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Two kinds of modified numerals*

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Abstract In this article, I show that there are two kinds of numeral modifiers: (Class A) those that express the comparison of a certain cardinality with the value expressed by the numeral and (Class B) those that express a bound on a degree property. The goal is, first of all, to provide empirical evidence for this claim and second to account for these data within a framework that treats modified numerals as degree quantifiers.

Keywords: modified numerals, scalar quantification, modality

1 Introduction

Modified numerals are most commonly exemplified by combinations of a numeral and a comparative, as in *more than 100*. Following Hackl (2001), I will refer to such expressions as *comparative quantifiers*. As (1) shows, however, apart from modification by a comparative, numerals combine with a striking diversity of expressions.

(1)	*more/fewer/less than 100*	comparative quantifiers
	no more than 100, *many more than 100*	differential quantifiers

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at least/most 100	superlative quantifiers
100 or more/fewer/less	disjunctive quantifiers
under/over 100, between 100 and 200	locative quantifiers
from/up to 100, from 100 to 200	directional quantifiers
minimally/maximally 100, 100 tops	other

For a long time, there seemed to be agreement in the formal semantic literature that there was little to be gained from a thorough investigation of these expressions. An especially dominant view, originating from generalised quantifier theory (Barwise & Cooper 1981), was that there was not much more to the semantics of such quantifiers than the expression of the numerical relations $>$, $<$, $\leq$ and $\geq$. In the past decade, however, several studies have shown that this is an overly simplistic assumption. Examples are Hackl 2001, Krifka 1999 and Takahashi 2006 on comparative quantifiers, Nouwen 2008b on negative comparative quantifiers, Solt 2007 on differential quantifiers, Geurts & Nouwen 2007, Umbach 2006, Corblin 2007, Büring 2008 and Krifka 2007b on superlative quantifiers, Corver & Zwarts 2006 on locative quantifiers and Nouwen 2008a on directional quantifiers.[1] Such investigations usually concern the specific quirks of a certain type of modified numeral. While I believe that it is important to have a semantic analysis of modified numerals on a case by case basis, I also believe that what is lacking from the literature so far is a view of to what extent the various modified numerals in (1) involve the same semantic structures. In this paper, I will attempt to reach a generalisation along this line by claiming that there are two kinds of modified numerals: (A) those that relate the numeral to some specific cardinality and (B) those that place a bound on the cardinality of some property. The difference will be made clear below. The main example of (A) are comparative quantifiers like *more/fewer than 100*. Most other kinds of modified numerals fall in the second class.

I will start by making clear what distinguishes the two classes of modified numerals by presenting a body of data that sets them apart. Then, in section 3, I introduce a well-founded decompositional treatment of comparative quantifiers, proposed by Hackl (2001), which I take to represent the proper treatment of class A modifiers. In section 4, I propose that class B modifiers are operators that indicate maxima/minima. I will then account for the distribution of these quantifiers by arguing that they are often blocked by unmodified numerals, which are capable of expressing equivalent meanings.

1 See also Nouwen 2010b for an overview.

Section 5 discusses a particular problem that occurs with the interaction of B-type quantifiers with modal operators. In section 6, I provide some more details on the empirical basis for the A/B distinction. Section 7 concludes.

2 Class A and class B modified numerals

It is a striking feature of comparative quantifiers that they can be used to assert extremely weak propositions. For instance, (2) is acceptable, even though it expresses a rather under-informative truth.

(2) A hexagon has fewer than 11 sides. A

This example contrasts strongly with the examples in (3), which are all unacceptable. (Or, alternatively, one might have the intuition that they are *false*).

(3) a. #A hexagon has at most 10 sides. B
b. #A hexagon has maximally 10 sides. B
c. #A hexagon has up to 10 sides. B

Why is this so? A naive theory might have it that (2) states that the number of sides in a hexagon is strictly smaller than 11 (i.e. <11), and that the only difference with (3) is that, there, it is stated that this number is smaller *or equal* to 10 (i.e.≤ 10). Clearly, 6 is both < 11 and ≤ 10. So why are not both kinds of examples under-informative but true? On the naive view, *having at most 10 sides* is expected to be equivalent to *having fewer than 11 sides*. That is, both these properties pick out objects with $n \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ sides. Semantically, no contrast is to be expected. Given this semantic equivalence, a pragmatic explanation of the contrast between (2) and (3) seems equally unlikely.[2]

Let us call quantifiers that are acceptable in such examples *class A* quantifiers and those that are like (3) *class B* quantifiers. As the contrast between (4) and (5) shows, the distinction is also visible with lower bound quantifiers.

2 A reviewer wondered whether the naive view could not be maintained if we assume that there is a pragmatic effect associated to the fact that $\leq n$ includes the possibility of n while $< n$ excludes it. It is very much unclear what kind of effect that would be, however. One could, for instance, base a pragmatic inference on the fact that, in (3a), the speaker seems to signal the possibility that a hexagon has 10 sides by using *at most 10*. However, one could equally argue that the same signal is given by the speaker of (2), simply by using *fewer than 11* instead of *fewer than 10*.

That is, (4) is under-informative, yet true and acceptable, while the examples in (5) are unacceptable/false.

(4) A hexagon has more than 3 sides. A

(5) #A hexagon has {at least / minimally} 3 sides. B

What I think is the underlying problem of examples involving class B expressions is that such quantifiers are incapable of expressing relations to definite amounts. Class A expressions, on the other hand, excel at doing so. Imagine, for instance, that we are talking about my new laptop and that we are concerned with how much internal memory it has. Say that it has 1GB of memory (and that I know that it has so much memory.) I can then assert (6) in a context where you, for instance, just told me that your laptop has 2GB of memory.

(6) My laptop has less than 2GB of memory.

Or, if your computer has a mere 512MB of memory, I can boast that:

(7) My laptop has more than 512MB of memory.

In these examples, I am comparing the definite amount of 1GB, i.e. the precise amount of memory I know my laptop has, to some given contrasting amount 2GB (512MB) by means of *less than* (*more than*). This is something class A quantifiers can do very well, but something that is unavailable for class B modified numerals:

(8) I know exactly how much memory my laptop has...
  a. ...and it is {#at most / #maximally / #up to} 2GB.
  b. ...and it is {#at least / #minimally} 512MB.

In contrast to (8), class B quantifiers are acceptable when what is 'under discussion' is not a definite amount, but rather a range of amounts, as in (9).

(9) a. Computers of this kind have {at most / maximally / up to} 2GB of memory.
  b. Computers of this kind have {at least / minimally} 512MB of memory.

In other words, it appears that class B quantifiers relate to ranges of values, rather than to a single specific cardinality.[3] This intuition is supported by (10).

(10) Jasper invited maximally 50 people to his party.

We normally interpret (10) to indicate that the speaker does not know how many people Jasper invited. That is, it is unacceptable for a speaker to utter (10) if s/he has a definite amount in mind, which is why the addition of *43, to be precise* in (11) is infelicitous.[4]

(11) Jasper invited maximally 50 people to his party. #43, to be precise.

By assuming that the speaker does not know the exact amount, (10) is interpreted as being about the range of values possible from the speaker's perspective. The speaker thus states that there is a bound on that range. The same intuition occurs if we substitute *maximally 50* by any other class B quantifier.

In sum, I showed that the landscape of modified numerals can be divided into two separate classes of expressions. What distinguishes class B quantifiers from other modified numerals is that they are incompatible with definite amounts and are always interpreted with respect to a range of values. Below, I will present a semantics of class B expressions that makes this intuition

3 In his comments on this article, David Beaver pointed out examples like (i), where the number appears to be a variable quantified over.

(i) There were maximally 50 people there at any one time.

Although I will not attempt a compositional analysis of cases like (i), such examples do appear to support the main intuition that class B quantifiers express relations between amounts and ranges. An example like (i) states that 50 is the maximum of the *range* formed by the different number of people present at different times. This is different from (ii), which states that at any time the number of people present did not exceed 50. (This is true, for instance, in case from start to finish there were always 20 people present.) So while (i) expresses a maximum on a range of values created by quantification, (ii) quantifies over different times and compares the number of people present at that time with 50.

(ii) There were fewer than 50 people there at any one time.

4 Compare this to (i), which forms a minimal pair with (11).

(i) Jasper invited fewer than 50 people to his party. 43, to be precise.

precise. Before I can do so, however, I will need to discuss the semantics of A-type numeral modifiers.

3 Hackl's semantics for comparative modifiers

In this section, I discuss the semantics for comparative modified numerals as developed in Hackl 2001. I will assume that this represents the proper treatment of class A numeral modifiers. I also extend the framework slightly by adding a way to account for the ambiguity of non-modified numerals.

3.1 Class A modifiers as degree quantifiers

What is the semantics of a class A quantifier? It is tempting to think that class A quantifiers correspond to the well-known generalised quantifier-style determiner denotations such as the ones in (12).[5]

(12) $[\![\text{more than 10}]\!] = \lambda P.\lambda Q.\ \exists x[\#x > 10 \ \&\ P(x) \ \&\ Q(x)]$
$[\![\text{fewer than 10}]\!] = \lambda P.\lambda Q.\neg\exists x[\#x \geq 10 \ \&\ P(x) \ \&\ Q(x)]$

In the past decade it has become clear that it is important to have a closer look at these modified numerals (Krifka 1999; Hackl 2001). In what follows, I will assume the following semantics of *fewer than*, which is based on the arguments in Hackl 2001.

(13) $[\![\text{more than 10}]\!] = \lambda M.\ \max_n(M(n)) > 10$
$[\![\text{fewer than 10}]\!] = \lambda M.\ \max_n(M(n)) < 10$

The workings of this definition will become clear below, but one of the main motivations for an analysis along this line can be pointed out immediately. The semantics in (13) is simply that of a comparative construction, where cardinalities are seen as a special kind of degrees. That is, like the comparative, it involves a degree predicate M and a maximality operator that applies to

5 In a set-theoretic approach (12) would correspond to the perhaps more familiar (i). I discuss (12) rather than (i) since, in what follows, I will assume a framework that makes use of sum individuals. It is easy to see that, within their own respective frameworks, (12) and (i) ultimately yield the same truth-conditions.

(i) $[\![\text{more than 10}]\!] = \lambda X.\lambda Y.|X \cap Y| > 10$
$[\![\text{fewer than 10}]\!] = \lambda X.\lambda Y.|X \cap Y| < 10$

this predicate (Heim 2000). In other words, (13) is completely parallel to other comparatives, like (14). While in (13), M is a predicate like *being a number* n *such that Jasper invited* n *people to his party*, in (14) M could, for instance, be filled in with something like *being a degree* d *such that Jasper is tall to degree* d.

(14) $[\![\text{-er than } d]\!] \quad = \quad \lambda M.\max_{d'}(M(d')) > d$

Hackl assumes that argument DPs containing a (modified) numeral always contain a silent counting quantifier *many*:

(15) $[\![\text{many}]\!] = \lambda n \lambda P \lambda Q.\exists x[\#x = n \ \& \ P(x) \ \& \ Q(x)]$

(16) 10 sushis ⇝ $[_{\text{DP}}$ [10 many] sushis]

In this framework, the numeral (of type d, of degrees) is an argument of the silent quantifier *many* (of type $\langle d, \langle\langle e,t\rangle, \langle\langle e,t\rangle, t\rangle\rangle\rangle$, of generalised quantifier-style determiners parameterised for degrees). By applying [10 many] to the noun (phrase), the standard generalised quantifier denotation of *10 sushis* is derived: $\lambda Q.\exists x[\#x = 10 \ \& \ \text{sushi}(x) \ \& \ Q(x)]$. The structure of a DP containing a modified numeral does not differ essentially. Modified numerals are also the argument of a counting quantifier, as illustrated in (17).

(17) fewer than 10 sushis ⇝ $[_{\text{DP}}$ [[fewer than 10] many] sushis]

As was stated above, *many* is parametrised for cardinalities, which we take to be degrees. *Fewer than 10*, however, denotes a degree quantifier, not a degree constant. Thus, to avoid a type clash, the modified numeral in (17) has to move, leaving a degree trace and creating a degree property.

(18) Jasper ate fewer than 10 sushis.
⇝ [[fewer than 10] [λn [Jasper ate [[n many] sushis]]]]

This leads to the following interpretation, which results in the desired simple truth-conditions.

(19) $[\lambda M.\max_n(M(n)) < 10] \ (\ \lambda n.\exists x[\#x = n \ \& \ \text{sushi}(x) \ \& \ \text{ate}(j,x)])$
$=_\beta$
$\max_n(\exists x[\#x = n \ \& \ \text{sushi}(x) \ \& \ \text{ate}(j,x)]) < 10$

This might seem like a rather elaborate way of deriving the truth-conditions for such simple sentences. Using (12), we would have derived as truth-conditions $\neg\exists x[\#x \geq 10 \,\&\, \text{sushi}(x) \,\&\, \text{ate}(j,x)]$, which is equivalent to (19), but which does not require resorting to (moving) degree quantifiers and silent counting quantifiers. Importantly, however, Hackl's theory makes some crucial predictions which are not made by theories assuming a semantics as in (12).

If, like degree operators, modified numeral operators can take scope, we expect to find scope alternations that resemble those found with degree operators (Heim 2000). As Hackl observed, this prediction is borne out. For reasons explained in Heim 2000, structural ambiguity arising from degree quantifiers and intensional operators like modals is only visible with non-upward entailing quantifiers, which is why all the following examples are with upper-bounded modified numerals.

The example in (20), for instance is ambiguous, with (20a) and (20b) as its two readings.

(20) (Bill has to read 6 books.) John is required to read fewer than 6 books.

a. 'John shouldn't read more than 5 books'
b. 'The minimal number of books John should read is fewer than 6'

One of the readings of (20) states that there is an upper bound on what John is allowed to read. The more natural interpretation, however, is a minimality reading, which is about the minimal number of books John is required to read. (That is, (20) would, for instance, be true if John meets the requirements as soon as he reads 3 or more books.)

Following Heim (2000), Hackl analyses this ambiguity as resulting from alternative scope orderings of the modal and the comparative quantifier. The upper bound reading, (20a), corresponds to a logical form where the modal takes wide scope. The minimality reading involves the maximality operator intrinsic to the comparative construction taking wide scope over the modal (Heim 2000).

(21) $\Box[\text{max}_n(\exists x[\#x = n \,\&\, \text{book}(x) \,\&\, \text{read}(j,x)]) < 6]$
[require [[fewer than 6] [λn [John read n-many books]]]]

(22) $\text{max}_n(\Box\exists x[\#x = n \,\&\, \text{book}(x) \,\&\, \text{read}(j,x)]) < 6$
[[fewer than 6] [λn [require [John read n-many books]]]]

A similar structural ambiguity can be observed with existential modals. The two readings of (23) are an upper bound interpretation as well as a reading which is very weak, stating simply that values below the numeral are within what is permitted, without stating anything about the permissions for higher values. (That is, the reading intended in (23b) is, for instance, verified by a situation where there are no restrictions whatsoever on what John is allowed to read. Clearly, (23a) would be false in such a situation.)

(23) John is allowed to bring fewer than 10 friends.
  a. 'John shouldn't bring more than 9 friends'
  b. 'It's OK if John brings 9 or fewer friends (and it might also be OK if he brings more)'

As before, these readings can be predicted to exist on the basis of the relative scope of modal and comparative quantifiers.

(24) $\max_n(\Diamond\exists x[\#x = n \,\&\, \text{friend}(x) \,\&\, \text{bring}(j,x)]) < 6$
[[fewer than 6] [λn [allow [John invite n-many friends]]]]

(25) $\Diamond[\max_n(\exists x[\#x = n \,\&\, \text{friend}(x) \,\&\, \text{bring}(j,x)]) < 6]$
[allow [[fewer than 6] [λn [John invite n-many friends]]]]

The reader may check that Hackl's predicted readings in (24) and (25) are indeed the attested ones.

3.2 Class B modifiers are different

These analyses are strongly supportive of an approach which treats comparative quantifiers as comparative constructions. The question now is whether class B quantifiers should be given a similar treatment. In other words, will the semantics in (26) do?

(26) $[\![\text{up to / maximally / at most / etc... 10}]\!] =^? \lambda M.\ \max_n(M(n)) \leq 10$

Can we really be sure about this???

Choosing a semantics that is parallel to that of *fewer than* is partly unintuitive since the class B quantifiers are not comparative constructions. Yet, cases like *maximally 10* suggest that the crucial ingredient of the semantics is the same, namely a maximality operator. The unsuitability of the analysis in (26) becomes immediately apparent, however, if we investigate examples with class B modified numerals embedded under an existential modal: these turn out not to be ambiguous (cf. Geurts & Nouwen 2007). Class B modifiers like *maximally*, *up to* and *at most* always yield an upper bound on what is allowed and resist the weaker reading that was found with comparative modifiers, as the contrast between (27) and (28) makes clear.

(27) John is allowed to bring fewer than 10 friends.
But more is fine too.

(28) John is allowed to bring {up to / at most / maximally} 10 friends.
#But more is fine too.

A further interesting property of the interaction of class B modified numeral quantifiers and modals is that existential modals interfere with the inferences about speaker knowledge that we found for simple sentences. Above, I observed that (29) licenses the inference that the speaker does not know how many friends Jasper invited. In contrast, (30) does not license any such inference; it is compatible with the speaker knowing exactly what is and what is not allowed.

(29) Jasper invited maximally 50 friends.

(30) Jasper is allowed to invite maximally 50 friends.

These observations add to the data separating class A from class B quantifiers. Summarising, the distinctions are then as follows. First of all, class B quantifiers, but not class A quantifiers, resist definite amounts, except when embedded under an existential modal. Second, class B quantifiers, but not class A quantifiers, resist weak readings when embedded under an existential modal.

In the next section I will argue that the peculiarities of class B quantifiers can be explained if we assume that they are quite simply maxima and minima indicators. Basically, what I propose is that the semantics of *maximally* (*minimally*) is simply the operator $\max_d$ ($\min_d$). This might be perceived as stating the obvious. What is not obvious, however, is how such a proposal accounts for the difference between class A and class B quantifiers. I will argue that the limited distribution of class B modifiers is due to the fact that they give rise to readings that are in competition with readings available for non-modified structures. I will show that, in many circumstances, the application of a class B modifier to a numeral yields an interpretation which is equivalent to one that was already available for the bare numeral. Before I can explain the proposal in detail, I therefore need to include an account of bare numerals in the framework.

3.3 The semantics of numerals

Above, I adopted the semantics of Hackl 2001 for comparative modified numerals. An important part in that framework is played by the counting quantifier *many*. I will re-name this operator $many_1$, for, in what follows, I assume that for any numeral there are two counting quantifiers available. These two options are to account for the two meanings of numerals that may be observed: on the one hand the existential / weak / lower-bounded meaning and, on the other hand, the doubly bound / strong meaning. An example like (31), for instance, is ambiguous between (31a) and (31b).

(31) Jasper read 10 books.

a. the number of books read by Jasper ≥ 10
b. the number of books read by Jasper = 10

I assume that, like the meaning in (31a), the meaning in (31b) is semantic and not the result of a scalar implicature that results from (31a). See e.g. Geurts 2006 for a detailed ambiguity account, and for some compelling arguments in favour of it.[6]

In the current framework, that of Hackl 2001, the weak reading in (31a) is due to a weak semantics for the counting quantifier: i.e. $many_1$. I propose that the strong reading, (31b), is accounted for by an alternative quantifier $many_2$ (taking inspiration from Geurts 2006.)[7]

(32) $[\![\text{many}_1]\!] = \lambda n \lambda P \lambda Q.\exists x[\#x = n \;\&\; P(x) \;\&\; Q(x)]$
$[\![\text{many}_2]\!] = \lambda n \lambda P \lambda Q.\exists! x[\#x = n \;\&\; P(x) \;\&\; Q(x)]$

Here, $\exists! x[\varphi]$ abbreviates $\exists x[\varphi \;\&\; \forall x'[x' \neq x \rightarrow \neg\varphi_{[x/x']}]]$.[8] In other words, $\exists! x$ stands for 'exactly one ...'. When x ranges over groups of individuals, $\exists! x[\#x = n \;\&\; P(x)]$ is verified by assigning to x the maximal group of individuals with property P, where n is the cardinality of that group. This is because any smaller group will not be the unique group with property P of its cardinality. For instance, if our domain is $\{a, b, c, d\}$, all of which satisfy P, then $\exists! x[\#x = 3 \;\&\; P(x)]$ is false, since several groups have three atoms and property P, among which $a \oplus b \oplus c$ and $a \oplus c \oplus d$. However, $\exists![\#x = 4 \;\&\; P(x)]$

6 But see Breheny 2008 for a dissenting view.

7 Here is a mnemonic. The 1 in $many_1$ represents the fact that this operator is *unilaterally* bound, namely lower-bounded only. $Many_2$ on the other hand is bilaterally bound.

8 Here, $\varphi_{[x/x']}$ is the formula that is exactly like φ except that free occurrences of x have been replaced by x'. Moreover, it is assumed that φ contains no free occurrences of x'.

is true, since apart from $a \oplus b \oplus c \oplus d$ there is no other group that has 4 atoms while satisfying P. Consequently, $\exists! x[\#x = n \ldots]$ stands for 'exactly $n \ldots$'. For instance, the doubly bound reading of *Jasper read 10 books* is (33). The truth-conditions of (33) are such that it is false if Jasper read fewer than 10 books (for then there would not be 10 books he read), but also false if Jasper read more than 10 books (for then there would be many groups of 10 books he read).

(33) $\exists! x[\#x = 10 \mathbin{\&} \text{book}(x) \mathbin{\&} \text{read}(j, x)]$

Not only does the option of two counting quantifiers, *many*$_1$ and *many*$_2$, suffice to account for the ambiguity of bare numerals, it is moreover harmless with respect to the semantics of comparative quantifiers. A sentence like *Jasper read more than 10 books* is not ambiguous. It is important to show that the availability of two distinct counting quantifiers does not predict ambiguities in such examples. It will be instructive to see in somewhat more detail why this is indeed the case.

The structure in (34) is exemplary of a simple sentence with a modified numeral object. As explained earlier, the modified numeral applies to the degree predicate that is created by moving the quantifier out of the DP.

(34) [MOD n [λd [Jasper read d many$_{1/2}$ books]]]

Now that there is a choice between two counting quantifiers, the denotation of the degree predicate depends on which of *many*$_1$ and *many*$_2$ is chosen. The predicate in (35) is the result of a structure containing *many*$_1$; the predicate in (36) is based on *many*$_2$. If, in the actual world, Jasper read 10 books, then (35) denotes $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. When, however, the predicate contains the *many*$_2$ quantifier, the denotation is a singleton set: $\{10\}$ if Jasper reads 10 books. This is because only the maximal group of books read by Jasper is such that it is the unique group of that kind of a certain cardinality. In general, the *many*$_2$-based degree predicate extension is a singleton set containing the maximum of the values in the denotation of the *many*$_1$-based degree predicate.

(35) $\lambda d.\exists x[\#x = d \mathbin{\&} \text{book}(x) \mathbin{\&} \text{read}(j, x)]$

(36) $\lambda d.\exists! x[\#x = d \mathbin{\&} \text{book}(x) \mathbin{\&} \text{read}(j, x)]$

As discussed above, comparative quantifiers involve maximality operators. However, the maximal values for degree predicates like (35) and (36) are always equivalent. In simple sentences based on a structure like (34), the option of having two distinct counting quantifiers does therefore not result in any ambiguity.

When we turn to cases where the degree predicate is formed by moving the modified numerals over a modal operator with universal force, something similar can be observed. If Jasper is required to read (exactly) 10 books, then the structure in (37) yields, again, the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Once more, the structure which contains the bilateral counting quantifier, the one in (38), yields the set containing the maximum of its weaker counterpart.

(37) [λd [require [Jasper read d many$_1$ books]]]
$\rightsquigarrow \lambda d.\Box\exists x[\#x = d \;\&\; \text{book}(x) \;\&\; \text{read}(j, x)]$

(38) [λd [require [Jasper read d many$_2$ books]]]
$\rightsquigarrow \lambda d.\Box\exists! x[\#x = d \;\&\; \text{book}(x) \;\&\; \text{read}(j, x)]$

Given that the relation between (38) and (37) is once again one of a set and its maximal value, no ambiguities can be expected to arise when comparative quantifiers are applied to these two predicates. This is as is desired.

Of course, it could be that the actual situation is not one containing a specific requirement, but one with for instance a minimality requirement. Say, for instance, Jasper has to read at least 4 books. In that case, (37) denotes the set $\{1, 2, 3, 4\}$. The extension of (38), however, is the empty set. (In such a context, there is no specific n such that Jasper has to read exactly n books.) Clearly, the maximal value for the predicate is undefined in such a case. This means that the logical form based on many$_2$ will not lead to a sensible interpretation and, so, we again do not expect to find ambiguity.

The case of predicates that are formed by abstracting over an existential modal operator is illustrated in (39) and (40). If Jasper is allowed to read a maximum of 10 books, then the two predicates are equivalent, both denoting the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.[9]

(39) $\lambda d.\Diamond\exists x[\#x = d \;\&\; \text{book}(x) \;\&\; \text{read}(j, x)]$

(40) $\lambda d.\Diamond\exists! x[\#x = d \;\&\; \text{book}(x) \;\&\; \text{read}(j, x)]$

In sum, the option of two counting quantifiers *many*$_1$ and *many*$_2$ is irrelevant when combined with a comparative quantifier. This is because the compara-

9 If there is in addition a lower bound, the two predicates are no longer equivalent, but their maximum will be.

tive quantifier is based on maximality and the degree predicates containing the different counting quantifiers do not differ in their maximum value.

4 The semantics of class B quantifiers

I now turn to the main proposal: class B quantifiers are maxima/minima indicators. I start with the upper-bounded modifiers.

4.1 Upper bound class B modifiers

In the formula in (41), $\mathrm{MOD}^{\downarrow}_{B}$ generalises over any of the class B modifiers *at most*, *maximally*, *up to*, etc.[10]

(41) $[\![\mathrm{MOD}^{\downarrow}_{B}]\!] = \lambda d.\lambda M.\, \max_n(M(n)) = d$

If the semantics of upper bound class B quantifiers is as in (41), then why is their distribution so limited? What I think is the reason for the awkwardness of a lot of examples with class B quantifiers is the fact that, in many cases, (41) is a vacuous operator. To be precise, the two propostions in (42) are equivalent whenever the cardinality predicate M denotes a singleton set. In such a case, a bare numeral form is to be preferred over a numeral modified by a class B modifier, since the latter derives the same meaning from a much more complex linguistic form.

(42) a. $\max_n(M(n)) = d$
b. $M(d)$

What I have in mind exactly is the kind of reasoning underlying Horn's division of pragmatic labour (Horn 1984). The idea is that a maxim of brevity,

10 For modifiers like *at most* and *maximally*, one might wonder whether (41) is not too restricted, given that they are capable of modifying DPs more generally. However, it appears that there is a common mechanism to all uses of such modifiers. For instance, (i) could be assigned its intuitive meaning if we assume that *at most* has the semantics in (ii), where the operator 'max' compares properties on the rank order [assistant professor < associate professor < full professor]:

(i) Jasper is at most an associate professor.

(ii) $[\![\text{at most}]\!] = \lambda P.\lambda x.\max_{P'}(P'(x)) = P$

It goes beyond the scope of this article to implement a formal connection between (ii) and (41), but it should be clear that the underlying mechanism is the same.

part of Grice's maxim of Manner (Grice 1975), steers toward minimising the form used to express something. This causes simple (unmarked) meanings to be typically expressed by means of simple (unmarked) forms. Marked forms which by convention could be given the same unmarked meaning as some unmarked form are instead given a more marked interpretation. There are many variations and implementations of this idea (McCawley 1978; Atlas & Levinson 1981; Blutner 2000; van Rooij 2004),[11] but what is most relevant for this paper is the general idea that an unmarked meaning is blocked as an interpretation for the marked form.

With this in mind, the equivalence of (42a) and (42b) whenever M denotes a singleton set has profound consequences for when it actually makes sense to state that the maximum of a degree predicate equals a certain value. That is, in cases where (42a) equals (42b), we expect that the use of *maximally* does not lead to an interpretation based solely on (42a), since the use of the bare numeral form would result in the same meaning. To illustrate this in some more detail let us carefully go through the following examples.

We know from the discussion above that one of the interpretations available for (43) is (44).

(43) Jasper invited 10 people.

(44) $\exists! x[\#x = 10 \;\&\; \text{people}(x) \;\&\; \text{invite}(x)]$

Now consider (45), which is interpreted either as (46) or as (47).

(45) Jasper invited maximally 10 people.

(46) [maximally 10 [λd [Jasper invited d many$_1$ people]]]
$\rightsquigarrow \max_n(\exists x[\#x = n \;\&\; \text{people}(x) \;\&\; \text{invite}(j, x)]) = 10$

(47) [maximally 10 [λd [Jasper invited d many$_2$ people]]]
$\rightsquigarrow \max_n(\exists! x[\#x = n \;\&\; \text{people}(x) \;\&\; \text{invite}(j, x)]) = 10$

The interpretations in (46) and (47) are equivalent. In fact, just like we do not expect ambiguities to arise with comparative quantifiers on the basis of the *many*$_1$/*many*$_2$ choice, we do not expect any ambiguities to arise with $\text{MOD}_B^{!}$ quantifiers, for the simple reason that both such operators involve

11 In fact, there is a close resemblance between this prevalent idea in pragmatics and blocking principles in other parts of linguistics. The commonality is that two different expressions cannot have identical meanings. See, for instance, the Elsewhere Condition (Kiparsky 1973) in phonology or the Avoid Synonymy principle (Kiparsky 1983) in morphology.

a maximality operator and that the maximal values of predicates based on $many_1$ are always those of predicates based on $many_2$. In what follows, we will therefore gloss over the two equivalent options by representing the semantics following the general scheme in (48).

(48) [maximally 10 [λd [Jasper invited d $\text{many}_{1/2}$ people]]]
$\rightsquigarrow \max_n(\exists(!)x[\#x = n \;\&\; \text{people}(x) \;\&\; \text{invite}(j, x)]) = 10$

Importantly, the single reading of (45) is equivalent to (44), the strong reading of (43). The example in (43), however, reaches this interpretation by means of a much simpler linguistic form, one which does not involve a numeral modifier. I propose that this is why the reading in (48) of (45) does not surface: it is blocked by (43).[12]

As observed above, we can nevertheless make sense of (45) once we interpret the sentence to be about what the speaker holds possible. So, a further possible reading for (45) is that in (49).

(49) $\max_n(\Diamond\exists(!)x[\#x = n \;\&\; \text{people}(x) \;\&\; \text{invite}(j, x)]) = 10$

Crucially, this interpretation is not equivalent to (50), which is the result of interpreting (43) from the perspective of speaker possibility.

(50) $\Diamond\exists!x[\#x = 10 \;\&\; \text{people}(x) \;\&\; \text{invite}(j, x)]$

12 An anonymous reviewer notes two complications with the proposed blocking mechanism. First of all, s/he wonders why *exactly 10* is not blocked in a similar way to *minimally 10*, since the same reasoning seems to apply. I acknowledge that this is something that needs to be explained. Interestingly, this is something any theory that believes in the existence of an 'exactly' sense for numerals has to explain. One promising route has been proposed by Geurts (2006), who suggests that *exactly* is semantically empty and that its only function is "to reduce pragmatic slack" (p. 320). That is, whereas bare *100* allows for an imprecise rough construal (Krifka 2007a), *exactly 100* enforces precision. If Geurts is on the right track, then there is no reason to expect that *exactly 100* is blocked by *100*.

A further complication noted by the same anonymous reviewer is that if we assume that the 'max' operator is presuppositional, we might come to expect that *maximally 100* blocks *100* instead of the other way around. This prediction appears to be made when at the same time we assume the *Maximize Presupposition* principle (Heim 1991). Since *maximally 100* and *100* share the same meaning, but the former triggers a presupposition, the use of *100* would be blocked. This is a very interesting scenario, but since I have little to say about the kind of presuppositions (if any) expressions like *maximally* trigger and I furthermore have no thoughts on how *maximize presupposition* would interact with a brevity maxim, I will leave this issue to further research.

In other words, the meaning in (49) for (45) is not blocked by the bare numeral form in (43) since (43) lacks this reading.

To be sure, I do not claim that (50) would be an available reading for (43). That is, the particular kind of interpretation that examples like (45) receive is available only as a last resort strategy. Underlying this analysis is the assumption that there exist silent modal operators. I can offer no independent evidence for this assumption, but stress that the intuitions regarding examples like (45) quite clearly point into the direction of some sort of speaker modality. In work on superlative quantifiers, we find some alternatives to the present account. Such approaches are meant to deal with *at most* and *at least* only, but if my arguments above are on the right track, then we could reinterpret these proposals for the semantics of superlative quantifiers as applying to the whole of class B. For instance, the analysis of class B expressions presented here differs from that of superlative modifiers in Geurts & Nouwen 2007. According to the present proposal, the modal flavour of (45) is due to a silent existential modal operator. In Geurts & Nouwen, however, the modal was taken to be part of the lexical content of superlative quantifiers. Another alternative, proposed for superlative modifiers in Krifka 2007b and which is closer to the present proposal, is to analyse examples like (45) not as involving a modal operator, but rather a speech act predicate, like *assert*. In that framework, the analysis of (45) would say that n=10 is the maximal value for which $\exists(!)x[\#x = n \,\&\, \text{people}(x) \,\&\, \text{invite}(j, x)]$ is assertable, rather than possible.[13] That is, according to Krifka, (45) is interpreted by assigning the modified numeral scope over an illocutionary force operator, rather than over a modal operator.

I will return to a comparison of these approaches below. I would like to point out immediately, however, what I think are the major disadvantages of both alternatives. The main problem is with examples like (51), which contain an overt existential modal.

(51) Jasper is allowed to invite maximally/at most 10 people.

13 In his comments on the first version of this paper, David Beaver observed that it it is not necessarily the speaker's knowledge that matters, as can be seen from (his) example (i).

(i) I know how many people were at the party, but I've been told not to reveal that number to the press. However, there were maximally 50 there.

It would be interesting to see if data like these help in reaching a synthesis of Krifka's account and the present proposal.

Its most salient reading is one in which 10 is said to be the maximum number of people Jasper is allowed to invite. That is, it places an upper bound on what is allowed. For Krifka, this is problematic since, here, the modified numeral is quite obviously not a speech act operator. For the proposal in Geurts & Nouwen 2007, such examples are problematic since the modal lexical semantics of *at most* predicts a reading with a double modal operator, one originating from the verb and one from the numeral modifier. To remedy this, Geurts and Nouwen provide an essentially non-compositional analysis of such examples as *modal concord*.[14]

In contrast, the current proposal deals effortlessly with examples, such as (51). What was crucial to my explanation of how (45) gets to be interpreted is that degree predicates based on modals with existential force denote non-singleton sets even when the counting quantifier associated with the numeral is *many*$_2$. This entails that saying that the maximum value for such a predicate is n is not equivalent to saying that the predicate holds for n. More formally, there is a contrast between (52a) and (52b).

(52) a. $\max_n(\exists(!)x[\#x = n \;\&\; \text{people}(x) \;\&\; \text{invite}(j, x)]) = 10$
$\Leftrightarrow\ \exists!x[\#x = 10 \;\&\; \text{people}(x) \;\&\; \text{invite}(j, x)])$

b. $\max_n(\Diamond\exists(!)x[\#x = n \;\&\; \text{people}(x) \;\&\; \text{invite}(j, x)]) = 10$
$\not\Leftrightarrow\ \Diamond\exists!x[\#x = 10 \;\&\; \text{people}(x) \;\&\; \text{invite}(j, x)])$

As a result, whenever an upper bound class B modifier scopes over an existential modal, no blocking from the simpler bare numeral form will be able to take place. The application of an upper bound class B quantifier to a degree predicate is only felicitous if the resulting readings are not readings that can be expressed just as well by omitting the class B modifier. This is the case when a modal with existential force has scope inside the degree

14 A further problem I see with the proposal in Krifka 2007b is that the analysis does not appear to extend straightforwardly to illocutionary forces other than assertion, although in fairness this might be because (at the time of writing) no detailed exposition of this theory exists. For instance, nothing suggests that superlative modified numerals can scope over a question operator in questions.

An additional disadvantage for the proposal of Geurts and Nouwen is that it does not yield an explanation of the lexical form of class B modifiers. Whereas the current proposal assigns to a modifier like *maximally* the semantics of a maximality operator, an extension of Geurts and Nouwen's approach would have to take it to be a modal, thereby disassociating it from the intuitive meaning of *maximal*.

predicate.[15] Treating upper bound class B quantifiers as maxima indicators thus also predicts the absence of weak readings for examples like (51). Given the flexible scope of the numeral modifier we expect this sentence to have two corresponding logical forms, (54a) and (54b). (From here on, $\diamonddot$ indicates deontic modality, to distinguish it from the (epistemic) speaker possibility $\Diamond$).

(53) Jasper is allowed to invite maximally/at most 10 people.

(54) a. $\max_n(\diamonddot\exists(!)x[\#x = n \,\&\, \text{people}(x) \,\&\, \text{invite}(j,x)]) = 10$
b. $\diamonddot[\max_n(\exists(!)x[\#x = n \,\&\, \text{people}(x) \,\&\, \text{invite}(j,x)]) = 10]$

If *maximally 10* is taken to have wide scope over the modal, then we arrive at (54a), the reading that says that the maximum number of people Jasper is allowed to invite equals 10. This is not a semantic interpretation that is available for (55). Its many$_2$ reading, for instance, says that inviting exactly 10 people is something that Jasper is allowed to do. This is much weaker than (54a). (The only way we can arrive at an equally strong reading for (55) is by means of implicature.)

(55) Jasper is allowed to invite 10 people.

If we take the modal in (51) to have widest scope, as in (54b), the resulting interpretation is one in which inviting exactly 10 people is allowed for Jasper. This is the reading for (55) discussed above, and so it is blocked. As a result, (54a) is the only interpretation available.

An interesting side to the account presented here is that the upper bound class B quantifiers do not encode the ≤ relation. As maxima indicators, their application only makes sense if what they apply to denotes a range of values. Otherwise, using the strong reading of the bare numeral form will do just as well.

Interestingly, the approach also predicts that some of the examples I discussed above do not only result in a blocking effect, but could moreover be predicted to be false. For instance, according to the approach set out above, the meaning of (56a) is that in (56b).

15 As far as I can see, assertability would have the same (crucially weak) properties as possibility. So, should a silent speech act predicate seem more plausible than a silent modal operator, then $\Diamond$ can just as well be interpreted as expressing assertability. It appears that such a move would be largely compatible with the proposal of Krifka 2007b.

(56) a. #A triangle has maximally 10 sides.
b. '*the maximum number of sides in a triangle is 10*'

The reading in (56b) is not only blocked by *A triangle has 10 sides*, but it is moreover plainly false. I believe that this predicts that (56a) should be expected to have a somewhat different status from (57), which strictly speaking has a true interpretation, but one that can be expressed by simpler means.

(57) #A triangle has maximally 3 sides.

It is difficult to establish whether this difference in status is borne out, or even how this difference can be recognised. However, my own intuition tells me that while (56) is never acceptable, (57) could be used in a joking fashion. Native speakers inform me that (58) is marginally acceptable:

(58) ?A triangle has minimally and maximally 3 sides.

4.2 Lower-bound class B modifiers

Lower-bound class B modifiers correspond to minimality operators. Let $\mathrm{MOD}_B^{\uparrow}$ correspond to any of the class B expressions *at least*, *from*, *minimally*, etc.

(59) $[\![\mathrm{MOD}_B^{\uparrow}]\!] = \lambda d.\lambda M.\ \mathrm{min}_n(M(n)) = d$

Note first that minimality operators are sensitive to the $many_1$ / $many_2$ distinction. Consider the degree predicate [λd. John read d $\mathrm{many}_{1/2}$ books] and, say, that John read 10 books. In the $many_1$ version of the logical form, the minimal degree equals 1. In fact, independent of how many books John read, as long as he read books, the minimal degree will *always* be 1. In the many_2 version of the logical form, the predicate denotes a singleton set, $\{10\}$ if John read 10 books. The minimal degree in that case is, of course, 10.

These observations already straightforwardly account for our intuitions for an example like (60).

(60) John read minimally 10 books.

The $many_1$ interpretation of (60) will be rejected, for it will always be false. The minimal value for any simple $many_1$-based degree predicate is always 1. The many_2 interpretation of (60) will be rejected too, for it will correspond to an interpretation saying that John read (exactly) 10 books. This reading is blocked by the bare numeral. (In fact, (60) in the many$_2$ variant is equivalent to *John read maximally 10 books*, which, as was explained above, is blocked for the same reasons.)

We can save (60) by interpreting it with respect to an existential modal operator. This yields two readings:

(61) a. $\min_d(\Diamond\exists x[\#x = d \;\&\; \text{read}(j, x) \;\&\; \text{book}(x)]) = 10$
b. $\min_d(\Diamond\exists! x[\#x = d \;\&\; \text{read}(j, x) \;\&\; \text{book}(x)]) = 10$

The form in (61a) is once more a contradiction: the minimal degree for which it is deemed possible that John read d-many$_1$ books is always 1. The reading in (61b) is much more informative. It says that that the minimal number for which it is thought possible that John read exactly so many books is 10. In other words, this says that it is regarded as impossible that John read fewer than 10 books. This is exactly the reading that is available.

4.3 Beyond modals

Some words are in order on the interaction of numeral modifiers with non-modal operators. Given the current proposal, any property that involves existential quantification would license the use of a class B modifier. However, it is known that degree operators (which we take modified numerals to be) cannot move to take scope over nominal quantifiers (cf. Kennedy 1997; Heim 2000).[16] This explains why (62) does not have the reading in (63).

(62) Someone is allowed to invite maximally 50 friends.

(63) the person who is allowed to invite most friends is allowed to invite 50 friends

As observed above, however, bare plurals do interact with class B quantifiers, as in for instance example (9). This would suggest that some intensional/modal analysis of the readings involved in such examples is in order. (Thanks to Maribel Romero for pointing this out to me.) I will leave a detailed analysis of these cases for further research.

16 In Heim's formulation: If the scope of a quantificational DP contains the trace of a degree phrase, it also contains that degree phrase itself. See Heim 2000 for details.

5 Maximal and minimal requirements

As Hackl (2001) observed, there is an interesting interaction between modified numerals and modals. I have extended these observations by showing how existential modals have a tight connection to class B modifiers in that they license their (otherwise blocked) existence. What I have not discussed so far is how class B modifiers interact with universal modals. It turns out that this part of the story is not straightforward at all.

Given my proposal in the previous section, we expect that there are in principle four logical forms that correspond to (64).[17]

(64) Jasper should read minimally 10 books.

(65) $\boxdot >$min:
The minimum n such that Jasper will read n books should be 10
a. $\boxdot[\text{min}_n(\exists x[\#x = n \,\&\, \text{book}(x) \,\&\, \text{read}(j,x)]) = 10]$ many$_1$
b. $\boxdot[\text{min}_n(\exists! x[\#x = n \,\&\, \text{book}(x) \,\&\, \text{read}(j,x)]) = 10]$ many$_2$

(66) min$> \boxdot$:
The minimum n such that Jasper should read n books is 10
a. $\text{min}_n(\boxdot\exists x[\#x = n \,\&\, \text{book}(x) \,\&\, \text{read}(j,x)]) = 10$ many$_1$
b. $\text{min}_n(\boxdot\exists! x[\#x = n \,\&\, \text{book}(x) \,\&\, \text{read}(j,x)]) = 10$ many$_2$

It turns out that none of these logical forms provide a reading that is in accordance to our intuitions regarding (64). First of all, notice that $\text{min}_n(\exists x[\#x = n \,\&\, \text{book}(x) \,\&\, \text{read}(j,x)]) = 10$ is a contradiction. If there are 10 books that Jasper read, then there is also a singleton group containing a book Jasper read. The minimum number of books Jasper read is therefore either 1 (in case he read something) or 0 (in case he did not read anything). It could never be 10. Consequently, (65a) is a contradiction. For a similar reason, (66a) is a contradiction too. If there needs to be a group of 10 books

17 In this paper, I ignore readings which (for the case of *at least*) Büring (2008) calls *speaker insecurity* readings and which Geurts & Nouwen (2007) discuss extensively. Basically, this reading amounts to interpreting the modal statement with respect to speaker's knowledge. Such readings are especially prominent with superlative quantifiers. For instance, the speaker insecurity reading of *Jasper should read at least 10 books* is: the speaker knows that there is a lower bound on the number of books that Jasper should read, s/he does not know what that lower bound is, but she does know that it exceeds 9.

Furthermore, I also ignore a reading of (64) in which *10 books* is construed as a specific indefinite. In that reading, (64) states that there are 10 specific books such that only if Jasper reads these books will he comply with what is minimally required.

read by Jasper, then there also need to exist groups containing just a single book read by Jasper. Once again, the minimum number referred to in (66a) is either 0 or 1, never 10.

Turning to (65b), notice that the $\min_n$-operator is vacuous here, since there is just a single n such that Jasper read exactly n books. This renders (65b) equivalent to the many_2 reading of *Jasper should read 10 books*, and so we predict it to be blocked. The interpretation in (66b) does not fare any better. In fact, the $\min_n$-operator is vacuous here as well. This means that (65b) is equivalent to (66b) and that it is consequently also blocked. Even if no blocking were to take place, (65b)/(66b) offer the wrong interpretation anyway. They state that Jasper must read exactly 10 books (no more, no fewer), which is not what (64) means.

One might think that the problems with (65b) and (66b) can be remedied by abandoning quantification over sums and instead using reference to (maximal) sums. For instance, (67) represents the truth-conditions we are after. (Here σ_x returns the maximal sum that when assigned to x verifies the scope of σ).

(67) $\min_n(\boxdot[\#\sigma_x(\text{book}(x)\ \&\ \text{read}(j,x)) \geq n]) = 10$

Still, here too the application of $\min_n$ is not meaningful, since there is only a single n such that $\boxdot[\#\sigma_x(\text{book}(x)\ \&\ \text{read}(j,x)) \geq n]$ holds, which is 10 if (64) is true. As a consequence, it would not matter whether we applied a maximality or a minimality operator. We then wrongly predict that (68) should share a reading with (64). (Note that (65b) and (66b) suffer from the same odd prediction, given that the operator $\min_n$ has no semantic impact there either.)

(68) Jasper should read maximally 10 books.

It appears then that the proposal defended in this article fails hopelessly on sentences like (64). As I will show, however, things are not so dire as they appear. In fact, I will argue that what we stumble upon here is a general, but poorly understood property of modals, which could be summarised as follows:

(69) Generalisation: universal modal operators are interpreted as operators with existential modal force when minimality is a stake

An illustration of (69) is (70), which is a satisfactory paraphrase for (64).

What is striking is that this paraphrase contains *allow* instead of *should*.

(70) 10 is the smallest number of books John is allowed to read

I will not offer an explanation for this generalisation (but see Nouwen 2010a for an attempt). I will simply show that if we look a bit closer at the interpretation of modal operators, then we come to understand that my theory actually yields a welcome analysis.

5.1 Previous analyses

There is a precedent. In an earlier theory of *at least*, Geurts & Nouwen 2007, the correct predictions regarding its relation to universal modals are arrived at by an essentially non-compositional mechanism. A central claim made in that paper is that superlative quantifiers are modal expressions themselves. For instance, (71a) was proposed to correspond to (71b).[18] Furthermore, it was assumed that there may be a non-compositional interaction between the modal that is implicitly contributed by a modified numeral and an explicit modal operator. For instance, (72a) is interpreted as an instance of *modal concord*, as in (72b), where the two modals fuse and the modal takes on the deontic flavour of *need*.[19]

(71) a. John read at least 10 books.
  b. $\Box\exists x[\#x = 10 \,\&\, \text{book}(x) \& \,\text{read}(j, x)]$

(72) a. John needs to read at least 10 books.
  b. $\boxdot\exists x[\#x = 10 \,\&\, \text{book}(x) \& \,\text{read}(j, x)]$

18 This is how I see the theoretical landscape: Although not immediately obvious, the proposal by Geurts and Nouwen already carries in it the idea that superlative quantifiers are minimality and maximality operators. For instance, (71b) is equivalent to stating that 10 is the minimal number of books John is allowed to read. Given the basic idea of treating class B operators as min/max-operators, one has a range of options to account for the distribution of such quantifiers and for their behaviour in intensional contexts. Geurts and Nouwen represent one extreme, where the lexicon specifies the exact behaviour of such quantifiers (together with the rule of modal concord). The present proposal puts forward the other extreme, where the lexical entry for superlative (and other class B) quantifiers is rather minimal, and where pragmatic mechanisms account for distribution and behaviour in intensional contexts.

19 I am simplifying the analysis here a little bit. Geurts & Nouwen (2007) propose that there is an additional conjunct to the meaning of sentences containing superlative quantifiers, for which they leave implicit whether it is entailed or implicated. For (71), for instance, there would be an additional condition in the truth-conditions saying: $\neg\Box\exists x[\#x > 10 \& \text{ book}(x) \,\&\, \text{read}(j, x)]$. Similarly for (72).

The approach of Geurts and Nouwen is the most broadly applicable approach to superlative quantifiers in the (admittedly small body of) literature on that topic. There are alternatives on the market, but they do not handle examples like these very well. As I mentioned above, Krifka (2007b) takes *at least* to be a *speech act modifier*. Basically, an example like (71) is analysed by Krifka in terms of what the speaker finds assertable and is paraphrased as follows: the lowest n such that it is assertable *that John read* n *books* is 10. When *at least* is embedded in an intensional context, however, it does not modify the strength of assertability, but rather the intensional operator. So, taking Krifka's analysis as suitable not just for superlative, but rather for all class B quantifiers, (72a) would be paraphrased as (73).

(73) 10 is the smallest value for n such that John should read n books

In such cases, Krifka's analysis is identical to the one I have set out above and it runs in exactly the same problem: (73) is not the reading we are after. Rather, (72a) means that 10 is the smallest number of books John is *allowed* to read.

5.2 Minimal requirements

Geurts & Nouwen (2007) and Krifka (2007b) say nothing about the distinction between class A and class B expressions. However, if we extend their proposals for superlative quantifiers to cover all B-type quantifiers, then we have an interesting trio of competing characterisations of such expressions. At face value, the observations made so far in this section would appear to speak in favour of the modal concord proposal of Geurts & Nouwen (2007) (generalised to all class B quantifiers) and against the account defended here or in Krifka 2007b. As I will argue now, however, there are reasons to believe that the problematic predictions made by the latter two theories are not due to the semantics of the modified numeral, but are actually the result of an overly simplistic understanding of requirements. What I will do is discuss in some detail examples like (74).

(74) The minimum number of books John needs to read to please his mother is 10.

Notice, first of all, that on an intuitive level, (74) is equivalent to (75).

(75) John needs to read minimally 10 books to please his mother.

Note, secondly, that (74) spells out the semantics I have proposed for (75). What I will show now is that when we look into the semantic details of (74), we will run into exactly the same problems as we did for (75). What this shows is that rather than thinking that my account of class B quantifiers is on the wrong track, there are actually reasons to believe that the proposal lays bare a hitherto unexplored problem for the semantics of modals like *need*, *require*, etc.

Let us consider the semantics of (74). Say that, in fact, the minimal requirements for pleasing John's mother are indeed John reading 10 books. That is, if John reads 10 or more books, she is happy. If he reads fewer, she will not be pleased. Standard accounts of goal-directed modality (von Fintel & Iatridou 2005) assume that statements of the form *to q, need to p* are true if and only if p holds in all worlds in which the goal q holds. Below, I refer to the worlds in which John pleases his mother as *the goal worlds*. It is instructive to see what we know about the propositions that are true in such worlds. The following is consistent with the context described above.

(76) a. In all goal worlds: $\exists x[\#x = 10 \,\&\, \text{book}(x) \,\&\, \text{read}(j,x)]$
b. In all goal worlds: $\exists x[\#x = 9 \,\&\, \text{book}(x) \,\&\, \text{read}(j,x)]$
c. In all goal worlds: $\exists x[\#x = 1 \,\&\, \text{book}(x) \,\&\, \text{read}(j,x)]$
d. In some (not all) goal worlds: $\exists x[\#x = 11 \,\&\, \text{book}(x) \,\&\, \text{read}(j,x)]$
e. In some (not all) goal worlds: $\exists x[\#x = 12 \,\&\, \text{book}(x) \,\&\, \text{read}(j,x)]$
f. In no goal world: $\neg\exists x[\text{book}(x) \,\&\, \text{read}(j,x)]$

Let us now analyse some examples. First of all, (77a) and (77b) are intuitively true and are also predicted to be true ((77a) by virtue of (76a) and (77b) by virtue of (76c).)

(77) a. To please his mother, John needs to read 10 books.
b. To please his mother, John needs to read a book.

The example in (78) is intuitively false, and is also predicted to be false, for the context is such that there are goal worlds in which John reads only 10, and not 11, books.

(78) To please his mother, John needs to read 11 books.

So far, so good. If we turn to examples that place a bound on what is required, however, then the theory makes a wrong prediction. The example in (79) is intuitively false. If interpreted as (80), however, it is predicted to be true (by virtue of (76c)).

(79) The minimum number of books John needs to read, to please his mother, is 1.

(80) $\min_n[\text{In all goal worlds: } \exists x[\#x = n \;\&\; \text{book}(x) \;\&\; \text{read}(j, x)]] = 1$

In general, theories such as that of von Fintel & Iatridou (2005) predict that if S is an entailment scale of propositions, and p is a proposition on this scale, then if p is a minimal requirement for some goal proposition q, then a statement of the form "the minimum requirement to q is p" is always predicted to be false, except when p is the minimal proposition of S. This makes a devastating prediction, namely that minimal requirements could never be expressed, since they would always correspond to the absolute minimum.

One might think that what is going wrong in the example above is that I assume that when we talk about how many books John read we should be talking about existential sentences, that is about *at least* how many books John read. The alternative would be to describe the number of books John read by means of the counting quantifier *many*$_2$, that is, how many books John read *exactly*. I'm afraid this only makes the problem worse. Here is a description of the relevant context in terms of the exact number of books that were read by John.

(81) a. In some but not all goal worlds: John read exactly 10 books.
b. In no goal world: John read exactly 9 books.
c. In no goal world: John read exactly 1 book.
d. In some but not all goal worlds: John read exactly 11 books.
e. In some but not all goal worlds: John read exactly 12 books.

Now, there is no number n such that John read exactly n books in all goal worlds. So, *the smallest number of books John needs to read* does not refer.

The upshot is that there is no satisfactory analysis of examples like (74) under the assumptions made here. In general, it seems that, under standard assumptions, there is no satisfactory analysis of minimal requirements. Whatever way we find to fix the semantics of cases like (74), however, this fix will work to save the account of class B quantifiers too, for (74) was a literal spell-out of the proposed interpretation of similar sentences with *at least*, *minimally*, etc. It goes beyond the scope of this article to provide such a fix. The overview in (81), however, can help to indicate where we should look for a solution.[20] Given that there is no goal world in which John read exactly n books for n's smaller than 10, it follows that 10 is the minimal number of books John *could* read to please his mother. In other words, examples like (74) show that, in the scope of a minimality operator, modals that are lexically universal quantifiers get a weaker interpretation.

That said, it is time to revisit example (64), repeated here as (82).

(82) Jasper should read minimally 10 books.

My proposal generated four logical forms, two of which were contradictory and two of which were blocked by a non-modified form. Let us revisit one of these logical forms, namely the one with a narrow scope modal and a doubly bound counting quantifier, represented in (83). The resulting truth-conditions were presented above as (84).

(83) [minimally 10 λn [should [Jasper read [n-many$_2$ books]]]]

(84) $\min_n(\boxdot\exists!x[\#x = n \;\&\; \text{book}(x) \;\&\; \text{read}(j,x)]) = 10$

What the discussion in the current section suggests is that it is a misunderstanding to assume that (83) is interpreted as (84), and that it looks like there is a mapping to a form like (85), instead.

(85) $\min_n(\diamondsuit\exists!x[\#x = n \;\&\; \text{book}(x) \;\&\; \text{read}(j,x)]) = 10$

This captures the intuitive meaning of (82).

At this point I do not have anything to offer which provides the mechanism behind the generalisation that the combination of a universal modal and a minimality operator leads to a semantics which is existential in nature. What is relevant for the present purposes is that this is a general phenomenon. Interestingly, this means there are noteworthy connections to other areas where the semantics of a modal statement appear mysterious. Schwager (2005), for instance, notices that certain imperatives, which are standardly considered to have universal modal force, require a weaker semantics. Her key examples are German imperatives containing *for example*.

(86) Q: How can I save money?
A: *Kauf zum Beispiel keine Zigaretten!*
Buy for instance no cigarettes
"For example, don't buy any cigarettes!"

20 See Nouwen 2010a for a proposal along these lines.

In the context of the question asked in (86), the imperative does not convey that to comply with the advice, the hearer has to stop buying cigarettes. Instead, it is interpreted as stating that one of the things one *could* do to save money is to stop buying cigarettes. Thus, examples like these display a mechanism that is similar to the interaction of numeral modifiers and modality.

The mysterious interaction of modified numerals and modals is moreover reminiscent of the interaction of modals and disjunction (Zimmermann 2000; Geurts 2005; Aloni 2007), especially since, on an intuitive level at least, a class B modified numeral like *minimally 10* (and, quite obviously, *10 or more*) appears to correspond to a disjunction of alternative cardinalities, with 10 as the minimal disjunct.[21] A central issue in the literature on modals and disjunction is that classical semantic assumptions fail to capture the entailments of sentences where a disjunctive statement is embedded under a modal operator (Kamp 1973). A detailed comparison of this complex issue with the discussion of minimal requirements that I presented here, however, will be left to further research.

6 More about the A/B distinction

In this section, I will attempt to give some initial answers to three empirical questions concerning the distinction between class A and B modified numerals that is central to this article. First of all, I turn to the issue of which expressions go with which class. So far, I have restricted my attention mostly to, on the one hand, comparative quantifiers (as proto-typical class A expressions) and, on the other hand, superlative, minimality/maximality and *up to*-modified numerals (as representatives of class B). What about expressions like the prepositional *over n* or *under n* or the double bound *between n and m* or *from n to m*? Below, I will turn briefly to such expressions.

A second empirical question concerns the validity of the examples used so far. Although I believe that the intuitions concerning the constructed examples in this article are rather clear, my plea for two kinds of modified numerals would still benefit from some independent objective support. Below, I present the results of a small corpus study that clearly reflects the distinction argued for in this article.

Finally, this section will turn to the cross-linguistic generality of the

21 See Nilsen 2007 and Büring 2008 for suggestions along this line for the modifier *at least* only.

proposal. I will provide data from a more or less random set of languages that suggest that the class A/B distinction is not a quirk of English or Germanic, or even Indo-European, but is, in fact, quite general.

6.1 Filling in class A and B

I will leave it an open question exactly which quantifiers belong to which class. Nevertheless, I can already offer some speculations on several quantifiers that I have so far not discussed. To start with disjunctive quantifiers, it appears that these are clear cases of class B expressions.

(87) a. #A triangle has 3 or more sides.
b. #A triangle has 3 or fewer sides.

With disjunctive quantifiers in class B, one might wonder whether there are any examples of class A expressions which are not the familiar comparative quantifiers *more/fewer/less than n*. I think that locative prepositional modifiers are a likely candidate for class A membership, however. In fact, I believe that the locative/directional distinction in spatial prepositions corresponds to the class A/B distinction when these prepositions are used as numeral modifiers.

Roughly, locative prepositions express the location of an object and are compatible with the absence of directionality or motion. Directional prepositions, on the other hand, cannot be used as mere indicators of location.

(88) Locative:
a. John was standing under a tree.
b. That cloud is hanging over San Francisco.
c. Breukelen is located between Utrecht and Amsterdam.

(89) Directional:
a. #John was standing up to here.
b. #John was standing from here.
c. #Breukelen is located from Utrecht to Amsterdam.

Now, compare (90a) and (90b).

(90) a. You can get a car for under €1000.
b. You can get a car for maximally €1000.

The example in (90b) is somewhat strange, since it claims that the most expensive car you can buy is €1000. The example in (89a), in contrast, makes no such claim. It clearly has a weak reading: there are cars that are cheaper than €1000 and there might be more expensive ones too. As explained above, such weak readings are typical for class A quantifiers and do not occur with class B quantifiers.[22] Furthermore, *under* seems perfectly compatible with definite amounts, such as in (91).

(91) The total number of guests is under 100. To be precise, it's 87.

Class A is then not restricted to comparative constructions only. In fact, other locative prepositions seem to behave similarly to *under*.

(92) The total number of guests is between 100 and 150. It's 122.

The locative complex preposition *between ... and ...* contrasts with its directional counterpart *from ... (up) to ...*, which behaves like a class B modifier: it is incompatible with definite amounts, as in (93), but felicitous if it relates to a range of values.

(93) #The ticket to the Stevie Wonder concert that I bought yesterday cost from €100 to €800.

(94) Tickets to the Stevie Wonder concert cost from €100 to €800.

It appears then that locative prepositions turn into class A modifiers, while directional ones turn into class B modifiers. A potential counterexample, however, is *over*, which apart from a (relatively rarely used) locative sense, as in (88b), has a directional sense, such as exemplified in (95).

(95) The bird flew over the bridge.

As a numeral modifier, however, *over* looks like a class A element. In (96), *over 100* is clearly relating the precise weight 104kg with 100kg. Note in (97) how this contrasts with the directional *100 ... and up*, which is made

22 An anonymous reviewer notes a complication. It appears that *under* cannot take wide scope with respect to a modal. That is, it fails to display scope ambiguities such as the one in (20) above. For instance, (i) (which is an example given by the reviewer) is odd, since it misses an interpretation where the modified numeral has scope over *require*.

(i) #John is required to come up with under 6 brilliant ideas.

felicitous by embedding it under an existential modal.

(96) He weighs over 100 kg. To be precise, he weighs 104 kg.

(97) a. #He weighs 100 kg and up.
b. He is allowed to weigh 100 kg and up.

A potential explanation for why the numeral modifier *over* lacks a directional/class B sense[23] is that the use of prepositions in numeral quantifiers is restricted to prepositions that are vertically oriented. This is connected to the observation of Lakoff & Johnson 1980 that cardinality is *metaphorically* vertical: more is higher (as in *a high number*), less is lower (as in *a low number*). Prepositions in modified numerals follow this metaphor.[24] What is interesting about *over*, however, is that only its locative sense is vertical. Its directional sense, as in (95), rather expresses a mainly horizontal motion. This could explain why there is no class B sense numeral modifier *over*.

Further clues that this analysis is on the right track come from Dutch, where the preposition *over* lacks a locative sense.

(98) *#De wolk hangt over San Francisco.*
The cloud hangs over San Francisco.

(99) *De vogel vloog over de brug.*
The bird flew over the bridge.

Instead of *over* in (98), *boven* (above) should be used for locative meanings.

(100) *De wolk hangt boven San Francisco.*
The cloud hangs above San Francisco.
'*The cloud hangs over San Francisco.*'

In Dutch, only *boven* can modify numerals. *Over*, which lacks a vertical sense, is unacceptable in modified numerals.

(101) *Inflatie kan {boven / #over} de 10% zijn.*
Inflation can {above / over} the 10% be.
'*Inflation can be over 10%*'

23 Thanks to Joost Zwarts for discussing this matter with me.

24 *Up (to)* and *under* are clearly vertical. *Between* and *from . . . to* are compatible with all possible axes.

I will refrain from attempting to offer further evidence for my suggestion that there is a correspondence between the locative/directional and the A/B distinction. In any case, it should be clear that the set of prepositional quantifiers offers an interesting range of contrasts that support the existence of two classes of modified numerals.

To summarise this subsection, I tentatively put forward the following classification for English modified numerals.

(102) Class A

(Positive:) *more than —*, *over —*
(Negative:) *fewer than —*, *less than —*, *under —*
(Neutral:) *between — and —*

(103) Class B

(Positive:) *at least —*, *minimally —*, *from — (up)*, *— or more*
(Negative:) *at most —*, *maximally —* , *up to —*, *— or fewer*, *— or less*
(Neutral:) *from — and —*

Missing from this classification are the negative comparative quantifiers like *no more/fewer than 10*. The reason for this is that the occurrence of negation complicates the comparison with other quantifiers. In fact, I think that such quantifiers are best treated as the compositional combination of a class A comparative modifier with a negative differential *no*. See Nouwen 2008b for the consequences of such a move and for more details on the interpretations available for sentences containing such quantifiers.

6.2 Support for the A/B distinction from a corpus study

I now turn to a small corpus study I conducted which supports the division between class A and class B modifiers. Recall that one of the central observations in favour of the distinction connected to contrasts such as (104). Whereas (104a) can be interpreted with respect to a definite actual number of people invited by Jasper, (104b) does not allow such an interpretation and instead is evaluated in relation to what the speaker holds possible.

(104) a. Jasper invited fewer than 100 people. 87, to be precise.
b. Jasper invited maximally 100 people. #87, to be precise.

I explained this contrast by proposing that upper bound class B quantifiers are indicators of maxima. The indication of the maximum of a single value leads to infelicity. Existential modals, however, introduce a range of (possible) values, which thereby license the application of the maxima indicator. For examples like (104b), where no overt modal is present, the hearer will have to accommodate an interpretation with respect to speaker possibility. Given that $\Diamond$-modals licenses the application of an upper bound class B modifier, one would expect, however, that class B modifiers co-occur with an overt modal operator relatively often. I conducted a corpus study to find out whether this expectation is fulfilled.

6.2.1 Method

I used the free service for searching *the Corpus of Contemporary American English* (COCA, 385 million words, a mix of fiction, science, newspaper and entertainment texts and spoken word transcripts) at americancorpus.org (Davies 2008). For each numeral modifier I took 100 quasi-random[25] occurrences of the modifier with a numeral. For each of these cases I examined whether the modified numeral was in the scope of an explicit existential modal operator (such as *can*, *could*, *might*, *possibly*, *allow*, etc.) In other words, I only looked at the surface form and only counted the number of cases where a modal expression has a scope relation with a modified numeral. Given the theory presented in this article, the prediction is that this number is significantly higher with class B numerals than with class A expressions.

I compared five modifiers: *fewer than*, *under*, *between*, *at most* and *up to*. Not all occurrences of these modifiers with a numeral in the corpus were taken into consideration. For instance, (105) was ignored because in this example *up to* is probably not a constituent.[26] That is, this example contains the particle verb *to lift up*, rather than the verb *to lift*.

(105) Periodically we'd lift up to 60 kilometers where the temperatures and pressures are more like Earth's.

I similarly disregarded occurrences of *under n* where *under* is a regular preposition rather than a preposition in a role of numeral modifier. (For instance, examples resembling *He was known under 2 different names*.)

25 'Quasi', since the results are given in chronological order and I would just take the earliest hits.

26 From: "To boldly go…", Donald Robertson (1994), *Astronomy*, Vol. 22, Iss. 12; pg. 34, 8 pgs.

6.2.2 Results

The results, summarised in the table in (106), support the proposal in this article. Here, *P* is the percentage of occurrences within a existential modal context, within a sample of 100 occurrences of that modifer.[27]

(106)

	Class A			Class B	
	fewer than	under	between	at most	up to
P	4%	3%	4%	23%	21%

The corpus thus shows a clear preference for combining class B quantifiers with existential modal operators, as was predicted.[28] Whether the data are as clear as (106) for other expressions too remains to be seen. It will be difficult to extend this type of study to other modifiers. *Maximally* and *from. . . to*, for instance, were included in the present corpus search, but did not yield enough occurrences to make a meaningful comparison.

6.3 The cross-linguistic generality of the distinction

The class A/B distinction is not a peculiarity of the English language. I will suggest in this subsection that, in fact, the distinction is quite general and that languages seem to fill in the two classes in roughly the same way. Dutch, for instance, mirrors the English data perfectly. To illustrate, (107) and (108) shows the A/B distinction in a contrast between comparative and superlative quantifiers.

(107) *Een driehoek heeft meer dan 1 zijde.*
A triangle has more than 1 side.

(108) *#Een driehoek heeft minstens 2 zijdes.*
A triangle has at least 2 sides.

There are similar contrasts for other numeral modifiers. In a nutshell, the Dutch data suggests the two classes in (109), which is parallel to English.

27 I also counted the number of occurrences in a universal modal context. As would be predicted, this yielded no significant difference between class A and class B modifiers. For all modified numerals, this number was between 1 and 5.

28 The contrast between the Class A and Class B data is significant (χ^2=41.2, df=1, p = 1.375×10^{-10}.)

(109) Dutch Class A

(Positive:) *meer dan* — (more than), *boven de* — (above the)
(Negative:) *minder dan* — (fewer/less than), *onder de* — (under the)
(Neutral:) *tussen de* — *en de* — (between the. . . and. . .)

(110) Dutch Class B

(Positive:) *ten minste* —, *minstens* —, *op z'n minst* — (at least), *vanaf* — (from off), *zeker* — (certain), *minimaal* — (minimal)
(Negative:) *ten hoogste* —, *hoogstens* —, *op z'n hoogst* — (at most),*tot* — (up to), *maximaal* — (maximal)
(Neutral:) *van* — *tot* — (from — to —)

In other languages, we find similar data. For instance, the division between comparative and superlative modifiers appears to be cross-linguistically quite general. In Italian, for instance, the following contrast exists.

(111) *Un triangolo ha piú di 1 lato.*
A triangle has more than 1 side.

(112) #*Un triangolo ha almeno 2 lati.*
A triangle has at least 2 sides.

In Chinese, there also exists a superlative form that behaves like a class B modifier.

(113) #*Sanjiaoxing zui-shao you liang-tiao bian.*
triangle most-little have 2-CL side

On the other hand, there also exists an alternative form resembling English *at least*, which behaves differently. The form *zhi-shao* can be used as in a similar way as English *at least* is in sentences like *At least it doesn't rain!*. Despite this parallel to the English superlative modifiers, the example in (114) appears to be fine, which suggests *zhi-shao* is of type A.

(114) *Sanjiaoxing zhi-shao you liang-tiao bian.*
triangles to-little have 2-CL side

I leave a more detailed investigation of such data for further research. Whatever the outcome, however, the data first and foremost reveal that the type of contrasts that have been the central focus of this paper occur in Chinese and that, thereby, Chinese also appears to have the class A/B distinction.

Above, I suggested that prepositional numeral modifiers are to be divided in two classes in accordance with the locative/directional distinction that exists for their spatial meanings. The clearest case of a class B directional prepositional modifier in English is *up to*. In many other languages, one and the same particle is used for indicating spatial, numerical and temporal extremes. (In English, *up to* cannot be used as a temporal operator, for which *until* exists.) In Dutch, for instance, the preposition *tot* has these three functions. Crucially, in all these three domains *tot* displays class B characteristics.

(115) #*Een driehoek heeft tot 10 zijdes.*
A triangle has up to/until 10 sides.

(116) #*Je auto stond tot hier geparkeerd.*
Your car stood up to/until here parked.
'#Your car was parked up to here'

(117) *Je auto mag tot hier geparkeerd worden.*
Your car may up to/until here parked be.
'You may park your car up to here'

(118) #*Jasper kwam tot middernacht de kamer binnengelopen.*
J. came up to/until midnight the room inside-walked.
'#J. entered the room until midnight'

(119) *Jasper mag tot middernacht de kamer binnen komen lopen.*
J. may up to/until midnight the room inside come walk.
'J. is allowed to enter the room until midnight'

Similar data exist for German *bis (zu)*, Hebrew *'ad*, Catalan *fins a*, Spanish *hasta* and Italian *fino a*. In fact, in Italian it appears that (120) is generally awkward, resisting a reading that connects to speaker's possibility. However, it becomes acceptable if an overt modal verb is inserted.

(120) ??*John ha invitato {al massimo / fino a} 50 amici.*
John has invited {at most / until} 50 friends.

(121) *John può invitare {al massimo / fino a} 50 amici.*
John can invite {at most / until} 50 friends.

7 Conclusion

The central aim of this article has been to put forward the empirical observation that numeral modifiers come in two classes: those that relate to definite amounts (class A) and those that resist association with definite cardinality (class B). Theoretically, I proposed that underlying this distinction is a difference in the kind of relations numeral modifiers encode: either a simple comparison relation between numbers (class A) or a relation between a range of values and its minimum or maximum (class B). I furthermore showed how this theory can be implemented in a framework where numeral modifiers are treated as degree quantifiers.

While there already existed analyses of both type A and type B modifiers, the class difference that was the central focus of this article has not yet been discussed. For the treatment of class A quantifiers in this article I adopted the proposal of Hackl 2001. My account of class B modifiers, on the other hand, is original. It can be compared to two closely related proposals on the semantics of superlative modifiers: Geurts & Nouwen 2007, where superlative modified numerals are proposed to lexically specify modal operators, and Krifka 2007b, where superlative quantifiers are proposed to be speech act modifiers. Both works do not discuss the class A/B distinction, but I take it that both these proposals, in view of the main observations of this article, can be viewed as accounts not just of superlative quantifiers, but of class B members in general. As suggested in section 5, my proposal is in certain respects quite close to Krifka's. It differs greatly, however, from Geurts & Nouwen 2007 in the way the interaction between modified numerals and modality is accounted for. In a way, the current article as well as Krifka 2007b represent a position where quantifiers lexically specify quite minimal functions, which consequently leads to much of the work being done by pragmatic mechanisms (such as *blocking*). For the proposal in Geurts & Nouwen 2007, on the other hand, the balance is different in that a much greater burden is placed on semantics. An in-depth comparison of these accounts of class B quantifiers, however, is left for further research.

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