggered when a scalar item that is 'high' on its scale (e.g. *and*) is in the scope of a DE element, *indirect scalar implicatures*. So in the case of (21), the neo-Gricean approach predicts the possibility of an indirect scalar implicature, which is not in fact perceived. We should note, however, that in other syntactic DE environments (i.e. syntactic environments, which, like negation, 'reverse' the direction of entailments), the neo-Gricean predictions seem to be at least partly borne out in the following sense: in such environments, one possible interpretation is the 'at least' reading, and the predicted indirect scalar implicature is observed. This is illustrated by the following examples:

(23) a. Every student who solved three problems passed.
b. You will necessarily pass if you solve three problems.
c. In this country, one is not allowed to have three children.

These sentences can be interpreted as follows, respectively:

(24) a. Every student who solved at least three problems passed, but it is not the case that every student who solved at least two problems passed.
b. You will pass if you solve three problems or more, but it is not necessarily the case that you will pass if you solve two problems or more.
c. In this country, one is not allowed to have three children or more, but one is allowed to have two children.

These interpretations are equivalent to the conjunction of two propositions: the proposition that is expressed by the literal meaning of the relevant sentences under an 'at least' construal of *three*, and the indirect implicatures that they are expected to trigger, i.e. the negation of the sentences that result from replacing *three* with *two*.

On the assumption that numerals can have an 'at least' reading and that, under this reading, they can give rise to scalar implicatures in 'exactly' the same way as other scalar items, we seem to face a puzzle: numerals are able to trigger indirect scalar implicatures when embedded in *some*, but not all, DE environments. I refer the reader to Fox et al. (2007) for an account of this puzzle within a non-Gricean theory of scalar implicatures.

3. The 'Underspecification' View and the Status of 'At Most' Readings

Given the shortcomings of the neo-Gricean approach, it is tempting to turn to a theory according to which the meaning of numerals is in itself radically underspecified. Carston, in various papers (Carston 1988; Carston & Carston 1998), proposes such an approach. According to her, numerals can be interpreted under either the 'exactly', 'at least', or 'at most' meaning, and the interpretation of a particular occurrence of a numeral is determined by general pragmatic considerations. Let us focus on Carston's claim that besides the 'exactly' reading and the 'at least' reading, numerals can also receive an 'at most' reading.

In fact, as discussed by Geurts (2006) and Breheny (2008), some of the alleged examples of 'at most' readings, on closer inspection, are not in fact genuine examples of 'at most' readings. Here are some of the relevant examples used by Carston:

(25) a. Sue can have 2000 calories without putting on weight.
b. You may attend six courses (and must attend three).

Now, it is true that the intended meaning of the above sentences *could* be paraphrased by adjoining 'at most' to the left of the relevant numerals. However, one must be aware that intuitions about paraphrases do not provide us with direct arguments regarding the semantics of the words, which occur in the relevant sentence (such as numerals), because one would need to have a good analysis of the paraphrases themselves in the first place. Now, the English phrase 'at most' is notoriously difficult to analyze (see Geurts et al. 2007; Nouwen 2010; Cohen & Krifka 2011). The real question to ask is how the sentences in (25) would be interpreted if numeral determiners were interpreted according to the following lexical entry, which is, arguably, what we have in mind when we talk about 'at most' readings:

(26) '*At most n' lexical entry for bare numerals*. For any numeral *n*, *n Xs are Ys* is true if and only if the number of *Xs* that are *Ys* is smaller than or equal to *n* (i.e. is at most *n*).

From such a lexical entry, the predicted meanings of (25a) and (25b), respectively, would be (under standard possible-worlds treatments of possibility modals) the following:

(27) a. There is an accessible possible world in which Sue does not put on weight and in which the number of calories she has is smaller than or equal to 2000.
b. There is a permissible world in which the number of courses you attend is smaller than or equal to six.

Now, it seems clear that these are not the intended meaning of the sentences. Note, for instance, that (27a) is true as soon as it is possible not to put on weight by having, say, *one* calorie and no more. So it is true, in particular, in a situation where Sue is doomed to put on weight if she has more than 1000 calories but is sure not to put on weight if she has just one calorie. Plainly, this conflicts with the intended reading, which seems to be stronger, as it entails, in particular, that if Sue had 'exactly' 2000 calories, she would not put on weight. Regarding (27b), the intended meaning, i.e. the one we would get by inserting the expression 'at most' ('you may attend at most six courses'), entails that there is *no* permissible world in which the number of course you attend is higher than six. But note that this reading is in fact 'exactly' the one predicted by the neo-Gricean approach. According to the neo-Gricean approach, the literal meaning of (25b) is the following:

(28) There is a permissible world in which the number of classes you attend is equal to or greater than six.

Now, this is strictly less informative than the following alternative statement:

(29) There is a permissible world in which the number of classes you attend is equal to or greater than seven.

Hence, (25b) is predicted to implicate the negation of (29), i.e. that there is no permissible world in which the number of classes you attend is equal to or bigger than seven. The resulting meaning is indeed equivalent to what you get with an informal 'at most' paraphrase, but, crucially, it cannot be derived if we start from a lexical entry for numerals such as the one given in (26). Rather, this particular example seems to argue *for*, and not against, the neo-Gricean approach.

According to the underspecification view, numerals can have an 'at most' reading as well as an 'at least' reading (putting aside the 'exactly' reading), and the choice between these various readings is entirely determined by considerations of relevance. It is thus expected that if a given sentence type is uttered in a context that is strongly biased in favor of one reading or the other, this reading should be selected. Given this prediction, the contrast between the two following sentences is highly problematic for the underspecification view:

(30) a. In the USA, one must be 18 years old in order to be allowed to vote.
b. # One must be 40 years old in order to be eligible to the Fields medal.

(30a) can be considered true, and this shows that the 'at least' interpretation is available: under the 'exact' or 'at most' interpretation, the sentence would be false, since the age requirement for voting in the USA is that voters should be 18 years old or more. As to (30b), if it licensed an 'at most' interpretation for the numeral, it should be considered true under this interpretation. Indeed, what it would then convey is that one must be 40 years old *or younger* in order to be eligible to the Fields medal, which is true. But it is clear that, once we know that there is a *maximal* age but no minimal age for being eligible to the Fields medal, we judge (30b) as necessarily false (while if 'at most' is added just to the left of '40', the sentence becomes true: 'One must be at most 40 years old in order to be eligible to the Fields medal'). This shows, at the very least, that the 'at least' reading and the putative 'at most' reading of numerals are not on par.

There are cases, however, where we seem to get a genuine 'at most' reading. Thus, in the case of (19), repeated below as (31), we tend to interpret the sentence as implying (32):

(31) If you have three children, you do not qualify for tax exemptions.
(32) If the number of children you have is smaller than or equal to three, then you do not qualify for tax exemptions.

However, as observed by Breheny (2008), this inference can be easily accounted for by assuming that the numeral in (31) receives an 'exactly' interpretation. Breheny's point is the following. Suppose that *three* in (31) is interpreted as equivalent to 'exactly three', i. e. to *three and no more than three*, then by virtue of its linguistic meaning alone, (31) just says that if you have 'exactly' three children, you do not qualify for tax exemptions and says nothing about cases where you have 'exactly' two or 'exactly' four children. However, we know from our general knowledge about the logic of taxation that tax exemptions, when they are related to the number of children in a household, are an increasing function of this number (i.e. a family with few children benefits from smaller tax breaks – if any – than a family with many children, everything else being equal). In other words, if a family with three kids does not benefit from any child-related tax break, then for sure a family with two kids does not either (everything else being equal). So if one accepts the truth of (31), one is going to deduce from the combination of (31) and one's prior general knowledge about taxation that if you have three children *or fewer than three children*, you do not qualify for tax exemptions. In other words, what happens in this case is that the reading based on an 'exact' interpretation of numerals is *contextually equivalent* to the reading based on an 'at most' interpretation: if we only consider possible worlds, which are compatible with our general knowledge about the logic of children-related tax exemptions, both readings are true in 'exactly' the same possible worlds. So while (31) provides additional evidence for the existence of a lexical entry for numerals, which does not assign to them an 'at least' reading (given that, as discussed in Section 2.2.2 the inference in (31) *contradicts* what (31) would mean on an 'at least' construal), it provides no clear evidence for the existence of the 'at most' reading.

There is thus no compelling evidence that a bare numeral could ever receive an 'at most' interpretation, and there is in fact evidence against such a view [cf. our discussion of the contrast in (30)].

4. An 'Exactly'-based Semantics: Breheny (2008)

The preceding discussion leaves us with the 'exact' reading and the 'at least' reading. As we will discuss in Section 5, an account in which bare numerals are taken to be ambiguous between these two readings can account for all the data we have reviewed so far. However, the problems of the neo-Gricean approach led some linguists to reconsider one of its basic assumptions, namely that the *at least* reading is one of the readings of bare numerals. Thus, both Geurts (2006) and Breheny (2008) take the primary meaning of numerals to be the 'exactly' meaning. The other apparent readings should then be derived from this basic reading, and not the other way around. While Geurts (2006) (to which we will return in Section 6.1) proposes a number of type-shifting operations which can generate new readings on the basis of the 'exactly' reading, Breheny claimed that the only genuine reading of numerals is the 'exactly' reading and that all other *apparent* readings are the by-products of pragmatic mechanisms.

Breheny's starting point is the observation that the 'exactly' reading is in fact available for a numeral across the board, i.e. no matter what its syntactic environment is (see also Horn 1992). As we already discussed, the neo-Gricean approach predicts the 'exactly' reading to be impossible, or at least marked, in DE environments. A second important point is that the 'exactly' reading often seems to be the preferred reading. Furthermore, some psycholinguistic developmental studies (Noveck 2001; Papafragou & Musolino 2003; Musolino 2004; Guasti et al. 2005; Pouscoulous et al. 2007; Huang & Snedeker 2009) suggest that the 'exact' reading of numerals is not acquired in the way scalar implicatures are: these studies have shown that young children tend to compute 'exact' readings for numerals to a much greater extent than they compute standard scalar implicatures. (On the other hand, Panizza et al. (2009) provide experimental evidence that, in adults, the 'exactly' reading of numerals is more easily accessed in upward-entailing environments, i.e. syntactic environments, which preserve the direction of entailments, than in DE environments, as is expected under the neo-Gricean approach). On the basis of these observations, Breheny argues, *contra* the traditional neo-Gricean approach, that the primary linguistic meaning of numerals ensures that they give rise to an 'exact' reading even when they combine with distributive predicates. Recall that we saw that the 'standard' rule for interpreting numerals when used as determiners, given in (7), results in an 'at least' reading when the numerical Determiner Phrase combines with a distributive predicate. Breheny thus proposes that we dispense with this rule and replace it with an alternative one, which, in distributive contexts, is equivalent to the following statement.

(33) A sentence of the form *three P Q*, where *P* and *Q* are one-place predicates (noun-phrases or verb-phrases) is true if and only if the set of individuals that have both property *P* and property *Q* contains 'exactly' three individuals.[2]

This rule ensures that the literal meaning of 'Three girls came in' is that the total number of girls who came in is three (we should note, however, that there exist other ways to ensure the same result – see also Geurts 2006, which we discuss in Section 6.1).

Of course, Breheny does not deny that sometimes we *seem* to get an 'at least' or 'at most' reading, and he offers an account for these cases. In order to understand Breheny's account, we should distinguish two types of environments. First, when a numeral occurs in a DE environment, it can often be paraphrased, depending on context, as '*n* or more' or '*n* or fewer than *n*'. Second, there are cases where a numeral occurs unembedded or in an upward-entailing context (UE context for short) and where it seems to be interpreted as equivalent to '*n* or more' (but, as we have already noted, not to '*n* or fewer than *n*').

4.1. APPARENT 'AT LEAST' AND 'AT MOST' READINGS IN DE ENVIRONMENTS

Let us start with DE environments. We have already seen a case, namely (19), repeated below as (34), where a numeral occurs within an if-clause and seems to receive an '*n* or fewer than *n*' reading.

(34) If you have three children, you do not qualify for tax exemptions.

As we saw, in this case, it is possible to account for the perceived 'at most' reading on the basis of the 'exactly' reading of numerals, once contextual background knowledge is taken into account.

In the very same syntactic environment, a numeral can also get an '*n* or more' reading. In fact, it is sufficient to remove the negation from (34) to get a sentence where the numeral is interpreted as equivalent to 'at least *n*':

(35) If you have three children, you qualify for tax exemptions.

(35) tends to imply that if you have three children *or more*, you qualify for tax exemptions. In the standard neo-Gricean approach, nothing special needs to be said for this case, since 'you have three children', as far its literal meaning is concerned, is assumed to be equivalent to 'you have at least three children'. When this sentence is embedded in an if-clause, i.e. in a DE environment, it is furthermore expected that the 'at least' reading cannot be strengthened into the 'exactly' reading. Now, Breheny's observation is that it is possible to account for the 'at least' reading in this case in 'exactly' the same way as the 'at most' reading for (34) can be accounted for. In the case of (34), the apparent 'at most' reading can be derived from the 'exact' reading once our background knowledge is taken into account.

'Exactly' the same explanation can be given for (35). According to Breheny, (35) only says that if you have 'exactly' three children (no more, no less), you qualify for tax exemptions. As such, it does not cover the cases where you have two or four children. Nevertheless, you know from your background knowledge that if you are entitled to a tax exemption in case you have 'exactly *n*' children, then this is also the case if you have 'exactly *n*' + 1 children. It follows that, given this background knowledge, (35) implies that if you have three *or more than three* children, then you qualify for tax exemptions.

Breheny's approach makes a very clear prediction: that 'at least' and 'at most' readings of numerals in DE contexts are always contingent on the existence of some background general, 'law-like' knowledge, which ensures contextual equivalence between the 'exact' reading and the relevant 'at least' or 'at most' readings. In particular, in a case where there is no underlying law that could give rise to the illusion of an 'at most' or 'at least' reading on the basis of the 'exactly' reading, only the 'exactly' reading should be available. In contrast with this prediction, the neo-Gricean account predicts that the 'at least' reading, but not the 'at most' reading, will be available 'for free', i.e. even in contexts which do not create any specific bias towards that reading. To my knowledge, whether this is the case has not been systematically investigated so far.

4.2. 'AT LEAST' INTERPRETATIONS IN UPWARD-ENTAILING CONTEXTS

In the absence of further mechanisms, Breheny's proposal is unable to account for the availability of 'at least' readings in environments such as the following:

(36) a. We need four chairs. Who could provide us with that?
b. I have four chairs. In fact, I have five.

(37) For this class, we are required to read two journal papers in phonology - but we can choose which papers

If bare numerals only have an 'exactly' meaning, then the discourse in (36b) is expected to be contradictory, since the first sentence means that I have four chairs and no more, while the second one entails that I have more than four chairs. Only under an 'at least' construal of the relevant numerals can the discourse be deemed consistent. As to (37), as we already pointed out for a completely parallel example [(11)], it clearly does not necessarily entail that we are required to read two journal papers and no more. Rather, it means that our obligation is to read *at least* two journal papers in phonology.

Now, in these cases, it is not possible to appeal only to contextual background knowledge in order to explain the availability of the 'at least' reading. The reason is the following. In both (36b) and (27), the 'at least' interpretation is asymmetrically entailed by the 'exact' interpretation. If I have 'exactly' four chairs, then it is true that I have four chairs or more;

if in all the worlds compatible with our obligations, we read 'exactly' two phonology papers, then in all the worlds compatible with our obligations we read at least two phonology papers. As a result, there is no way that adding some information could help us go from the 'exact' reading to the 'at least' reading. From Breheny's point of view, what is needed here is a mechanism whereby the 'exact' meaning gets *weakened*. Put differently, the problem in these cases is that the relevant sentences can be judged as true even in some contexts where, under the 'exact' reading, they are false.

Breheny thus posits a specific weakening mechanism to handle these cases. Breheny's starting point is that, quite generally, nouns can be implicitly restricted. When this is the case, the noun 'chairs', for instance, means something such as 'chairs that are *P*s', where *P* is an arbitrary property. As a result, when interpreting an utterance of 'I have four chairs', one can hypothesize that the speaker has in mind something like 'I have 'exactly' four chairs that are *P*s', for some property *P*. If the content of *P* cannot be determined by the addressee of the sentence, then the only thing that she can deduce is that, if the sentence is true, then there exists some property *P* in the mind of the speaker such that the speaker believes that he has 'exactly' four chairs which are *P*s. In a sense, thus, the *pragmatic* interpretation of the sentence becomes just this: that for some property *P*, I have 'exactly' four chairs with that property. Now, this is in fact 'exactly' equivalent to 'I have four chairs or more'. Suppose indeed that I have more than four chairs. Then for some property *P* that is true of only four of these chairs, it is true that I have 'exactly' four chairs with property *P*, and therefore the sentence 'I have four chairs' can be considered true.

Breheny extends this approach to modal contexts, in order to deal with cases like (37).

Breheny's account, however, is not without problems. One such potential problem is the following: if bare numerals only have an 'exact' interpretation, we might expect them to behave just like phrases of the form 'exactly *n*' or '*n* and no more than *n*'. In particular, one might expect that the mechanism whereby numerals can give rise to an apparently 'at least' reading will also be able to apply to phrases of the form 'exactly *n*' and '*n* and no more than *n*'. This is however not so. For instance, a sentence such as 'I have 'exactly' four chairs' or 'I have four and no more than four chairs' cannot be followed by 'In fact I have five' without contradiction. Breheny discusses this problem explicitly in the case of 'exactly'. For cases such as 'I have 'exactly' four chairs' (which he takes to be incompatible with the 'at least' reading), Breheny sketches a pragmatic analysis whereby the use of 'exactly' signals that the interpretative process whereby the 'at least' reading could in principle arise (namely, a reasoning about the implicit restrictions that the speaker has in mind) is blocked. Another potential problem is the following. Breheny's paper focuses on cases where a bare numeral is used as a quantifier, in which case it has a 'cardinal' meaning, i.e. it is used to talk about the *number* of objects, which have a certain property. However, many of the relevant facts about the interpretation of numerals that we have reviewed so far can be replicated by using constructions in which a numeral has, informally speaking, an 'ordinal' reading, typically when it is used in the context a of 'measurement scale', such as the scales of height and age (what we called the 'degree use' of numerals). Consider for instance the following sentence:

(38) Mary is 18 years old.

(38) seems to have only an 'exact' reading, i.e. to be necessarily false if Mary is, say, 20 years old. Consider however what happens when a sentence such as (38) is embedded under a necessity modal, as in (39):

(39) In order to be allowed to vote, one has to be 18 years old.

As we have already noted, (39) means that one has to be 18 years old *or more* in order to be able to vote, but that it is not necessary to be *19 years old or more*, i.e. that the minimal required age is 18. As we have seen, this is fully expected under an 'at least' interpretation for '18 years old', but not under an 'exact' interpretation. However, it is entirely unclear how Breheny's weakening mechanism (which involves an implicit restrictions on the set of objects quantified over by numerals when they are used for counting) can apply in this case. A similar observation holds for the following case:

(40) One must be 6 feet tall in order to be hired in the military.

(40) can be understood as meaning that the minimal height for being hired in the military is 6 feet, while on an 'exact' interpretation, the sentence would state that the military has a very weird rule, namely that it only hires people who are 'exactly' 6 feet tall. Again, it is unclear how Breheny's mechanism for deriving 'at least' readings would work in this case.[3]

5. The Ambiguity Account

It turns out that all the data we have discussed so far can be accounted for by assuming that bare numerals are ambiguous between the 'exact' and 'at least' interpretations, as pointed out, for instance, by Geurts (2006). Specifically, if coupled with a standard, neo-Gricean approach to scalar implicatures, such an account is able to capture, with no further stipulation, three basic generalizations about the interpretation of bare numerals:

(41) Three generalizations about the interpretation of bare numerals:
  a. 'At least' readings are available in all embedded environments, marginal in simple, unembedded contexts.
  b. 'Exactly' readings are available in all syntactic environments.
  c. 'At most' interpretations are available only in DE environments.

Let us start with (41a). If numerals have an 'at least' reading, it is no surprise to find that the 'at least' reading is in principle available in every syntactic environment. However, it is also expected that numerals can give rise to scalar implicatures, which is why, even on their 'at least' reading, they will tend to be pragmatically strengthened into the 'exactly' reading when they occur unembedded. More generally, when interpreted under their 'at least' readings, numerals will behave 'exactly' as predicted by the neo-Gricean approach discussed in Section 2. In particular, as we discussed in Section 2.2.1 in relation to example (11), when a numeral is embedded under an operator with universal force, such as a necessity modal, it is expected to give rise to a 'wide-scope' scalar implicature, and the resulting reading is not in general equivalent to the 'exact' reading. Consider now (41b). Since, by assumption, numerals do have an 'exactly' reading, we expect this reading to be available in every syntactic environment. But note that, on such an account, the 'exactly' reading should nevertheless be somewhat more salient in an unembedded context than in a DE environment. This is so because, in unembedded contexts, even if the numeral, as far as its literal meaning is concerned, is interpreted under its 'at least' interpretation, it will be strengthened into the 'exactly' meaning by way of scalar implicature. This prediction receives some support from a recent experimental paper (Panizza et al. 2009) that shows that speakers are less likely to select the 'exactly' interpretation when a numeral occurs in a DE environment than in an unembedded context. Finally, (41c) is also expected: if bare numerals are ambiguous between the 'at least' and 'exact' readings, then there will be no genuine 'at most' interpretations. The only thing that could happen is that, in some cases, the 'exact' reading will turn out to be contextually equivalent to the 'at most' reading, following the logic of Breheny's analysis, discussed in Section 4. But this could only happen if the numeral occurs in a DE context. The reason is the following: the 'at most' reading is obtained as the *conjunction* of the sentence with an 'exactly reading' for the numeral and some proposition that belongs to our background knowledge. The resulting reading, therefore, has to be logically stronger than (i.e. has to entail) the 'exactly reading'. Now, in a simple, unembedded context, this cannot be the case, because the 'exact' interpretation is not entailed by the 'at most' interpretation (in fact, the reverse is true). For instance, the proposition that John read four or fewer than four books does not entail the proposition that John read 'exactly' four books (but the proposition that John read 'exactly' four books entails the proposition that he red four or fewer than four books). In order to get an entailment in the other direction, one has to embed a sentence of this type in a DE context (for instance, 'Nobody read four books or fewer' entails 'Nobody read 'exactly' four books'). Only then is it possible to derive the 'at most' reading by some form of *strengthening* of the 'exact' reading.

6. Lexical Ambiguity or Embedded Scalar Implicatures?

Given the empirical adequacy of such an ambiguity-based account, we should also wonder about the source of this ambiguity. One natural possibility is that bare numerals are lexically ambiguous between the 'at least' reading and the 'exact' reading. This is the view advocated by Geurts (2006). But there exists another possibility, which is suggested by Chierchia et al. (forthcoming) and is closer in spirit to the neo-Gricean approach to bare numerals. As we will see, the predictions of this second type of account are slightly different from a lexical ambiguity account, in an interesting way.

6.1. LEXICAL AMBIGUITY (GEURTS 2006)

Geurts (2006) proposes that numerals, when used as determiners, are primarily associated with two distinct lexical entries. One lexical entry is the one corresponding to the syncategorematic rule given in (7), which gives rise to an 'at least' reading. Another lexical entry, the one which gives rise to the 'exactly' meaning, is associated with the following semantic rule:

(42) A sentence of the form *three P Q*, where *P* and *Q* are one-place predicates (noun-phrases or verb-phrases) is true if and only if there is a *unique* collection *C* of individuals, which contains 'exactly' three members, and such that *C* belongs to the extension of both *P* and *Q*.

The only difference with the lexical entry given in (7) is the addition of the requirement that the plurality whose existence is asserted should be *unique*. Let us apply this lexical entry to 'Three men came in'. What we get is the following:

(43) There is a unique collection made up three men such that that collection came in.

Now, it is easy to see that (43) is false in case more than three men came in. For in such a case, there are *several* collections of three men such that this collection of men came in.

According to Geurts, the lexical entry corresponding to (42) is basic, but a number of so-called type-shifting operations can apply to this lexical entry and create new meanings. One such type-shifting operation turns this lexical entry into another one which corresponds to the predicative use of numerals (as in 'We are three'). Then a second type-shifting operation can turn the 'predicative' lexical entry into the entry corresponding to the lexical rule given (7), i.e. the one corresponding to the 'at least' interpretation. The details of the account are not crucial to our purposes. It is also possible to start from the predicative use and to define two distinct type-shifting operations, which, when applied to the lexical entry corresponding to the predicative use, turn it into two distinct lexical entries corresponding to (7) and (42), respectively. We should note, however, that a basic feature of Geurt's account is that it only applies to 'cardinal' uses of numerals and not to cases where a numeral is used for expressing measurement (i.e. as a degree-denoting expression). If it is true that in such uses, the distribution of 'exactly' and 'at least' meanings parallels what is observed for the 'determiner' uses (as we discussed at the end of Section 4, in relation to Breheny's account), then something is missing. But it is of course possible to define a lexical entry for both the 'at least' and 'exactly' readings that can cover degree uses as well as 'cardinal' uses.

6.2. AN AMBIGUITY ACCOUNT BASED ON EXHAUSTIVITY OPERATORS

In this section, I would like to sketch another ambiguity approach where numerals (when used as determiners or in degree constructions) are not taken to be *lexically* ambiguous but nevertheless give rise to ambiguities due to their interactions with other operators. To do this, let me first provide the following paraphrases for the two readings of 'Three men came in'.

(44) a. 'At least' reading: At least three men came in
b. 'Exactly' reading: The proposition that at least three men came in is the most informative true proposition of the form 'at least *n* men came in'.

It is quite clear that the paraphrase in (44b) is related to the neo-Gricean account of 'exactly' readings, according to which they arise as quantity implicatures. Now, in recent years, there have been arguments for the view that the neo-Gricean account of scalar implicatures, which is based on considerations of informativity, should be either replaced or supplemented with a 'grammatical' account in which the computations, which are assumed to give rise to scalar implicatures, are 'encoded' in a so-called exhaustivity operator, which can apply, more or less freely, to any constituent in a sentence included in embedded positions, giving rise to so-called embedded implicatures (Chierchia et al., forthcoming, and references cited therein, as well as Sauerland 2012 for a recent discussion). To make things concrete, let us define the following exhaustivity operator:

(45) a. If ϕ is a sentence associated with a set of alternatives C, which contains ϕ, then *exh* (ϕ) is true if and only if ϕ is the most informative true sentence in C.
b. A sentence ϕ counts as the most informative in a set if and only if it entails all the members of the set.

Now, applied to 'Three men came in', on the assumption that the relevant set C of alternatives contains all sentences of the form '*n* men came in', we get the following result:

(46) '*exh*(Three men came in)' is true if and only if 'Three men came in' is the most informative true sentence among the sentences of the form '*n* men came in'.

If we assume that, in the absence of *exh*, numerals give rise to an 'at least' reading, (46) is equivalent to (47), i.e. to the 'exactly' reading

(47) '*exh*(Three men came in)' is true if at least three men came and no more than three men came in.

We can make the further assumption that a numeral has a strong preference for being in the scope of *exh*. This captures both the fact that, in unembedded contexts, the 'exactly' reading is clearly preferred and that when a numeral is embedded, under, say, a necessity modal, it can easily be interpreted under the 'at least' reading. Let us see this. Consider the following logical form:

(48) *exh*(we are required to solve three problems)

According to the present analysis, the meaning of (48) is the following:

(49) 'We are required to solve at least three problems' is the most informative true sentence of the form 'We are required to solve at least *n* problems'.

Now, (49) is equivalent to (50):

(50) We are required to solve at least three problems, and we are not required to solve at least four problems.

This is 'exactly' the same prediction as the standard neo-Gricean account, and it is a correct prediction. However, if the operator *exh* can be freely inserted when it is associated with numerals, it is also expected that 'exactly' readings will be available in every syntactic environment. Consider for instance the following logical form, meant as a possible representation for 'John did not read three books'.

(51) NOT[*exh*(John read three books)]

Given that '*exh*(John read three books)' is equivalent to 'John read 'exactly' three books', (51) above means that John did not read 'exactly' three books. In such an account, where the exhaustivity operator is free to apply at any site, we get all the predictions of a lexical ambiguity approach: the 'at least' reading is always available (since the exhaustivity operator does not have to be present); the 'wide-scope' implicatures, which are expected in the neo-Gricean approach, are still predicted; and the 'exactly' reading is expected to be available in any syntactic environment, due to the possibility of inserting an exhaustivity operator under the scope of every operator within which the numeral is embedded. However, we should note that, in general, scalar implicatures cannot be embedded in every possible syntactic environments. In particular, as discussed in Section 2, a standard scalar item such as 'or' cannot retain its 'strong' reading (i.e. the exclusive reading) in the scope of negation or other DE operators, unless it is stressed. This fact is one of the empirical motivations for the traditional, neo-Gricean account. In order to account for these restrictions, the grammatical approach to scalar implicatures, based on *exh*, has to posit a constraint, which prevents *exh* from occurring in a DE environment. But then, even from the point of view of the grammatical approach, the fact that numerals can be interpreted under an 'exactly' reading under negation without being prosodically marked provides an argument for the lexical ambiguity account. So an account in which the ambiguities triggered by numerals are explained in terms of their association with *exh* has to assume that the constraint that normally bars *exh* from DE environments is not relevant in the case where the scalar alternatives on which *exh* operates are triggered by a numeral. This exceptional behavior of numerals could follow from the idea that numerals are intrinsically focused, in the sense that they automatically activate their alternatives (i.e. other numerals). This could make sense of the fact that numerals apparently do not need to be stressed in contexts where other scalar items need to be stressed (Breheny 2008, Section 3, 99–100 – see also Horn 1992; Horn & Horn 2006 for related observations), on the assumption that stress, for other scalar items, is a phonological reflex of focus. We should also note that even with numerals (Panizza et al. 2009), results suggest that the 'exactly' interpretation is less accessible in DE contexts than in upward-entailing contexts.

So far, the exhaustivity-based approach does not seem to have much appeal, given that it seems to require a number of poorly motivated assumptions. However, as was pointed out to me by Danny Fox (p.c.), this approach predicts *more* readings than a lexical ambiguity approach. If these readings appear to be attested, they provide evidence for the exhaustivity-based approach. To see this, consider the following schematic representation:

(52) $Op_1[\ldots exh\ldots Op_2[\ldots \text{numeral}\ldots]]$

In this representation, the exhaustivity operator occurs in an embedded position, but not immediately above the numeral, since an operator (Op_2) intervenes between the exhaustivity operator and the numeral. As a result, the numeral will not be interpreted under its 'exactly' reading but will give rise to what we called a 'wide-scope' implicature in Section 2.2.1, and this implicature itself will be integrated in the meaning of the constituent on which the higher operator (Op_1) operates, thus giving rise to what we can call an 'intermediate embedded implicature'. Such an intermediate embedded implicature could not be predicted on the sole basis of a Gricean inference taking as input the 'at least' reading (the argument from 'intermediate embedded scalar implicatures' has been used to argue for the existence of embedded scalar implicatures against a lexical ambiguity account for all scalar items – see Sauerland 2012), which builds on the work of Fox et al. (2008). Here is a sentence that seems to be interpreted in 'exactly' this way:

(53) Context: for each exam, there is a number such that the professor demands that we solve at least that number of problems – and we are always allowed to solve more than the minimal required number.

a. Whenever the professor demanded that we solve three problems, I managed to do what she asked, but not when she asked us to solve more than three problems.
b. Intended parse: Whenever *exh*(the professor demanded that we solve three problems), I managed to do what she asked, but not when she asked us to solve more than three problems.

In (53b), where *whenever* is Op_1 and *demanded* is Op_2, to see what reading is predicted for (53b), consider first the meaning of the clause, which is embedded under 'whenever', i.e. '*exh* (the professor demanded that we solve three problems)'. This meaning can be paraphrased as follows: 'The professor demanded that we solved at least three problems and did not demand that we solve at least four problems'. The resulting reading for (53) is then as follows:

(54) Whenever the professor demanded that we solve at least three problems and did not demand that we solve at least four, I managed to do what she asked, but not when she asked us to solve more than three problems.

The crucial observation here is that, in the absence of the embedded exhaustivity operator and on the basis of the 'at least' reading, (53a) would express a contradiction, since the first conjunct would state that in every situation where the professor demanded that we solve at least three problems, including those where he demanded that we solve more than three problems, I managed to do what she asked, and this contradicts the second conjunct. On an 'exactly' reading for the numeral ('Whenever the professor's demand was that we solve 'exactly' three problem, …'), the sentence would only talk about situations where the professor demanded that we solve three problems and that we do not solve more than three problems, i.e. situations that are excluded by the specified context. So the fact that a sentence like (53a) is natural and, most importantly, seems to be able to receive the interpretation corresponding to (53b) provides evidence for the availability of structures such as (52). There is one further issue to discuss, though. As we have seen, in negative contexts, standard scalar items can marginally give rise to embedded scalar implicatures, if they are prosodically marked. In the case of negative environments, this is taken to follow from the so-called metalinguistic use of negation, discussed in Section 2.2.2, but one can also posit, in principle, that other operators as well can have a metalinguistic use. Now, under the lexical ambiguity account, a numeral, which is interpreted under its 'at least' meaning, is expected to behave just like other scalar items. In particular, such a numeral is expected to marginally license an embedded scalar implicature in a DE environment if it is prosodically marked. Under a lexical ambiguity account, therefore, while the 'exactly' reading of numerals would never require any kind of prosodic marking on the numeral, readings involving an intermediate embedded scalar implicature in a DE environment are expected to correlate with stress on the relevant numeral. If it turns out that, under the intended interpretation for (53a), the numeral needs to be prosodically marked, then this example would in fact argue *against* an account based on exhaustivity operators [since then it would be a complete mystery why no prosodic marking is needed in the case of (51), while this would be fully expected under a lexical ambiguity account]. So does the numeral in (53a) need to be stressed? To my knowledge, there has not been any empirical work dealing with this specific question. One possible way to address it is to contrast (53a) with an example in which the numeral is replaced by some other, more standard scalar item, and see whether the resulting sentence requires the relevant scalar item to be more prosodically marked than a numeral in the same position (thanks to Philippe Schlenker for relevant discussions). For instance, we can contrast (53a) with (55) (in the case of 'some' and its 'competitor' 'all', with the result that 'the professor demanded that we solve some of the problems' implicates 'the professor did not demand that we solve all of the problems'):

(55) Whenever the professor demanded that we solve some of the difficult problems, I managed to do what she asked, but not when she asked us to solve all of the difficult problems.

In order to reach a firm conclusion on this point, experimental work would probably be called for. In the context of the present discussion, what is important is to note that an approach in which the ambiguities triggered by numerals do not originate in a lexical ambiguity but rather on the optional presence of an exhaustivity operator predicts more readings than a lexical ambiguity account, so that both approaches can in principle be distinguished on the basis of some (admittedly complex) data.[4]

7. *Conclusion*

In this paper, I explored a number of theoretical options in order to deal with the apparent ambiguities triggered by bare numerals. I argued that in order to predict the full range of data, it is necessary to posit that the linguistic meaning of numerals creates a systematic ambiguity between the 'exactly' meaning and the 'at least' reading. I discussed two different ways of generating these ambiguities. On one approach, numerals are lexically ambiguous. This lexical approach can itself receive very different implementations, and the choice of a particular implementation depends in part on issues that I have not dealt with at all in this paper, having to do with the semantics of plurality and that of degree constructions. On a second view, the basic meaning of numerals is the one that gives rise to the 'at least' reading, and the 'exactly' meaning arises through the interactions of the relevant numeral and a covert operator that can be inserted in embedded positions. I showed that these two approaches make different predictions in some complex cases, which calls for further investigation.

Short Biography

Benjamin Spector obtained a PhD in linguistics in January 2006 from Université Paris 7, after having studied at École Normale Supérieure. From 2006 to 2008, he was a Junior Fellow at the Harvard Society of Fellows. Since 2008, he is a research scientist at Institut Jean Nicod (Centre National de la Recherche Scientifique), which is part of the Institut d'Étude de la Cognition of École Normale Supérieure. His research is concerned with the interfaces between syntax, semantics, and pragmatics. An important aspect of his work aims to understand the division of labor between grammatical mechanisms and pragmatic inferential processes, with a focus on scalar implicatures, exhaustivity effects, and related phenomena. Benjamin. has also worked on the semantics and pragmatics of interrogative sentences, scope ambiguities, and the semantics and pragmatics of number features. He regularly teaches at École Normale Supérieure, and has been invited to teach classes in several universities (University of Vienna, and UCLA) and summer schools (ESSLLI, and LSA Institute).

Notes

* Correspondence address: Benjamin Spector, CNRS-ENS-EHESS, Institut Jean Nicod, Pavillon jardin, École Normale Supérieure, 29 rue d'Ulm, 75005 Paris, France. E-mail: benjamin.spector@gmail.com

[1] We are only interested in the reading where disjunction takes scope below 'required', i.e. in the reading 'Fred's obligations include the following: solve the first or second problem', as opposed to the wide-scope reading 'Either Fred is required to solve the first problem, or he is required to solve the second problem'. The variant with a 'displaced' *either* is supposed to disambiguate in favor of the narrow-scope reading in this particular case, for reasons discussed by Larson (1985).

[2] Breheny's statement is slightly more complicated so that it can take care of cases where numerical phrases combine with non-distributive predicates, even though he does not elaborate. What Breheny seems to intend with his statement (29) is that 'Three P Q' is true if and only if the set of all atomic individuals x that are parts of (atomic or plural) individuals X that belong to both the denotations of P and Q has cardinality 3. This ensures that 'Three boys lifted the piano' is true if, say, Sam lifted the piano on his own, Peter and John lifted the piano together, and no other piano lifting took place.

[3] What would be needed would be to treat the unit 'foot' as a standard predicate, so that a covert restriction P could be added to it, so that 'being 6 feet tall' could be interpreted as 'being 6 (feet that are Ps) tall'. Besides being intuitively non-sensical, such a possibility is, as far as I can see, incompatible with standard treatments of degree constructions, in which '6 feet' is treated as denoting a specific degree on the scale of height, i.e. not in relation with the cardinal uses of numerals.

[4] In a recent presentation (Kennedy 2010), Chris Kennedy offered a proposal which is able to capture what we called 'wide-scope' and 'intermediate scalar implicatures' in a different way. He proposes that number words have a 'maximal' meaning, which in simple, unembedded contexts amounts to an 'exactly' reading. That is, 'Three men came in' is analyzed as stating that is the maximal number n such that at least n men came in. But when a numeral occurs in the scope of a modal, it can covertly move over the modal (without the noun), take scope over it, and leave a numerical variable in its base position. The sentence 'We are required to solve three problems', under the 'at least' meaning, can then be analyzed as follows:

(56) a. Three$_n$ (We must read at least n books)
b. Three is the maximal number n such that we must read at least n books.
$\rightsquigarrow$ We must read at least three books, and we do not have to read more than n books.

As Kennedy points out, this is analogous to the standard treatment of comparative phrases in modal contexts (cf. Heim 2000). In the proposal based on exhaustivity operators, the exhaustivity operator has to be introduced at the very site where, in Kennedy's account, the numeral has to move. It is worth noting that, on Kennedy's account, the numerical variable itself is assumed to have an 'at least' meaning. Similar to Breheny (2008), Kennedy needs to assume an independent mechanism in order to account for the existence of genuine 'at least' readings. As an anonymous reviewer points out, a potential issue for such an account is that wide-scope construals for comparatives under modals seem to be syntactically constrained and that it is not clear that similar constraints hold for 'intermediate' readings with numerals. Consider for instance the following:

(57) Whenever the professor made the request that we solve three problems, I managed to do what she asked, but not when she asked us to solve more than three problems.

(57) can receive the same 'intermediate' reading as (53a). In contrast with this, comparatives in the same position do not seem to license what would be, under Kennedy's approach, a parallel reading.

(58) Whenever the professor made the request that we solve fewer than three problems, I managed to do what she asked.

If the comparative 'fewer than three' were able to outscope 'made the request', (58) could be interpreted as (59b), under the parse (59a):

(59) a. Whenever [(fewer than three)$_n$ (the professor made the request that we solve at least *n* problems)], . . .
b. Whenever the number of problems such that the professor demanded that we solve at least that number of problems was less than three, I managed to do what she asked.

Yet it is far from clear that such a reading is available.

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