Semantic composition and linear order

The toy semantics we built in class this week contains lexical items that denote individuals (e.g., names like *Kim*), sets of individuals (e.g., intransitive verbs like *smoke* and *run*), 1-place set-theoretic operations (e.g., negation = set complementation) and 2-place set-theoretic operations (e.g., conjunction = set intersection). It also contains the composition rules in (1).

(1)  
   a. **PREDICATION**  
      If α is a constituent whose immediate subconstituents are β and γ, and if [β] is an individual and [γ] a set of individuals, then [α] = 1 if [β] is a member of [γ], and 0 otherwise.
   
   b. **1-PLACE APPLICATION**  
      If α is a constituent whose immediate subconstituents are β and γ, and if [β] is a 1-place set-theoretic operation and [γ] is a set of individuals, then [α] = [β](γ). ([α] is the result of applying [β] to [γ].)
   
   c. **2-PLACE APPLICATION**  
      If α is a constituent whose immediate subconstituents are β, γ, and δ and if [β] is a 2-place set-theoretic operation and [γ] and [δ] are sets of individuals, then [α] = [β](γ, δ). ([α] is the result of applying [β] to the pair (γ, δ).)

As we discussed in class, the toy semantics we devised has a couple of interesting properties. The following properties are particularly relevant:

**UNIQUENESS**
No input receives more than one interpretation. Our lexical entries provide a unique denotation for each word, and the composition rules are all written in such a way that if they apply to a given constituent at all, they will yield a unique extension for that constituent.

**INSENSITIVITY TO LINEAR ORDER**
The composition rules make reference to “the immediate constituents of” the phrase to be interpreted (i.e., its daughters), but they do not care about the linear ordering of these constituents. That is, when we say “α is a constituent whose immediate subconstituents are β and γ”, we are talking about constituents of either the form (2a) or (2b).

(2)  
   a.  
      \[ \alpha \quad \beta \quad \gamma \]
   
   b.  
      \[ \alpha \quad \gamma \quad \beta \]

This means that any two syntactic representations that differ only in permutations of linear order in (one or more of) their subtrees, but not in hierarchical relations, are semantically equivalent (have denotations that lead to the same truth conditions).
A. Given the insensitivity to linear order of the composition rules, it is not clear that our system is actually coherent. Why not? To answer this question, consider the hypothetical connective \( tw\) without, whose extension is the operation of set subtraction, which is defined in (3):

\[
(3) \quad \text{For any two sets } X \text{ and } Y: X - Y = \{x \mid x \in X \text{ and } x \notin Y\}
\]

Assume that structures with \( tw\) without are syntactically identical to those with \( and\), i.e., they are ternary branching, as shown in (4) for ‘Kim runs \( tw\) without smoking’. (For now, let’s continue to ignore verbal morphology.) What is the crucial property of \( tw\) without that makes it a problem for order-insensitive composition rules?

B. One way to fix this problem would be to introduce order-sensitive composition rules. Formulate an appropriate rule to handle the hypothetical word \( tw\) without (and others like it).

C. Another way to fix the problem would be to revise the syntax and assume that all structures are binary branching. Spell out the necessary revisions to our composition rules and lexical entries that this solution would require. (Hint: You should be able to implement this solution by adding just one new composition rule and getting rid of one old one.)

D. How does the real English word \( without\) compare to the hypothetical \( tw\) without? Can the real \( without\) be treated adequately in a theory that has no order-sensitive rules? For the purposes of this assignment, you should just worry about examples in which \( without\) is followed by a verb or VP, and you can assume for simplicity that complex VPs denote sets. For example, \( like\ tea\) denotes the set of individuals that like tea. (Don’t worry about how the direct object is interpreted for now.)

In thinking about the last question, you should be aware of the fact that verbs like \( smoke\) and \( run\) exhibit an ambiguity between so-called ‘episodic’ and ‘habitual’ interpretations. For example, \( Kim\ smoked\) can mean either that Kim smoked on some occasion, or that Kim was a habitual smoker. For reasons that we won’t go into (though it’s an interesting topic!), this ambiguity disappears in the simple present tense (only the habitual interpretation remains). Be sure that you keep the interpretations of the verbs you use constant in any examples you compare, since comparing habitual uses with episodic uses may introduce additional factors that are orthogonal to the question under consideration here.