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AT LEAST ET AL.: THE SEMANTICS OF SCALAR MODIFIERS

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On the naive account of scalar modifiers like *more than* and *at least*, *At least three girls snored* is synonymous with *More than two girls snored*, and both sentences mean that the number of snoring girls exceeded two (the same, mutatis mutandis, for sentences with *at most* and *less/fewer than*). We show that this is false and propose an alternative theory, according to which superlative modifiers (*at least/most*) are quite different from comparative ones (*more/less/fewer than*). Whereas the naive theory is basically right about comparative modifiers, it is wrong about superlative modifiers, which we claim have a MODAL meaning: an utterance of *At least three girls snored* conveys two things: first, that it is CERTAIN that there was a group of three snoring girls, and second, that more than four girls MAY have snored. We argue that this analysis explains various facts that are problematic for the naive view, which have to do with specificity, distributional differences between superlative and comparative modifiers, differential patterns of inference licensed by these expressions, and the way they interact with various operators, like modals and negation.*

1. INTRODUCTION. We are concerned in this article with what we propose to call SCALAR MODIFIERS. Expressions falling under this rubric come in two types: SUPERLATIVE (*at least*, *at most*) and COMPARATIVE (*more than*, *less/fewer than*). Our focus of attention is on superlative and comparative quantifiers, like *at most three beers* and *more than two vodkas*, though other uses of scalar modifiers are taken into account as well.

It might seem that the semantics of scalar modifiers is a fairly straightforward matter, but readers of Kay 1992 and Krifka 1999 will be aware that it is not. The problems we concentrate on all have to do with the fact that the distinction between comparative and superlative modifiers runs much deeper than is generally acknowledged. Our aim in this article is twofold: to establish that the differences between comparative and superlative modifiers are profound, and then to explain them. The keystone in our proposal is that superlative but not comparative modifiers are modal expressions.

For a long time, semantic research on quantification has concentrated its attention on commonalities between quantifying expressions and has tended to gloss over the internal makeup of composite quantifiers like the ones treated here. Recently, however, there has been an increasing awareness that all quantifiers are special (e.g. Ariel 2004 and Hackl & Acland 2006 on *most* and Jayez 2006 on the French determiner *plusieurs*) and that an adequate analysis of their compositional structure is essential to a proper understanding of nonlexical quantifiers (e.g. Krifka 1999, Hackl 2000). In both respects, this article continues a trend.

2. FOUR PUZZLES. On the face of it, it would appear that superlative and comparative quantifiers are interdefinable, as follows:

At least n A are B. $\Leftrightarrow$ More than $n - 1$ A are B.
At most n A are B. $\Leftrightarrow$ Fewer than $n + 1$ A are B.

If these equivalences obtained, 1a and 2a should be synonymous with 1b and 2b, respectively.

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(1) a. Fred had at least three beers.
 b. Fred had more than two beers.

(2) a. Fred had at most two beers.
 b. Fred had fewer than three beers.

Introspectively, these equivalences would seem entirely plausible, and therefore it does not come as a surprise that they are standardly held to be valid (thus e.g. Keenan & Stavi 1986, Krifka 1999, Landman 2004). This naive view is sanctioned by the standard practice in mathematics to read '>' as *more than*, '≥' as *at least*, and so on. However, on reflection this view soon proves to be untenable. For one thing, it entails that we could well do without either superlative or comparative quantifiers, and thus raises the question of why languages such as English or Dutch should employ both types of expression to begin with. And this issue is harmless in comparison to the following puzzles, which show quite conclusively that the naive view is fatally flawed.

2.1. PUZZLE #1: SPECIFICITY. If the naive view on comparative and superlative quantifiers were correct, 3a and 3b should be equivalent.

(3) a. I will invite at most two people, namely Jack and Jill.
 b. ?I will invite fewer than three people, namely Jack and Jill.

Yet, there are clear differences. First, for most speakers 3b is markedly less felicitous than 3a is. Second, while 3a does not entail that anyone will be invited, it follows from 3b that the number of people to be invited equals two (that is to say, the invitees will be Jack and Jill). Similarly, while 4a is perfectly acceptable, and allows for the possibility that more than two people will be invited, 4b is less felicitous and rules out that possibility (cf. Kadmon 1992, Corblin 2007).

(4) a. I will invite at least two people, namely Jack and Jill.
 b. ?I will invite more than one person, namely Jack and Jill.

These contrasts already suggest rather strongly that superlative and comparative quantifiers are not interdefinable. But how do they arise? Our suggestion is that the *namely* riders in the (a) and (b) sentences are licensed in entirely different ways (assuming that the latter sentences are acceptable at all). The superlative quantifiers in the (a) sentences contain an existential expression that admits of a specific construal, viz. 'two people'. Thus interpreted, the phrase conveys that there is a particular pair of persons the speaker has in mind, and the *namely* rider identifies this pair as Jack and Jill. If this analysis is correct as far as it goes, it does not extend to the (b) sentences, if only because the phrases *three people* and *one person* are of the wrong cardinality. Furthermore, it is clear that an indefinite embedded in a comparative quantifier never admits of a specific construal.

(5) *I will invite more/fewer than two people, namely Jack and Jill.

If the *namely* riders in 3b and 4b are not licensed by a specific construal of an antecedent expression, what is it that enables them? The answer, we conjecture, is that in these cases the preceding clause is understood as implying that SOME people will be invited, and the *namely* rider restricts this to two people, viz. Jack and Jill. The degraded quality of the sentences suggests that, for many speakers, this procedure is only marginally acceptable.

Assuming that the foregoing observations are on the right track, our first puzzle may now be formulated as follows: Why is it that an indefinite expression can have a specific reading if it is contained in a superlative quantifier but not if it is part of a comparative quantifier?

2.2. PUZZLE #2: INFERENCE PATTERNS. If the naive view were correct, one should expect superlative and comparative quantifiers always to pattern together in inference. The following examples show that this expectation is not borne out by the facts. Suppose that the premise in 6 is given.

(6) Beryl had three sherries.

Then it follows that 7a is true, of course. But it isn't nearly as evident that 7b follows, as well.[1]

(7) a. Beryl had more than two sherries.
 b. Beryl had at least three sherries.

Intuitively, the reason why 7b does not follow from 6 is that it conveys that Beryl may have had more than three sherries; 7a, however, suggests no such thing.

Turning to downward-entailing scalar quantifiers, we observe a similar contrast. Still supposing that 6 is true, we are entitled to conclude 8a but not 8b.

(8) a. Beryl had fewer than five sherries.
 b. Beryl had at most four sherries.

In this case, the difference is harder to pin down in intuitive terms, but in our view the contrast is due to the fact that, whereas 8b explicitly grants the possibility that Beryl may have had four sherries, 8a does not, and this is why the latter but not the former may be derived from 6. Such, at any rate, is the explanation we defend below.

2.3. PUZZLE #3: DISTRIBUTIONAL RESTRICTIONS. Generally speaking, superlative modifiers have a wider range of distribution than their comparative counterparts, as the examples in 9 show.[2]

(9) a. Betty had three martinis {at most/*fewer than}.
 b. {At least/*More than}, Betty had three martinis.[3]
 c. Wilma danced with {at most/*fewer than} every second man who asked her.
 d. Wilma danced with {at least/?more than} Fred and Barney.

In view of these observations, it is somewhat surprising that on some occasions it is the distribution of superlative expressions that is more restricted.

(10) a. Betty didn't have {?at least/most//more/fewer than} three martinis.
 b. Few of the girls had {?at least/most//more/fewer than} three martinis.

These examples suggest that superlative quantifiers are positive polarity items, but this is not quite right.

(11) a. {All/Most/?About five/?None} of the girls had at least/most three martinis.
 b. Before going to bed, Betty {always/usually/?often/?occasionally/*never} has at least/most three martinis.

[1] Some of the readers of earlier versions of this article disagreed with our intuition that 7b does not follow from 6. We address this issue in §6, where we present quantitative data in support of our judgment. Note, incidentally, that the crucial fact is that 7a and 7b aren't on a par, and this much seems uncontroversial.

[2] While the main pattern we try to establish here has not been contested so far, the data aren't entirely cut and dried. See n. 16 for further discussion of 9d. Some of the following examples become acceptable if the negation is used metalinguistically. For instance, (i), where *at least* is outscoped by negation, is fine.

(i) Betty didn't have at least THREE martinis: she had at least FIVE.

We do not attempt to sort out these matters here, but return to them in the aforementioned footnote and §10.

[3] Here, *at the least* might be preferred over *at least* (as was suggested to us by Brian Joseph). In what follows we assume that these two alternative forms are semantically equivalent, and we focus on *at least*, since this is the more common expression.

Examples 11a,b show that superlative quantifiers dislike being in the scope not only of downward-entailing expressions but also of some existential quantifiers, like *about five*, *often*, and *occasionally*. Comparative quantifiers, by contrast, would have been fine in all of these cases.

Our third puzzle, then, is why the distribution of superlative expressions is freer in general but more restricted in certain special cases.

2.4. PUZZLE #4: MISSING READINGS. Occasionally, sentences with comparative quantifiers are ambiguous in a way that their superlative counterparts are not.

(12) a. You may have at most two beers.
b. You may have fewer than three beers.

Someone uttering 12a grants the addressee permission to have two beers or fewer, but not more. Example 12b may be used to express the same message, but it can also have a weaker reading, on which permission is granted to have fewer than three beers, without ruling out the possibility that the addressee have more than two beers.[4]

The pair of sentences in 13 exhibits the same contrast.

(13) a. That waitress can carry at most nineteen glasses.
b. That waitress can carry fewer than twenty glasses.

Both sentences may be used to convey that the waitress in question cannot carry more than nineteen glasses. But unlike 13a, 13b can be read without this implication, as well.

3. OUTLINE OF THE PROPOSED ANALYSIS. Our four puzzles should suffice to dispel the naive view that superlative and comparative modifiers are interdefinable. In several ways, these two classes of expressions behave quite differently, and somehow the differences have to be accounted for. But how? The first idea that comes to mind, perhaps, is to turn to pragmatics, but we consider it unlikely that a viable solution can be found there. For one thing, we see no reason to believe that the presuppositions of superlative and comparative modifiers bifurcate in any relevant way. For another, it should be noted that conversational implicature is not going to be of help, either. Given that, on the naive view, *at least* n and *more than* n − *1* are semantically equivalent, and of the same order of complexity, we fail to see how any conversational implicature associated with one expression could fail to be associated with the other. The same, mutatis mutandis, for *at most* n and *fewer than* n + *1*.

On the naive view, the lexical meanings of superlative and comparative modifiers stand to each other as '≤' stands to '<'. Our observations show that, in the absence of adequate pragmatic support, things cannot be as simple as this; there have to be more pronounced differences among the lexical meanings of scalar modifiers.

Our analysis is informed by the following intuition. The utterance of a sentence like 14a simply commits the speaker to the claim that the number of dancing girls exceeded three.

(14) a. More than three girls danced.
b. At least four girls danced.

By contrast, an utterance of 14b conveys TWO things: first, that it is certain that there is a group of four girls, each of whom danced; and second, that more than four girls may have danced. The best way of capturing this intuition, as far as we can see, is by assuming that a superlative modifier like *at least* has a MODAL meaning. That is to say,

[4] Admittedly, this construal may seem far-fetched, but it would be appropriate in the context of certain drinking games, for example.

while the meaning of 14a may well be represented in the conventional way, as in 15a, the interpretation of 14b is rather more elaborate, as shown in 15b.

(15) a. $\exists x[\text{girl}(x) \wedge \#x > 3 \wedge \text{dance}(x)]$
b. $\Box\exists x[\text{girl}(x) \wedge 4(x) \wedge \text{dance}(x)] \wedge \Diamond\exists x[\text{girl}(x) \wedge \#x > 4 \wedge \text{dance}(x)]$

The variables in these representations range over groups of individuals.[5] (Individuals may be seen as singleton groups.) Hence, 15a says that there exists a group x that fits the following description: the members of x are girls, the cardinality of x is greater than three, and each of the members of x danced. The meaning of 14b, as represented by 15b, is twofold. The first conjunct of 15b says that there MUST be a group of four girls that danced. ('4(x)' is short for '#x = 4'.) The second conjunct expresses that it MAY be that more than four girls danced.

The contrast between *less/fewer than* and *at most* is accounted for along the same lines.

(16) a. Fewer than five girls danced.
b. $\neg\exists x[\text{girl}(x) \wedge 5(x) \wedge \text{dance}(x)]$

The formula in 16b says that there was no group of five dancing girls, which entails that any groups of dancing girls were of cardinality four or lower. Again, this interpretation contrasts with that of the corresponding sentence with *at most*.

(17) a. At most four girls danced.
b. $\Diamond\exists x[\text{girl}(x) \wedge 4(x) \wedge \text{dance}(x)] \wedge \neg\Diamond\exists x[\text{girl}(x) \wedge \#x > 4 \wedge \text{dance}(x)]$

The first conjunct of 17b grants the possibility that there may have been a group of four dancing girls, while the second conjunct rules out the possibility that there were more than four girls that danced.

Comparing 15b and 17b, we observe that, on the proposed analysis, *At least* n *A are B* and *At most* n *A are B* are similar in that:

(i) they designate a cutoff point by stating that there may or must be a group of ABs of cardinality *n*, and
(ii) they either allow or disallow for the existence of AB-groups beyond that cutoff point.

We call (i) the primary component of the meaning of *at least/most*, which makes (ii) the secondary component. For the time being, we assume that both components are part of the lexical meaning of *at least/most*, but without rejecting outright the alternative view that only the primary component is semantic, while the secondary component is pragmatically derived. The division of labor between semantics and pragmatics is discussed further in §9, where we tentatively suggest that the primary component constitutes the core meaning of a scalar modifier, while the secondary component is a conventionalized conversational implicature.

The remainder of this article is chiefly concerned with showing how the interpretations in 15–17 can be derived in a principled way. The puzzles of the last section are solved as we go along.

4. INDEFINITES, NUMERALS, AND SCALES. Our analysis of the combinatorics of scalar modifiers is based in part on that in Krifka 1999, which in its turn enlists a number of ideas that are widely used in the semantic literature (see Landman 2004 for an in-depth

[5] In the literature on plurals our groups are often called 'individuals', which we find odd. Our usage implies that individuals are singleton groups, which is admittedly odd, too, though perhaps less so. The sad truth is that English lacks a word for referring to individuals and groups, and nothing else.

survey). In this section we elucidate the key notions with the help of the simple example in 18.[6]

(18) Four girls were dancing.

As usual, nouns are taken to denote properties, and $[\![\text{girls}]\!]$, that is, the denotation of *girls*, may be represented by $\lambda x.\text{girl}(x)$, which is the property any group has iff it is made up of girls only. Number words are of the same semantic type, so $[\![\text{four}]\!] = \lambda x.\#x = 4$, or in slightly abbreviated form: $\lambda x.4(x)$. This is the property that any group has iff it consists of four individuals. If a numeral is prefixed to a noun, the resulting expression is interpreted by way of composition, so $[\![\text{four girls}]\!] = \lambda x[4(x) \wedge \text{girl}(x)]$, which is the property any group has iff it consists of four girls.

Now suppose 19 is stated.

(19) The dancers were four girls.

Here the indefinite *four girls* is applied as a predicate to the group (call it 'd') referred to as *the dancers*, so we get: $\lambda x[4(x) \wedge \text{girl}(x)](d)$, which reduces to: $4(d) \wedge \text{girl}(d)$. That is to say, the dancers were girls and they were four.

In 19, the indefinite is used as a predicate; it does not have existential force. If it has, as in 18, we employ existential closure, a type-shifting rule that transforms a predicate like $\lambda x[4(x) \wedge \text{girl}(x)]$ into an existential quantifier, as in 20.

(20) $[\![\exists[\lambda x[4(x) \wedge \text{girl}(x)]]]\!] = \lambda P \exists x[4(x) \wedge \text{girl}(x) \wedge P(x)]$

Although it is not really necessary, we assume for expository purposes that existential closure is triggered by a covert element at the appropriate level of syntactic structure, which is to say that the logical form of 18 is as in 21.

(21) [$\exists$[four girls] [were dancing]]

Our chief motive for introducing empty elements is to have a typographical marker for highlighting the contrast between existential and predicative construals: while [*four girls*] is a predicate, $\exists$[*four girls*] is an existential expression.

To finish the semantic derivation of 18: after existential closure of the predicate, the existential quantifier associated with $\exists$[*four girls*] combines with the property denoted by *dance*, resulting in 22.

(22) $\exists x[4(x) \wedge \text{girl}(x) \wedge \text{dance}(x)]$

This says that there is a group of four girls, each of whom danced. To a first approximation, 22 gives a fair representation of the meaning of 18, but it does not fully capture what the sentence would normally convey. For, whereas 18 would typically be understood as implying that the number of dancing girls didn't exceed four, 22 does not rule out that possibility. For the purposes of this article, we assume that the missing ingredient in the interpretation of 18 is a conversational implicature—a scalar implicature, to be more precise, which may be rendered as in 23.[7]

(23) $\neg\exists x[\#x > 4 \wedge \text{girl}(x) \wedge \text{dance}(x)]$

Between them, 22 and 23 entail that the number of dancing girls was four, neither more nor less.

[6] The following discussion is relatively informal and glosses over some problems of detail. The appendix contains a more rigorous presentation of the proposed analysis and discusses it in some greater depth.

[7] That is to say, we are supposing here that the orthodox neo-Gricean line on numerals is the right one. Although many authors still adhere to this view (e.g. Krifka 1999, Winter 2001), others have come to reject it (e.g. Horn 1992, Carston 1998, Geurts 1998, 2006, Breheny 2005), and we align ourselves with the latter party. The issue is irrelevant to our present purposes, however, and for expository reasons we find it more convenient to pretend that the neo-Gricean account is correct.

In order to explain how scalar implicatures arise, we need scales, obviously, and we follow Krifka 1999 in assuming that such scales are built from focus-induced alternatives. To explain how, consider 24.

(24) Ada_F danced.

It is usually assumed that focus on an expression α serves to induce a set of alternatives to α's denotation, $[\![\alpha]\!]$. (Rooth 1985 was the first to offer a formal implementation of this idea, and Krifka's analysis extends Rooth's.) For example, in 24 the relevant alternatives might be {Ada, Berta, Carla, . . . }, in which case the sentence is used to convey that it is Ada who danced, rather than Berta, Carla, ... In some cases, such alternatives line up to form a scale. For example, if a number word such as *four* is focused, the set of alternatives is $\{\ldots, \lambda x.4(x), \lambda x.5(x), \lambda x.6(x), \ldots\}$, and using '$\rhd$' to symbolize the precedence relation, the corresponding scale is as given in 25.

(25) $\ldots \lambda x.6(x) \rhd \lambda x.5(x) \rhd \lambda x.4(x) \rhd \ldots$

Note, incidentally, that this is not an entailment scale: a group with the property $\lambda x.6(x)$ does not have any other property of the form $\lambda x.n(x)$. Still, this particular scale is implicative in another sense, for if a group has the property of having six elements, it perforce has subgroups of cardinalities five, four, and so forth.

The scale in 25 is associated with a focused number word. If such a word combines with a noun, a similar scale should be associated with the resulting expression. Krifka introduces a mechanism for composing scales that does this. The basic idea is intuitive enough (see the appendix for a precise definition). Consider the indefinite $\textit{four}_F$ *girls*, and suppose that 25 is the scale induced by $\textit{four}_F$; then the scale associated with $\textit{four}_F$ *girls* will be that given in 26.

(26) $\ldots \lambda x[6(x) \wedge \text{girl}(x)] \rhd \lambda x[5(x) \wedge \text{girl}(x)] \rhd \lambda x[4(x) \wedge \text{girl}(x)] \rhd \ldots$

That is, the property of being a group of six girls outranks the property of being a group of five girls, and so on. Similarly, the scale associated with the existential quantifier Ⓔ[$\textit{four}_F$ *girls*] is given in 27.

(27) $\ldots \lambda P\exists x[6(x) \wedge \text{girl}(x) \wedge P(x)] \rhd$
$\lambda P\exists x[5(x) \wedge \text{girl}(x) \wedge P(x)] \rhd$
$\lambda P\exists x[4(x) \wedge \text{girl}(x) \wedge P(x)] \rhd \ldots$

Finally, once Ⓔ[$\textit{four}_F$ *girls*] has combined with *were dancing*, we obtain the scale in 28.

(28) $\ldots \exists x[6(x) \wedge \text{girl}(x) \wedge \text{dance}(x)] \rhd$
$\exists x[5(x) \wedge \text{girl}(x) \wedge \text{dance}(x)] \rhd$
$\exists x[4(x) \wedge \text{girl}(x) \wedge \text{dance}(x)] \rhd \ldots$

Now, the implicature licensed by 18 is obtained by ruling out all alternatives on this scale that are higher (and, as it happens, logically stronger) than the sentence's literal meaning, which was given in 22; see 29.

(29) $\neg\exists x[5(x) \wedge \text{girl}(x) \wedge \text{dance}(x)] \wedge \neg\exists x[6(x) \wedge \text{girl}(x) \wedge \text{dance}(x)] \wedge \ldots$

This is equivalent to 23.

Krifka's account predicts that scalar implicatures are constrained by focus, which seems to be right. It should be noted, however, that the derivation of a suitable implicature for a sentence like 18 does not require that the numeral be focused. For the same implicature will be obtained with focus on *four girls*, provided the relevant scale is 26. It seems reasonable to suppose that the scale induced by focusing on a numeral indefinite like *four girls* will be of this form by default.

The essential gain of building up scales by Krifka's method is that it associates scales not only with sentences but also with their constituents. Apart from the fact that this is intuitively satisfying, it also opens up the possibility that there are nonsentential operators whose interpretation is scale-dependent. There are several focus particles that fit this description, *only* and *even* being obvious cases in point. With Krifka, we believe that scalar modifiers fall into this category, as well.

Before concluding this section, we note that the analysis just outlined is partly motivated by expository convenience, so let us briefly indicate what is and what is not essential to our account of scalar modifiers. While it is crucial to our enterprise that numeral indefinites may be construed alternatively as predicative or existential, nothing hinges on the assumption that the latter sense is derived from the former. Instead of employing existential closure, we might as well have adopted a type-shifting rule that maps quantifiers onto predicates, or we could have done away with type shifting altogether, and stipulated that numerals are polysemous between the two senses. Nor is it relevant to our purposes that a numeral quantifier expresses an 'at least' meaning, which is subsequently restricted by scalar implicature (see n. 7). What IS important, though, is that scales may be associated with subsentential expressions. This and the distinction between predicative and existential construals of indefinites are essential prerequisites for our account of scalar modifiers.

5. COMPARATIVE MODIFIERS. In the literature on generalized quantification, expressions like *more than three highballs* are standardly taken to consist of a quantifying expression and a noun: [[*more than three*] *highballs*] (e.g. Barwise & Cooper 1981, Keenan & Stavi 1986). As observed by Krifka, there are several drawbacks to this analysis. For one thing, it rules out a uniform account of scalar quantifiers and phrases like *more than happy*, *less than satisfactory*, and so on. For another, it makes it difficult to explain why scalar modifiers should be sensitive to focus, as witness the contrast between 30a and 30b.

(30) a. Betty had more than three$_F$ highballs: she had seven.
b. Betty had more than [three highballs]$_F$: she had a couple of piña coladas, as well.

Observe that, unlike 30a, 30b does not imply that the number of highballs consumed by Betty exceeded three, which is difficult to reconcile with the proposed grammar of the quantifier. Accordingly, and still following Krifka's lead, we give up the assumption that *more than three* is a constituent. Rather, *more than* combines with *three highballs*, which is analyzed as a property-denoting expression. In other words, comparative modifiers take predicates as their arguments—more accurately, their arguments are first-order predicates.[8] This analysis allows for *more than three highballs* and *more than happy* to be interpreted uniformly, and enables us to do justice to the focus sensitivity of scalar modifiers, as we show below.

The lexical meaning we propose for *more than* is given in 31.

(31) $[\![\text{more than } \alpha]\!] = \lambda x \exists \beta[\beta \rhd \alpha \wedge \beta(x)]$, where α and β are of type $\langle e, t\rangle$

[8] In fact, it isn't necessary to suppose that *more than three highballs* is invariably parsed as [*more than* [*three highballs*]]: the analysis we advocate is consistent with the possibility (suggested by Kadmon 1992) that *more than three highballs* is syntactically ambiguous between [*more than* [*three highballs*]] and [[*more than three*] *highballs*], for it will deliver the same meaning on either analysis, provided the focus is on *highballs*.

To illustrate what *more than* does to its arguments, suppose that the alternatives to *warm* include *hot* and *scalding*, and that we have the temperature scale in 32.

(32) $\lambda x.\text{scalding}(x) \rhd \lambda x.\text{hot}(x) \rhd \lambda x.\text{warm}(x)$

Now, $[\![\text{more than warm}_F]\!] = \lambda x[\text{hot}(x) \bigvee \text{scalding}(x)]$. That is, when applied to $[\![\text{warm}_F]\!]$, $[\![\text{more than}]\!]$ selects the properties that outrank $\lambda x.\text{warm}(x)$ on the temperature scale, that is, $\lambda x.\text{hot}(x)$ and $\lambda x.\text{scalding}(x)$, and returns that property that an entity has iff $\lambda x.\text{hot}(x)$ or $\lambda x.\text{scalding}(x)$ applies to it.

Note that 31 requires that the argument of *more than* be a first-order predicate, that is, a predicate that applies only to entities of type *e*, which in our framework are groups (recall that individuals count as singleton groups). Consequently, assuming that *more than three highballs* is parsed as dividing into *more than* and *three highballs*, the latter must be predicative; it cannot be existential. Hence, the first half of 30a is of the form [*Betty had* ⓔ[*more than* [*three*$_F$ *highballs*]]]. The scale associated with *three*$_F$ *highballs* is given in 33 (cf. 26).

(33) $\ldots \lambda x[5(x) \wedge \text{highball}(x)] \rhd$
$\lambda x[4(x) \wedge \text{highball}(x)] \rhd$
$\lambda x[3(x) \wedge \text{highball}(x)] \rhd \ldots$

So, when *more than* is combined with *three*$_F$ *highballs*, we get 34.

(34) $\lambda x[[4(x) \wedge \text{highball}(x)] \bigvee [5(x) \wedge \text{highball}(x)] \bigvee \ldots]$
$= \lambda x[\#x > 3 \wedge \text{highball}(x)]$

Applying existential closure, this is type-shifted into a quantifier, which will combine with the remainder of the sentence to yield the interpretation of 30a given in 35.

(35) $\exists x[\#x > 3 \wedge \text{highball}(x) \wedge \text{have}(b, x)]$

There are no surprises here. Things become more interesting when we turn to 30b, where focus is not on the numeral but on the indefinite *three highballs*, as a consequence of which the relevant scale is not constrained by the phrase's internal structure, and is determined entirely by pragmatic factors. The scale might be the one shown in 33 (though, as it happens, this is ruled out by the second half of 30b), or it might be a scale on which the property of consisting of three highballs ranks below the property of consisting of three highballs and any number of piña coladas. In the latter case, we predict that 30b does not entail that the number of highballs Betty drank is greater than three—which is as it should be.

According to the scale in 36, having six or more margaritas outranks having three highballs.

(36) $\lambda x[\#x \geq 6 \wedge \text{margarita}(x)] \rhd \lambda x[3(x) \wedge \text{highball}(x)]$

Our account of *more than* predicts that, in a context in which this scale is firmly established, *Betty had more than* [*three highballs*]$_F$ need not imply that Betty had any highballs at all. We believe this is correct. Compare 37 also.

(37) She is more than a major: she is a lieutenant colonel.

If the woman in question is a lieutenant colonel, she is unlikely to be a major, as well. The proposed analysis allows for this possibility.

The meaning of *less/fewer than* mirrors that of *more than*, as one should expect.

(38) $[\![\text{less/fewer than } \alpha]\!] = \lambda x \exists \beta[\alpha \rhd \beta \wedge \beta(x)]$, where α and β are of type $\langle e, t\rangle$

When applied to a property α, $[\![\text{less/fewer than}]\!]$ selects the properties $\beta_1, \ldots, \beta_n$ that are outranked by α on the relevant scale, and returns that property that an entity has iff it is β_1 or $\ldots$ or β_n. Hence, with the temperature scale in 32, $[\![\text{less than scalding}_F]\!]$

$= \lambda x[\text{warm}(x) \vee \text{hot}(x)]$. Similarly, the predicative interpretation of *fewer than three*$_F$ *beers* is $\lambda x[\#x < 3 \wedge \text{beer}(x)]$.

Turning to the quantificational interpretation of expressions like *fewer than three*$_F$ *beers*, we run into an issue known as VAN BENTHEM'S PROBLEM (van Benthem 1986). With $[\![\text{fewer than [three}_F\text{ beers]}]\!] = \lambda x[\#x < 3 \wedge \text{beer}(x)]$, the existential closure of this expression yields $\lambda P\exists x[\#x < 3 \wedge \text{beer}(x) \wedge P(x)]$, and the interpretation of 39a becomes 39b.

(39) a. [Fred had ∃[fewer than [three$_F$ beers]]]

b. $\exists x[\#x < 3 \wedge \text{beer}(x) \wedge \text{have}(f, x)]$

What 39b says is that there is a group of zero, one, or two beers that were consumed by Fred, and in our logic, this is a trivial statement, which is true no matter how many beers Fred had, if he had any at all.

Faced with the same problem, Krifka (1999) proposes to deal with it by assuming that downward-entailing expressions are semantically empty and merely designate alternatives for elimination at a later stage in the interpretation process. In Krifka's analysis, it is the assertion of the matrix sentence that discards the marked alternatives. In order to implement this idea, Krifka introduces a system for polarity marking on scalar alternatives, and represents assertion by an operator that removes negative-marked alternatives. Instead of this rather heavy apparatus, we opt for the more conservative solution by de Swart (2001), who, in addition to existential closure, introduces a further rule for transforming predicates into quantifiers: universal closure.[9] When applied to *fewer than* [*three*$_F$ *beers*], universal closure produces the quantifier in 40.

(40) $[\![\forall\text{[fewer than [three}_F\text{ beers]]}]\!] = \lambda P[\forall x[[\text{beer}(x) \wedge P(x)] \rightarrow \#x < 3]]$

Replacing ∃ in 39a with ∀, we obtain 41a, whose interpretation is 41b.

(41) a. [Fred had ∀[fewer than [three$_F$ beers]]]

b. $\forall x[[\text{beer}(x) \wedge \text{have}(f, x)] \rightarrow \#x < 3]$

In words: no group of beers consumed by Fred consisted of more than two individuals (i.e. beers).

Now that we have at our disposal two rules for turning predicates into quantifiers, the question arises of when to apply which. The solution we adopt is simply to leave the application of the closure rules free; that is to say, the operators ∃ and ∀ may be inserted ad libitum, but more often than not at least one of them disqualifies because it produces a reading that is trivial (as in 39), contradictory, or just highly unlikely. To the best of our knowledge, this simple scheme is good enough for our purposes in this article. It may well be that it is insufficiently restricted in the general case, and if it is, other than purely pragmatic constraints may have to be introduced, but that is an issue we do not address here.

In §2 we observed that 42 appears to be ambiguous between a reading that merely gives permission to have fewer than three beers and another reading that, in addition, prohibits the consumption of more than two beers (this was one half of puzzle #4). This ambiguity is readily explained in terms of relative scope. At the level of logical form, the quantifier ∀[*fewer than* [*three beers*]] may have *may* in its scope, or vice versa, and the resulting readings are 42b and 42c. (We use '⊡' and '◇' for symbolizing deontic necessity (obligation) and possibility (permission), respectively.)

[9] It should be noted that de Swart does not introduce universal closure in order to avoid van Benthem's problem. Rather, it is motivated by her analysis of various kinds of indefinites in the flexible type-theoretical framework of Partee 1987.

(42) a. You may have fewer than three beers. (= 12b)
b. $\forall x[[\text{beer}(x) \wedge \Diamond\text{have}(f, x)] \rightarrow \#x < 3]$
c. $\Diamond\forall x[[\text{beer}(x) \wedge \text{have}(f, x)] \rightarrow \#x < 3]$

To conclude this section, we begin to address another of the puzzles raised above, viz. #1. We observed that, whereas in *at most/least* n *A*, the indefinite n *A* allows for a specific construal, a specific reading does not seem to be available if the same expression is embedded in a comparative quantifier. The latter restriction is due, we believe, to the fact that comparative modifiers take first-order predicates as arguments, while specific indefinites are always existential; or, put otherwise, a predicative indefinite cannot have a specific reading.[10]

(43) a. I didn't see two friends of mine: Jack and Jill.
b. ?I doubt that those people are two friends of mine: Jack and Jill.

In 43a, the indefinite *two friends of mine* occurs in argument position and therefore has to be an existential expression, which admits of a specific construal. In 43b, by contrast, a specific construal does not appear to be feasible, and this correlates with the fact that the indefinite is predicative. Hence, the reason why the argument of a comparative modifier cannot be specific is due to a selection restriction imposed by the modifier, which requires that its arguments be first-order predicates.

6. SUPERLATIVE MODIFIERS. Like their comparative cousins, superlative modifiers operate on scales associated with their arguments, but they are different in two respects. First, superlative modifiers express modal meanings.[11] Second, while the argument of a comparative modifier must be a first-order predicate, superlative modifiers freely take a wide range of argument types. Leaving the exact demarcation of this range for another occasion, we conjecture that superlative modifiers take arguments of any BOOLEAN type, that is, propositional arguments (type *t*) and predicative ones (type $\langle a, t\rangle$, where *a* is any type). Hence, our semantic entry for *at least* distinguishes between two cases, as seen in 44.

(44) a. If α is of type *t*, then $[\![\text{at least } \alpha]\!] = \Box\alpha \wedge \exists\beta[\beta \rhd \alpha \wedge \Diamond\beta]$
b. If α is of type $\langle a, t\rangle$, then $[\![\text{at least } \alpha]\!] = \lambda X[\Box\alpha(X) \wedge \exists\beta[\beta \rhd \alpha \wedge \Diamond\beta(X)]]$

It should be noted straightaway that this definition presupposes that the αs and βs live on the same entailment scale. For nonentailment scales, such as orders of rank, 44 is

[10] Why should a specific construal be contingent on an existential reading of the indefinite in question? Within the framework used in this article, that is not entirely obvious. Indeed, in our current logic, a predicative meaning is always equivalent to some existential meaning. To illustrate, *You are a fraud* can be rendered alternatively as (i) or (ii), and these formulae have the same truth conditions.

(i) fraud(y)
(ii) $\exists x[\text{fraud}(x) \wedge x = y]$

In order to bring out the link between specific and existential construals, we would have to move to a dynamic framework like DISCOURSE REPRESENTATION THEORY, in which existential (but not predicative) indefinites serve to introduce discourse referents (see Geurts 1999 for an analysis of specificity in these terms). In such a framework, it is natural to assume that a specific interpretation presupposes the availability of a discourse referent, and since predicative indefinites do not introduce discourse referents, they cannot have specific construals, either.

[11] As we were finishing the first draft of this article, Bert Bultinck referred us to a passage in his dissertation that prefigures our modal account of superlative modifiers (Bultinck 2002:229–31). Strictly speaking, the cited passage is concerned with the interpretation of bare numerals (*one*, *two*, . . .), but it is evident that, if Bultinck had addressed the semantics of *at least* and *at most*, his findings would have been in very much the same spirit as ours.

too strong. In the appendix we show how the definition can be relaxed so as to solve this problem, but for now we stick with 44, because it is simpler.

To illustrate the workings of 44a, consider 45.

(45) a. At least [it isn't raining]$_F$.

b. $\Box\neg$raining $\wedge$ $\exists$p[p $\rhd$ $\neg$raining $\wedge$ $\Diamond$p]

Assuming that the entire propositional argument of *at least* is focused, 45a conveys, according to our analysis, that the speaker is sure it isn't raining, and that he or she considers it possible that something 'better' than nonraining might be the case, as well.[12] What exactly this means depends very much on the context, of course, but our analysis is consistent with the 'all is not lost' feeling that 45a would typically express.[13]

Using 44b, ⟦at least warm$_F$⟧ comes out as λx[$\Box$warm(x) $\wedge$ $\Diamond$[hot(x) $\vee$ scalding(x)]]. That is, when applied to ⟦warm$_F$⟧, ⟦at least⟧ selects the properties that outrank λx.warm(x) on the temperature scale (we are still using the scale in 32), and returns that property that an entity has iff it MUST be warm and MAY be hot or even scalding. So, if someone utters 46a while pointing at a bowl of soup s, the resulting meaning is 46b.

(46) a. This is at least warm.

b. $\Box$warm(s) $\wedge$ $\Diamond$[hot(s) $\vee$ scalding(s)]

In words: the speaker is certain that the soup is warm and considers it possible that it is hot or even scalding.

Proceeding to the interpretation of superlative quantifiers, let us consider how the proposed analysis deals with 47a.

(47) a. Betty had at least four$_F$ highballs. (cf. 30a)

b. [Betty had ⓔ[at least [four$_F$ highballs]]]

c. [Betty had [at least ⓔ[four$_F$ highballs]]]

If 47a is parsed after the model of its comparative counterpart in 30a, its underlying structure is 47b, and existential closure is applied after the scalar modifier has been combined with its argument. It turns out, however, that on this construal 47a comes out being self-contradictory, stating as it does that there is a four-member group of highballs that may have more than four elements (see the appendix for details).[14] Fortunately, there is nothing to prevent existential closure from applying to *four highballs* before it combines with *at least*, as shown in 47c; this derivation results in the interpretation in 48.

(48) $\Box\exists$x[4(x) $\wedge$ highball(x) $\wedge$ have(b, x)] $\wedge$
$\Diamond\exists$x[#x $>$ 4 $\wedge$ highball(x) $\wedge$ have(b, x)]

In words: the speaker is certain that there is a group of four highballs each of which was drunk by Betty, and considers it possible that Betty drank more than four highballs.

The semantic entry we propose for *at most* is given in 49.

[12] Although the type of modality involved in the interpretation of scalar modifiers need not be epistemic, we pretend for the time being that it is.

[13] Larry Horn and Christopher Piñón have objected against the necessity operator that we include in our entry for *at least*. While we have to concede that this operator was inspired by considerations of symmetry, it is not just a matter of aesthetics, for the necessity operator figures essentially in our account of modal concord (§8).

[14] Applying universal closure is not an option, either, and for the same reason. In the remainder of this section, we no longer consider the applicability of universal closure, because, as it turns out, it never produces viable results for the cases under discussion here.

(49) a. If α is of type t, then $[\![\text{at most } \alpha]\!] = \Diamond\alpha \wedge \neg\exists\beta[\beta \rhd \alpha \wedge \Diamond\beta]$
b. If α is of type $\langle a, t\rangle$, then $[\![\text{at most } \alpha]\!] = \lambda X[\Diamond\alpha(X) \wedge \neg\exists\beta[\beta \rhd \alpha \wedge \Diamond\beta(X)]]$

In the passage in 50 culled from the 'Daily Kos' website, *at most* modifies a propositional argument.

(50) (a) But hanging out in a gay bar is not evidence that one is gay. (b) At most, it is evidence of thirstiness and a desire to get drunk.

Apparently, it is being presupposed here that evidence that someone is gay is more valuable than evidence of his being thirsty and wanting to get drunk. Let us label the former proposition 'e_{gay}', and the latter '$e_{thirsty}$', and let us assume, furthermore, that apart from e_{gay} there are no salient alternatives to $e_{thirsty}$. Then the meaning of 50b comes out as given in 51.

(51) $\Diamond e_{thirsty} \wedge \neg\Diamond e_{gay}$

That is, 50b is construed as conveying, in the context given, that hanging out in a gay bar may be evidence of thirstiness and a desire to get drunk, while ruling out the possibility that it is evidence that the person in question is gay.

When applied to the predicate $[\![\text{warm}_F]\!]$, $[\![\text{at most}]\!]$ selects the properties that are outranked by $\lambda x.\text{warm}(x)$ on the temperature scale, and returns that property that an entity has iff it may be warm but cannot be either hot or scalding: $[\![\text{at most warm}_F]\!] = \lambda x[\Diamond\text{warm}(x) \wedge \neg\Diamond[\text{hot}(x) \vee \text{scalding}(x)]]$, or equivalently: $\lambda x[\Diamond\text{warm}(x) \wedge \neg\Diamond\text{hot}(x) \wedge \neg\Diamond\text{scalding}(x)]$.

Our analysis of 52a parallels that of 47a.

(52) a. Betty had at most four$_F$ highballs.
b. [Betty had [at most ∃[four$_F$ highballs]]]
c. $\Diamond\exists x[4(x) \wedge \text{highball}(x) \wedge \text{have}(b, x)] \wedge \neg\Diamond\exists x[\#x > 4 \wedge \text{highball}(x) \wedge \text{have}(b, x)]$

Assuming that 52b is the underlying structure of 52a, the semantic representation we end up with is 52c. This formula says two things: it grants the possibility that Betty had four highballs, and it excludes the possibility that she had more than four. To our minds, this captures the meaning of 52a quite nicely.

In the upcoming sections, we develop our story of superlative modifiers a bit further, but what we have so far suffices for solving all but the fourth of the puzzles posed at the beginning of this article.

6.1. Puzzle #1: specificity (revisited). In the last section, we suggested that the reason why the arguments of comparative modifiers do not admit of a specific construal is that these modifiers require their arguments to be first-order predicates. We now can add to this that, since superlative modifiers will apply to predicates of any order, including quantifiers, they should allow their arguments to have a specific interpretation—which they do.

(53) a. I will invite at most two people, namely Jack and Jill. (= 3a)
b. [I will invite [at most ∃[two people]]]

(54) a. *I will invite fewer than two people, namely Jack and Jill. (= 5)
b. [I will invite ∃[fewer than [three people]]]

In 53a, the indefinite *two people* can have a specific construal because it may be parsed as an existential quantifier, as shown in 53b. In 54a, by contrast, the same expression can only be interpreted as a first-order predicate, and therefore does not acquire existential force, which is a prerequisite for specificity. Hence, it is because the selection restrictions they impose on their arguments are less stringent that superlative modifiers allow their arguments to have specific readings, while comparative modifiers do not.

6.2. PUZZLE #2: INFERENCE PATTERNS (REVISITED). We observed that the inferential patterns superlative quantifiers engage in are sometimes curiously resticted when compared to their comparative counterparts. Examples 55a,b and 55c,d are two minimal pairs that we used to illustrate this point (cf. 6–8).

(55) Beryl had three sherries.
$\Rightarrow$ a. Beryl had more than two sherries.
$\not\Rightarrow$ b. Beryl had at least three sherries.
$\Rightarrow$ c. Beryl had fewer than five sherries.
$\not\Rightarrow$ d. Beryl had at most four sherries.

These are our own judgments, but when it turned out that not all of our readers concurred, we decided to collect some quantitative data. In two paper-and-pencil experiments, we asked native speakers of Dutch to judge arguments like the above (presented in Dutch). The main results were that 55a and 55c were endorsed by nearly all subjects (i.e. 100% and 92%, respectively; $n = 29$), while 55d was rejected in 78% of the cases ($n = 41$). So far, these data accord with our intuitions, though it should be noted that there was a substantial minority of subjects who accepted 55d. Opinions were divided about 55b, however, which was accepted in half of the cases; more accurately, the rates of acceptance were 48% in one experiment and 51% in the other ($n = 29$ and 41, respectively).

That 55a and 55c should follow from 55 is unsurprising, nor is it particularly remarkable that our theory gets these facts right. What is surprising, and problematic for what we dubbed the naive view on superlative quantifiers, is that speakers are reluctant to accept that 55b and 55d follow from 55. Our theory explains why this should be so. According to the analysis we advocate, both 55b and 55d entail that it is possible that Beryl had four sherries, which is inconsistent with the premise in 55, so these inferences are simply not valid.

But it should be evident that this cannot be the whole story. For, if the conclusions in 55b and 55d are not valid, as our analysis predicts, we should expect them to be rejected in the great majority of cases, which is not what happened. However, there are a number of additional factors that may have helped to shape the response patterns we observed. To begin with, it should not be taken for granted that, if it is given that n individuals have property A, speakers will always reject the claim that more than n individuals might be As, for it could be that the claim is brought into line with the facts by construing it as a counterfactual, for instance. If we take this possibility into account, it explains why, in our experiments, subjects sometimes accepted conclusions 55b and 55d. It does not, however, explain why the former should be accepted more often than the latter. Here, two further factors may have played a role.

First, post hoc interviews revealed that, when presented with the conclusion in 55b, some subjects interpreted the PREMISE as saying that Beryl had AT LEAST three sherries. Second, we note that, although 55b and 55d both entail that it is possible that Beryl had four sherries (and thus clash with the premise—provided the numeral gets an 'exact' reading), the status of this entailment may be different between the two cases: while in 55d it is the primary component of the meaning of the sentence, it is the secondary component of 55b. It may be, therefore, that this information has more of a default status in 55b than in 55d, making it easier to overrule in the former case than in the latter.[15]

6.3. Puzzle #3: distributional restrictions (revisited). The distributional pattern of scalar modifiers, we have seen, is somewhat peculiar. On the one hand, there is a clear main trend, which is that superlative modifiers have a wider range of distribution than their comparative counterparts. On the other hand, there are also exceptions to this trend, environments in which comparative modifiers are felicitous, while superlative modifiers are not. How are these facts to be explained?

First off, we have to concede that we don't have anything like a full-fledged theory of the distribution of scalar modifiers. What we have to offer is merely an outline of an explanation, which turns on two elements in our analysis. First, we have assumed that comparative and superlative modifiers impose different restrictions on their arguments—more accurately: the former are more selective than the latter, in that they combine only with first-order predicates. This difference explains the contrasts observed in 9, which we repeat here for convenience.

(56) a. Betty had three martinis {at most/*fewer than}.
b. {At least/*More than}, Betty had three martinis.
c. Wilma danced with {at most/*fewer than} every second man who asked her.
d. Wilma danced with {at least/?more than} Fred and Barney.

Examples 56a and 56b show that superlative but not scalar modifiers may occur in adverbial (or adsentential) positions. Example 56c illustrates that *at most* but not *fewer than* combines with at least some quantifiers, and assuming that names can be interpreted as quantifiers, too, 56d shows that *at least* and *more than* contrast with each other in the same way.[16] On our account, these patterns are as expected.

If superlative modifiers come with less stringent selection restrictions than comparative ones, why should they be more restricted in their distribution in certain cases? The answer, we submit, is that superlative modifiers are modal expressions. It is a well-known fact that the distribution of modal expressions (and epistemic modals in particular) is restricted in various ways, and if superlative modifiers are modal expressions, too, then they should be similarly restricted. This prediction appears to be correct; see 57 and 58.

(57) a. {Each/Most/?About five/*None} of the guests may have dispatched the butler.

[15] The primary/secondary distinction was introduced in §3, and is taken up again in §9.

[16] Larry Horn observes that (i) is, at the very least, much better than 56d.

(i) On my trip to Europe, I'm planning to see more than Paris and Berlin.

It may be that, in order to account for such examples, we have to relax the constraint that *more than* imposes on its argument. Alternatively, one might explore the possibility that, in this case too, the argument of the comparative modifier is a predicate after all; the idea being that, at the semantic level, *more than* combines not with *Paris and Berlin* but rather with *see Paris and Berlin*, which is a first-order predicate. As evidence favoring this line of analysis one might cite examples like (ii) or (iii), in which the modifier appears to take scope over the verb also at the syntactic level.

(ii) dat ik meer heb gezien dan Parijs en Berlijn (Dutch)
that I more have seen than Paris and Berlin
'that I have seen more than Paris and Berlin'

(iii) There is more to be seen than Paris and Berlin.

Cases like these also raise the question of how comparative-modifier constructions relate to comparatives in the more standard sense of the word. See §10 for some discussion of this issue.

b. {Each/Most/?About five/*None} of the guests danced with at least/most three waitresses.
c. {Each/Most/About five/None} of the guests danced with more/fewer than three waitresses.

(58) a. Betty might not have had three martinis.
b. ?Betty didn't have perhaps three martinis.
c. Betty didn't have {?at least/most//more/fewer than} three martinis. (= 10a)

Example 57a illustrates that epistemic *may* doesn't mind being in the scope of strong quantifiers like *each* or *most*, but dislikes—to varying degrees—being outscoped by weak quantifiers like *about five* or *none*. Examples 57b,c show that superlative modifiers have the same likes and dislikes, while comparative modifiers are equally comfortable with all quantifiers. Similarly, 58a and 58b illustrate that epistemic modals dislike being outscoped by negation: the first sentence only has a reading on which the modal has wide scope, and the second is simply infelicitous. Again, as is shown by 58c, superlative but not comparative modifiers follow the same trend.

Recently, distribution patterns of modal expressions have been discussed by von Fintel and Iatridou (2003), Nuyts (2004), and Tancredi (2005), among others, and if anything has become clear it is that it is not even clear what the problem is, with some of the experts rejecting as ill-formed sentences that sound perfectly fine to others. We neither want nor need to get into this debate. It suffices for the purposes of this article if it is agreed that superlative modifiers pattern with modal expressions, and this much should be plausible enough.

7. Modals within np. One of our central claims is that superlative modifiers are modal expressions. Against this claim, it might be objected that, even if it is correct as far as it goes, the distribution of superlative modifiers is somewhat peculiar: while modality is usually expressed verbally, adverbally, or adsententially, our account entails that superlative modifiers routinely occur adnominally. It turns out that this contrast is misleading, however, since at least some bona fide modals occur in the very same positions.

(59) a restaurant with {at most/maybe} thirty tables

Notwithstanding the fact that, generally speaking, modal particles like *maybe* are confined to positions reserved for adverbial or adsentential modifiers, in 59 *maybe* must be part of the embedded NP. In this respect it patterns with *at most*, and the two expressions yield interpretations that are very similar. They are not quite the same, though, as the observations in 60 demonstrate.

(60) a. a restaurant with {*at most/maybe} as many as thirty tables
b. a restaurant with {*at most/maybe} thirty tables or even more

A speaker who is of the opinion that thirty tables is quite a lot would be less likely to say *at most thirty tables* than *maybe thirty tables*; *at most* has a negative mood that *maybe* lacks. By the same token, whereas *at most thirty tables* doesn't leave room for lifting the upper bound it imposes, *maybe thirty tables* does, as 60b shows. These observations may be explained by assuming that the lexical content of *maybe* is strictly included in that of *at most*.

(61) a. Mildred had maybe five sodas—and that's {not enough/a lot}.
b. $\Diamond\exists x[5(x) \wedge \text{soda}(x) \wedge \text{have}(m, x)]$

(62) a. Mildred had at most five sodas—and that's {not enough/?a lot}.
b. $\Diamond\exists x[5(x) \wedge \text{soda}(x) \wedge \text{have}(m, x)] \wedge$
$\neg\Diamond\exists x[\#x > 5 \wedge \text{soda}(x) \wedge \text{have}(m, x)]$

If 61a is interpreted as 61b, it allows for a positive uptake as well as a negative one. But if *maybe* is supplanted with *at most*, an upper bound is imposed by lexical (i.e. nonpragmatic) means, and a positive uptake is blocked.

The adnominal use of modal particles is somewhat idiosyncratic. The examples in 59 and 60 become infelicitous if *maybe* is replaced with *possibly*, or a stronger modal particle like *certainly* or *necessarily*. In Dutch, however, there is nothing wrong with an example like 63.

(63) een restaurant met zeker dertig tafels
a restaurant with certainly thirty tables
'a restaurant with at least thirty tables'

This is perfectly colloquial, and what is more, in this use the modal particle *zeker* is fully equivalent to *minstens* 'at least'. That is to say, their distribution, the construals they admit of, and the inferential patterns they give rise to parallel each other in all respects.

Therefore, we conclude that our observations about intra-NP modality lend further support to our claim that superlative modifiers have modal meanings.

8. Modal concord. Thus far, our analysis of the ways scalar modifiers combine with other expressions has been resolutely compositional. In this section, we argue that, due to the fact that they are modals, superlative modifiers are special in yet another respect: they can engage in what we call modal concord, which, prima facie at least, is a noncompositional mode of semantic combination.[17]

It is a familiar fact that, in many if not most languages of the world, double negation does not always cancel out, and several negative expressions may conspire to express a single denial. Salvatore's complaint, in Umberto Eco's *The name of the rose*, is a case in point.

(64) I don't know nothing.

On its intended concord reading, 64 is an emphatic way of expressing that the speaker doesn't know anything. But in standard English the sentence only admits of a compositional construal, and entails that there is something the speaker knows. In a negative-concord language like French, negative sentences may be ambiguous between concord readings and compositional readings, as example 65, from Corblin 1996, illustrates.

(65) Personne n'aime personne.
no.one NEG.loves no.one
Concord reading: 'No one loves anyone.' ($\neg\exists x\exists y L(x, y)$)
Compositional reading: 'Everyone loves someone.' ($\neg\exists x\neg\exists y L(x, y)$)

While negative concord has been widely discussed in the literature (see Corblin et al. 2005 for an overview), little attention has been given to the fact that modals exhibit

[17] More accurately: on the face of it modal concord APPEARS to be noncompositional. It is not essential to our purposes that it is, and if our treatment of modal concord can be recast in compositional terms, we won't mind at all. In fact, Geurts & Huitink 2006 proposes a compositional analysis of the *could you possibly* variety of modal concord, but since this proposal is still somewhat tentative, and it is not quite clear how it would extend to the case at hand, we do not adopt it here.

concord behavior, too.[18] We suspect that the phenomenon is widespread among the languages of the world, but illustrate it here with examples from Dutch.

(66) Hij moet zeker in Brussel zijn.
he must certainly in Brussels be
Compositional reading: 'I suppose he has to be in Brussels.' ($\Box\boxdot A$)
Concord reading: 'He (definitely) has to be in Brussels.' ($\Box A$)

On its compositional reading, 66 contains two modal operators: a deontic and an epistemic one, with the latter outscoping the former. On its concord reading, the same sentence contains just a single modal operator, and the expressions *must* and *certainly* join forces in conveying a single semantic constituent. Example 67 shows that modal concord may involve more than two modal expressions.

(67) Ze zou misschien wel eens dronken kunnen zijn.
she could maybe rather-likely drunk can be
Compositional reading: none
Concord reading: 'She might be drunk.' ($\Diamond A$)

Even though 67 contains up to four expressions that could be argued to be modal in nature, a reading of the form $\Diamond \ldots \Diamond A$ does not seem to be possible; the only construal that is readily available is the concord reading, on which all modal items in the sentence conspire to express a single operator.

It is only to be expected that, just as there are all sorts of restrictions on negative concord, there are restrictions on modal concord, as well. As far as we know, this is as yet unexplored territory, and charting it is rather too tall an order for the present article. However, there are two, fairly obvious, constraints that we should like to suggest. First, it is clear that whenever a concord reading is available, it tends to be preferred, and a compositional reading may be very hard if not impossible to obtain. Second, expressions engaging in modal concord need to be of the same sort. For 66, a concord reading is available because the two modal expressions the sentence contains both express necessity. Similarly, 67 allows a concord reading because all of its modal expressions may express possibility. But if expressions of possibility and necessity are mixed, a concord reading will not be forthcoming.

(68) Hij moet misschien in Brussel zijn.
he must perhaps in Brussels be
Compositional reading: 'Perhaps he has to be in Brussels.' ($\Diamond\boxdot A$)
Concord reading: none

Assuming as we do that superlative modifiers are modal expressions, it is to be expected that they engage in modal concord too. But how? For various reasons, the preceding observations about modal concord do not apply without further ado to superlative modifiers. The most obvious difficulty is that, on our analysis, a superlative modifier introduces not one but two modal operators, which don't have the same force, and therefore the question arises of which of the two is going to engage in modal concord. We propose to deal with this issue by making use of the distinction between the primary and secondary components of the meanings of scalar modifiers, which we introduced in §3. To elaborate on that distinction, consider the schematic representations of *at least* and *at most* sentences given in 69.

[18] The phenomenon has been intermittently observed since the 1970s, for example, by Halliday (1970), Lyons (1977), and Papafragou (2000). To the best of our knowledge, the only attempt so far at theoretical analysis is by Geurts and Huitink (2006).

(69) a. At least n A are B.
$\Box\exists x[A(x) \wedge n(x) \wedge B(x)] \wedge \Diamond\exists x[A(x) \wedge \#x > n \wedge B(x)]$
b. At most n A are B.
$\Diamond\exists x[A(x) \wedge n(x) \wedge B(x)] \wedge \neg\Diamond\exists x[A(x) \wedge \#x > n \wedge B(x)]$

Both types of superlative modifiers introduce a pair of modal propositions, but intuitively the two members of each pair do not have the same status. In both 69a and 69b, the principal message is conveyed by the first conjunct, which designates a cutoff point, while the second conjunct seems less central. (In fact, as we show in §9, it may plausibly be argued that the second conjunct is pragmatically derived from the first.) Accordingly, we hypothesize that it is the modal operator of the first conjunct (the primary operator, as we call it) that engages in modal concord. Combining this with the observation that the expressions involved in modal concord must have the same force, this entails that *at least* engages in modal concord with expressions of necessity (*must*, *have to*, *be certain*, etc.), while *at most* engages in modal concord with expressions of possibility (*may*, *can*, etc.).

Thus far we have assumed that the two modal operators introduced by a scalar modifier are always epistemic. For example, we would construe 69a as expressing that, for all the speaker knows, it is certain that n A are B, and that it is possible that more than n A are B. If this were to be our last word on the matter, our account would predict that scalar modifiers can engage in modal concord with epistemic modals only—which, as it turns out, is not the case. Hence, our official position is more nuanced. While some modal expressions, like English *be able*, have a quite specific meaning, others allow for a range of possible interpretations (*to have* is a case in point). What we would like to suggest is that scalar modifiers are like the latter, in that they can take on a range of modal interpretations and are construed as epistemic only by default. The lexical meaning of a scalar modifier specifies the force of the modal operators it contains, but it does not determine the kind of worlds they range over.[19]

To sum up the foregoing discussion, our rules of engagement for superlative quantifiers and modal expressions are the following:

- If a superlative modifier combines with a modal expression whose force matches that of its primary operator, the two modals may fuse to yield a concord reading, which is preferred, ceteris paribus, to a compositional construal.
- If there is no such match, there is no concord reading.
- The modal operators introduced by a superlative modifier are epistemic by default, and therefore generally tend to take wide scope.

In the following we explain the workings of these rules with the help of several examples.

(70) You must have at least two beers.
a. Concord reading:
$\boxdot\exists x[2(x) \wedge beer(x) \wedge have(y, x)] \wedge$
$\Diamonddot\exists x[\#x > 2 \wedge beer(x) \wedge have(y, x)]$
b. Compositional reading:
$\Box\boxdot\exists x[2(x) \wedge beer(x) \wedge have(y, x)] \wedge$
$\Diamond\boxdot\exists x[\#x > 2 \wedge beer(x) \wedge have(y, x)]$

[19] It almost goes without saying that the two modal operators introduced by a scalar modifier are always of the same type. That is to say, they are both epistemic, both deontic, or whatever. That this must be so follows from the view we cautiously endorse in §9, to the effect that the secondary component of a scalar modifier is a conventionalized conversational inference that derives from the primary component.

Since the primary operator of *at least* has the same force as *must* (both are necessity operators), the two can engage in modal concord, and if *must* is construed deontically, the modal operators introduced by the scalar modifier will follow suit. This yields a reading that may be paraphrased as follows: The hearer is called upon to bring about a state of affairs in which he or she has two beers, and is allowed to bring about a state of affairs in which he or she has more than two beers. In addition to this concord reading, we predict that the sentence has a compositional construal, as well, which is given in 70b. Here, the outer modal in each conjunct is epistemic (which is the default), so what the sentence says, on this reading, is something like this: For all the speaker knows, it must be the case that the hearer has to bring it about that he or she drinks two beers, and it may be the case that the hearer has to bring it about that he or she has more than two beers. This interpretation may be harder to obtain, but according to our intuitions it is available.

Replacing the strong modal verb in 70 with a weak one, we obtain 71.

(71) You may have at least two beers.
  a. Concord reading: none
  b. Compositional reading:
  $\Box\Diamonddot\exists x[2(x) \wedge \text{beer}(x) \wedge \text{have}(y, x)] \wedge$
  $\Diamond\Diamonddot\exists x[\#x > 2 \wedge \text{beer}(x) \wedge \text{have}(y, x)]$

In this case, modal concord is not possible, there being no match between the verb and the primary operator of *at least*, and therefore we get only a compositional reading.

Turning from *at least* to *at most*, the pattern of predicted readings reverses.

(72) You must have at most two beers.
  a. Concord reading: none
  b. Compositional reading:
  $\Diamond\boxdot\exists x[2(x) \wedge \text{beer}(x) \wedge \text{have}(y, x)] \wedge$
  $\neg\Diamond\boxdot\exists x[\#x > 2 \wedge \text{beer}(x) \wedge \text{have}(y, x)]$

Here, concord is not possible, because the primary operator of *at most* does not match with the modal verb, and therefore only a compositional reading is available, which appears to be correct. By contrast, a concord reading is available for 73, which we predict to be ambiguous between a construal on which the addressee is forbidden to have more than two beers, and one on which the sentence is used to make a statement ABOUT what is and is not allowed.

(73) You may have at most two beers. (= 12a)
  a. Concord reading:
  $\Diamonddot\exists x[2(x) \wedge \text{beer}(x) \wedge \text{have}(y, x)] \wedge$
  $\neg\Diamonddot\exists x[\#x > 2 \wedge \text{beer}(x) \wedge \text{have}(y, x)]$
  b. Compositional reading:
  $\Diamond\Diamonddot\exists x[2(x) \wedge \text{beer}(x) \wedge \text{have}(y, x)] \wedge$
  $\neg\Diamond\Diamonddot\exists x[\#x > 2 \wedge \text{beer}(x) \wedge \text{have}(y, x)]$

In 70–73, the compositional reading was derived on the assumption that the scalar modifier outscopes the modal verb. This assumption is motivated by the fact that, in these cases, the modal verb is construed deontically, while the modifier gets an epistemic construal, and epistemic operators generally dislike being in the scope of deontic ones. Thus we have solved our last puzzle, that is #4, which asked why 73 should lack one of the readings available for 12b, repeated here as 74.

(74) You may have fewer than three beers. (= 12b)

We have seen in the foregoing how to derive the key reading of 74, which merely allows the addressee to have fewer than three beers, without prohibiting the consumption of more than two beers. The point is that, in order to derive the same reading (or at least a parallel one) for 73, *may* would have to outscope *at most*, which doesn't seem to be possible.

9. Lexical content, pragmatic inference, or both? Let us have another look at the two example sentences we started with, repeated here, and compare their analyses.

(75) a. Fred had at least three beers. (= 1a)
b. $\Box\exists x[3(x) \wedge \text{beer}(x) \wedge \text{have}(f, x)] \wedge \Diamond\exists x[\#x > 3 \wedge \text{beer}(x) \wedge \text{have}(f, x)]$

(76) a. Fred had more than two beers. (= 1b)
b. $\exists x[\#x > 2 \wedge \text{beer}(x) \wedge \text{have}(f, x)]$

On reflection, it seems plausible that the conjunction in 75b might be derivable from 76b, which according to what we have been calling the naive view represents the meaning of 75a and 76a alike. Surely, if I commit myself to the truth of 76b, I therewith commit myself to the claim that, for me, 76b expresses an epistemic necessity, which is what the first conjunct of 75b says. Furthermore, if I say that Fred had more than two beers, and I thought he didn't have more than three, then surely I should have said that he had three beers—from which it seems to follow, à la Grice, that 76b implicates that, as far as I know, Fred may have had more than three beers, which is what the second conjunct of 75b says. (Similar observations can be made about *at most* and *less/fewer than* sentences.) Thus arises the question of why inferences that are pragmatic when licensed by comparative modifiers should be semantic when they associate with superlative modifiers. Why enshrine in the lexicon content that might just as well be derived by pragmatic reasoning?

To begin with, we reiterate an observation made in §3: it is highly unlikely that the differences between *at least* n and *more than* n − *1* are merely a matter of conversational implicature, for the simple reason that, if they were semantically equivalent, the two expressions should license the same implicatures. The same, mutatis mutandis, holds for *at most* n and *less/fewer than* n + *1*. In other words, the naive view that comparative and superlative modifiers stand to each other as '≤' stands to '<' is untenable in view of the puzzles presented in §2.

The assumption that superlative modifiers differ from comparative ones in that they contain modal elements played a crucial part in several of the explanations we offered in the foregoing. We called upon it to explain why the distribution of superlative modifiers is, in certain cases, more restricted than that of their comparative counterparts, it figured essentially in our account of the inferences licensed by superlative modifiers, and it was presupposed in our claim that superlative modifiers engage in modal concord. In other words, without the assumption that superlative modifiers are modals, three of our four puzzles would have remained unsolved.

We consider this strong evidence for a modal analysis of superlative modifiers, but admittedly it does not prove that ours is the right one, so we grant that there may be alternative modal analyses worth pursuing. For example, consider the possibility that the lexical content of a superlative modifier is only half of what we have supposed so far. On this account, the core meaning of 75a would be captured in its entirety by what we have called the primary component of its meaning, that is, the first conjunct of 75b, and the second conjunct would be recategorized as a pragmatic inference. This analysis shares two virtues with the one we have proposed, for the stories about modal concord and the distribution of superlative modifiers can remain the same. In fact, this version allows us to finesse our account of modal concord. Recall that we had to stipulate that there is a difference in status between the two parts of our bimodal analysis of sentences like 75a, as a consequence of which it is the modal operator of the first part that may engage in modal concord. On the monomodal analysis, this postulate becomes obsolete, which is an advantage.

Still, there are problems with this simplified version of our proposal. One is that our account of the inferential patterns characteristic of superlative modifiers does not go through anymore. Another problem with the simplified modal account relates to the discussion of §7, where we compared superlative modifiers with other modals that may appear NP-internally, like English *maybe*.

(77) a restaurant with {at most/maybe} thirty tables (= 59)

We argued that, though they are closely related, *maybe* and *at most* do not express exactly the same content, and that the meaning conveyed by *maybe* is more general. The simplified modal analysis fails to account for this, since according to it *maybe* and *at most* are semantically equivalent.

Yet another possibility is to assume that the secondary component of the meaning of *at least/most* is a conventional implicature, which is to say that it is lexically encoded but truth-conditionally inert (Potts 2005). This doesn't seem right to us, however, for the simple reason that the secondary component of the meaning of *at least/most* doesn't have the look and feel of a purely conventional piece of content; intuitively, it should be derivable from the primary component. Thus we are driven to conclude, if only tentatively, that the secondary component is a conventionalized conversational implicature: a pragmatic inference that has become part of the lexical content of *at least/most*. This way we can have our cake and eat it too: the two parts of the content of *at least/most* are both in the lexicon AND one is derived from the other. Still, as things stand, we have to concede that, even if it has been used before (e.g. by Horn 1989), the notion of conventionalized conversational implicature is a suggestive label rather than a well-understood theoretical construct. Calling an inference by this name is little more than a first stab at explanation.

To sum up, the evidence in favor of the view that superlative modifiers are modals seems quite compelling to us, and all things considered the bimodal analysis advocated here appears to be the best way of implementing that view. But the problem, or one of the problems, that remains is how to account for the difference in status between the two components of the content conveyed by way of superlative modifiers.

10. CONCLUDING REMARKS. To conclude, we want to briefly mention two issues our theory gives rise to. The first has to do with embedded occurrences of *at least* and *at most*.

(78) a. Betty had at least three martinis.
b. ?Betty didn't have at least three martinis.

We have argued that the contrast between 78a and 78b is due to the fact that superlative modifiers are modals. However, it should be noted that, despite its oddness, 78b is perfectly intelligible. What it says (even if it says it awkwardly) is simply that Betty had at most two martinis—and there is no way our analysis of *at least* will capture that reading. Similarly, as it stands, our theory fails to account for examples like 79.

(79) If Betty had at least three martinis, she must have been drunk.

The reading we predict here is something like: 'If it must be the case that Betty had three martinis and it may be that she had more than three, then she must have been drunk'—which is not what the sentence means. In brief, our modal analysis sometimes fails to produce the right interpretations when superlative modifiers occur in embedded positions.

We cannot tackle this issue here, but merely note that the same problem arises with the modal analysis of *or* that was first proposed by Zimmermann (2000) and revamped by Geurts (2005). According to this theory, the meaning of a disjunctive statement is actually a conjunction of possibilities. Simplifying matters somewhat, the idea is that the meaning expressed by 80a is something along the lines of 80b.

(80) a. Fred had a soda or milk.

b. $\Diamond$Fred had a soda $\wedge$ $\Diamond$Fred had milk $\wedge$ $\neg\Diamond$Fred had anything else

On this construal, a speaker who utters 80a says (in Grice's sense of the word) that s/he considers it possible that Fred had a soda, that s/he considers it possible that Fred had milk, and that s/he rejects the possibility that Fred had any beverages other than milk and soda. It should be evident that this gives the wrong results when a disjunction is embedded under negation or in the antecedent of a conditional (though not, as Geurts points out, when it occurs in the consequent). This problem has been discussed at some length by Zimmermann (2000) and Geurts (2005), and we conjecture that whatever turns out to be the right approach for *or* should also work for *at least* and *at most*.

As noted in the introduction, our proposal can be seen as an exponent of the growing awareness that compositional structure is essential to the semantics of quantifying expressions. Only by decomposing comparative and superlative quantifiers into their meaningful parts were we able to offer an alternative to the naive view that they are interdefinable. But shouldn't we have taken the decompositional stance one step further? Shouldn't a theory of comparative and superlative modifiers bring out their family ties with comparative and superlative adjectives and adverbials?

To be sure, our analyses hint at such connections.[20] We have proposed that *more than three beers* denotes the set of groups of beers whose cardinality exceeds that of the groups in the denotation of *three beers*, which is parallel to the standard view according to which *taller than Wilma* denotes the set of individuals whose height exceeds that of Wilma. Therefore, one might expect that our analysis should generalize to a full-blown theory of comparison, possibly one that is not (as is customary) based on the notion of degree, but in which comparisons are always between scalar alternatives.

However, it is by no means clear that all varieties of comparison involve the same underlying mechanism. There are some notable differences between *more than* as a scalar modifier and *more . . . than* as part of a comparative construction with a gradable adjective or adverb. First, in the cases of scalar modification we were concerned with, the complex modifier has to be a continuous phrase, as witness the contrast between *more (*beers) than three beers* and *more scared than Betty*. The same observation can be made in cases where the scale of alternatives is not one of amounts. For instance, 81a is felicitous, but inserting material between *more* and *than* produces oddities like 81b. What is more, if it is acceptable at all, the interpretation of 81b is markedly different from that of 81a, in that it involves metalinguistic comparison.

(81) a. I'm more than happy with the results.

b. ?I'm more ecstatic about the results than (merely) happy with them.

Second, in some languages the distinction between comparative modifiers and other forms of comparison is lexicalized. For instance, French distinguishes between *plus de*

[20] In the next few paragraphs we are heavily indebted to an anonymous referee, to whom we also owe the examples in 81.

and *plus . . . que*, as in *plus de trois bières* ‘more than three beers’ vs. *plus grand que Fred* ‘taller than Fred’. Again, this suggests that all comparatives are not alike.

Similar remarks apply to the relation between superlative modifiers and superlatives in the more common sense of the word: the interpretations of *at least/most* and *-est* seem to deviate in several ways. For one thing, superlative morphology does not generally give rise to modal meanings. For another, an expression like *the tallest glass* depends for its interpretation on the context in a way *at least/most three glasses* does not.

In short, it remains to be seen what exactly is the relationship between comparative and superlative modifiers, on the one hand, and comparative and superlative morphology, on the other.

Appendix

Compositional semantics and scales

Any parsed natural language string α is assigned an ORDINARY semantic value, $[\![\alpha]\!]$, as well as an ALTERNATIVE semantic value, alt(α). In the cases we are interested in, alt(α) is ordered: its elements line up along a scale. We refer to such a scale with $\trianglerighteq_\alpha$: $\beta \trianglerighteq_\alpha \gamma$ signifies that β is ranked at least as highly as γ on the scale of alternatives associated with α. The relations $\trianglelefteq_\alpha$, $\triangleleft_\alpha$, and $\triangleright_\alpha$ are defined in terms of $\trianglerighteq_\alpha$, as one would expect. If α is lexical and not in focus, then the scale associated with it is the trivial scale $\{\langle[\![\alpha]\!], [\![\alpha]\!]\rangle\}$.

Given a scale that is associated with an expression, we may derive a corresponding scale for phrases containing this expression (cf. Krifka 1999:11).

(1) For any mode of composition such that $[\![\alpha \cdot \beta]\!] = [\![\alpha]\!] * [\![\beta]\!]$: $X \trianglerighteq_{\alpha\cdot\beta} X'$ iff $X = A * B$, $X' = A' * B'$, $A \trianglerighteq_\alpha A'$, and $B \trianglerighteq_\beta B'$

For example, the scale $\trianglerighteq_{\text{three}_F}$ is simply the scale of the cardinality properties. (In fact, this scale is the same for all cardinals.) Consequently, there is a scale that is associated with a complex phrase like *three*$_F$ *beers*, namely the scale that orders groups of beers with respect to the number of individuals in each group. Formally, it is as given in 2.

(2)
$$\begin{array}{lll}
[\![\text{three}_F]\!] & = & \lambda x.3(x) \\
\trianglerighteq_{\text{three}_F} & = & \{\langle \lambda x.n(x), \lambda x.m(x)\rangle \mid n \geq m\} \\
[\![\text{beers}]\!] & = & \lambda x.\text{beer}(x) \\
\trianglerighteq_{\text{beers}} & = & \{\langle \lambda x.\text{beer}(x), \lambda x.\text{beer}(x)\rangle\} \\
[\![\text{three}_F \text{ beers}]\!] & = & \lambda x.3(x) \circ \lambda x.\text{beer}(x) = \lambda x[3(x) \wedge \text{beer}(x)] \\
\trianglerighteq_{\text{three}_F \text{ beers}} & = & \{\langle \lambda x[n(x) \wedge \text{beer}(x)], \lambda x[m(x) \wedge \text{beer}(x)]\rangle \mid n \geq m\}
\end{array}$$

The NP *three*$_F$ *beers* denotes a property of groups (type $\langle e, t\rangle$). Its alternative semantic value is a scale of properties of groups.

Closure

If $[\![A]\!]$ is of type $\langle e, t\rangle$, existential closure shifts $[\![A]\!]$ to the type of quantifiers, $\langle\langle e, t\rangle, t\rangle$.

(3) $[\![\exists[A]]\!] = \lambda P\exists x[[\![A]\!](x) \wedge P(x)]$, where $[\![A]\!]$ is of type $\langle e, t\rangle$

With this empty determiner the $\langle e, t\rangle$ denotation of [*more than* [*three*$_F$ *beers*]] may be shifted into an object of type $\langle\langle e, t\rangle, t\rangle$.

(4) $[\![\exists[\text{more than [three}_F \text{ beers]]}]\!]$
$= \exists[\lambda x[\#x > 3 \wedge \text{beer}(x)]]$
$= \lambda P\exists x[\#x > 3 \wedge \text{beer}(x) \wedge P(x)]$

Existential closure is not a suitable type-shift operation for downward-entailing quantifiers, and we propose that in these cases an alternative operation applies, namely universal closure (de Swart 2001).

(5) $[\![\forall[A]]\!] = \lambda P\forall x[P(x) \rightarrow [\![A]\!](x)]$

We furthermore assume that any backgrounded material in the predicate that is shifted by $\forall$ forms part of the restriction of the universal quantifier. Consequently, the quantifier *fewer than three*$_F$ *beers* is interpreted as in 6.

(6) $[\![\forall[\text{fewer than [three}_F \text{ beers]]}]\!]$
$= \forall[\lambda x[\#x < 3 \wedge \text{beer}(x)]]$
$= \lambda P\forall x[[P(x) \wedge \text{beer}(x)] \rightarrow \#x < 3]$

Applying this to an argument like *Barney* results in the proposition that any group of beers consumed by Barney consists of no more than two beers.

As remarked above, universal closure interacts with focus. The machinery introduced in the foregoing allows us to account for this, too, by defining $\textcircled{\exists}$ and $\textcircled{\forall}$ as in 7.

(7) a. $[\![\textcircled{\exists}[\alpha]]\!] = \lambda P \exists \beta \exists x[P(x) \wedge \beta \in alt(A) \wedge \beta(x) \wedge [\![A]\!](x)]$

b. $[\![\textcircled{\forall}[\alpha]]\!] = \lambda P \forall \beta \forall x[[P(x) \wedge \beta \in alt(A) \wedge \beta(x)] \rightarrow [\![A]\!](x)]$

Assuming $[\![A]\!] \in alt(A)$, 7a reduces to $\lambda P \exists x[P(x) \wedge [\![A]\!](x)]$, which is the definition of existential closure given in 3. We leave it to the reader to verify that the definition of universal closure in 7b gives the right result, too.

The locus of superlative modification

For comparative quantifiers, the order of modification and type-lifting is fixed, in effect, by the restrictions comparative modifiers impose on their arguments.

(8) a. $^*[\text{more than}]_{\langle\langle e,t\rangle,\langle e,t\rangle\rangle}\ [\textcircled{\exists}_{\langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle}\ [n\ \text{N}]_{\langle e,t\rangle}]_{\langle\langle e,t\rangle,t\rangle}]$

b. $[\textcircled{\exists}_{\langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle}\ [[\text{more than}]_{\langle\langle e,t\rangle,\langle e,t\rangle\rangle}\ [n\ \text{N}]_{\langle e,t\rangle}]_{\langle e,t\rangle}]_{\langle\langle e,t\rangle,t\rangle}$

Superlative modifiers are analyzed as free modifiers, which apply to arguments of any Boolean type. Given this polymorphic typing of superlative modifiers, they will often give rise to several alternative derivations, not all of which need be acceptable. For instance, $\textcircled{\exists}$[*at least* [n_F *N*]] always denotes the empty set. This is because it describes the set of groups of N whose cardinality must be *n* and may be greater than *n*. There is no such group, obviously, and therefore this structure is outlawed on pragmatic grounds. It is possible, however, to interpret *at least* in a higher position, and thus arrive at a viable reading. To illustrate, compare the following two structures and their interpretations.

(9) [Betty had $\textcircled{\exists}$[at least [four$_F$ beers]]]

a. $[\![\text{at least [four}_F \text{ beers]}]\!]$
$= \lambda x[\Box[4(x) \wedge beer(x)] \wedge \Diamond[\#x > 4 \wedge beer(x)]]$

b. $[\![\text{Betty had } \textcircled{\exists}\text{[at least [four}_F\text{ beers]]}]\!]$
$= \exists x[\Box[4(x) \wedge beer(x)] \wedge \Diamond[\#x > 4 \wedge beer(x)] \wedge have(b, x)] = \bot$

(10) [Betty had [at least $\textcircled{\exists}$[four$_F$ beers]]]

a. $[\![\text{at least } \textcircled{\exists}\text{[four}_F\text{ beers]}]\!]$
$= \lambda P[\Box\exists x[4(x) \wedge beer(x) \wedge P(x)] \wedge \Diamond\exists x[\#x > 4 \wedge beer(x) \wedge P(x)]]$

b. $[\![\text{[Betty had [at least } \textcircled{\exists}\text{[four}_F\text{ beers]]]}]\!]$
$= \Box\exists x[4(x) \wedge beer(x) \wedge have(b, x)] \wedge$
$\Diamond\exists x[\#x > 4 \wedge beer(x) \wedge have(b, x)]$

Whereas the structure that interprets *at least* as applying to a predicate results in absurdity, the structure where it is applied to a quantifier yields a perfectly plausible proposition. Superlative modifiers are not restricted to application to quantifiers, however. Sometimes *at least* can be sensibly evaluated within an NP, as in 11, for example.

(11) $[\![\text{an [[at least four-star] hotel]}]\!]$
$= [\![an]\!]([\![\text{at least}]\!](\lambda x.4\text{-}star(x)) \circ \lambda x.hotel(x))$
$= [\![an]\!](\lambda x[\Box 4\text{-}star(x) \wedge [\Diamond 5\text{-}star(x) \vee \Diamond 6\text{-}star(x) \vee \ldots\ ]] \circ \lambda x.hotel(x))$
$= [\![an]\!](\lambda x[\Box 4\text{-}star(x) \wedge [\Diamond 5\text{-}star(x) \vee \Diamond 6\text{-}star(x) \vee \ldots\ ] \wedge hotel(x)]$
$= \lambda x[1(x) \wedge \Box 4\text{-}star(x) \wedge [\Diamond 5\text{-}star(x) \vee \Diamond 6\text{-}star(x) \vee \ldots\ ] \wedge hotel(x)]$

On this construal, *an at least four-star hotel* is a hotel of which it is certain that it has four stars, but which may have more.

Cases like these are exceptional, though. Generally speaking, superlative modifiers tend to be cashed out relatively late in the semantic derivation, as in the compositional construal of 12a, for instance.

(12) a. You may have at most two beers. (= 12a in main text)

b. $[\![\text{[at most [you may have } \textcircled{\exists}\text{[two}_F\text{ beers]]]]}]\!]$
$= [\![\text{at most}]\!](\Diamond\exists x[2(x) \wedge beer(x) \wedge have(y, x)])$
$= \Diamond\Diamond\exists x[2(x) \wedge beer(x) \wedge have(y, x)] \wedge$
$\neg\Diamond\Diamond\exists x[\#x > 2 \wedge beer(x) \wedge have(y, x)]$

In principle, 12a might be parsed as [*may* [*you have* [*at most* $\textcircled{\exists}$[*two$_F$ beers*]]]], but on this analysis the epistemic modal contributed by *at most* would end up with the scope of the deontic modal *may*, yielding an incoherent reading. In order to avoid this, the deontic modal is evaluated first, and the result is 12b.

Nonentailment scales

As we have analyzed it, the interpretation of *at least* is always factive in the sense that *At least* α entails that α is the case.[21] For the examples discussed so far, this prediction is correct. However, all of the examples

[21] Assuming, that is, that the modality conveyed by *at least* is epistemic, which is the default.

we have seen involved entailment scales, that is, scales with the property that objects higher in the order contain any objects further down. There are scales that do not have this property, like rank orders, for example. Being a lieutenant colonel outranks being a major, but one cannot be both a lieutenant colonel and a major. Similarly, gold is more expensive than silver, and in this respect outranks it; but being gold precludes being silver.

Our semantics for *at least* presupposes that scales are ordered by entailment rather than rank, for it only makes sense to state that something is necessarily α and possibly β, where $\beta \rhd \alpha$, if being α does not preclude being β. However, it isn't too difficult to accommodate nonentailment scales, along the lines of 13.

(13) $[\![\text{at least } \alpha]\!] = \exists\beta[\beta \unrhd \alpha \wedge \Box\beta] \wedge \exists\beta[\beta \rhd \alpha \wedge \Diamond\beta]$

This generalizes the definition given in the main text in 44a in a straightforward way; the $\langle a, t\rangle$ sense of *at least* is revised accordingly. To illustrate, 14a is now construed as 14b.

(14) a. Betty is at least a major.
b. $\Box$[major(b) $\vee$ lieutenant-colonel(b) $\vee$ general(b) $\vee$ . . .] $\wedge$
$\Diamond$[lieutenant-colonel(b) $\vee$ general(b) $\vee$ . . .]

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