## A SEMANTICS

FOR POSITIVE AND COMPARATIVE ADJECTIVES*

## Contents

1. Introduction
1.1. The Degree Theory of Comparatives
1.2. Fuzzy Semantics
2. Vague Predicates
2.1. Adjectives as Predicates
2.2. Graduality and Indeterminacy
2.3. Partial Models
3. Comparison Classes
3.1. The Basic Notion
3.2. Successive Application of Predicates
3.3. Orderings
4. Degrees
4.1. Degree Modifiers
4.2. Measure Phrases
5. The Comparative Construction
5.1. Rules for Simple Comparatives
5.2. Interpretation
5.3. Comparatives with less and as
5.4. Supervaluations and Indeterminacy

NOTES
Bibliography

## 1. Introduction

The purpose of this paper is to develop a model-theoretic semantics for positive and comparative adjectives. I shall be primarily concerned with sentences of a simple kind, such as those in (1)-(2):
(1)(a) Sean is tall.
(b) How tall is Sean?
(c) Sean is very tall.
(2)(a) Jude is taller than Leo.
(b) Jude is taller than Leo is.
(c) Jude is more happy than Leo is sad.

Since the literature already contains more or less detailed proposals for dealing with such constructions, it might be questioned whether there is any need for a new theory. I think it is fairly easy to show that most
existing proposals are fundamentally inadequate. Consider the following claim:'
(3) If A is a positive adjective, then the meaning of [AP A-er than $\mathrm{X}]$ is a function of the meaning of A .

This is merely a special case of Frege's principle of compositionality, applied to a class of English surface structures. For example, [AP taller than Jude is] is a complex expression, and taller is derived by a regular morphological process from [A tall]. If Frege's principle is accepted at all, then (3) seems to follow automatically. It is difficult to escape the conclusion that the meaning of the positive adjective is a basic component of the comparative. Nevertheless, this conclusion is often denied. Indeed, a number of writers have adopted the reverse strategy, and attempted to explain the positive in terms of the comparative. For example, it is sometimes suggested that the predicate is a tall man should be analysed as 'is taller than the average man/most men'. It is not hard to see why this course is tempting. Adjectives which allow comparatives seem to be inherently relational. That is, a sentence like (la) appears to make little sense unless we are comparing Sean's height to the height of other individuals. It has been assumed, therefore, that the relation implicit in the positive adjective A is just the relation represented by [ap A-er than ...].

A common way of carrying out this idea is to assign sentences containing positive adjectives an underlying structure which is rather abstract. That is, adjectives are not mapped into one-place predicates, but rather into relational expressions of the sort that are also assigned to explicit comparatives. In this way the letter, but not the spirit, of Frege's law can be observed. But it is hard to see how otherwise unmotivated appeals to complex underlying structures can be defended. Moreover, such proposals totally fail to account for the fact that across a wide variety of languages the positive is formally unmarked in relation to the comparative. ${ }^{2}$

I conclude, therefore, that a semantic theory of adjectives will be minimally adequate only if it is compatible with (3). It is fairly straightforward to show that the great majority of existing proposals fail to reach this level of adequacy. I shall consider just two examples.

### 1.1. The Degree Theory of Comparatives

Cresswell's (1976) study of comparative constructions is noteworthy for its precision and detail. Furthermore, he adopts the strategy of taking a
phrase such as tall man to be relational in logical structure; in particular, it is taken to represent a relation between an individual and a degree (of tallness). Consequently, his paper can be taken as a useful prototype of the approach that I am criticizing.
Cresswell assigns to the expression tall man the logical structure (pos(tall))(man). ${ }^{3}$ The operator pos combines with tall to form a complex noun modifier. This is then applied to man. The resulting expression denotes a function that yields the value True for an individual $u$ iff
(4)(i) $u$ is a man,
(ii) $u$ is tall to degree $d$,
(iii) $d$ lies at the top end of the scale which results when an ordering $>$ on degrees of height is restricted to the set of degrees to which men are tall.
Cresswell does not say very much about the way in which adjectives in predicative position are to be treated. But if the claim (3) is modified in an obvious way to cover prenominal adjectives, it will clearly be incompatible with Cresswell's major assumptions. The meaning of [ , taller man than Sean] will not be a function of the meaning of [ N itall man] since the operator pos will play no role in the interpretation of the former structure, but is crucially involved in the interpretation of the latter. As far as I can tell, there is no independent justification for introducing pos; it is merely a device for fixing up the semantics.

The decision to treat positive adjectives in the way just outlined has unfortunate repercussions for Cresswell's analysis of comparatives. His basic claim - if I can simplify somewhat - is that a sentence like
(5) Bill is taller than Tom is
is true just in case the degree to which Bill is tall precedes the degree to which Tom is tall in the relevant ordering on heights. This raises the question: what exactly is a degree? It is a virtue of Cresswell's paper that he provides an explicit formal definition (1976: 281). The idea is simple and, I think, basically correct. Suppose $d$ is the degree to which Bill is tall. Then $d$ is simply the equivalence class consisting of all things which are neither less tall nor more tall than Bill. For any given adjective, say tall, it is possible to define an equivalence relation, $\approx_{\text {tall }}$, as follows. ${ }^{4.5}$

$$
\begin{equation*}
u \approx \text { tall } u^{\prime} \text { iff for all } v \tag{6}
\end{equation*}
$$

(i) $u$ is taller than $v$ iff $u^{\prime}$ is taller than $v$,
(ii) $v$ is taller than $u$ iff $v$ is taller than $u^{\prime}$.

Bill's degree of tallness is thus $\left\{u: u \approx{ }_{\text {tall }}\right.$ Bill $\}$.

The striking thing about this definition is that it invokes the comparative relation 'is taller than' as an unanalyzed primitive. That is, in order to say anything precise about degrees, Cresswell utilizes a semantic metalanguage in which comparatives occur. Now, as we saw above, the formal interpretation of (5) in Cresswell's system presupposes the notion of a degree; but in order to account for the latter, we must presuppose an understanding of comparative sentences. This seems utterly circular.
Of course, it might be argued that all truth conditional semantics is circular in this way. A truth definition for a language just allows us to state biconditionals such as: $u$ satisfies the formula cat $(x)$ iff $u$ is a cat. This seems to be Cresswell's line of thought in the following remark:

We take as basic data that we can and do make comparisons, i.e., that comparative
sentences can be true or false $\ldots$ It is not, in my opinion, the business of logic or
linguistics (at least syntax) to explain how it is that we make the comparisons we do make
or what the principles are by which we make them. (Cresswell 1976: 281)
This statement is unexceptionable if we take it to mean that speakers have certain psychological abilities - such as comparing the lengths of two sticks - whose explanation is outside the domain of semantics. Yet we also require a semantic theory for English to analyse the interpretation of complex expressions in terms of the interpretations of their components. An expression of the form [aP A-er than X] is clearly complex. How do its components contribute to the meaning of the whole? The introduction of degrees helps not at all in answering this question. Once degrees are defined along the lines of (6), the proposed truth conditions for (5) amount to nothing more than this: (5) is true iff Bill is taller than Tom is. In fact the situation is even worse. Degrees are not only redundant, they introduce unjustified complexity. Intuitively, (5) could be verified by directly comparing Bill and Tom. Yet according to (6), we can only determine the degree to which Bill (or Tom) is tall by first partitioning the universe of discourse $U$ under the equivalence relation $\approx_{\text {tall }}$; that is, for each pair $\left\langle u, u^{\prime}\right\rangle$, testing whether the relation 'is taller than' holds. Thus, a degree analysis of comparatives makes the rather implausible claim that (5) can only be verified by first evaluating $x$ is taller than $y$ for each value of $x$ and $y$ in $U$.

In conclusion, let me repeat that there does not seem to be any way of analyzing degrees other than that suggested by (6). Consequently, it is extremely unlikely that any alternative theory which relied on degrees or extents could fare much better than Cresswell's.

### 1.2. Fuzzy Semantics

There does not seem to have been much attention paid to comparative constructions by fuzzy semanticists. The topic receives no mention by Lakoff (1972, 1973), and only a couple of casual references in Zadeh (1971, 1977). Nevertheless, it might be thought that fuzzy set theory could provide the basis for a semantic account of comparatives which observed the compositionality principle (3) and also avoided the difficulties indicated in Cresswell's approach. Let me indicate briefly why I think that this optimism would be misplaced.
In classical model theory, the extension of a one-place predicate is always a subset $X$ of the universe of discourse $U$. Members of $U$ will either definitely belong or definitely not belong to $U$. In place of $X$, we can equally well use a function $f_{X}$ on $U$ which yields the value 1 (true) for an argument $u$ if $u \in X$, and the value 0 (false) otherwise. This function is known as the characteristic function of $X$. In fuzzy semantics, the extension of a predicate like tall will be a fuzzy set to which individuals will belong to a certain degree. This fuzzy set can be most easily understood in terms of its characteristic function $\mu_{x}$. Instead of taking just two values, 1 or $0, \mu_{X}$ can take indefinitely many values, each representing a degree of membership in $X$. Usually, the range of $\mu_{X}$ is taken to be the real interval $[0,1]$.

At first sight, it might seem that the introduction of this membership function circumvents all the difficulties attendant on equivalence classes. However, this impression is false. Let $\mu_{\text {tall }}$ be the membership function associated with the predicate tall, and consider the following condition on it:

For all $u, u^{\prime} \in U, \mu_{\text {tall }}(u)=\mu_{\text {tall }}\left(u^{\prime}\right)$ iff $u$ is exactly as tall as $u^{\prime}$.
Could $\mu_{\text {tal }}$ fail to observe (7)? The answer is obviously no. There can be no consistent assignment of numerical values to degrees of membership until degrees as equivalence classes have been constructed by means of the appropriate equivalence relation. Fuzzy semantics holds no greater prospects for an adequate account of comparatives than does Cresswell's theory. For it is just a degree analysis with some numerical icing.
It is perhaps worth briefly mentioning another difficulty which would be encountered by fuzzy semantics (at least in Lakoff's (1972) formulation) if it were to be developed into a theory of comparatives. Suppose we accepted that the interpretation of the positive adjective tall was successfully captured by the function $\mu_{\text {tall }}$. In order to conform to (3), we would want the interpretation of (5) to be a function of the
information $\mu_{\text {tal }}$ (Bill) and $\mu_{\text {tal }}($ Tom). The obvious answer is to let (5) be true just in case $\mu_{\text {tall }}($ Bill $)>\mu_{\text {tall }}($ Tom) (where $>$ is the natural ordering on the real numbers). But this conflicts with the reasonable assumption that if an individual $u$ reaches a certain height, say six foot three, then $u$ is definitely tall and hence $\mu_{\text {tall }}(u)=1$. For if Bill is six foot four, while Tom is six foot three, $\mu_{\text {tall }}($ Bill $)=\mu_{\text {tall }}$ (Tom), and (5) comes out false.

## 2. Vague predicates

### 2.1. Adjectives as Predicates

There is a well-known theory, first advanced by Montague (1970) and Parsons (1970), according to which adjectives are basically noun modifiers. On this approach, the predicative use of an adjective is to be analysed in terms of its prenominal use. Thus, Nat is big is taken to mean something like Nat is a big entity or, in some contexts, Nat is a big flea. However, Kamp (1975) has defended the traditional idea that adjectives are one-place predicates, and suggested that some of the familiar difficulties encountered by this approach can be overcome in a semantics where contextual factors are accorded an important role. Unfortunately, there is not space to review the relevant arguments here; instead, I shall simply assume that the traditional view is fundamentally correct as far as degree adjectives are concerned. An adjective belongs to this class iff
(8)(i) it can occur in predicative position, i.e. after copular verbs such as be, seem, become,
(ii) it can be preceded by degree modifiers such as very and fairly.

In this paper, I shall be concerned solely with degree adjectives. Moreover, again for reasons of space, the topic of positive and comparative adjectives in prenominal position will have to be postponed to a subsequent paper. But there do not seem to be any major obstacles to extending the treatment adopted in this study to such constructions.

### 2.2. Graduality and Indeterminacy ${ }^{6}$

Let us say that an adjective A is Linear iff it satisfies the following condition:
(9) Whenever $c$ is a context of use, and $\mathrm{NP}_{1}, \mathrm{NP}_{2}$ denote in-
dividuals within the sortal range of $A$, then the sentence $N P_{1}$ is A-er than $\mathrm{NP}_{2}$ has a definite truth value in $c$.
That is, from the comparative of a linear adjective, it is possible to construct a linear ordering ${ }^{7}$ of all objects in the domain of application of the adjective. For instance, taller than represents a relation which linearly orders any set $X$ of vertically extended objects: if $u, u^{\prime} \in X$, then either $u$ is taller than $u^{\prime}$, or $u^{\prime}$ is taller than $u$, or $u$ is exactly as tall as $u^{\prime}$.

However, as McConnell-Ginet (1973) and Kamp (1975) have observed, a large proportion of adjectives in English are nonlinear; that is, they fail to satisfy (9). A good example is clever. There is no single criterion of application which alone determines whether a person is clever. Instead, the adjective is associated with a number of criteria, and these fail to constitute a necessary and sufficient set of conditions for cleverness. ${ }^{8}$ Let us suppose, for the sake of argument, that there are only two properties associated with being clever: an ability to manipulate numbers, and an ability to manipulate people. Anyone who possesses both these properties will certainly be clever, and anyone who possesses neither will certainly not be. In some contexts of use, possession of just one of these properties will suffice for being clever, while in other contexts it will not. Why are such adjectives nonlinear? Suppose that Sue is better than Dick at manipulating numbers, whereas Dick is better than Sue at manipulating people. In a context $c$ where both criteria are potentially relevant and where there is no accepted method for weighing them against one another, it is difficult to see what the truth value of ( 10 ) should be.
(10) Sue is cleverer than Dick.

In a sense it is true, but in another sense it is false. Indeed, it would be possible to change $c$ into a new context $c_{1}$ where the manipulation of numbers was the only criterion relevant to assessing cleverness, in which case (10) would be definitely true. But it would be equally possible to stipulate that only the manipulation of people was relevant, and in this context $c_{2}$, (10) would be false. The fact that both these options are open at $c$ seems a conclusive reason for taking (10) to be undefined in truth value at $c$.
This conclusion has, nevertheless, been attacked by Heny (1978). While conceding that conflicting criteria may be relevant in a particular context, he claims that an adjective cannot be evaluated on the basis of different criteria when it occurs more than once within the same sentence. If this were correct, then (11) would be false in $c$. Sue is clever, and so is Dick.

Admittedly, someone who heard (11) and knew nothing about Sue and Dick might well assume that a single criterion of cleverness was being invoked. But a better informed addressee would probably infer that the speaker was using clever in two different ways, rather than conclude that something false had been asserted. (Notice that an utterance of (11) could of course be followed by the qualification but in two different respects.) Suppose, furthermore, that the speaker followed his utterance of (11) with a question: which do you think is cleverer? The second addressee would presumably be at a loss to answer, in the absence of any clue as to which was the relevant criterion. In the light of these considerations, it seems to me that Heny's objection cannot be upheld and that we should indeed conclude that there are nonlinear adjectives in English.

Linear adjectives exhibit a particular kind of vagueness: graduality. That is, the fuzzy boundary between objects of which the adjective is definitely true and those of which it is definitely false can be conceptualized as a gradual transition. Nonlinear adjectives exhibit a second kind of vagueness: indeterminacy. It is indeterminate which particular criteria have to be met for the adjective to be true of an object. Indeterminacy is superordinate to graduality in the following sense. An indeterminate predicate can be regarded as a function of (the meanings of) a set of predicates which are not indeterminate; i.e. predicates which are either not vague at all, or are only gradual. Thus, in order to understand linear adjectives, we need to have some account of graduality, while in order to understand nonlinear adjectives we also need to know what kinds of functions of determinate predicates are involved. In the sequel, I shall mainly concentrate on linear adjectives, since they are more amenable to analysis. However, I shall have something more to say about nonlinear adjectives in Sections 3.3 and 5.4.

### 2.3. Partial Models

I shall take for granted Montague's (1973) definition of the language IL of intensional logic. But for simplicity of exposition, I shall completely ignore intensional constructions in English, and expressions of IL which denote intensions. This extensional fragment of IL will simply be called ' $L$ '. As before, I shall use boldface English expressions to represent constants of L. Although the models for $L$ will extend in various ways the models which Montague defined for IL, I shall not attempt to present
full definitions here, since that would involve me in excessive formal detail.
Expressions of L are assigned to different syntactic categories. These categories can be identified with the TYPES of $L$, and are chosen in such a way that there is a precise correspondence between the type of an expression and its semantic interpretation. That is, there is a function $D$ which associates with each type $\tau$ a set $D_{r}$ of possible denotations for expressions of type $\tau$. The basic types are $e$ (for expressions that denote 'entities', i.e. individuals) and $t$ (for expressions that denote truth values). Thus, $D_{e}$ is a set of individuals, namely the universe of discourse $U$, while $D_{t}$ is the set $\{0,1\}$ of truth values. Complex types are of the form $\langle\sigma, \tau\rangle$, where $\sigma$ and $\tau$ are themselves types, and $D_{\langle\sigma, \tau\rangle}$ is the set of functions from $D_{\sigma}$ to $D_{r}$ We write this set as $D_{r}^{D_{\sigma}}$. So, for example, $D_{(e, t)}$ is $D_{l}^{D_{e}}$, i.e. the set of functions from $U$ to $\{0,1\}$.
For each type $\tau$, there is a set $\mathrm{ME}_{\tau}$ of meaningful expressions of type $\tau$. Thus, $\mathrm{ME}_{e}$ is the set of expressions that denote individuals and $\mathrm{ME}_{(e, t)}$ is the set of expressions denoting functions from individuals to truth values; i.e. $\mathrm{ME}_{(e, t)}$ can be regarded as the set of one-place predicates. In the sequel, I shall be particularly concerned with those basic members of $\mathrm{ME}_{(e, t)}$ that correspond to adjectives in English. I shall use ' $A d j$ ' to refer to the relevant subset.

A model for $L$ based on $U$ will consist of the family of sets in the range of $D$, together with another function $F$ which assigns an appropriate denotation to each nonlogical constant. Let us assume, for convenience, that proper names in English are translated in $L$ as constants of type $e$. Then (since L contains no intensional expressions) we might expect $F_{\text {sen }}$ to simply be an element of $D_{e}$ and $F_{\text {tal }}$ to be an element of $D_{(e, t)}$. But this predicts that the formula tall(Sean) will always. receive a definite truth value, and thus fails to capture the vagueness of tall.
In any given context of use, there are some people whom we consider to be definitely tall, others who are definitely not tall, and yet others who are somewhere in between. This suggests that the extension of tall, at any context, should yield the value 1 for members of the first group, 0 for members of the second group, and be undefined for members of the third group. In other words, it should be a partial function from $U$ to $\{0,1\}$. We write the set of such functions as $\{0,1\}^{(U)}$. In order to capture the context-dependence just mentioned, we will want $F$ to assign extensions relative to contexts of use. That is, if $C$ is a nonempty set of contexts, and $\alpha$ is a constant of type $\tau$, then $F_{\alpha}$ will be a function from $C$ to $D_{r}$ In the case where $\alpha$ belongs to $\operatorname{Adj}, F$ must also meet
condition (12):
(12) Whenever $\alpha \in \operatorname{Adj}$ and $c \in C, F_{\alpha}(c) \in\{0,1\}^{(U)}$.

So, for any context $c, F_{\text {tan }}(c)$ will be a partial function on $U$.
Following Kamp (1975), I shall say that the positive extension of a predicate $\zeta$ in a context $c$ is the set of things in $U$ of which it is definitely true, and its negative extension is the set of things of which it is definitely false. In notation, we have, for any $c$,

$$
\begin{equation*}
\operatorname{pos}_{6}(c)=\left\{u \in U: F_{\zeta}(c)(u)=1\right\} \tag{13}
\end{equation*}
$$

(ii) $\operatorname{neg}_{6}(c)=\left\{u \in U: F_{b}(c)(u)=0\right\}$

Individuals who fail to belong to either the positive or negative extension of $\zeta$ are said to belong to the EXTENSION GAP of $\zeta$. ${ }^{9}$

Suppose Sean is five foot eight. Then according to the present analysis, there will be contexts $c$ at which the sentence

## Sean is tall

lacks a truth value. If Sean is on the borderline between tall and not tall, then (14) should be neither definitely true nor definitely false. It may nevertheless seem that (14) will be true to some extent. In order to capture this idea, Kamp introduces a set of new valuations of tall which close up the extension gap in a consistent manner. There are various ways of presenting this idea formally. ${ }^{10}$ For our purposes, it is simplest to introduce a two-place function $\mathscr{S}$ which assigns to any $c \in C$ and $\zeta \in \operatorname{Adj}$ a set of new contexts. Relative to any $c^{+} \in \mathscr{\mathscr { C }}(c, \zeta), F$ assigns to $\zeta$ a classical extension; i.e. $F_{\zeta}\left(c^{+}\right)$is a total function. The underlying intuition is this. Vague predicates have the virtue that we can use them without having to draw a clear boundary between the positive and negative extensions. Nevertheless, there are occasions on which clarity is required, and it is characteristic of vague predicates that they can be made more precise. So, for example, one can imagine contexts where the boundary between tall and not tall is sharply drawn by stipulation; at five foot seven, say. $\mathscr{(}(c, \zeta)$ is intended to represent the set of contexts in which $\zeta$ has been made precise. ${ }^{\text {" }}$ Of course, there will be lots of different ways of making it precise, but not all of them will be acceptable. Suppose that Sean is five foot eight, as before, while Jude is five foot seven, and that they both belong to the extension gap of tall in $c$. Then there should be no way of making tall precise so that Jude but not Sean belongs to the new positive extension; formally, there must be no $c^{+} \in \mathscr{S}(c$, tall $)$ such that $F_{\text {un }}\left(c^{+}\right)($Jude $)=1$ while $F_{\text {ur }}\left(c^{+}\right)($Sean $)=0$.

It is now possible to define truth for sentences of $L$ relative to the set
of classical valuations induced by $\mathscr{S}$. Suppose for simplicity that $A d j=$ \{tall\}. Then a sentence of L will be $\mathscr{S}$-true at $c$ if it is true at all $c^{+} \in \mathscr{P}\left(c\right.$, tall); $\mathscr{P}$-false at $c$ if it is false at all $c^{+} \in \mathscr{C}(c$, tall $)$; and partially true at $c$ if it is true at some but not all $c^{+} \in \mathscr{C}(c$, tall $)$.

From the logician's point of view, $\mathscr{\mathscr { S }}$-truth is useful because it allows the classical notion of logical truth to be preserved. In order to indicate the reason for this result, it will be helpful to introduce some more formal apparatus.

A PARTIAL CONTEXT-DEPENDENT INTERPRETATION FOR L, BASED ON $U$ and $C$, is a pair $\langle D, F\rangle . D$ is a function of the sort already discussed; it assigns sets of denotations to types. $F$ is a function which assigns meanings to constants of L . That is, whenever $\alpha$ is a constant of type $\tau$, $F_{\alpha}$ is a function in the set $D_{r}^{c}$. Moreover, $F$ must also meet condition (12), according to which the extension of a member of Adj at any $c$ is a partial function from $U$ to $\{0,1\}$.

An ASSIGNMENT to variables is a function $a$ such that if $v$ is a variable of type $\tau, a(v)$ is a member of $D_{r}$ Furthermore, $a[z / v]$ is that function $a^{\prime}$ just like $a$ except that $a^{\prime}(v)$ is $z$.

Suppose now that $\alpha$ is an arbitrary expression of L , and $\mathfrak{d}$ is a partial context dependent interpretation. We write $[\alpha]_{c a}^{\alpha}$ for the extension of $\alpha$ Under 9 , at a context $c$ and assignment $a$. I will not set down a complete truth definition for L ; however, some of the important clauses are the following: ${ }^{12}$
(15)(i) If $\alpha$ is a constant, then $[\alpha]_{c a}^{\mathrm{a}}=F_{\alpha}(c)$.
(ii) If $\alpha$ is a variable, then $\llbracket \alpha \rrbracket_{c a}^{\alpha}=a(\alpha)$.
(iii) $\left.\llbracket \alpha(\beta) \rrbracket_{c a}^{\boldsymbol{a}}=\llbracket \alpha \rrbracket_{c a}^{\mathbf{n}} \llbracket \beta \rrbracket_{c a}^{\mathrm{n}}\right)$.
(iv) If $\varphi$ is a formula, then
$\left\lceil\mid \varphi \rrbracket_{c a}^{\boldsymbol{u}}=1\right.$ if $\lceil\varphi\rceil_{c a}^{\underline{\alpha}}=0$, $[ \urcorner \varphi]_{c a}^{\alpha}=0$ if $[\varphi]_{c a}^{\alpha}=1$.
(v) If $\varphi, \psi$ are formulae, then
[ $\varphi \vee \psi]_{c a}^{\mathbb{\alpha}}=1$ if $[\varphi]_{c a}^{\tilde{a}}=1$ or $[\psi]_{c a}^{\mathbb{a}}=1$, $\llbracket \varphi \vee \psi]_{c a}^{\mathbb{x}}=\mathbf{0}$ if $\left\lceil\varphi \rrbracket_{c a}^{\mathbb{a}}=\llbracket \psi \rrbracket_{c a}^{\mathbb{a}}=\mathbf{0}\right.$.

Relative to a partial interpretation of the kind just introduced, the schema $\varphi \vee\urcorner \varphi$ will fail to be valid. Take a particular instance:

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tall(Sean) v 7tall(Sean)
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If (16) is evaluated at a context $c$ such that $F_{\text {tul }}(c)$ is undefined for the argument Sean, then the whole sentence will also be undefined, by clauses (iv) and (v) above. Nevertheless, (16) will be $\mathscr{C}$-true. For tall(Sean) will receive a definite truth value at every $c^{+} \in \mathscr{S}(c$, tall).

It seems that $\mathscr{\varphi}$-truth also provides a basis for interpreting comparatives. Without going into details, ${ }^{13}$ we can say that taller than denotes a relation which holds of any $u, u^{\prime}$ in the extension gap of tall at $c$ iff the class of new contexts at which $u^{\prime}$ satisfies tall $\left(x_{0}\right)$ is a proper subset of the class of new contexts at which $u$ satisfies $\operatorname{tall}\left(x_{0}\right)$. More formally, $\left\langle u, u^{\prime}\right\rangle$ will belong to the extension of taller than at $c$ iff

$$
\begin{align*}
& \left\{c^{+} \in \mathscr{P}(c, \text { tall }): F_{\text {tall }}\left(c^{+}\right)\left(u^{\prime}\right)=1\right\} \subset  \tag{17}\\
& \left\{c^{+} \in \mathscr{P}(c, \text { tall }): F_{\text {tall }}\left(c^{+}\right)(u)=1\right\}
\end{align*}
$$

This idea is attractive because it does satisfy the principle (3) which I discussed in Section 1. For (17) only appeals to the meaning of the positive adjective. Unfortunately, it is difficult to generalize the proposal in a non ad hoc way to cover those cases where $u$ and $u^{\prime}$ do not both belong to the extension gap of the predicate in question. It is plausible to suppose that a context $c$ which leaves a predicate $\zeta$ vague can be modified to a new context $c^{+}$in which the extension gap of $\zeta$ is reduced or eliminated. For $c^{+}$decides cases which were left undetermined in $c$ while preserving all assignments of truth and falsity already made there. It is much less plausible, however, to suppose that $c$ can be modified to a new sort of context $c^{\prime}$ in which previous assignments of truth and falsity are revoked. Yet something like this seems to be required if (17) were to hold, for instance, of a pair $u, u^{\prime}$ both of which belonged to the positive extension of tall at $c$. For another example of the same problem, consider the following sentences. ${ }^{14}$
(18)(a) ' $a n$ ' is a longer word than ' $a$ '.
(b) 'an' is a long word.
(c) ' a ' is a long word.

On Kamp's approach, (18a) will only be true in a context $c$ if there is some context $c^{\prime}$ in which the standards for what counts as a long word have been so altered that (b) comes out true in $c^{\prime}$ but (c) comes out false. But it is difficult to see how the mechanisms for making vague predicates more precise can lead us to such a context $c^{\prime}$.

## 3. Comparisons classes

### 3.1. The Basic Notion

In this section, I want to develop the idea of partial interpretations so as to overcome the difficulty encountered by (17). The key concept involved in the revised approach is that of comparison class. ${ }^{15}$ In
interpreting a sentence like Lana is clever at a context $c$ in which Lana is presupposed to be a chimp, we would naturally understand clever to mean clever for a chimp. We would be concerned with Lana's cleverness relative to the set of chimps. The latter constitutes the comparison class for clever in $c$. It is, I think, fairly uncontroversial that something like a comparison class does figure in the background assumptions against which sentences containing vague predicates are evaluated. Presumably it is related to the rather amorphous idea of a 'topic of conversation'; in many cases, the comparison class is just the set of things that the participants in a conversation happen to be talking about at a given time. In formal terms, a comparison class is a subset of the universe of discourse which is picked out relative to a context of use. A similar idea is involved in restricting the range of quantifier phrases. The truth, in a context $c$, of everybody is having a good time depends on the domain of quantification, and this will typically be a small subset of all the things that could possibly be talked about in $c$. However, I shall not attempt to develop the notion of a comparison class within a wider pragmatic theory. Instead, I shall just introduce by fiat a function $q_{l}$ such that whenever $c \in C, \mathscr{U}(c) \subseteq U$.

Before adding more details to the formal definition of $q u$, it is worth considering how it affects the valuation of adjectives. Suppose $U$ is the set of all the physical objects on earth. According to our previous notion of a partial context-dependent interpretation, $F_{\text {tun }}(c)$ has the effect, for any $c$, of carving $U$ into three sets, $\operatorname{pos}_{\text {tan }}(c)$, neg ${ }_{\text {and }}(c)$ and an extension gap. On this global scale of things, some mountains will turn out to be tall, while most other objects will turn out not to be tall. In effect, the comparison class in $c$ is $U$ itself. What happens when $c$ is changed to a new context $c^{\prime}$ such that $\mathscr{\ell}\left(c^{\prime}\right)$ is the set $X$ of human beings? Figuratively speaking, we want the extension of tall in $c^{\prime}$ to become FOCUSSED on $X$. It is not sufficient for $F_{\text {tan }}\left(c^{\prime}\right)(u)$ to agree in value with $F_{\text {tan }}(c)(u)$ whenever $u \in X$, since every $u \in X$ belongs to the negative extension of tall in $c$. Instead, the valuation of tall in $c^{\prime}$ has to be restructured so that it carves $X$ into three subsets exactly as though $X$ were the whole universe of discourse. In this way tall will end up being true of a sizeable proportion of $X$.

We also need to ask what the value of $F_{6}(c)(u)$ should be when $u$ falls outside the comparison class determined by $c$. Although the evidence on this point is not clear cut, it seems to me that $F_{f}(c)$ should be undefined for every object outside $\mathscr{U}(c)$. Let me try to justify this. Suppose
is uttered in the context of watching jockeys weigh in before a horse race. The appropriate comparison class for tall will be the set of jockeys taken into account in $c$. The question is then what value should be assigned in $c$ to (20), where Sam is no jockey, but a rather tall basketball player:
(20) Sam is tall.

The first option is to take (20) as true in $c$. But this will nullify the introduction of comparison classes, for though it may make good sense to say that Bill is tall relative to the set of jockeys in $c$, it would not be consistent with our usual understanding of tall to say that both Bill and Sam are tall. As soon as we take Sam into consideration, (19) will be judged false.

On the other hand, it also seems odd to say that (20) is false in $c$. For Sam is much taller than Bill. Thus the only plausible option is to let (20) be undefined in $c$.

It is important, in considering this issue, to assume that (20) has nOT been uttered in $c$. For we cannot rule out the possibility that this would lead to the relevant comparison class being extended to admit Sam. In general, if someone utters a sentence $\varphi$ that would be undefined in the current context $c$, then conversational principles tend to ensure that $c$ is modified in such a way that $\varphi$ comes out true (or at least defined); cf. Lewis (1979). Thus, the discussion of (20) is made rather abstract by the requirement that we consider its semantic value only under the condition that Sam is left entirely out of consideration.

Let me summarize the discussion of comparison classes so far. First, $\mathscr{Q}$ picks out, for every context $c$, a subset of $U$. Second, for any vague predicate $\zeta, F_{\zeta}(c)$ is a partial function from $\mathscr{U}(c)$ to $\{0,1\}$. It follows from this that everything outside $\mathscr{U}(c)$ will be outside the domain of $F_{6}(c)$. These two conditions are presented formally in (21):
(21) $\mathscr{U}$ is a function such that whenever $c \in C$ and $\zeta \in A d j$,
(i) $\quad \mathscr{U}(c) \subseteq U$,
(ii) $\quad F_{\zeta}(c) \in\{0,1\}^{(\boldsymbol{( x}(c))}$.

The truth value of a sentence like (19) is given by

## (22) $[\text { tall }(B i l l)]_{c a}^{2}$

and depends not just on the physical properties of Bill, but also on the comparison class determined by $c$.

It might be objected at this point that adjectives turn out to be no less relational on my approach than they are in the degree theory that I
criticised earlier (cf. Section 1.1); the main difference is that the extra argument is a comparison class rather than a degree, and it has been shunted out of the logical structure into the contextual coordinate. ${ }^{16}$

The objector might develop his point by arguing that (22) could equally well be formulated as (23):

$$
\begin{equation*}
\| \operatorname{tall}(\mathbf{X})(\mathbf{B i l i l})]_{c a}^{\mathbf{a}} \tag{23}
\end{equation*}
$$

On this proposal, $\mathbf{X}$ is an indexical expression of type $\langle e, t$, and its value at a given context $c$ is just the comparison class determined by $c$. Since tall will be of type $\langle\langle e, t\rangle,\langle e, t\rangle\rangle$, this analysis seems to mesh in rather neatly with the hypothesis, briefly mentioned earlier, that adjectives are basically common noun modifiers. For $\mathbf{X}$ could be regarded as the counterpart of an indexical nominal proform.

Nevertheless, there are at least two reasons for preferring my original formulation. First, the logical structure in (22) is simply less abstract than that in (23), and according to the kind of criteria developed by Keenan and Faltz (1978), it is clearly to be preferred. If we stick to (22), then we have a semantics which can be applied directly to English adjectives without the mediation of a formal language such as $L$; if, on the other hand, the selection of comparison classes is made dependent on the presence of an extra argument in logical structure, this option is barred to us.
Second, predicate adjectives do not behave in the way we would expect if they were combined with an indexical nominal proform in logical structure. Consider the following sentence, where an indexical pronoun occurs in a VP that has triggered VP Deletion: ${ }^{17}$
(24) Jude drank some of that, and Leo did too.

It is impossible to interpret the second conjunct in such a way that the missing indexical is assigned a referent which differs from that assigned to the indexical occurring the trigger VP. For example, one could not use (24) to mean that Jude drank some whiskey and Leo drank some orange squash, however hard he pointed first at one bottle and then at another. Suppose now that A and B are in a furniture shop, discussing the merits of various items. On the view 1 am attacking, when $A$ says to B this is comfortable, he is in effect saying something like this is a comfortable one, where one is interpreted exophorically. Now, in a similar way to before, A cannot use (25), pointing first to an armchair, then to a sofa, intending to convey (26):
(25) This is a comfortable one, and that is too.
(26) This is a comfortable chair, and that is a comfortable sofa.

But counter to the prediction made by the objector's analysis, (27) could be used by $A$ in the same situation, in order to convey (26):
(27) This is comfortable, and that is too.

To the extent that the indexical hypothesis makes empirical claims, it is strikingly disconfirmed by this data. What we have seen is that comparison classes can switch across VP deletion, while the reference of indexicals cannot. Notice also that contextual restriction of the universe of quantification, which I earlier suggested could be handled in terms of comparison classes, behaves in the way we would expect, i.e. like contextual restriction of the domain of adjectives. Consider (28) for example.

Leo gave a bridge party at home yesterday and Jude took the kids swimming. Leo thought everyone had a good time, and so did Jude.

One possible interpretation of the second sentence is that Leo thought everyone at the bridge party had a good time, while Jude thought that everyone who went swimming had a good time. The general conclusion to be drawn, I think, is that there are certain kinds of context-dependent phenomena in semantics which cannot be forced into the pattern of indexical reference by pronouns.

### 3.2. Successive Application of Predicates

Let me start by introducing a terminological convention. I have claimed that if $X$ is the comparison class for a predicate $\zeta$ in a context $c$, then the function $F_{\zeta}(c)$ has the effect of partitioning $X$ into three disjoint subsets. In such a case I shall say that the predicate $\zeta$ has been APPLIED to the set $X$. Now it is an interesting characteristic of degree adjectives that they can be applied, in the sense just introduced, to a wide variety of sets; there does not seem to be any general constraint on which subsets of $U$ can serve as comparison classes.

Bearing this point in mind, imagine that we are told to sort a group $X$ of people into tall and not tall members, that is, we have to apply tall to $X$. We start to work, and after a while we have divided $X$ into three smaller groups: those who are definitely tall, according to our standards, those who are definitely not tall, and third group of people that we can't quite decide about. We can either stop at this point, or try to make our categorization more precise. Suppose we go on; how should we proceed? On Kamp's approach, we could either choose some arbitrary
cut-off point, or else just relax our standards for applying the predicate; in either case, we would somehow stretch the positive and negative extensions so that they encompassed individuals who were previously excluded. However, I think there is a more persuasive way of conceptualizing the required procedure: we simply reapply tall to the extension gap. ${ }^{18}$ The meaning of the predicate stays the same, but the comparison class is changed. If this reapplication again leaves us with some cases that we can't decide about, we just repeat the procedure until, in the end, we have made up our minds about everybody. In order to make a linear adjective more precise, therefore, we must systematically modify the comparison class in a series of stages, refocussing at each stage on the extension gap left at the previous stage.
It will be useful at this point to introduce some further notation. First, if $f$ is a partial function on a set $X$, then the domain of $f, \operatorname{dom}(f)$, is a subset $Y$ of $X$ such that $f$ is a total function on $Y$. Note, in particular, that for any $c \in C$ and $\zeta \in \operatorname{Adj}, \operatorname{dom}\left(F_{\zeta}(c)\right)=\operatorname{pos}_{\zeta}(c) \cup$ neg $_{\zeta}(c)$. Second, $c[X]$ is to be that context $c^{\prime}$ just like $c$ except that the comparison class in $c^{\prime}$ is $X$. Thus, if $X \subseteq \mathscr{U}(c)$, then $F_{6}(c[X])$ will be the partial function which results when the comparison class for $\zeta$ in $c$ has been narrowed down from $\mathscr{U}(c)$ to $X$. The sorting process that I described informally in the previous paragraph can now be formulated as follows. We first partition a given set $X$ by means of the function $F_{\zeta}(c[X])$. Then let $Y$ be the extension gap that remains, i.e. $X-\operatorname{dom}\left(F_{\zeta}(c[X])\right)$. We take $Y$ as our new comparison class, and partition it by means of the function $F_{5}(c[Y])$. Then once again we take the remaining extension gap, and focus on that. And so on. This process is illustrated in Figure 1. Here,

Fig. 1.

$X_{1}=\operatorname{pos}_{\zeta}(c[X]), \quad X_{2}=\operatorname{neg}_{\zeta}(c[X]), \quad$ and $\quad Y=X-\operatorname{dom}\left(F_{\zeta}(c[X])\right) ;$ and similarly for the other sets.

Let me return now to the problem of comparatives. I pointed out, when discussing Kamp's proposal, that it was difficult to see how the operation of making a vague predicate more precise could lead naturally to a context in which 'an' was considered to be an orthographically long word. I want to claim that nevertheless there is such a context: namely, that in which the comparison class for long is a set of word forms consisting of either one or two letters. The word 'an' is long, relative to a set such as $X=\{$ 'an', ' I ', 'we', 'on', 'a'\}. Suppose $Y$ is the set of all English word forms. Then my proposal, put formally, is that while $F_{\text {long }}(c[Y])$ ('an') $=0$, this semantical decision becomes irrelevant if we focus on $X$. That is, $F_{\text {long }}(c[X])\left({ }^{\prime} a n '\right)=1$, while $F_{\text {long }}(c[X])\left({ }^{\prime} a^{\prime}\right)=0$.

This claim presupposes that, from a semantical point of view, it is possible to exclude the larger set of all words from consideration. Of course as speakers we may find it psychologically difficult to move from $c[Y]$ to the more restrictive $c[X]$ without carrying over the judgements that were appropriate relative to $Y$. But I don't think too much importance should be attached to this difficulty; it can be seen in a different light if we consider the following situation. A speaker $s$ is presented with a pair of blocks and asked to classify them as tall and short. This

pair is then augmented by a number of taller blocks and $s$ is asked to classify the new set into tall and short members. Relative to the set $X$, the block $u$ will be classified as tall by $s$; relative to $X^{\prime}$, on the other

hand, $u$ will be classified as short. It would be wrong, surely, to say that $s$ made an error in his first judgement; the falsity of $u$ is tall in the
second context doesn't in any way render that sentence less true in the first context. The psychological difficulty involved in narrowing down a comparison class, rather than expanding it, falls within the domain of a more general problem discussed by Lewis (1979) in connection with 'rules of accommodation'. These are conversational principles which allow the set of possibilities taken into account in a context $c$ to expand if what is said in $c$ is thereby rendered true. ${ }^{19}$

For some reason, I know not what, the boundary readily shifts outward if what is said requires it, but does not so readily shift inward if what is said requires that. Because of this asymmetry, we may think that what is true with respect to the outward-shifted boundary must be somehow more true than what is true with respect to the original boundary. I see no reason to respect this impression. Let us hope, by all means, that the advance toward truth is irreversible. That is no reason to think that just any change that resists reversal is an advance toward truth. (Lewis 1979: 355)

### 3.3. Orderings

It is now possible to reformulate clause (17) with comparison classes playing a central role. I shall first present the basic idea, and then try to make the underlying assumptions more explicit.
Let $>_{c, 5}$ be the relation which is denoted, at a context $c$, by the expression is $\zeta$-er than. Then a pair $\left\langle u, u^{\prime}\right\rangle$ belongs to $\rangle_{c, \zeta}$ iff there is some subset $X$ of $\mathscr{U}(c)$ such that $u$ belongs to the positive extension of $\zeta$, relative to $c[X]$, while $u^{\prime}$ belongs to the negative extension of $\zeta$, relative to $c[X]:{ }^{20}$

$$
\begin{align*}
& \left.\left\langle u, u^{\prime}\right\rangle \in\right\rangle_{c, 5} \text { iff }(\exists X \subseteq \mathscr{Q}(c))\left(F_{5}(c[X])(u)\right.  \tag{29}\\
& \left.=1 \& F_{5}(c[X])\left(u^{\prime}\right)=0\right]
\end{align*}
$$

To make this idea more plausible, let me present a diagram which indicates how a function $F_{\zeta}$, when applied to various subsets of $\mathscr{U}(c)$, induces the ordering $>_{c, \zeta}$ on $\mathscr{U ( c ) \text { . It turns out that we do not have to }}$ consider all possible subsets of $q(c)$. It suffices to apply $\zeta$ at each stage $n$ to the following sets: for each $X$ introduced at stage $n-1,(X-$ $\operatorname{neg}_{\xi}(c[X])$ ) and ( $X-\operatorname{pos}_{\zeta}(c[X])$ ).
Suppose for example that $\zeta$ is the expression tall and that $\mathscr{U}(c)$ is the set of blocks illustrated on the top line of Figure 2. Then the rest of the diagram portrays a reasonably plausible way of sorting the blocks by length. (The groupings within the sets represent, from left to right, the positive extension, extension gap if any, and negative extension of tall with respect to those sets.) Notice for example that $\langle a, e\rangle \in\rangle_{c, \text { tan }}$ by virtue of the extension of tall relative to $\{a, c, d, b, e, f\}$, while $\langle b, d\rangle \in\rangle_{\text {c.adu }}$ relative to the comparison class $\{c, b, d\}$. Moreover, neither


Fig. 2.
$\langle e, f\rangle$ nor $\langle f, e\rangle \in\rangle_{c, \text { tall }}$ since $e$ is exactly as tall as $f$. That is, I have assumed that $F_{\text {tall }}$ is undefined for a comparison class like $\{e, f\}$ whose members do not differ with respect to the property of tallness.

What are the principles that govern the interpretation of degree adjectives and which allow sets to be progressively partitioned in this way? As part of the answer to this question, I want to introduce two new relations based on $\zeta$ and $c$.

First, a relation $\sim_{c, \zeta}$ of NONDISTINCTNESS with respect to $\zeta$ at $c$. This holds between two objects $u$ and $u^{\prime}$ just in case there is no subset $X$ of $\mathscr{U}(c)$ such that $u$ and $u^{\prime}$ both belong to $X$ and such that $F_{\zeta}$ assigns $u$ and $u^{\prime}$ different values relative to $X$.

$$
\begin{align*}
& \left\langle u, u^{\prime}\right\rangle \in \sim_{c, \zeta} \text { iff }(\forall X \subseteq q(c))\left[u, u^{\prime} \in X\right.  \tag{30}\\
& \left.\Rightarrow F_{\zeta}(c[X])(u)=F_{l}(c[X])\left(u^{\prime}\right)\right]
\end{align*}
$$

Second, there is a stronger relation $\approx_{c, \zeta}$ of equivalence with respect to $\zeta$ at $c$. Roughly speaking, two individuals $u$ and $u^{\prime}$ will stand in this relation at $c$ iff there is no context $c^{\prime}$ which is more determinate than $c$ and in which $u$ and $u^{\prime}$ are distinct with respect to $\zeta$. In order to define this second relation, it is necessary to spell out what I mean by 'determinate' here. I want to say that $c$ ' is at least as determinate as $c$
with respect to $\zeta$ just in case (a) $c$ and $c^{\prime}$ specify the same comparison class $X$, and (b) for all $Y \subseteq X$ and for all $u \in Y$, if $F_{f}(c[Y])(u)$ is defined, then $F_{6}\left(c^{\prime}[Y]\right)(u)$ is defined and agrees in value with $F_{5}(c[Y])(u)$. So all assignments of truth made in $c$ are preserved in $c^{\prime}$, and possibly some further assignments are made in $c^{\prime}$ to cases which were left undetermined in $c$. Given the standard set theoretical treatment of functions, it therefore suffices to say, for condition (b), that whenever $Y \subseteq X, F_{\zeta}(c[Y]) \subseteq F_{\zeta}\left(c^{\prime}[Y]\right)$. To see this, let $f$ and $f^{\prime}$ be characteristic functions and suppose $f$ is defined for $u$. In this case, and only in this case, $f$ will contain a pair $\langle u, i\rangle$, where $i=0$ or $i=1$. If, therefore, for every such $u, f^{\prime}(u)$ is defined and agrees in value with $f(u)$, then for every pair $\langle u, i\rangle,\langle u, i\rangle \in f \Rightarrow\langle u, i\rangle \in f^{\prime}$, and hence $f \subseteq f^{\prime}$. On this basis, we define the set $\mathscr{D}(c, \zeta)$ of contexts which are at least as determinate as $c$ with respect to $\zeta$.

$$
\begin{align*}
& \mathscr{D}(c, \zeta)=\left\{c^{\prime}: \mathscr{U}(c)=\mathscr{U}\left(c^{\prime}\right) \&\right.  \tag{3}\\
& \left.(\forall X \subseteq \mathscr{U}(c))\left[F_{\zeta}(c[X]) \subseteq F_{\zeta}\left(c^{\prime}[X]\right)\right]\right\}
\end{align*}
$$

Let me give a concrete example (see Figure 3). This figure is to be understood as follows. The set $X=\{j, m, d, s\}=\{$ Jude, Mona, Dick, Sue $\}$ is partitioned by $F_{\text {ckere }}$ at the various contexts. These differ in terms of the criteria for applying the predicate. $c_{0}$ is a context at which both interpersonal and mathematical skills are relevant. $c_{1}$ is a context at which interpersonal skills are exclusively relevant, and $c_{2}$ is one at which mathematical skills are exclusively relevant. The contexts $c_{21}$ and $c_{22}$ are further refinements of $c_{2}$; the first specifies an ability to do mental arithmetic as the sole criterion, while the second specifies an aptitude for proof theory. It can be seen, for instance, that $c_{1}$ is at least as determinate as $c_{0}$, and so is $c_{2}$, though this relation is undefined between $c_{1}$ and $c_{2}$. The function $\mathscr{D}$ gives the values in (32):

$$
\begin{align*}
& \mathscr{D}\left(c_{0}, \text { clever }\right)=\left\{c_{0}, c_{1}, c_{2}, c_{21}, c_{22}\right\}  \tag{32}\\
& \mathscr{D}\left(c_{1}, \text { clever }\right)=\left\{c_{1}\right\} \\
& \mathscr{D}\left(c_{2}, \text { clever }\right)=\left\{c_{2}, c_{21}, c_{22}\right\} \\
& \mathscr{D}\left(c_{21}, \text { clever }\right)=\left\{c_{21}\right\} \\
& \mathscr{D}\left(c_{22}, \text { clever }\right)=\left\{c_{22}\right\}
\end{align*}
$$

Having introduced $\mathscr{D}$, it is now straightforward to define $\approx_{c, z}$ :

$$
\begin{equation*}
\left\langle u, u^{\prime}\right\rangle \in \approx_{c_{c, l}} \text { iff }\left(\forall c^{\prime} \in \mathscr{D}(c, \zeta)\right)\left[\left\langle u, u^{\prime}\right\rangle \in \sim_{c, l}\right] \tag{33}
\end{equation*}
$$

Thus, in Figure 3, every $\left\langle u, u^{\prime}\right\rangle \in X^{2}$ belongs to $\sim_{c_{0}, \text { ckerer }}$, but no $\left\langle u, u^{\prime}\right\rangle$ belongs to $\approx c_{c_{0} \text { dever }}$. And though $\langle j, m\rangle$ belongs to $\sim c_{c_{1} \text {,derar }}$ and $\sim c_{c_{2} \text {, cevrer }}$, it belongs to $\approx_{c_{1}, \text { ckrex }}$ but not $\approx_{c_{2} \text {, dverer }}$.


Fig. 3.
It may be unclear why both the relations of nondistinctness and equivalence are required. The latter relation is the important one; it is intended to correspond to the relation expressed by is exactly as $\zeta$ as. By contrast, $\sim_{c, \zeta}$ is introduced into the semantic metalanguage in order to allow nonlinear adjectives to be distinguished from linear ones. If $\zeta$ is a linear adjective, then every pair which belongs to $\sim_{c, \zeta}$ will also belong to $\approx_{c, \zeta}$. Suppose, however, that $\zeta$ is the translation of a nonlinear adjective like clever. As we have just seen, two individuals can be nondistinct with respect to this adjective at a particular context merely because they are incommensurable. That is, neither of the next two sentences is true at $\boldsymbol{c}_{2}$.
(34) Jude is cleverer than Mona.
(35) Mona is cleverer than Jude.

But this does not mean that Jude is exactly as clever as Mona should be taken to be true at $c_{2}$. For there are more determinate contexts at which (34) and (35) do come out true, namely $c_{21}$ and $c_{22}$ respectively. In view of this, Jude and Mona cannot be considered equivalent at $c_{2}$ with respect to the predicate clever.

I now want to formulate two conditions which will be placed on the interpretation of all members of Adj. ${ }^{21}$ For the first one, consider an
arbitrary subset $X$ of $\mathscr{U ( c ) \text { . In principle, one of the following three }}$ situations might obtain: (a) the domain of $F_{l}(c[X])$ is empty; (b) one and only one of $\operatorname{pos}_{\boldsymbol{f}}(c[X])$ and neg $(c[X])$ is nonempty; (c) both $\operatorname{pos}_{6}(c[X])$ and neg ${ }_{6}(c[X])$ are nonempty. Under what conditions should we allow (a) to arise? If and only if $X$ is empty or all members of $X$ are nondistinct with respect to $\zeta$. Under what conditions should we allow (b) to arise? In my opinion, none. Suppose only two members of $X, u$ and $u^{\prime}$, are distinct with respect to $\zeta$; i.e. there is at least one subset $X^{\prime}$ of $u(c)$ such that $F_{\zeta}\left(c\left[X^{\prime}\right]\right)(u)$ differs from $F_{\zeta}\left(c\left[X^{\prime}\right]\right)\left(u^{\prime}\right)$. Then one of them should go in the positive extension of $\zeta$ at $c[X]$, while the other should go in the negative extension. More generally, if there are at least two members of $X$ which are distinct with respect to $\zeta$, then situation (c) must obtain. This latter constraint can be expressed formally as follows:

$$
\begin{align*}
& (\forall X \subseteq \mathscr{Q}(c))\left[( \exists u , u ^ { \prime } \in X ) ( \exists X ^ { \prime } \subseteq \mathscr { G } ( c ) ) \left[F_{\zeta}\left(c\left[X^{\prime}\right]\right)(u) \neq\right.\right.  \tag{36}\\
& F_{\zeta}\left(c\left[X^{\prime}\right]\right)\left(u^{\prime}\right) \Rightarrow(\exists u)\left[u \in \operatorname{pos}_{\zeta}(c[X])\right] \& \\
& \left.(\exists u)\left[u \in \operatorname{neg}_{\zeta}(c[X])\right]\right]
\end{align*}
$$

The second condition concerns the consistency with which individuals are allocated to the positive and negative extensions of an adjective in various comparison classes. Suppose there is some class relative to which $\zeta$ is judged to be true of $u$ but false of $u^{\prime}$. Then for every other comparison class (in that context) to which $u$ and $u^{\prime}$ belong, $u^{\prime}$ cannot consistently be assigned to the positive extension of $\zeta$ unless $u$ is too, and $u$ cannot consistently be assigned to the negative extension of unless $u^{\prime}$ is too.

$$
\begin{align*}
& \left(\forall u, u^{\prime}\right)\left[\left\langle u, y^{\prime}\right\rangle \in\right\rangle_{c, b} \Rightarrow  \tag{37}\\
& (\forall X \subseteq थ(c))\left[u^{\prime} \in \operatorname{pos}_{6}(c[X]) \Rightarrow u \in \operatorname{pos}_{l}(c[X]) \&\right. \\
& \left.\left.u \in \operatorname{neg}_{6}(c[X]) \Rightarrow u^{\prime} \in \operatorname{neg}_{6}(c[X])\right]\right]
\end{align*}
$$

These two conditions, let me repeat, are intended to govern the interpretation of all degree adjectives. It is possible to prove that if $\zeta$ satisfies (36) and (37), then for any $c,\rangle_{c, \delta}$ is a strict partial ordering of $\mathscr{Q}(c)$, i.e. an asymmetric, transitive relation in $Q_{(c)}$. Let me indicate briefly why this is so. The asymmetry of $\rangle_{c, 6}$ follows immediately from (37). For transitivity, suppose $\left\langle u, u^{\prime}\right\rangle$ and $\left\langle u^{\prime}, u^{\prime \prime}\right\rangle$ belong to $\rangle_{c, 6}$. We have to show that there is some $X \subseteq \mathscr{Q}(c)$ such that $u \in \operatorname{pos}_{q}(c[X])$ and $u^{\prime \prime} \in$ neg $_{6}(c[X])$. It follows from (36) and (37) that $\left\{u, u^{\prime}, u^{\prime \prime}\right\}$ is such a set; I leave it to the reader to work out the details.
A relation $R$ is usually said to be connected in a set $X$ iff for every $u$, $u^{\prime} \in X$, if $u \neq u^{\prime}$, then either $\left\langle u, u^{\prime}\right\rangle$ or $\left\langle u^{\prime}, u\right\rangle$ belongs to $R$. In the present framework it is natural to adopt a slightly different notion of con-
nectedness in which the identity relation $=$ is replaced by our equivalence relation $\approx$ :

$$
\begin{align*}
& \rangle_{c, \zeta} \text { is CONNECTED in } \mathscr{Q}(c) \text { iff }  \tag{38}\\
& \left(\forall u, u^{\prime} \in \mathscr{U}(c)\right)\left[\left\langle u, u^{\prime}\right\rangle \not \approx_{c, \xi} \Rightarrow\right. \\
& \left.\left.\left.\left\langle u, u^{\prime}\right\rangle \in\right\rangle_{c, \xi} \vee\left\langle u^{\prime}, u\right\rangle \in\right\rangle_{c, \xi}\right]
\end{align*}
$$

Any two individuals which are equivalent with respect to $\zeta$ at $c$ are also nondistinct there, for any $\zeta \in \operatorname{Adj}$; but the converse only holds for some members of $A d j$. Consider all the subsets of $\mathscr{U}(c)$ which cannot be further partitioned by $\zeta$. If each of these is an equivalence class under $\approx_{c, \zeta}$ then $\zeta$ can be regarded as having completely partitioned $\mathscr{U}(c)$ :
(39) $\quad \zeta$ is COMPLETE AT $c$ iff $\sim_{c, \zeta} \subseteq \approx_{c, \xi}$.

If $\zeta$ is complete at $c$, it follows from the definition of $\sim$ that any two individuals in $\mathscr{U}(c)$ are either equivalent or distinct with respect to $\zeta$ at $c$. In conjunction with (36) and (37), we can then conclude that $>_{c, \zeta}$ is connected in $\mathscr{U}(c)$. Since an asymmetric, transitive, connected relation is a linear ordering, this gives us the following result:

THEOREM If $\zeta$ is complete at $c$, then $>_{c, \zeta}$ is a linear ordering of $\mathscr{q}(c)$.
We can now also give a more succinct definition of the class of linear adjectives:
(40) $\zeta$ is LINEAR iff $\zeta$ is complete at every $c \in C$.

## 4. Degrees

Before I develop a proposal for comparative constructions based on the results of the last section, it will be useful to consider the interpretation of degree modifiers such as very and fairly and of measure phrases such as six foot. This will form the basis for a slightly more systematic consideration of the role played by contexts in the semantics for $L$.

### 4.1. Degree Modifiers

What are the truth conditions of the next sentence?
(41) Mary is very tall.

According to Wheeler (1972), this will be true if Mary is tall compared to the set of tall people. This insight can be incorporated rather naturally into the present framework.

Let us first introduce a convention for referring to translations from

English into L: for any category $\mathbf{X}, \mathbf{X}^{\prime}$ is to be the translation in L of a tree rooted by a node labelled X. Having adopted this convention, I shall henceforth write $\mathrm{A}^{\prime}$ in place of $\zeta$ for arbitrary members of $\operatorname{Adj}$.

Returning now to very, we simply require an evaluation rule of the following sort:

$$
\begin{equation*}
\left[\operatorname{ver} y\left(\mathrm{~A}^{\prime}\right)\right]_{c a}^{\mathbb{Z}}=\left[\mathrm{A}^{\prime}\right]_{c \mid X] a}^{\mathbb{\alpha}} \text {, where } X=\operatorname{pos}_{A^{\prime}}(c) \text {. } \tag{42}
\end{equation*}
$$

When very is applied to $A^{\prime}$, the resulting expression is itself a vague predicate. The value of the latter at a context $c$ is equivalent to the value of $\mathrm{A}^{\prime}$ at a context $c[X]$ whose comparison class is just the set of things in the positive extension of $\mathrm{A}^{\prime}$ at $c$.
A similar treatment is possible for fairly. According to Wheeler, Bill is fairly tall will be true if Bill is tall but not very tall. However, this does not seem quite correct to me; surely someone can be fairly tall without being tall. A more plausible suggestion is that Bill will be in the positive extension of fairly tall at a context $c$ if he is tall relative to everyone in $\mathscr{U}(c)$ except those who are very tall. On this analysis, we get the following evaluation rule:

$$
\begin{align*}
& {\left[\text { fairly }\left(\mathrm{A}^{\prime}\right)\right]_{c a}^{\mathbb{\alpha}}=\left[\mathrm{A}^{\prime}\right]_{c a \mid X] a}^{\mathbb{\alpha}},}  \tag{4}\\
& \text { where } X=\left(\mathscr{U}(c)-\operatorname{pos}_{A^{\prime}}\left(c\left[\operatorname{pos}_{\Lambda^{\prime}}(c)\right]\right)\right) \text {. }
\end{align*}
$$

My general claim is that degree modifiers serve the function of introducing a new comparison class which is narrower than the prevailing one. This has the effect of shifting the boundary of the positive extension of the head adjective either upwards or downwards in relation to its previous position. ${ }^{22}$
In the clauses (42), (43), degree modifiers have been treated as syncategorematic expressions. It is interesting to ask what the alternative would be. What is the set of possible denotations appropriate to an expression like very? Consider for a moment the standard evaluation rule for the S5 possibility operator $\diamond$.

$$
\begin{align*}
& {[\delta \varphi]_{w}=1 \text { iff }[\varphi]_{w^{\prime}}=1 \text { for some } w^{\prime} \in W \text { (the set of }}  \tag{44}\\
& \text { possible worlds) }
\end{align*}
$$

In (44), $\diamond$ has the effect of changing the world of evaluation of $\varphi$, but is assigned no interpretation of its own. Nevertheless, $\diamond$ can be given an interpretation in isolation by treating it as a function from propositions to propositions. In order to do this, we must first have something like Montague's operator * which allows us to construct expressions that denote intensions. We will then be able to form formulae $\diamond\left({ }^{\wedge} \varphi\right)$. The evaluation rule for $\diamond$ will go as follows.

Whenever $w \in W$ and $P \in\{0,1\}^{W}, \quad F_{\circ}(w)(P)(w)=1$ iff $P\left(w^{\prime}\right)=1$ for some $w^{\prime} \in W$.
(45) brings out clearly that $\diamond$ needs to know the intension of its argument. In parallel fashion, very needs to know more than just the extension of its argument, though this is concealed by (42). In fact, we must be able to specify something analogous to an intension, namely a function from contexts to extensions. Such functions have been called CHARACTERS by Kaplan (1977). ${ }^{23}$ It is straightforward to represent characters in the metalanguage by using an abstraction operator $\lambda$. Thus, the character of tall is simply $\lambda c$ आtall $]_{c a}^{\boldsymbol{Z}}$.

However, in order to assign very an interpretation in isolation, we need to be able to denote the character of tall in the object language. This requires some important modifications in Montague's system of intensional logic which I do not want to undertake here. Let me just indicate the main points.

First, it is necessary to introduce a character operator, say ${ }^{n}$, together with its inverse ${ }^{U}$. That is, for any expression $\alpha,{ }^{n} \alpha$ denotes the character of $\alpha$, while ${ }^{\text {un }} \alpha$ denotes exactly what $\alpha$ denotes.

Second, the types of intensional logic have to be augmented so that for any type $\tau,\langle k, \tau\rangle$ is also a type (' $k$ ' for karacter). Expressions of type $\langle k, \tau\rangle$ denote functions from contexts to elements of $D_{r}$; i.e. $D_{(k, \tau)}=D_{\tau}^{C} .^{24}$

Third, let $\mathfrak{A}$ be an interpretation for $L$. Then in order to extend it to an interpretation for the augmented language, we must add the following clauses:
(46) If $\alpha$ is a constant of type $\tau$, then $F_{\alpha} \in D_{(k, \tau)}$.
(47)(a) $\llbracket{ }^{n} \alpha \rrbracket_{c a}^{q}$ is $\lambda c \llbracket \alpha \rrbracket_{c a}^{n}$.
(b) If $\alpha \in \mathrm{ME}_{(k, \tau)}$, then $\llbracket^{U} \alpha \rrbracket_{c a}^{\alpha}$ is $\llbracket \alpha \rrbracket_{c a}^{\mathfrak{\mu}}(c)$.

Given these modifications (which unfortunately I do not have space to discuss in detail), very can be taken to denote a function from characters of predicates to sets. Thus, if we set $\tau_{\mathrm{A}}$ (the type associated with the category A) to be $\langle e, t\rangle$, very will be an expression which takes an argument of type $\left\langle k, \tau_{A}\right\rangle$ and yields an expression of type $\tau_{A}$. The evaluation rule for very can be reformulated as

Whenever $c \in C$ and $z \in D_{\left(k, r_{A}\right)}, F_{\text {very }}(c)(z)=z(c[X])$, where $X=\{u: z(c)(u)=1\}$.

Following Jackendoff (1977), I shall introduce a syntactic category Deg (though I do not want to commit myself to specific details of his analysis). Modifiers like very and fairly will be assigned to Deg, as will
expressions like so, too, as, more, how, etc. The type $\tau_{\text {Deg }}$ will be $\left\langle\left\langle k, \tau_{A}\right\rangle, \tau_{A}\right\rangle$, and a structure such as $\left[{ }_{A P}\left[D_{\text {Deg }} v e r y\right]\left[{ }_{A}\right.\right.$ tall $]$ will be translated as very ${ }^{\wedge}$ tall)
These considerations suggest a rather plausible semantic analysis of interrogatives in which an AP has been fronted by wh-movement. A question of the form

$$
\begin{equation*}
\left[s\left[A_{A P}\left[D_{\text {ceg }} H o w\right] A\right] \text { is } N P\right] \tag{49}
\end{equation*}
$$

requires an answer of the form [s NP is [AP[Deg $\alpha] \mathrm{A}]$ ], where $\left[_{\text {Deg }} \alpha\right.$ ] determines some modification of the current comparison class relative to which [s NP is A] is true. In order to capture this formally, we need to introduce 'degree variables', that is, expressions which range over functions of the sort expressed by degree modifiers. Thus, let $\mathcal{N}$ be a variable of type $\left\langle k, \tau_{\text {Deg }}\right\rangle$; and let us adapt Montague's (1973) brace convention, so that if $\alpha$ is of type $\langle k,\langle\sigma, \tau\rangle\rangle$ and $\beta$ is of type $\sigma$, then $\alpha\{\beta\}$ is to be taken as $\left[{ }^{[ } \alpha\right](\beta)$. Then, for example, $\mathcal{N}\left\{{ }^{n}\right.$ tall\} will be a well-formed expression of type $\tau_{A}$. Let's suppose in addition that Karttunen's (1977) semantics for questions is correct (and temporarily import all the familiar paraphernalia of intensionality). Then if $\mathrm{NP}^{\prime}$ is an expression of type $\langle\langle e, t\rangle, t\rangle$ and $p$ is a variable of type $\langle s, t\rangle,(49)$ will translate as follows:

$$
\begin{equation*}
\lambda p \vee \mathcal{N}\left[{ }^{v} p \wedge p=\left[{ }^{\wedge} \mathrm{NP}^{\prime}\left(\lambda x\left[\mathcal{N}\left\{{ }^{n} \mathrm{~A}^{\prime}\right\}(x)\right]\right)\right]\right] \tag{50}
\end{equation*}
$$

This denotes, at any world $w$, the set of propositions $\left\lfloor^{\wedge} \mathrm{NP}^{\prime}\left(\lambda x\left[\mathcal{N}\left\{{ }^{n} \mathrm{~A}^{\prime}\right\}\right.\right.\right.$ $(x)])]_{\mathrm{cwa}}$ which are true at $w$ under some assignment to the variable $\mathcal{N}$.

In the rest of the paper, I shall revert to syncategorematic rules where they are easier to follow. Nevertheless, I shall also make free use of character-denoting expressions, and consider them a part of $L$.

### 4.2. Measure Phrases

I shall assume for convenience that measure phrases such as six foot in (51) are also members of Deg:
(51) Mona is six foot tall.

In order to state the semantics of this phrase, I want to first discuss briefly some basic notions to do with measurement.

Suppose that $u$ is a rod. What kind of thing is the length of $u$ ? The most plausible answer parallels the Frege-Russell analysis of cardinality: the length of $u$ is the set of extended physical objects which are exactly as long as $u$. Let $L$ be the set of objects to which the adjective long can be meaningfully applied, and let $\approx_{c, m m}$ be the equivalence
relation based on long, as defined in (33). If we assume that comparisons are carried out with equal precision in all contexts, the $c$ argument in this relation can be suppressed without loss. The length of an object $u \in L$ will be the set $\bar{u}=\left\{u^{\prime} \in L:\left\langle u, u^{\prime}\right\rangle \in \approx{ }_{\text {rong }}\right\}$. The set of lengths of all objects in $L$ will be the partition $\mathscr{L}$ of $L$ generated by the relation $\approx{ }_{\text {ong }}$.

I now want to construct a rudimentary system for measuring length. To begin with, we need to select some particular object as a standard. The British system of measures apparently takes the yard as the basic unit of length, and the prototype is a bronze bar, known as "No. 1 standard yard", which was cast in the early part of the nineteenth century. However, from the naive point of view it seems more natural to take the foot as the basic unit, so let us suppose that $L$ contains a suitable prototype $f$. Members of $L$ can be laid end to end, so let us take con as a primitive concatenation operation on $L$. The next step is to define a similar operation $\overline{c o n}$ on $\mathscr{L}$. This is done as follows:

$$
\begin{align*}
& \text { Whenever } \bar{u}, \bar{u}^{\prime} \in \mathscr{L}, \overline{\operatorname{con}}\left(\bar{u}, \bar{u}^{\prime}\right)=  \tag{52}\\
& \left\{y \in L:(\exists u \in \tilde{u})\left(\exists u^{\prime} \in \bar{u}^{\prime}\right)\left[\left\{\operatorname{con}\left(u, u^{\prime}\right), y\right\rangle \in \approx_{\text {long }}\right]\right\}
\end{align*}
$$

That is, the concatenation of two lengths $\bar{u}$ and $\bar{u}^{\prime}$ is the length which consists of all those objects which are exactly as long as the concatenation of two objects drawn respectively from $\bar{u}$ and $\bar{u}^{\prime}$. Once $\overline{c o n}$ has been defined, we can construct multiples of a given unit of length. For example, if $\bar{u} \in \mathscr{L}$, then $2 \bar{u}=\overline{\operatorname{con}}(\bar{u}, \bar{u})$, and more generally, for $n>1, n \bar{u}=\overline{\operatorname{con}}((n-1) \bar{u}, \bar{u})$. The STANDARD SEQUENCE based on $\bar{u}$ (cf. Krantz et al (1971)) is the sequence $\langle\bar{u}, 2 \bar{u}, 3 \bar{u}, \ldots\rangle$. According to this construction, then, the length six foot will be an element of the standard sequence based on $\overline{\mathbf{f}}$, namely $\mathbf{6 \overline { f }}{ }^{25}$

Let us return now to (51). There seem to be two possible ways of taking the adjective phrase six foot tall: either as at least six foot tall or as exactly six foot tall. Pragmatic considerations might incline us towards the former. By Grice's maxim of quantity, an utterance of (51) will carry, in most contexts, the conversational implicature that the speaker doesn't know Mona's height to be any greater than six foot. On this approach then, (51) will convey, but not entail, that Mona is no taller than six foot. Support for this view comes from the fact that the implicature can be cancelled.
(53) A: The minimum height for applicants for this job is six foot.

B: Well, Mona is six foot tall; in fact she's six foot three.
On the other hand, exchanges like (54) suggest that (51) can be regarded as simply false if Mona is more than six foot tall.
(54) A: How tall is Mona?

B: Six foot tall.
C: No she's not, she's six foot three.
It is not clear that such data can be naturally encompassed within the Gricean theory. ${ }^{26}$ Possibly measure phrases are in general ambiguous between the at least reading and the exactly reading. As the latter is slightly easier to formulate, I shall ignore the at least reading, though nothing hangs on this decision.

Measure phrases can only precede a small number of adjectives, among them being long, tall, broad, wide, thick, deep, high, and old. When they are so preceded, these vague predicates are rendered precise. ${ }^{27}$ For example, on the exactly reading, six foot tall is true of $u$ iff $u$ is exactly as tall as some object which measures six feet, i.e. some object in the set $6 \overline{\mathbf{f}}$. Thus we get the following sort of evaluation rule for ${ }_{\text {[Deg }}$ six foot $]^{\prime}$ :
(55) Whenever $u \in L$,

$$
\llbracket \operatorname{six} \operatorname{foot}\left(\mathrm{A}^{\prime}\right) \rrbracket_{c a}^{\Psi}(u)=1 \text { iff }(\exists u \in 6 \overline{\mathbf{f}})\left[\left\langle u, u^{\prime}\right\rangle \in \approx_{c, A^{\prime}}\right]
$$

In sentences such as (56), we can simply let six foot denote the set $\mathbf{6} \overline{\mathbf{f}}$ :
(56)(a) Mona is six foot.
(b) Mona's height is six foot.

However, the first asserts that Mona is one of the elements of that set, while the second asserts that the set of objects exactly as tall as Mona is identical to that set.

Consider now the following example:
Mona is taller than six foot.
Unlike (58), (57) appears to have no well-formed version in which than is followed by a clause.
(58) Mona is taller than Jude.
(59)(a) Mona is taller than Jude is.
(b) *Mona is taller than six foot is.

Heny (1978) takes the unacceptability of (59b) to be evidence for Hankamer's (1973) hypothesis that than is both a complementizer and a preposition (see also Jackendoff 1977: 208). On this view, (57) and (58) have the common structure NP is [ap taller [pp than NP]]. While I do not wish to argue that (57) is derived by deletion from a source like (59b), it seems unlikely to me that syntactic factors are responsible for the
latter's unacceptability. The problem is that tall cannot be predicated of lengths in the same way that it can of individuals. Although (60) is acceptable - and paraphraseable as 'anyone who is six foot tall is tall' the sentences in (61) are not. (Example (61c) is adapted from McCon-nell-Ginet (1973).)
(60) Six foot is tall.
(61)(a) *Jude is tall and so is six foot.
(b) *Jude wants to be what six foot is: tall.
(c) *Six foot is taller than Jude (is).

I suspect that (57) is on a par with sentences like Mona is nicer than just nice and he comes as frequently as every day (Lees 1961), rather than (58). It should be glossed as 'Mona is taller than six foot tall', not as 'Mona is taller than the length six foot'.

If we regard $\mathbf{6} \overline{\mathbf{f}}$ as a degree of tallness, then the present treatment of degrees turns out to be very similar to that proposed by Cresswell (1976). A general definition of the following sort suggests itself:
(62) Whenever $u \in \mathscr{U}(c)$, the degree to which A holds of $u$ in $c=\left\{u^{\prime}:\left\langle u, u^{\prime}\right\rangle \in \approx{ }_{c, \mathrm{~A}^{\prime}}\right\}$

The main difference is that Cresswell attempts to base his semantics for comparatives on degrees even though he has no satisfactory analysis of the equivalence relation that generates them. However, as I shall show later, degrees do not play any essential role in the interpretation of comparatives; they fall out as a natural by-product of the analysis of comparatives in terms of comparison classes.

## 5. The comparative construction

### 5.1. Rules for Simple Comparatives

In this section, I shall present some rules which indicate how simple comparative constructions are to be generated, and how they are to be translated into L. Subsequently, I shall show how the resulting logical structures are to be interpreted, using the ideas developed in foregoing sections.

For the sake of definiteness, I shall formulate the syntactic rules within the framework of context-free phrase structure grammars developed by Gazdar (forthcoming (a)). The obvious attraction of this approach is its extremely restrictive metatheory. However, my semantic proposals could just as well be incorporated into any of the main
syntactic theories currently being developed (e.g. the revised Extended Standard Theory of Chomsky (1973, 1977), the framework of Bresnan (1977), Bresnan and Grimshaw (1978), or some version of Montague Grammar).

At this point, I shall briefly describe those aspects of Gazdar's approach which are most relevant to my present task. Following McCawley (1968), phrase structure (PS) rules are interpreted as tree admissibility conditions, as opposed to productions in a rewrite system. Accordingly, in place of the familiar S $\rightarrow$ NP VP, we write [s NP VP], and similarly for other rules.

A complete rule of the grammar will specify not just a PS rule, but also a semantic rule which translates the tree admitted by the PS rule into a suitable formal language. In the present case, given a PS rule of the form $\left[a \beta_{1} \ldots \beta_{n}\right]$, the translation rule will take the translations of $\beta_{1} \ldots \beta_{n}$ and convert them into an expression of L of type $\tau_{\alpha}$ (where $\tau_{\alpha}$ is the type associated with category $\alpha$ ). This expression will then serve as the translation of $\alpha{ }^{28} \mathrm{So}$, for example, a possible rule of the grammar might be the pair $\left\langle[\mathrm{s} ~ \mathrm{NP} V P], \mathrm{VP}^{\prime}\left(\mathrm{NP}^{\prime}\right)\right\rangle$.
In addition, the set of nonterminal symbols of the grammar is enriched in two ways. The first is fairly minor. Among the set of features which can occur on node labels are counted not only abstract morpho-syntactic elements, but also a restricted set of lexical items. So, for example, $\mathrm{AP}_{\text {[more] }}$ is a possible node label.

Second, this set of 'basic' node labels is augmented by a set of 'derived' node labels. If $\alpha$ and $\beta$ are any basic node labels, then the pair containing $\alpha$ and $\beta$, written as $\alpha / \beta$, is a derived node label. A node which bears the label $\alpha / \beta$ will dominate exactly the trees that can be dominated by $\alpha$, except that in each such tree there must occur a node $\beta / \beta$ which in turn immediately dominates a phonologically null symbol $t{ }^{29}$ Intuitively, any tree rooted by the node $\alpha / \beta$ will contain a 'gap' of category $\beta$; it will thus correspond to a structure from which a constituent $\beta$ has been extracted on a movement or deletion analysis. So, for example, S/AP will represent a sentence which lacks an AP somewhere.

Suppose there is already a rule of the form $\left[{ }_{\alpha} \gamma_{1} \ldots \gamma_{n}\right]$ in the grammar (where $\alpha$ and $\gamma$ are basic nodes). Then if there is a derived label of the form $\alpha / \beta$, there will also be a derived rule of the form $\left[_{\alpha / \beta}\right.$ $\left.\gamma_{1} \ldots \gamma_{i} / \beta \ldots \gamma_{n}\right]$. That is, one of the nodes $\gamma_{i}$ that can be dominated by $\alpha$ must bear the derived label $\gamma_{i} / \beta$ when it is dominated by $\alpha / \beta$. Rules of this sort allow the 'gap' information to be carried progressively down the tree.

In addition, there are 'linking' rules which introduce and eliminate
derived nodes. An example of an introduction rule is [AP AP S/AP]. This says that an AP can expand as an AP with an S/AP complement. The only elimination rules we need consider fall under the following schema:

$$
\begin{equation*}
\left\langle\left[_{\alpha / \alpha} t\right], v^{\cup} v_{0}^{\left\langle k_{,}, \tau_{a}\right\rangle}\right\rangle . \tag{63}
\end{equation*}
$$

Thus, for any category $\alpha, \alpha / \alpha$ can dominate the 'trace' $t$.
According to the semantic part of the rule, this trace will correspond in logical structure to the 'extension' of the first variable of type $\left\langle k, \tau_{\alpha}\right\rangle$ (where $\tau_{\alpha}$ is the type associated with category $\tau_{\alpha}$, as before). ${ }^{30}$ Since $v^{\left(k^{k}, \tau_{\alpha}\right)}$ ranges over characters of expressions of type $\tau_{\alpha}$, the value of ${ }^{\cup} v_{0}^{\left(k, r_{\alpha}\right)}$ relative to a context $c$ and assignment $a$ will be an element $a\left(v_{0}^{\left(k, \tau_{\alpha}\right)}\right)(c)$ of $D_{\tau_{\alpha}}$. In general, the translation of a constituent $\alpha / \beta$ will be just like the translation of $\alpha$ except that it will contain a free variable of type $\left\langle k, \tau_{\beta}\right\rangle$.

Recall that $\tau_{\mathrm{A}}=\tau_{\mathrm{AP}}=\langle e, t\rangle$ and $\tau_{\mathrm{Deg}}=\left\langle\left\langle k, \tau_{\mathrm{A}},\right\rangle \tau_{\mathrm{A}}\right\rangle$. As an abbreviatory convention, I shall use ' 2 ' to stand for $v_{0}^{\left(k, \tau_{A}\right\rangle}$ and ' $\mathcal{N}$ ' to stand


Let $\gamma$ range over the two-place sequences in $\Gamma$ :

$$
\begin{equation*}
\Gamma=\{(\text { more }, \text { than }\rangle,\langle l e s s, \text { than }\rangle,\langle a s, a s\rangle\} \tag{64}
\end{equation*}
$$

Following a proposal of Gazdar (forthcoming b), these sequences are used like agreement features in the following schema to ensure that members of Deg in the head AP cooccur with the appropriate complementizers.

$$
\begin{align*}
& \left\langle\left[\text { AP }^{\text {AP }} \underset{\left[\gamma_{0}\right]\left[\gamma_{1}\right]}{ } \alpha\right], \mathrm{AP}^{\prime}(\lambda 2 \lambda \mathcal{N}[\beta])\right\rangle \text {, where } \alpha \text { is S/Deg, S/AP or NP, }  \tag{65}\\
& \text { and } \\
& \text { if } \alpha=\mathrm{S} / \mathrm{Deg}, \beta=\mathrm{S} / \mathrm{Deg}^{\prime}, \\
& \text { if } \alpha=\mathrm{S} / \mathrm{AP}, \beta=\lambda 2\left[\mathrm{~S} / \mathrm{AP}^{\prime}\right]\left(^{\cap} \mathcal{N}\{2\}\right) \\
& \text { if } \alpha=\mathrm{NP}, \beta=\operatorname{NP}^{\prime}(\lambda x[\mathcal{N}\{2\}(x)]) .
\end{align*}
$$

According to this rule, the complement of a comparative AP can be a clause containing a Deg or an AP 'gap', or else a phrase of the form $\gamma_{1}$ NP.

The first stage in expanding these complements is governed by the following schema:

$$
\begin{equation*}
\left.\left.\underset{\left[\gamma_{1}\right]}{\langle[ } \alpha \gamma_{1} \alpha\right], \alpha^{\prime}\right\rangle \tag{66}
\end{equation*}
$$

In effect, we follow Chomsky and Lasnik's (1977: 495) opinion that than and as should be assigned to no lexical category, but "simply belong to the comparative phrase".

The additional rules which would be required for a full analysis of these complements would take me too far from the main topic of this paper；however，their general nature should be reasonably clear from the example trees displayed later．

Rules（67），（68）expand the head AP（though for reasons of space，I shall ignore the problems of recursion within this constituent）．

The semantic core of the comparative construction is contained in the rules for individual members of Deg：
（69）〈［Des $m o r e], \lambda 2 \lambda \mathscr{Z} \lambda x \vee \mathcal{N}[\mathcal{N}\{2\}(x) \wedge\urcorner \cup \mathscr{Z}(2)(\mathcal{N})]\rangle$
（70）〈［Deg less］，$\lambda 2 \lambda \mathscr{Z} \lambda x \wedge \wedge \mathcal{N}[7 \mathcal{N}\{2\}(x) \wedge \cup \mathscr{Z}(2)(\mathcal{N})]\rangle$ ．
（71）〈［Deg $\left.a s], \lambda 2 \lambda \mathscr{Z} \lambda x \vee \mathcal{N}\left[{ }^{\cup}(2)(\mathcal{N}) \rightarrow \mathcal{N}\{2\}(x)\right]\right\rangle$
It is generally agreed that（72）contains an AP＇gap’，and（73）a Deg ＇gap＇，as indicated by the dashes（see Bresnan（1976）for arguments to this effect）．${ }^{31}$
（72）Jude is taller than Mona is $\qquad$ .
（73）Mona is more happy than Jude is $\qquad$ sad．
On Bresnan＇s analysis，the first of these sentences involves Comparative Deletion（or CD），while the second involves Comparative Subdeletion （or CS）．By contrast，as Hankamer（1973）has argued，there is at least one derivation of（74）on which no deletion or ellipsis has taken place．${ }^{32}$
（74）Jude is taller than Mona．
The trees below illustrate how these sentences would be generated on the present approach．
（75）

(76)

(77)

(75) and (77) induce the same translation. This is given in reduced form in (78):

$$
\begin{equation*}
\vee \mathcal{N}\left[\mathcal{N}\left\{^{\cap} \text { tall }\right\}(\text { Jude }) \wedge \neg \mathcal{N}\left\{^{n} \text { tall }\right\}(\text { Mona })\right] \tag{78}
\end{equation*}
$$

(76), on the other hand, induces the following (reduced) translation:

$$
\begin{equation*}
\left.\vee \mathcal{N}\left[\mathcal{N}\left\{{ }^{n} \text { happy }\right\}(\text { Mona }) \wedge\right\urcorner \mathcal{N}\left\{{ }^{n} \text { sad }\right\}(\text { Jude })\right] \tag{79}
\end{equation*}
$$

(78) is strikingly reminiscent of Seuren's (1973) proposal, according to which (72) would have a semantic representation along the lines of (80) (where $e$ is a variable ranging over 'extents'):
(80) $\quad \exists e$ [Jude is tall to $e \&$ Mona is not tall to $e$ ]

The main difference, of course, is my use of the degree variable $\mathcal{N}$. As I indicated earlier, this ranges over functions of the sort expressed by
degree modifiers like very, quite, and fairly. Indeed one might regard $\mathcal{N}$ as the formal counterpart of that, which seems to function as an indexical Deg.

The idea of using a degree variable for the interpretation of CS constructions, as in (79), seems to have been first proposed by McCon-nell-Ginet (1973: 191). She suggests that a sentence like (73) should be analysed as in (81) (where 'operatory' is roughly equivalent to my $\mathcal{N}$ ):
(81) For some operator ${ }_{y}$, Mona is operator ${ }_{y}$ (happy) \& Jude is not operatory (sad).
The novel features of my approach are (a) that the degree variable $\mathcal{N}$ is used for both the CD and the CS constructions, and (b) that the resulting logical structures are given a model-theoretic interpretation in terms of comparison classes. It is to this latter topic that I now turn.

### 5.2. Interpretation

Let us begin by considering (79), the translation of (73). It will be true in a context $c$ just in case there is some value of $\mathcal{N}$ which satisfies the matrix formula. Consider a particular value, say the function expressed by very. In this case, we have to determine the truth conditions of the formula $\quad\left[v e r y\left({ }^{\wedge}\right.\right.$ happy)(Mona) $\wedge 7 v e r y\left({ }^{n}\right.$ sad)(Jude)]at $c$. When the two phrases very $\left({ }^{( }\right.$happy) and very $\left({ }^{n}\right.$ sad) are evaluated at $c$, the interpretation rule for very (cf. (42), section 4.1) dictates that happy and sad must be evaluated at two new contexts, say $c^{\prime}$ and $c^{\prime \prime}$ respectively. Suppose that $\operatorname{pos}_{\text {mppy }}(c)=X_{1}$ and $\operatorname{pos}_{\text {sed }}(c)=Y_{1}$. Then $c^{\prime}=c\left[X_{1}\right]$, while $c^{\prime \prime}=c\left[Y_{1}\right]$. Now let pos happp $\left(c^{\prime}\right)=X_{11}$ and neg $\operatorname{sad}\left(c^{\prime \prime}\right)=Y_{12}$. Then very ( ${ }^{( }$happy)(Mona) will be true at $c$ iff Mona is in $X_{11}$, while very $\left({ }^{( }\right.$sad) (Jude) will be false at $c$ iff Jude is in $Y_{12}{ }^{33}$ This situation is illustrated in Figure 4.
Of course, the truth of (79) does not depend on $\mathcal{N}$ taking the value $F_{\text {ver }}$ - I am certainly not claiming that (73) entails Mona is very happy. But it is important that the value of $\mathcal{N}$ be a function of the general kind


Fig. 4.
which is expressed by very. That is, a function $h$ such that for any context $c$ and predicate meaning $z, h(c)(z)=z\left(c^{\prime}\right)$, where $c^{\prime}$ specifies a new comparison class $X \subseteq \mathscr{U}(c)$, and $X$ is itself determined as a function of the value of $z$ in $c$. In the special case where $h=F_{\text {very }}$, the new comparison class $X$ is determined by the function $\lambda c \lambda z\{u \in$ $थ(c): z(c)(u)=1\}$. This may look a bit strange at first; but for the arguments $c$ and $F_{\text {happy }}$, it simply yields the set $\left\{u \in \mathscr{U}(c): F_{\text {happy }}(c)(u)=\right.$ $1\}$, which in turn is just pos $_{\text {happy }}(c)$.
What I want to do now is define a class $G$ of such functions which map arguments $c, z\left(c \in C\right.$ and $\left.z \in D_{\tau_{A}}\right)$ into subsets of $\mathscr{U}(c)$. Let $\varphi(z, c, u)$ be a metalanguage formula constructed out of the two basic formulae $' z(c)(u)=1$ ' and ' $z(c)(u)=0$ ' using only the connectives ' $\sim$ ', ' $v$ ' and ' $\&$ ' (under their standard two-valued interpretation). So, for example, one particular instance of $\varphi(z, c, u)$ would be $z(c)(u)=1 \vee(\sim(z(c)(u)=1)$ $\& \sim(z(c)(u)=0)$ ). For given values of $z$ and $c$, this formula would be satisfied by all those $u \in \mathscr{U}(c)$, which belonged either to the positive extension of $z$ and $c$, or to the extension gap of $z$ at $c$.
(82) $G$ is the smallest set such that
(i) $\lambda c \lambda z\{u \in \mathscr{U}(c): \varphi(z, c, u)\} \in G$,
(ii) if $g \in G$ and for some $c, z, g(c, z)=X$, then $\lambda c \lambda z\{u \in \mathscr{U}(c): \varphi(z, c[X], u)\} \in G$.

Perhaps the purpose of $G$ can best be grasped by considering the family of sets which are in the range of the functions in $G$. Let $\mathbf{G}(c, z)$ be defined, for any $c$ and $z$, as $\{X:(\exists g \in G)[g(c, z)=X]\}$. One can visualize this family of sets as being recursively constructed along the lines illustrated in Figure 1 (Section 3.2). Given a predicate $\zeta$ and a context $c$, we first form the sets pos $_{l}(c)$ and neg $_{l}(c)$. We can then construct from these two a field $\mathscr{F}_{थ(c)}$ of sets, using the operations of complementation, union, and intersection. Every set $X$ in $\mathscr{F}_{\boldsymbol{F}_{(c)}}$ can then be taken as a comparison class, allowing the sets $\operatorname{pos}_{6}(c[X])$ and neg ${ }_{6}(c[X])$ to be formed; and a field $\mathscr{F}_{X}$ over $X$ can then be constructed; and so on.
I now want to stipulate that the variable $\mathcal{N}$ is to range over the set of functions $h$ such that for some $g \in G$, and any $c, z, h(c)(z)=z(c[X])$, where $X=g(c, z)$. Thus, we define $H \subseteq D_{\left(l, T_{D E g}\right)}$ in the following way:

$$
\begin{align*}
& (\forall h)\left(h \in H \Leftrightarrow(\exists g \in G)\left(\forall z \in D_{\left(k, \tau_{N}\right)}\right)(\forall c \in C)\right.  \tag{83}\\
& {[h(c)(z)=z(c[g(c z)])]}
\end{align*}
$$

The truth conditions of (82) can thus be spelt out as follows (again ignoring the problem of cross-cutting criteria): ${ }^{34}$

$$
\begin{align*}
& \llbracket \vee \mathcal{N}\left[\mathcal{N}\left\{{ }^{n} \text { happy }\right\}(\text { Mona }) \wedge \neg \mathcal{N}\left\{{ }^{n} \text { sad }\right\}(\text { Jude }) \rrbracket_{c a}^{\mathfrak{a}}=1\right.  \tag{84}\\
& \text { iff there is some } h \in H \text { such that } \\
& \llbracket \mathcal{N}\left\{^{n} \text { happy }\right\}(\text { Mona }) \wedge \mathcal{N}\left\{\left\{^{n} \text { sad }\right\}(\text { Jude })\right]_{c a[h / \mathcal{N}]}^{\{ }=1 \\
& \text { iff there is some } h \in H \text { such that } \\
& h(c) F_{\text {happy }}(\text { Mona })=1 \text { and } h(c) F_{\text {sad }}(\text { Jude })=0 \\
& \text { iff there is some } g \in G \text { such that } \\
& F_{\text {happy }}\left(c\left[g\left(c, F_{\text {happy }}\right)\right](\text { Mona })=1\right. \text { and } \\
& F_{\text {sed }}\left(c\left[g\left(c, F_{\text {ssod }}\right)\right](J u d e)=0 .\right.
\end{align*}
$$

One attractive feature of this analysis is that the interpretation of CD constructions falls out as a special case. Consider (78) for instance. The evaluation procedure is exactly the same as that given in (84). Consequently, (78) will be true at an index $\langle c, a\rangle$ iff (85) holds.
(85) There is some $g \in G$ such that $F_{\text {tall }}\left(c\left[g\left(c, F_{\text {tall }}\right)\right]\right)(\mathrm{Jude})=1$ and $F_{\text {tall }}\left(c\left[g\left(c, F_{\text {tun }}\right)\right]\right)($ Mona $)=0$.

But here we only have to consider one new context $c\left[g\left(c, F_{\text {tall }}\right)\right]$. So (85) is equivalent to
(86) There is some $X \in \mathbf{G}\left(c, F_{\text {tull }}\right)$ such that $F_{\text {tall }}(c[X])($ Jude $)=1$ and $F_{\text {tull }}(c[X])($ Mona $)=0$.

By virtue of definition (29), (86) entails that the pair (Jude, Mona) belongs to the ordering $>_{c, \text { anll }}$. Thus, we come back, in the end, to the kind of orderings constructed from comparison classes that I discussed in section 3.

By way of summary, let me indicate the semantic relations which hold between the three kinds of construction I have considered here. The CD construction can be defined semantically in terms of the CS construction, and the NP complement construction can be defined in terms of the CD construction. For, on the semantics I have given, the following equivalences provably hold:
(87) Jude is taller than Mona is tall $\Leftrightarrow$

Jude is taller than Mona is $\Leftrightarrow$ Jude is taller than Mona

### 5.3. Comparatives with less and as

I want to turn briefly at this point to comparatives involving less and as. The two sentences in (88) will receive the corresponding logical structures in (89) as translations:
(88)(a) Jude is less tall than Mona.
(b) Jude is as tall as Mona.
(89)(a) $\mathcal{V} \mathcal{N}\left[7 \mathcal{N}\left\{{ }^{n}\right.\right.$ tall $\}($ Jude) $) \wedge\left\{^{n}\right.$ tall $\}$ (Mona) $]$
(b) $\wedge \mathcal{N}\left[\mathcal{N}\left\{{ }^{n}\right.\right.$ tall $\}$ (Mona) $\rightarrow\left\{{ }^{n}\right.$ talll (Jude)]

I am assuming that (88b) has the truth conditions of Jude is at least as tall as Mona. The fact that an utterance of (88b) will often convey that Jude is exactly as tall as Mona is to be explained along standard Gricean lines: in the absence of conflicting factors, a speaker who utters the sentence will conversationally implicate that he is not in a position to make the stronger assertion that Jude is taller than Mona.
The advantage of assigning (88) the logical structures in (89) should be evident. The equivalences in (90) will be provable just on the basis of standard logic; cf. (91):
(90)(a) Jude is taller than Mona $\Leftrightarrow$
(b) Mona is less tall than Jude $\Leftrightarrow$
(c) Mona is not as tall as Jude
(91)(a) $\vee \mathcal{N}\left[\mathcal{N}\{n\right.$ tall $\}(J u d e) \wedge ~ ᄀ \mathcal{N}\left\{{ }^{n}\right.$ tall $\}($ Mona $\left.)\right] \Leftrightarrow$
(b) $\mathcal{V} \mathcal{N}\left[7 \mathcal{N}\left\{{ }^{n}\right.\right.$ tall $\}\left(\right.$ Mona) $\wedge \mathcal{N}\left\{{ }^{n}\right.$ tall $\}($ Jude $\left.)\right] \Leftrightarrow$
(c) $\urcorner \wedge \mathcal{N}\left[\mathcal{N}\left\{{ }^{\cap}\right.\right.$ tall\}(Jude) $\rightarrow \mathcal{N}\left\{{ }^{\cap}\right.$ tall\}(Mona)]

Another inference that we want to be able to capture is that from (90a) to (92):
(92) Jude is (at least) as tall as Mona

However, the standard truth-conditions for connectives and quantifiers certainly do not justify the inference from (91a) to (93)

$$
\begin{equation*}
\wedge \mathcal{N}\left[\mathcal{N}\left\{^{n} \text { tall }\right\}\left(\text { Mona } \rightarrow \mathcal{N}\left\{{ }^{n} \text { tall }\right\}(\text { Jude })\right]\right. \tag{93}
\end{equation*}
$$

Instead, we have to appeal here to condition (37) (section 3.3) which was earlier imposed on the interpretation of all degree adjectives. According to this, if there is some $X$ such that $F_{6}(c[X])($ Jude $)=1$ and $F_{\zeta}(c[X])$ (Mona) $=0$, then for every $Y \subseteq \mathscr{U}(c)$, if $F_{f}(c[Y])(M o n a)=1$ then $F_{\ell}(c[Y])(J u d e)=1$. Thus, the inference from (91a) to (93) will only be valid relative to the class of models which satisfy (37).

### 5.4. Supervaluations and Indeterminacy

I want to conclude this study by returning to the topic of supervaluations. The treatment of vagueness which is embodied in the device of supervaluation involves a two-stage truth definition. First, sentences
are given a partial interpretation. The clauses for comparatives which I have indicated in (84) are to be regarded as part of this first stage. Second, the concept of $\mathscr{C}$-truth for L is defined altoig the lines sketched in Section 2. I shall now add some further details to the characterization of this latter stage.
Whenever $\varphi$ is a formula of L , let $Z(\varphi)$ be the set of predicates occurring in $\varphi$. (This set will be defined by induction on the length of $\varphi$ in the usual way.) It will be recalled that $\mathscr{S}(c, \zeta)$ was earlier introduced as the set of contexts $c^{\prime}$ such that $F_{6}\left(c^{\prime}\right)$ was a total function. An analogous but more precise notion can be defined with the help of the function $\mathscr{D}$, as defined in (31) (section 3.3).

$$
\begin{equation*}
\mathscr{S}^{*}(c, Z(\varphi))=\bigcup_{\zeta \in Z(\varphi)}\left\{c^{\prime}: c^{\prime} \in \mathscr{D}(c, \zeta) \& \operatorname{dom}\left(F_{\zeta}\left(c^{\prime}\right)\right)=\mathscr{U}\left(c^{\prime}\right)\right\} \tag{9}
\end{equation*}
$$

That is, $\mathscr{C}^{*}$ gives us, for any $\zeta \in Z(\varphi)$, the set of contexts $c^{\prime}$ which are at least as determinate as $c$ with respect to $\zeta$, and which eliminate any truth value gap associated with $\zeta$ in $c$.
Whenever $\varphi$ is a formula of $L$ and $\mathfrak{A}$ is a partial interpretation for $L$, then the supervaluation of $\varphi$ at an index $\langle c, a\rangle,\lfloor\varphi\rfloor_{c a}^{s}$, is defined in the following manner:
(95)(a) $\llbracket \varphi \rrbracket_{c a}^{S_{\alpha}^{*}}=1$ if $\llbracket \varphi \rrbracket_{c^{\prime} a}^{\mathrm{T}}=1$ for all $c^{\prime} \in \mathscr{S}^{*}(c, Z(\varphi)$ ),

(c) $\left\lfloor\varphi \rrbracket_{c a}^{S *}\right.$ is undefined otherwise.

Again, this will allow us to retain a classical notion of logical truth despite the presence of vague predicates in L. However, there is a further interesting consequence. Under a partial interpretation, sentences of the form (96) will always have a definite truth value, regardless of whether $\zeta$ is linear or not: ${ }^{35}$

$$
\begin{equation*}
\wedge \mathcal{N}\left[\mathcal { N } \left\{\{ ^ { n } \zeta \} ( \text { Mona } ) \rightarrow \mathcal { N } \left\{\left\{^{n} \zeta\right\}\right.\right.\right. \text { (Jude)] } \tag{96}
\end{equation*}
$$

Let's consider one particular state of affairs under which this sentence will be true.
(97) For every $X \in \mathbf{G}\left(c, F_{f}\right), F_{f}(c[X])($ Mona $)=F_{f}(c[X])($ Jude $)$.

That is to say, there is no relevant comparison class in which Mona and Jude are distinct with respect to the property expressed by $\zeta$. Suppose $\zeta$ is a linear predicate, say heavy. Then we can conclude from (97) that Mona and Jude are equivalent with respect to the property of being heavy, and indeed that Jude is exactly as heavy as Mona. Suppose, on the other hand that $\zeta$ is a nonlinear predicate such as clever. Then, as I
have already pointed out, we cannot reliably take (97) as evidence that (98) is true.

Jude is as exactly as clever as Mona.
For these two people may simply be incommensurable with respect to the criteria for clever which are specified by $c$. In order to be sure that (98) is true, it is necessary to ascertain whether (97) holds for all contexts $c^{\prime}$ which are at least as determinate as $c$ with respect to clever; for these may introduce further criteria which discriminate between Jude and Mona. It seems, therefore, that we may wrongly conclude that (99) is true at $c$ in the partial truth definition because of negative information in $c$; i.e. because clever is too indeterminate at $c$.

$$
\begin{equation*}
\wedge \mathcal{N}\left[\mathcal{N}\left\{{ }^{\cap} \text { clever }\right\}(\text { Mona }) \rightarrow \mathcal{N}\left\{{ }^{n} \text { clever }\right\}(\text { Jude })\right] \tag{99}
\end{equation*}
$$

But what happens when we consider the supervaluation of this formula? $\mathscr{S}^{*}$ will introduce exactly those more determinate contexts which allow us to arrive at the correct truth conditions. (99) will be $\mathscr{S}^{*}$-true at $c$ if it is true under a partial interpretation $\mathfrak{A}$ at every $c^{\prime} \in \mathscr{P}^{*}$ ( $c$, \{clever\}). It will be $\mathscr{S}^{*}$-false at $c$ if it is false - or equivalently (100) is true-under $\mathfrak{A}$ at every $c^{\prime} \in \mathscr{S}^{*}(c,\{$ clever $\}$ ).

$$
\begin{equation*}
\vee \mathcal{N}\left[\mathcal{N}\left\{{ }^{n} \text { clever }\right\}(\text { Mona }) \wedge \neg \mathcal{N}\left\{{ }^{\cap} \text { clever }\right\}(\text { Jude })\right] \tag{100}
\end{equation*}
$$

And it will be $\mathscr{S}^{*}$-undefined at $c$ if (99) is true at some $c^{\prime}$ introduced by $\mathscr{P}^{*}$, while (100) is true at other $c^{\prime}$. That is, it will be $\mathscr{S}^{*}$-undefined at $c$ just in case the nondistinctness of Jude and Mona with respect to the property of being clever at $c$ conceals the conflicting judgements that would be made if the criteria were more selective or more effectively weighted.

We have seen that supervaluations play their usual role with sentences containing positive adjectives: the partial interpretation is 'filled out' in a consistent manner so as to restore classical logic. But as I have just shown, they also play a further role of introducing truth value gaps in the interpretation of comparative sentences containing nonlinear adjectives. It is important to note that, by contrast, the interpretation of comparatives containing linear adjectives will be completely unaffected by the supervaluation procedure. For if $\zeta$ is linear, then every context is fully determinate with respect to $\zeta$.

Finally, consider (101) and its translation, (102).
(101) Either Jude is as clever as Mona, or Jude isn't as clever as Mona.

$$
\begin{aligned}
& \wedge \mathcal{N}[\mathcal{N}\{\cap \text { clever }\}(\text { Mona }) \rightarrow \mathcal{N}\{n \text { clever }\}(\text { Jude })] \vee \\
& \neg \wedge \mathcal{N}\left[\mathcal{N}\{\cap \text { clever }\}(\text { Mona }) \rightarrow \mathcal{N}\left\{{ }^{\cap} \text { clever }\right\}(\text { Jude })\right]
\end{aligned}
$$

(102) is equivalent to the disjunction of (99) and (100), and we have seen that both of these may be $\mathscr{S}^{*}$-undefined. Yet if they are $\mathscr{S}^{*}$-undefined, it also follows that one or other is definitely true at every new $c^{\prime}$ introduced by $\mathscr{P}^{*}$; hence their disjunction, and equally (102), must be $\mathscr{S}^{*}$-true. Thus supervaluation can be used to restore classical logic at this 'higher level' too.

## NOTES

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Of the many people who have contributed to the ideas expressed here, Hans Kamp has been undoubtedly the most important. Over the last few years, I have profited enormously from many conversations with him on the topic of comparatives. I am also particularly grateful for the comments and criticisms I have received from Gerald Gazdar, Frank Heny, Ruth Kempson, Christopher Longuet-Higgins, Barry Richards, Ivan Sag, and an anonymous referee of this journal. Last of all, I would like to thank Jane Blackett for cheerfully undertaking to type the penultimate draft.
${ }^{1}$ This formulation should also be taken to cover constructions of the form [ap more A than X].
${ }^{2}$ In a survey of 123 languages, Ultan (1972) found that the comparative was either unmarked, or else marked in relation to the positive, but never vice versa.
${ }^{3}$ I have slightly modified Cresswell's notation to conform to usage elsewhere in this paper. Thus where Cresswell writes $\langle\alpha, \beta\rangle$ for the application of a one-place functor $\alpha$ to an argument $\beta$, I write $\alpha(\beta)$. I am also assuming that expressions of English will be translated into expressions of intensional logic before being model-theoretically interpreted. Only one aspect of this translation procedure is relevant for the moment: lexical items will be translated as constants of intensional logic, and these constants will be represented as boldface versions of the corresponding English expressions.
${ }^{4}$ Cresswell's equivalence relation in fact holds between pairs consisting of individuals and possible worlds, but this extra detail does not affect the basic idea.
${ }^{3}$ Intuitively, $u$ and $u$ ' should belong to the same equivalence class under $\approx_{\text {and }}$ iff $u$ is exactly as tall as $u^{\prime}$, and this is presumably what Cresswell intends. However, there is a problem in his formulation. Cresswell allows that for some adjectives $\zeta$, the relation 'more $\zeta$ than' will be a partial ordering. That is, there may be distinct individuals $u$ and $u$ ' such that neither $\left(u, u^{\prime}\right)$ nor $\left(u^{\prime}, u\right.$ ) belongs to the ordering. In such a case we would want to say that $u$ and $u^{\prime}$ are incommensurable with respect to $\zeta$. (I discuss this issue at some length in following sections.) But since (6) sets the pattern for constructing equivalence relations $\approx$ 。 for arbitrary adjectives $\zeta$, it may well turn out that $u \approx \zeta u^{\prime}$ even though $u$ and $u^{\prime}$ are incommensurable with respect to $\zeta$. Hence, we would not be justified in claiming (i):
(i) $\quad u$ and $u$ ' belong to the relation 'exactly as $\zeta$ as' iff $u \approx_{\boldsymbol{c}} u$ '.

Cresswell seems to have overlooked this difficulty, since (i) is just what he does claim; see Cresswell (1976: 273-274).
${ }^{6}$ The useful terminology of graduality vs. indeterminacy was suggested by Östen Dahl during a lecture at University College, London, in Autumn 1978.
${ }^{7}$ A linear ordering is an asymmetric, transitive and connected relation. Both linear and nonlinear adjectives induce asymmetric and transitive relations in comparatives, but only the former induce connected relations.
${ }^{8}$ This closely parallels the kind of vagueness which occurs in the meaning of common nouns and which has been discussed by several authors; see, for instance, Labov (1973), Lakoff (1972) and Putman (1975). It is perhaps worth remarking that the set of possible criteria which determine the extension of nonlinear adjectives will be open-ended. The meaning of such expressions depends heavily on use.
${ }^{9}$ It is sometimes claimed that there can be no clear dividing line between, on the one hand, individuals of whom a predicate is definitely true or false, and on the other, individuals who are borderline cases; see, for instance, Haack (1978: 165) and Putnam (1975: 217). The objection is quite plausible. For we may, with some confidence, assign Mary, who is six foot three, to the positive extension of tall, and Bill, who is five foot seven, to the extension gap. But exactly how much taller would Bill have to be in order to be definitely tall? There seems to be no nonarbitrary answer.

Nevertheless, this does not constitute conclusive evidence for postulating a second order vagueness affecting the boundaries between the boundary area and the positive and negative extensions. Suppose $c$ is a context in which the dividing line between the positive extension and extension gap of tall lies at five foot eleven. If you choose to assert in $c$ that John, who is five foot ten, is tall, then I will have no motivated basis for rejecting your claim. Rather than taking your assertion to be without truth value, I will try to accommodate your presuppositions by simply moving to a new context $c^{\prime}$ which is just like $c$ except that the boundary separating tall from sort of tall has shifted downwards a little. The fact that there is no particular reason for defending one boundary rather than another does not mean that there is no boundary. (This is roughly the kind of argument adopted by Kamp (1978). There is a general discussion of the phenomenon of 'accommodation' in Lewis (1979).) ${ }^{10}$ The basic idea is that of a supervaluation (see, for example, van Fraassen (1966). The insight that vagueness can be analysed in terms of supervaluations is also found in the work of Dummett (1975) and Fine (1975). The use of delineations by Lewis (1972) for handling vagueness, though superficially rather different, has some underlying resemblances to the supervaluation approach. Although Lewis does not use the notions of positive and negative extension, they will arise by default. For in a given context, certain atomic sentences containing vague predicates will be judged to be definitely true or definitely false, i.e. true under all delineations or true under none. This can only be explained in terms of the relevant vague predicates possessing positive and negative extensions.
" In fact, Kamp's analysis is somewhat more complex. He introduces a relation 'at least as vague as' between valuations. A classical valuation is then the limit of a sequence of valuations under this relation. His paper should be consulted for further details.
${ }^{12}$ The valuations for these connectives and other logical constants are the same as those stated in Kamp (1975: 135). In fact, they follow Kleene's (1952) truth tables for the strong connectives.
${ }^{13}$ Again, for more details, see Kamp (1975). The basic idea is stated in the appendix to Lewis (1972), where it is attributed to David Kaplan.
${ }^{14}$ I am grateful to Frank Heny for suggesting this example to me.
${ }^{15}$ The earliest published reference to this term that I have been able to find is in Hare (1952). (I am grateful to Aaron Sloman for bringing this work to my attention.) More recently, comparison classes have been discussed by Wheeler (1972) and Siegel (1977). It was the latter paper that first set me thinking along present lines.
${ }^{16}$ This point was made by an anonymous referee.
${ }^{17}$ I am indebted to Ivan Sag for this argument.
${ }^{18}$ This crucial idea is due to Morgan and Pelletier (1977: 92).
${ }^{19}$ Although there is a passage in his article where Lewis explicitly relates this phenomenon to the interpretation of comparatives, I have not cited it since Lewis's slightly different approach to the latter might obscure the basic point.
${ }^{20}$ Unbound variables in all the following definitions are to be taken as universally quantified.
${ }^{21}$ I am assuming throughout this discussion that $\mathscr{U}(c)$ is included in the sortal range of the predicates in question.
${ }_{22}$ As things stand at present, (i) will be undefined at $c$ if Sean is outside the positive extension of tall at $c$, though intuitively it should be true:
(i) Sean isn't very tall.

While it would be possible to adjust the semantics to get the right result here. I would prefer to invoke again one of Lewis's (1969) 'rules of accommodation'. Suppose that $c[X]$ is the context introduced by very at $c$. Then the utterance of (i) in $c$ induces a new context $c^{\prime}$ which is just like $c$ except that negom $\left(c^{\prime}[X]\right)=\operatorname{neg}_{\tan }(c[X]) \cup(Q u(c)-X)$. If Sean is in ( $थ(c)-X$ ), (i) will be true in $c^{\prime}$.
${ }^{23}$ More precisely, Kaplan takes a character to be a function from contexts to 'contents'. In an intensional framework, a content would be identified with a sense; e.g. the character of tall would be the function $\lambda c \lambda w[\text { tall }]_{\text {cwa }}^{n}$.

I should also point out that Kaplan denies that there are any expressions in English which operate on characters. This is clearly incompatible with the position adopted here. Independent reasons for accepting character operators are proposed in Klein (1978).
${ }^{24}$ This is not quite accurate, since we cannot always guarantee that expressions of type ( $k, \tau$ ) will denote total functions on $C$. For example, if $\varphi$ contains a vague predicate, then ${ }^{n} \varphi$ will denote a partial function from $C$ to $\{0,1\}$.

In an intensional framework, one might set $D_{\langle k, t\rangle}=\left(D_{\tau}^{\mu}\right)^{c}$.
${ }^{25}$ By basing the semantics of measure terms on a standard, it is possible to account for the fact that they are rigid designators. That is, at any world $w$, one foot denotes the set of objects in $w$ which are exactly as long as $f$ is in the actual world; cf. Kripke (1972: 275). ${ }^{26}$ This problem is not special to measure phrases. For a general discussion, see Wilson (1975: 148-153).
${ }^{27}$ Of course, this is an oversimplification, since numerical expressions are typically used as approximations; cf. Sadock (1977). However, I think that is a separate issue.
${ }^{28}$ The idea of formulating rules in this way seems to have originated with Bach (1978). A Montague grammar version is discussed in the Appendix of Klein (1978).
${ }^{29}$ In some languages, $\alpha / \alpha$ may be realized as a resumptive pronoun.
${ }^{30}$ The use of a type schema for variables of this sort is proposed by McCloskey (1979). In a purely intensional fragment, one would have ${ }^{\mathrm{V}} \mathrm{v}_{0}^{(s, t e)}$ instead.
${ }^{31}$ Bresnan uses the category QP rather than Deg.
${ }^{32}$ I am assuming that a proper name like Jude will translate into $\lambda P P$ (Jude), where $P$ is a variable of type $\langle e, t\rangle$.
${ }^{33}$ I am ignoring the issue of indeterminacy here; it is taken up in the final section.
${ }^{34}$ The approach adopted here might be criticized for failing to distinguish between (82), which is reasonably intelligible, and (i), which is not:
(i) Sean is more curt than Leo is heavy.

Nevertheless, I think it is possible in the present framework to characterize the difference between the two sorts of sentence. Consider the general case of relations of the form (ii):
(ii) $\ldots$ is more $\zeta$ than $\qquad$ is $\eta$

In order for a sentence $N P_{1}$ is more $\zeta$ than $\mathrm{NP}_{2}$ is $\eta$ to be informative, it must be possible to construe (ii) as an ordering of the appropriate domain. But this will be so only for a small set of predicates $\zeta$ and $\eta$. Roughly speaking, $\zeta$ and $\eta$ must be such that we can construct an order-preserving mapping between the two sets of equivalence classes constructed from the respective relations $\approx_{c, \xi}$ and $\approx_{c, n}$. Unfortunately, I do not have space to enter into this interesting problem at greater length here.
${ }^{35}$ This claim has to be qualified slightly: it only holds good for those contexts $c$ and predicates $\zeta$ which satisfy ( $\mathbf{i}$ ):
(i) $\quad\left(\exists u, u^{\prime} \in \mathscr{U}(c)\right)\left[\left(u, u^{\prime}\right) \notin \approx_{c, 5}\right]$

Let us call such contexts admissible (with respect to $\zeta$ ). Then it seems that we must only consider truth in admissible contexts. However, this restriction is justifiable, I think, since inadmissible contexts will not contribute in any conceptually meaningful way to our understanding of vague predicates.

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