HURFORD'S CONSTRAINT AND THE THEORY OF SCALAR IMPLICATURES : EVIDENCE FOR EMBEDDED IMPLICATURES*

GENNARO CHIERCHIA[1]
DANNY FOX[2]
BENJAMIN SPECTOR[1,3]
Harvard University[1]*, MIT*[2]*, and Institut Jean-Nicod*[1]

1 Introduction

According to the neo-Gricean approach to scalar implicatures, the scalar implicatures of a given sentence S are computed on the basis of S's litteral meaning and that of its *competitors*, i.e. its scalar alternatives. It is thus possible (see for instance Spector 2003, van Rooij and Schulz 2004, Fox 2007) to define an operator which takes two arguments, a set of alternatives *ALT* and a sentence S, and returns the conjunction of S and of its scalar implicature relatively to *ALT*, what we may call the *strengthened meaning* of S.

From a neo-Gricean perspective, such an operator is viewed merely as a shortcut, i.e. as a tool that the linguist can use in order to quickly compute the strengthened meaning of a given sentence, but not as part of the grammatical system. However, another possibility exists: the implicature-computing operator might be part of grammar, and be present in the logical form of sentences. Such a *grammatical view* of scalar implicatures has been defended in several recent works, on various grounds (e.g. Chierchia 2006, Fox 2007; see Chierchia et al. – to appear – for a review[1]). If something like this is correct, then in principle the implicature-computing operator could be inserted not only at the matrix level, but also in embedded positions, thus giving rise to *embedded scalar implicatures*. In contrast with this, the standard neo-Gricean view does not predict the existence of embedded scalar implicatures, because under this view scalar implicatures are derived on the basis of the maxim of quantity which is only relevant to complete speech acts, and not to subconstituents of a sentence. Several recent works have argued for the existence of embedded implicatures (see in particular, besides the papers cited above, Landman 1998,

*Authors' names are listed alphabetically.

[1]Chierchia's (2004) also argued that scalar implicatures are not derived by purely pragmatic and non-compositional mechanisms, but are computed compositionally, on a par with other aspects of sentence meaning. But Chierchia's (2004) precise implementation did not resort to an implicature-computing operator present in the logical forms of the relevant sentences.

Chierchia 2004, and also Magri 2007), hence for a radical reassesment of the standard neo-Gricean approach. In response to these arguments, there have been various attempts to show that the alleged examples of embedded implicatures could in fact be analyzed as'global implicatures', once a careful formalization of the underlying pragmatic reasoning is given (Sauerland 2004, Spector 2003, van Rooij and Schulz 2004, Russell 2006).

In this paper, we are going to present a new argument[2] for the existence of embedded implicatures, and, more specifically, for the possibility of inserting an implicature-computing operator in an embedded position. We'll show that a complex set of phenomena involving the interactions between a constraint on disjunctive phrases (Hurford's Constraint) and the interpretation of scalar items can be accounted for in an enlightening way once we assume that an implicature-computing operator can (and sometimes must) be inserted in embedded positions.

2 Hurford's Constraint

Hurford (1974) points to the following generalization:

> **Hurford's Constraint**: A sentence that contains a disjunctive phrase of the form 'S or T' is infelicitous if S entails T or T entails S.[3]

This constraint is illustrated by the infelicity of the following sentences:

(1) a. #Mary saw a dog or an animal.
b. #Mary saw an animal or a dog.
c. #Every girl who saw an animal or a dog talked to Jack.

However, the following example, which is felicitous, seems to be a counterexample to Hurford's Constraint:

(2) Mary solved the first problem or the second problem or both problems

If *or* is interpreted inclusively, then clearly (2) violates Hurford's Constraint, since 'Mary solved both problems' entails 'Mary solved the first problem or the second problem'. On the basis of such examples, Hurford reasoned that *or* has to be ambiguous, and that one of its readings is the exclusive reading. On an exclusive construal of the first disjunction in (2), the sentence no longer violates Hurford's Constraint. Gazdar (1979) noticed other cases where Hurford's Constraint appears to be obviated, such as (3):

(3) Mary read some or all of the books

If *some* simply expresses existential quantification and if *all* is universal quantification, then 'all of the books' entails 'some of the books', and (3) violates Hurford's Constraint. By analogy

[2]Or rather several closely interrelated arguments, which were first developed independently a) in a seminar taught by D. Fox in 2004 (especially subsection 5.2 of the present paper, see Fox 2004) and in a MIT/Harvard seminar jointly taught by D. Fox and G. Chierchia in 2006 (especially sections 2, 3 and 4 of the present paper) and b) in B. Spector's dissertion (especially sections 2 and 5 of the present paper, see Spector 2006). In Chierchia et al. (To appear) we present these arguments in a broader context.

[3]The relation 'entail' has to be understood under its generalized version, i.e. 'is included in', so as to be applicable to pairs of non-propositional constituents.

with Hurford's reasoning about disjunction, one might conclude that *some* is ambiguous as well, and means *some but not all* on one of its readings. But Gazdar argued that multiplying lexical ambiguities in order to maintain Hurford's Constraint misses an obvious generalization. Namely, the items that have to be analyzed as ambiguous in order to maintain Hurford's Constraint are all scalar items, and the new meanings that are introduced correspond to the scalar implicatures that these items induce in simple contexts. Instead of assuming that these scalar items are ambiguous (which would, in effect, amount to a rejection of the whole neo-Gricean enterprise), Gazdar proposed to weaken Hurford's generalization in the following way:

> **Gazdar's generalization**:
> A sentence containing a disjunctive phrase 'S or T' is infelicitous if S entails T or if T entails S, unless T contradicts the conjunction of S and the implicatures of S.

Let us look at a schematized version of (2):

(4) (A or B) or (A and B)

Let S be *A or B* and T be *A and B*. The conjunction of S and its scalar implicatures is *A or B but not both*, which contradicts T. So the felicity of (4) is predicted by Gazdar's generalization.

Even though Gazdar did not spell-out an account for this generalization, one could interpret his observations as suggesting that violations of Hurford's Constraint involve some kind of 'implicature cancellation mechanism', in the sense that the second disjunct is used, so to speak, to cancel an implicature of the first disjunct (see Sharvit and Gajewski 2007). Instead of resorting to such a mechanism, we'll argue for the following:

- Hurford's Constraint is correct as originally stated.
- All the apparent violations of Hurford's Constraint involve the presence of an implicature-computing operator within the first disjunct, ensuring that Hurford's Constraint is met - hence the presence of a 'local implicature'.[4]

Following previous works (Groenendijk and Stockhof 1984, van Rooij and Schulz 2004, Spector 2003) we call the implicature-computing operator an *exhaustivity operator*, which we represent as *exh*.

It is clear that something close to Gazdar's generalization follows from these two assumptions: suppose S2 entails S1; then 'S1 or S2' violates Hurford's Constraint; yet 'exh(S1) or S2' may happen to satisfy Hurford's Constraint; this will be so if S1 together with its implicatures is no longer entailed by S2, which will be the case, in particular, if S2 contradicts S1 together with its implicatures. For instance, a sentence of the form 'A or B or both A and B' has to have the following underlying structure for it to satisfy Hurford's Constraint: [exh(A or B)] or [both A and

[4] In a way, we are extending Hurford's original account based on ambiguity to all scalar items. But, in contrast with Hurford, we do not assume any kind of *lexical* ambiguity. Rather, on our view, scalar items appear to be ambiguous because of the optional presence of an embedded implicature-computing operator.

B].[5] Such a logical form satisfies HC because the first disjunct, i.e. *exh(A or B)* means *A or B but not both* and is therefore not entailed by the second conjunct (*(A and B)*).

The assumption that all the apparent violations of Hurford's Constraint involve the presence of *exh* in the first disjunct turns out to make very precise predictions in a number of cases - predictions that do not fall out from Gazdar's proposal. In the next subsections, we'll spell out these predictions and corroborate them in various ways. Since they crucially depend on the assumption that an embedded implicature-computing operator is obligatorily present in all the relevant cases, these predictions provide important evidence, in our view, for the claim that SIs can be computed in embedded positions.

Before moving to these detailed arguments, however, we should be explicit about the meaning of *exh*. For the time being, we take the semantics of *exh* to be essentially the same as the one proposed by Rooth (1992) and Krifka (1993) for *only*.[6] That is, *exh* applies to a sentence S (which we call the *prejacent*) and a set of alternative propositions *ALT* (which we represent below as a subscript to *exh*), and returns the conjunction of S and the negation of all the members of ALT that are not entailed by S. For instance, (5a) below is equivalent to (5b), because *Jack read all of the books* is the only member of the alternative set which is not entailed by the prejacent (i.e. by *Jack read most of the books*).

(5) a. $\text{exh}_{\{\text{J. read some of the books, J. read most of the books, J. read all of the books}\}}$(Jack read most of the books)
b. Jack read most of the books, and he did not read all of the books

Equivalently, exh_{ALT}(S) states that S is true and that the only members of ALT that are true are those entailed by S (i.e. nothing is true in ALT besides S and what follows from S), which corresponds to the following definition:

(6) $[\![\text{exh}_{\text{ALT}}(\text{S})]\!]^{w} = 1$ iff $[\![\text{S}]\!]^{w} = 1$ and $\forall\phi \in \text{ALT}\ (\phi(w) = 1 \rightarrow ([\![\text{S}]\!] \subseteq \phi))$

In the rest of this paper, though, we will generally omit the first argument of the exhaustivity operator, i.e. the set of alternatives with respect to which the prejacent's implicatures are computed. Hence we'll simply write *exh*(S) instead of exh_{ALT}(S). We will make the simplifying assumption that the set of alternatives is fully determined by the logical form of the prejacent (in particular, by the scalar items that occur in the prejacent).

3 Forcing embedded implicatures

Gazdar's generalization, as such, does not make any particular prediction regarding the reading that obtains when there is an apparent violation of Hurford's Constraint. But consider now the following sentence (in a context where it has been asked which problems Peter solved within a certain set of problems):

[5] In this paper, we do not account for the following asymmetry, pointed out by Singh (2007):

(i) a. Mary saw Peter or Sue or both Peter and Sue
b. #Mary saw both Peter and Sue or Peter or Sue

See Singh (2007) for an interesting proposal. See also Fox and Spector (2008).

[6] But see subsection 5.2 and the references cited therein, in which it is shown that the exhausitivity operator must in fact be defined in a more complex way.

(7) Peter either solved both the first and the second problem or all of the problems.

In the absence of an exhaustivity operator, (7) would violate Hurford's Constraint, since solving all of the problems entails solving the first one and the second one. And (7) would then be equivalent to (8):

(8) Peter solved the first problem and the second problem.

Therefore, we predict that an exhaustivity operator has to be present, with the effect that (7)'s logical form is the following:

(9) exh(Peter solved the first problem and the second problem) or he solved all of the problems

Recall that the meaning of our operator is supposed to be, on first approximation, the same as that of *only*. If we are right, (7) should therefore be understood as equivalent to the following:

(10) a. Peter only solved the first problem and the second problem, or he solved all of the problems

b. Either Peter solved the first problem and the second problem and no other problem, or he solved all the problems

It turns out that this is indeed the only possible reading of (7). In other words, (7) is clearly judged false in a situation in which Peter solved, say, the first problem, the second problem and also the third problem (out of a set of more than three problems).[7] It is hard to see how an analysis based on the notion of 'implicature cancellation' could account for the particular interpretation that such a sentence triggers. Under such a theory, the presence of an implicature cancellation device is not expected to yield any new implicatures.[8] So (7) seems to be a clear case of an embedded implicature.

It is worth noticing that the reading we observe is maintained when the whole sentence is itself embedded, say, in a DE-context:

(11) Whoever solved the first and the second problems or solved all of the problems will pass.

This sentence is understood as equivalent to:

(12) Whoever solved only the first and the second problems or solved all of the problems will pass.

[7] In the case at hand, we assume that the alternatives of the first disjunct (i.e. 'Peter solved the first problem and the second problem') consist of all the propositions that can be expressed by sentences of the form *Peter solved X*, where X denotes one of the problems or a plurality made up of some of the problems. So the alternatives, in this case, are not solely defined in terms of scales.

[8] Note indeed that no implicature gets truly cancelled; in the absence of the second disjunct, the first disjunct ('Peter solved the first problem and the second problem') implicates that Peter didn't solve any other problem; once the second disjunct is added, the implicature doesn't really disappear; rather, it is integrated into the meaning of the first disjunct. If no exhaustivity operator applied to the first disjunct, there would be no simple way to derive the reading we get by applying an exhaustivity operator to the whole sentence, because (ia) below is equivalent to (ib):

(i) a. exh(Jack solved the first problem and the second problem or all of the problems)

b. Jack solved the first problem and the second problem and no other problem

In short, our argument can be summed up as follows: if Hurford's Constraint is correct as originally formulated, and if the implicature-computing operator *exh* can occur in embedded positions, then in some cases, the only way to satisfy Hurford's Constraint is to insert *exh* locally, and this gives rise to particular readings which turn out to be the only possible readings.

4 Forcing even more embedded implicatures

So far, we have discussed sentences of the form 'S or T' such that T entails S but T is incompatible with *exh(S)*, to the effect that 'exh(S) or T' does not violate Hurford's Constraint. Such sentences force the insertion of *exh* within the scope of the main disjunction, thus making a case for embedded implicatures in UE contexts. We would like to show that there may also be cases where the only way to satisfy Hurford's Constraint is to insert an exhaustivity operator in an even more embedded position, namely within a subconstituent of the first disjunct.

Consider the following sentence:

(13) Every student solved some of the problems

There have been discussions in the recent literature as to whether *some* in a sentence like (13) can be read as equivalent to *some but not all* (e.g. Chierchia 2004, Spector 2006, Geurts, to appear -a). On the grammatical view we are considering, the possibility of such an interpretation is expected, since inserting *exh* within the scope of *every student* would generate exactly this reading.[9] Using the same technique as before, we are going to show that we can force *some* in (13) to be read as *some but not all* by embedding (13) itself in a bigger structure.

Before going through the argument, let us make some preliminary points. Suppose (13) only competes with (14):

(14) Every student solved all of the problems

Then 'exh((14))' is equivalent to '$(13) \wedge \neg(14)$', i.e:

(15) Every student solved some of the problems and at least one student did not solve them all

This is clearly a natural interpretation of (13).

Inserting *exh* below every student gives rise to a stronger reading:

(16) a. (Every student)$_x$ (exh(x solved some of the problems)
b. (Every student)$_x$ (x solved some of the problems $\wedge$ $\neg$(x solved all of the problems))
c. Every student read some but not all of the problems.

Let us now construct a sentence in which (13) occurs as a constituent and is forced to correspond to the logical form in (16) for the whole sentence to comply with Hurford's Constraint. We claim that the sentence given in (17) achieves exactly that:[10]

[9]This interpretation is however not necessarily ruled out from the neo-Gricean standpoint. Some theories in which scalar implicatures are derived pragmatically on the basis of the global meaning of the relevant sentence (van Rooij and Schulz 2004, Spector 2003, Spector 2006) are able to generate this 'strengthened' meaning, provided the following also count as alternatives for (13): 'some students solved some of the problems', 'some students solved all of the problems'. Fox (2007) proposes a constraint on alternatives that rules out such a large set of alternatives for (13) (cf. his footnote 35). For the sake of this discussion, we assume that the *some but not all* reading under a universal quantifier can only be derived as a local implicature.

[10]Some speakers might feel that in (17) *some* has to be stressed.

(17) Every student solved some of the problems, or Jack solved all of them and all the other students solved only some of them.

Our empirical claim is that (17)'s only reading is one in which *some of the problems* in the first disjunct is interpreted under its strengthened meaning, i.e. *some but not all*. In other words, we claim that (17) is equivalent to (18), so that it is judged false if a student other than Jack solved all of the problems:

(18) Every student solved some of the problems, and no student except maybe Jack solved all of the problems.

This equivalence follows directly from the combination of Hurford's Constraint and the possibility of inserting *exh* locally. Let us show why.

First consider what happens if (17) contains no exhaustivity operator at all: then clearly Hurford's Constraint is violated since the second disjunct entails the first one. What if we apply *exh* to the first disjunct, as in the previous examples?

(19) exh(Every student solved some_{F} of the problems), or Jack solved all of them and every other student solved only some of them

This logical form is equivalent to the following sentence:

(20) Every student solved some of the problems and not all of the students solved them all, or Jack solved all of them and every other student solved only some of them

It turns out that this still violates Hurford's Constraint: the second disjunct does in fact entail that not all of the students solved all of the problems, hence entails the first disjunct.

So the only possible analysis is the following:

(21) $(\text{Every student})_{\text{x}}$ (exh(x solved some_{F} of the problems) or Jack solved all of them and all the other students solved only some of them.

(21) yields exactly the reading that we in fact observe, which, notice, is neither equivalent to the 'literal' reading of the sentence (the reading that we would see if there were no operator at all) nor to the 'globally strengthened' meaning of the sentence (the reading that we would see if there were just one operator scoping over the entire sentence).[11] Not only have we constructed an example where an embedded implicature clearly arises,[12] but we have also provided a precise account of the manner by which this reading comes about. This account, if successful, provides support to our two assumptions, i.e. that a covert exhaustivity operator can be inserted locally and that Hurford's Constraint is correct as originally formulated.

[11] This last claim cannot be rigorously defended without being quite explicit about the alternatives of such a complex sentence, and we do not provide a complete argument in this paper. Intuitively, what happens is that (17) (in the absence of a covert exhaustivity operator) and (13) are equivalent and, given certain reasonable assumptions about how alternatives are computed, they remain equivalent even after global exhaustification, i.e. exh((17)) = exh((13)) (= *Every student solved some of the problems, and not all students solved all of the problems*).

[12] Note that the claim that (17)'s interpretation involves an embedded implicature does not in fact depend on our assumption that the *some but not all* reading of *some* requires the presence of the operator within the scope of the universal quantifier. As mentioned in fn. 9, it may be possible to derive this reading by applying the operator to the first disjunct as a whole. What is crucial here is that there must be an exhaustivity operator *within* the first disjunct, hence in an embedded position.

5 Hurford's Constraint and recursive exhaustification

In this section, we are going to show that even in cases where the obligatory presence of an embedded implicature-computing operator does not have any direct effect on the literal truth-conditions of a given sentence, it nevertheless has observable effects that can be detected by looking at the implicatures (or lack thereof) that the sentence in question triggers. In our terms, the presence of the embedded implicature-computing operator turns out to have a truth-conditional effect when the sentence is itself embedded under another implicature-computing operator.

First, we'll look at the interpretation of disjunctions of the form 'A or B or both' in the scope of necessity modals (5.1). Then we'll offer an account of the 'cancellation effect' triggered by *or both* in non-embedded contexts (5.2).

5.1 *Or both* in the scope of necessity modals

Consider the following two sentences:

(22) We are required to either read *Ulysses* or *Madame Bovary*

(23) We are required to either read *Ulysses* or *Madame Bovary* or both

What is clear about these sentences is that they both implicate that we are not required to read both *Ulysses* and *Madame Bovary*.[13] Otherwise, at first sight, they do not seem to trigger different implicatures. But upon reflection, they, in fact, do. Suppose that we are actually required to read *Ulysses* or *Madame Bovary* and that we are not allowed to read both of them. Then (22) would be an appropriate statement, while (23) would not. (22), on its most natural reading, is silent as to whether or not we are allowed to read both novels. But (23) strongly suggests that we are allowed to read both novels. So (23) seems to trigger an implicature that (22) does not.

This is further confirmed by looking at the following dialogues:[14]

(24) A: We are required to either read *Ulysses* or *Madame Bovary*
B: No! we have to read both

(25) A: We are required to either read *Ulysses* or *Madame Bovary*
B: # No! We are not allowed to read both

(26) A: We are required to either read *Ulysses* or *Madame Bovary* or both
B: No! We are not allowed to read both

(24)B shows that in a denial context, negation can target an implicature: B, in (24), is objecting to an implicature of the previous sentence, namely, that we are not required to read both novels. (25)B is clearly deviant. This shows that (25)A does not implicate that we are allowed to read both novels; indeed, if (25)A did trigger this implicature, then B's objection would be perfectly felicitous, since it would count as an objection to an implicature of A's utterance. What is important

[13]Interestingly, the presence of *or both* does not cancel the implicature that is normally triggered in the absence of *or both*. This, in our view, is unexpected if or both is viewed as an implicature-cancelling device, a view that might be attributed to Gazdar (1979), as we mentioned above.

[14]The presence of *either* is not crucial to these judgments; its function here is simply to rule out the wide scope interpretation of disjunction, which is irrelevant here (see Larson 1985). In particular, note that *either . . . or* does not generally force an exclusive reading, as illustrated by the fact that the following sentence is perfectly consistent: 'We are required to either read Ulysses or Madame Bovary, and we may read both'.

in the current context is that (25) clearly contrasts with (26). B's objection in (26) is completely natural, and hence confirms that (26)A does implicate that we are allowed to read both novels.

How are these facts to be explained? How come (23) has an implicature that (22) doesn't have? Note that (23) and (22) have the same truth-conditions. Yet they trigger different implicatures. We are going to show that this phenomenon is in fact entirely expected from our perspective. The implicatures associated with (23) and (22) are, in fact, instances of the following generalization:

(27) A sentence of the form $\Box$ *(A or B)* triggers the following implicatures:[15]

- $\neg\Box A$
- $\neg\Box B$

To illustrate this generalization let us begin with (22), repeated here as (28), which implicates that we have a choice as to how to satisfy our obligations.

(28) We are required to either read *Ulysses* or *Madame Bovary*

The 'strengthened' reading of (28), on the basis of the generalization in (27), is given in (29a), which is equivalent to (29b):[16]

(29) a. We are required to either read *Ulysses* or *Madame Bovary*, we are not required to read *Madame Bovary*, and we are not required to read *Ulysses*.

b. We are required to either read *Ulysses* or *Madame Bovary*, we are allowed to read *Ulysses* without reading *Madame Bovary*, and we are allowed to read *Madame Bovary* without reading *Ulysses*.

Now let's see the consequences of this generalization for the case of (23) ('We are required to read *Ulysses* or *Madame Bovary* or both'), schematized as follows:

(30) $\Box$[exh(A or B) or (A and B)]

(30) is predicted to implicate the following:

(31) a. $\neg\Box$(exh(A or B))

b. $\neg\Box$(A and B))

i.e. :

[15]The symbol '$\Box$' stands for any modal operator with universal force (*we are required to* ..., *Necessarily* ... , *Jack demanded that* ..., etc.).

[16]Proof:

- (29a) entails (29b): suppose that (29a) is true. Then in all the permissible worlds we read one of the two novels; since we are not required to read *Ulysses*, there is a permissible world w_1 in which we don't read *Ulysses*; but then in w_1 we read *Madame Bovary*. So in w_1 we read *Madame Bovary* without reading *Ulysses*, and therefore we are allowed to read *Madame Bovary* without reading *Ulysses*; by symmetry, we are also allowed to read *Ulysses* without reading *Madame Bovary*.
- (29b) entails (29a): suppose (29b) is true. Then a) we are required to read one of the two novels and b) since we are allowed to read either one without reading the other one, we are not required to read *Ulysses* and we are not required to read *Madame Bovary*, i.e. (29a) is true.

(32) a. We are not required to read *Ulysses* or *Madame Bovary* and not both
b. We are not required to read both *Ulysses* and *Madame Bovary*

The end-result is the following proposition, which indeed corresponds to the most natural reading of (23):

(33) We are required to read *Ulysses* or *Madame Bovary*, we are not required to read only one of the two novels, we are not required to read both novels.

From (33) it follows that we are allowed to read both novels, which was the desired result.

So far we have shown that given the generalization in (27), the observed interpretation of (23) follows directly from the assumption that *exh* is present in the first disjunct. Of course, it is also desirable to understand why this generalization holds, in order to give a complete account. It turns out that the exhaustivity operator as we have defined it so far (adding the negations of all non-weaker alternatives) can derive all the inferences that we observed, provided we now assume that the scalar alternatives of a disjunctive phrase *X or Y* include not only the phrase *X and Y* but also each disjunct X and Y independently. If so, then (22) has the following alternatives (using again the sign '$\Box$' to abbreviate 'we are required to', 'U' to abbreviate 'read *Ulysses*', and 'MB' to stand for 'read *Madame Bovary*'):[17]

(34) ALT((22))= $\{\Box$(U or MB), $\Box$U, $\Box$MB, $\Box$(U and MB)$\}$

To get the 'strengthened meaning' of (22), we simply have to add the negation of each of its alternatives that is not weaker than it. In this case, this results in the following, from which the generalization in (27) follows:

(35) exh((22)) = $\Box$(U or MB) $\wedge$ $\neg\Box$U $\wedge$ $\neg\Box$MB

Let us now go back to our original example:

(36) a. We are required to either read *Ulysses* or *Madame Bovary*, or both
b. $\Box$[exh(U or MB) or (U and MB)]

By assumption, the logical form of (36a) must be (36b) (i.e. the exhaustivity operator must have been introduced for Hurford's Constraint to be satisfied). Given our new assumptions about the alternatives of a disjunctive phrase, (36)'s alternatives include, among others, $\Box$*exh(U or MB)* (i.e. 'we are required to read either *Ulysses* or *Madame Bovary* and we are forbidden to read both') and $\Box$*(U and MB)*. To prevent the discussion from getting too long, let us make the simplifying assumption that these are the only alternatives. As these two alternatives are not logically weaker

[17] Following standard assumptions, the scalar alternatives of a complex expression are derived by possibly multiple substitutions of scalar terms with one of their scale-mates. For any sentence ϕ containing scalar terms $< t_1, \ldots, t_n >$ (noted $\phi(t_1, \ldots, t_n)$), the alternatives of ϕ are all the sentences of the form $\phi(t'_1, \ldots, t'_n)$, where t'_j belongs to the scale of t_j, for any j. What we need here is that the alternatives of any sentence ϕ containing *X* or *Y* (noted ϕ*(X or Y)* include $\{\phi(X \text{ or } Y), \phi(X), \phi(Y), \phi(X \text{ and } Y)\}$. Technically, one possible way of deriving this alternative set is to assume, as in Sauerland (2004), that the scale of *or* is the following partially ordered set : *{or, L, R, and}* where *L* is the binary connective such that *X L Y* is equivalent to *X*, and *R* is the one such that *X R Y* is equivalent to *Y*. For alternatives, see Spector (2006) and Katzir (2007).

than (36) (which is actually equivalent to 'we are required to read *Ulysses* or *Madame Bovary*'), the strengthened meaning of (36), i.e. what results from an application of the operator *exh* to the whole sentence, has to entail the negation of these two alternatives, which gives rise to a reading which is paraphrased in three different ways in (37). This is exactly what we wanted to derive.

(37) a. $\Box$(U or MB) $\wedge$ $\neg\Box$exh(U or MB)) $\wedge$ $\neg\Box$(U and MB)
b. We are required to either read *Ulysses* or *Madame Bovary*, and we are not required to read only one of them, and we are not required to read both
c. We are required to read *Ulysses* or *Madame Bovary* and we are allowed to read both of them and we are allowed to read only one of them

Let us sum up what has been shown: the obligatory presence of an exhaustivity operator applying to the first disjunct in sentences like (23), together with the assumption that each member of a disjunctive phrase contributes to the alternatives of the disjunctive phrase, immediately predicts that (23), though equivalent to (22), has more alternatives. The existence of these additional alternatives predicts, in turn, that the strengthened meaning of (23) is different from that of (22), and the precise prediction that is made seems to be corroborated.[18]

One crucial assumption for this account was that the alternatives of a disjunctive phrase include each of the disjuncts separately. This move, which is independently motivated,[19] gives rise to a certain problem: once this assumption is made, the exhaustivity operator as we have defined it so far leads to the contradictory proposition when applied to a simple, non-embedded disjunctive sentence.[20] We'll return to this problem in the next sub-section, in which we'll introduce a new, more realistic, exhaustivity operator, which does not run into the same difficulty. But what is important so far is that this more realistic operator allows us to maintain the account that has just been presented: it turns out that for all the cases that we have considered in this subsection, the operator *exh*, as we have defined it initially, yields the same result as the more realistic operators that have been proposed in the recent literature.[21]

5.2 Cancellation

In this section, we show that the assumptions that we have developed in the previous sections are also able to predict the most well-known observation about the behavior of or both. Namely,

[18]Recall that we made the simplifying assumption that (30) had only two alternatives. When we add the alternatives that are induced by the more embedded disjunctions and conjunctions, we end up with a much bigger set (even after having eliminating some alternatives that are equivalent to other ones): {$\Box$(U or MB), $\Box$(exh(U or MB)), $\Box$(exh(U)), $\Box$(exh(MB), $\Box$(U and MB), $\Box$U, $\Box$MB}. As a result, the following inferences are also predicted: 'we are not required to read both novels' , 'we are not required to read *Madame Bovary*', 'we are not required to read *Ulysses*'. These are good results as well. Note however that we could get a different result if we considered even more alternatives, namely, those that are triggered by the scale <*allowed*, *required*>. Fox (2007) proposes a constraint on the computation of alternatives that rules out these additional alternatives (his footnote 35).

[19]This assumption has been shown to be able to solve a much discussed puzzle regarding sentences in which a scalar term occurs under the scope of a disjunction. The puzzle was first discussed in Chierchia (2004). Sauerland (2004) offered a solution based on a modified scale for disjunction. Sauerland's ideas will be presented in the next subsection.

[20]Here is why: if the alternatives of 'p or q' are {p or q, p, q, p and q}, then negating all the non-weaker alternatives amounts to adding $\neg$p, $\neg$q, $\neg$(p and q). But '(p or q) $\wedge$ $\neg$p $\wedge$ $\neg$q $\wedge$ $\neg$(p $\wedge$ q)' is contradictory.

[21]This is so thanks to the following fact: let *exh* be the exhaustivity operator as we have defined it and let *exh** be one of the operators defined in the recent literature (Spector 2003, Spector 2006, van Rooij and Schulz 2004, Fox 2007). Then, for any given sentence ϕ, *if exh(ϕ) is not contradictory, then exh(ϕ) = exh*(ϕ).*

in non-embedded contexts, 'or both' removes the exclusive reading of disjunction ((38) below, schematized in (39)), i.e. seems to 'cancel' an implicature.

(38) Peter or Jack or both came

(39) exh(p or j) or (p and j)

In our terms, this is again a case where the presence of a local exhaustivity operator has no direct detectable truth-conditional effect, since (39) (*(p* or_{excl} *q)* or_{incl} *(p and q)*) is equivalent to *p* or_{incl} *q*. As before, though, the presence of an embedded exhaustivity operator creates more alternatives. We will now show that these new alternatives affect the interpretation that result from applying the operator (again) to (39). Specifically, applying *exh* to (39) is a vacuous move, which accounts for the 'cancellation effect' triggered by *or both*. In order to show how this prediction comes about, we will use a more realistic exhaustivity operator, one that does not yield contradictions when it applies to simple disjunctive sentences (see fn. 20).

Before doing so, though, let us notice that once it is assumed that the presence of *or both* entails the presence of an implicature-computing operator applying to the first disjunct, the 'cancellation' effect of or both happens to be an instance of the following generalization (which is due to Gazdar 1979):

(40) A sentence of the form ϕ *or* ψ triggers the inference that its author does not know whether ϕ is true and does not know whether ψ is true.

Applied to (39), this results in the following: the author of (39) does not know whether *p* or_{excl} *q* is true and does not know either whether *p and q* is true.[22] Now this conclusion is obviously incompatible with the derivation of an 'exclusive' implicature (i.e. an inference that the speaker believes that *p and q* is false). Hence the cancellation effect of *or both* follows.

In order to explain why the generalization in (40) is in fact correct, and what role the presence of the embedded implicature-computing operator plays in deriving the right result, it is necessary to move to a more sophisticated theory of implicature computation.

Let us first have a look at a simple disjunctive sentence, adding the crucial assumption that each disjunct is an alternative of the initial sentence:

(41) a. Peter or Jack came
b. p or j

The alternatives for *p or j* are now assumed to be {*p or j*, *p*, *j*, *p and j*}. As explained in fn. 32, applying *exh* to (41) relatively to this set of alternatives results in the contradictory proposition (because *p* and *j* are both strictly stronger than *p or j*, and we end up with *exh(p or j) = (p* $\wedge$ *j)* $\wedge$ $\neg$*p* $\wedge$ $\neg$*j*). Plainly, some modification is needed.

In order to understand how the operator should be modified, it is useful to take a small digression and see how this problem can be dealt with within a neo-Gricean setting (We are now presenting the analysis found in Sauerland 2004 - related proposals can be found in Spector 2003, Spector 2006 and van Rooij and Schulz 2004):

[22] Note that in the absence of the operator *exh*, applying the generalization in (40) to *(p* or_{incl} *q)* or_{incl} *(p and q)* (i.e. to an LF that does not contain *exh*) results in a contradiction, if the speaker is taken to believe what he says; for the speaker, on the one hand, would have to believe that *(p* or_{incl} *q)* or_{incl} *(p and q)* is true. i.e. that *p* or_{incl} *q* is true, and, on the other hand, would have to be undecided as to whether that very proposition is true.

- The speaker has said 'p or j', so she believes that 'p or j' is true.
- It follows from the maxim of quantity that that the speaker does not believe more than this relative to the set of alternatives: in other words, she only believes 'p or j', i.e. she does not have the belief that p is true, she does not have the belief that j is true and she does not have the belief that 'p and j' is true (this last statement is actually entailed by the first two).

At this point we have derived inferences of the form 'the speaker does not have the belief that S', which Sauerland terms *primary implicatures* (*secondary implicatures* are inferences of the form 'the speaker believes that not-S'). Now comes a crucial observation, based on the assumption that the speaker is logically coherent: since the speaker believes that 'p or j' is true and does not have the belief that p is true (primary implicature), she cannot have the belief that j is false. Indeed, if she believed 'not j', then given that she believes 'p or j', she would have to believe p. Symmetrically, the speaker does not have the belief that p is false. The end result of this reasoning is that the speaker has no opinion as to whether p is true or false and as to whether j is true or false. It is a fact that these 'ignorance inferences', which Gazdar (1979) called 'clausal implicatures', are indeed triggered by disjunctive statements in normal contexts.

'Secondary implicatures' of the form "the speaker believes that not S", where S is an alternative of the sentence uttered, require an additional step, a move from "The speaker does not believe that p" to "The speaker believes that not-p" (what Sauerland 2005 calls the "epistemic step" - see also Chemla 2008 for an interesting discussion). But it is now clear that sometimes this move will contradict previously established primary implicatures. For instance, in the above case, moving from "the speaker does not have the belief that p" to "the speaker believes that not-p" actually conflicts with the already established conclusion that "the speaker does not have the belief that not-p". Sauerland proposal is that the move from "the speaker does not believe that p" to "the speaker believes that not-p" occurs only if this does not contradict the primary implicatures that have been derived in the first step. In the case of 'p or j', this move can apply to the alternative 'p and j', but not to p and j taken in isolation. We end up with an exclusive reading for disjunction, together with two "ignorance inferences", according to which the speaker does not know whether p is true and does not know whether j is true.

Can we achieve the same results in a theory in which SIs are generated by an implicature-computing operator, rather than by purely pragmatic reasoning? What is needed is a more sophisticated definition of the operator *exh*, one that ensures that the application of *exh* to a given sentence S returns the proposition that *would* have resulted from the application of Sauerland's purely pragmatic procedure.[23] Various and related proposals in the recent literature provide us with such a definition.[24] We are not going to present these fairly technical proposals,

[23] Though *exh* does not derive primary implicatures. Those would probably be pragmatic even in a theory that incorporates *exh*. See Fox (2007) for cases where predictions can diverge due to this difference between Sauerland's procedure and a theory derives many of the same results by a particular meaning for *exh*.

[24] See Spector (2003, 2006, 2007), van Rooij and Schulz (2004), Fox (2007). Spector's and van Rooij & Schulz's proposals are heirs to Groenendijk and Stockhof's (1984) exhaustivity operator. For the time being, it is enough to assume that a non-weaker alternative gets negated if and only if negating it is consistent with negating any other non-weaker alternative. In the case of 'p or j', the only such "innocently excludable" alternative is actually 'p and j'. An actual formalization in these terms is given by Fox (2007). Fox's definition of *exh* is the following:

(i) a. Preliminary definition: An alternative of ϕ is *innocently ϕ-excludable* if its negation belongs to all maximal-consistent sets that include only ϕ and negations of an alternative of ϕ.

b. $exh_{\mathrm{ALT}}(\phi) = \phi \wedge \neg a_1 \wedge \neg a_2 \wedge \ldots \wedge \neg a_n$, where $\{a_1, \ldots, a_n\}$ is the set of all innocently ϕ-excludable members of ALT.

and we will simply assume that the proposition that *exh* returns when applied to a sentence S is what would result from applying Sauerland's procedure to S.

Let us therefore apply Sauerland's procedure to (38) and (39), repeated as (42) and (43):

(42) Peter or Jack came, or both came

(43) exh(p or j) or (p and j)

(43)'s alternatives include, among others, the following two sentences: *exh(p or j)* (which is equivalent to *p or j but not both*) and *p and j*. Both these sentences are strictly stronger than (43) (which is equivalent to 'Peter or_{incl} Jack came'). We therefore derive the following primary implicatures: the speaker does not have the belief that *exh(p or j)* is true, and the speaker does not have the belief that *p and j* is true. Can we strengthen (via the epistemic step) these primary implicatures into secondary implicatures without generating a contradiction? We would end up with the following propositions:

(44) a. $p \vee j$ (= the literal meaning of the (43))
b. $\neg(p \wedge j)$
c. $\neg(p \vee_{\text{excl}} j)$

But these three statements, taken together, are contradictory (if both (44a) and (44b) are true, then '$p \vee_{\text{excl}} j$' is true and therefore (44c) is false). It follows that neither of the two alternatives *exh($p \vee j$)* and $p \wedge j$ is 'innocently excludable' (following Fox's 2007 terminology). So the epistemic step will actually be vacuous, and no secondary implicature will be derived. The reader will check that this remains true even when we take into considerations the other alternatives (namely: *p*, *j*, *p or j*, *exh(p)*, *exh(j)*, *exh(p and j)*). So we have shown the following:

(45) exh((44)) = (44)

Note that in the absence of an exhaustivity operator applying to the first disjunct, we would in fact derive '$\neg$(p and j)' as a secondary implicature: for the alternatives of 'p or q or both' would then be {p, q, p or q, p and q},[25] i.e. exactly the same as the ones for 'p or q' .

6 Conclusion

We have offered a theory of the interactions of SIs and Hurford's constraint. The hypothesis that embedded implicatures can be generated in principle in any position[26] seems to us to be needed in order to predict a wide array of facts pertaining to the interpretation of sentences which apparently violate Hurford's Constraint but are in fact perfectly acceptable.

We used Hurford's Constraint to force the presence of an exhaustivity operator in an embedded position, and were able to show that in some cases, very specific readings were predicted which turned out to be the only possible readings. In other cases, the obligatory presence of a local exhaustivity operator in the logical form of a given sentence S did not alter the truth-conditions of S, but did have a truth-conditional effect once S was itself embedded under another exhaustivity

[25] $(p \vee q) \wedge (p \wedge q)$ can be eliminated, as it is equivalent to $(p \vee q)$.

[26] Which is not to say that there are no constraints whatsoever on the insertion of *exh*. See Fox and Spector (2008) for a specific proposal.

operator: the presence of the embedded operator sometimes yields additional implicatures, or, in other cases, prevents some implicatures from arising. From this perspective, the phenomenon of 'implicature cancellation' by means of (apparently) redundant disjuncts like 'or both' or 'or all' turned out to be a sub-case of a wider observation: the presence of an embedded operator, even when it does not have direct truth-conditional effects, actually affects the sentence's alternatives, with possible consequences for the resulting interpretation once an additional alternative-sensitive operator is introduced. The corroboration of these complex predictions supports the main premise, i.e. that an implicature-computing operator can (and sometimes must) be inserted in embedded positions.

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