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Conditionals

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1. Grice

In his William James Lectures, Paul Grice defends an analysis of indicative conditionals in terms of material implication. The observation that sometimes, there seem to be non-truth functional reasons for accepting these conditionals is explained by invoking conversational implicatures adding to the meaning proper of this construction. Consider a sentence like (1):

(1) If my hen has laid eggs today, then the Cologne cathedral will collapse tomorrow morning [1]

On a Gricean account, (1) is interpreted as material implication. My utterance of it, however, implicates that there is some connection between my hen laying eggs and a possible collapsing of the Cologne Cathedral. That is, what I ultimately convey to my audience is some stricter form of implication. Very roughly, this implicature arises in the following way: Suppose I just happen to know that my hen laid eggs today as well as that the Cologne cathedral will collapse tomorrow, and these are my only grounds for uttering (1). Given a material analysis, what I said would be true. Although being true, my utterance would be misleading. I could have said:

(2) My hen has laid eggs today and the Cologne Cathedral will collapse tomorrow.

If I say (1) to you, you don't expect me to be in the position of uttering (2). Hence you will assume that I have other reasons for asserting (1). What could these other reasons be? It couldn't just be that I know that my hen hasn't laid any eggs today. Or just that I am convinced that the Cologne Cathedral will collapse tomorrow. If this were so, I should have uttered (2) or (3) respectively:

(2) My hen hasn't laid any eggs today.
(3) The Cologne Cathedral will collapse tomorrow.

If you believe that I won't lead you astray and that for all I know, I told you the truth, you conclude that I must believe that there is some non truth-functional connection between the antecedent and the consequent of (1).
A Gricean analysis of indicative conditionals is appealing in that - unlike so many other accounts replacing material implication - it is able to explain why it is that material implication has cropped up again and again in the history of logic and mathematics. The material implication interpretation would be <u>the</u> interpretation of conditionals as soon as we abstract away from certain principles regulating everyday conversation. It would be an interpretation that is accessible to all of us, not some arbitrary invention created for the purposes of an eccentric group of scientists. Learning logic would consist in dropping a few rules for cooperative interaction.

2. <u>Gibbard</u>

In <u>Two Recent Theories of Conditionals</u>, Allan Gibbard proves that <u>any</u> conditional operator --> satisfying (4) and the additional rather obvious principles (5) and (6) is in fact material implication:

(4) p --> (q --> r) and (p & q) --> r are logically equivalent.
(5) p --> q entails the corresponding material implication, that is p --> q is false in all worlds in which p is true and q is false.
(6) If q follows from p , then p --> q is a logical truth.

(4) is a principle that holds for "if...then"-clauses in English. (7) and (8) are logically equivalent:

(7) If you are back before eight, then if the roast is ready, we will have dinner together.

(8) If you are back before eight and the roast is ready, we will have dinner together.

Gibbards proof, then, seems to exclude any sort of stricter implication as a candidate for the interpretation of indicative conditionals in English. It gives further support to a material implication analysis in the spirit of Grice.

3. <u>The gradual decline of material implication</u>

The two preceding sections built a strong case in favor of a material implication interpretation of indicative conditionals. This glorious picture cannot withstand closer scrutiny, however. The recent history of semantics can be seen as a history of the gradual decline of the material conditional. There was a time, for example, when even sentences like

(9) All porches have screens

were formalized with the help of an "if...then"-clause interpreted in the material mode:

(10) For all x [If x is a porch, then x has screens]

These times are gone. Formalizations like (10) disappeared for reasons of generality. Realizing that

(11) Some porches have screens

couldn't be formalized as

(12) There is an x [If x is a porch, then x has screens]

was not yet deadly. But attempts to formalize sentences like

(13) Most porches have screens
(14) Many porches have screens
(15) Few porches have screens

made it very clear that material conditionals had no role to play in the formalization of sentences with quantifiers. Another two-place connective, conjunction, was successful in the case of (11), but then again was an absolute failure with (13), (14) and (15). Paying close attention to quantifiers like "most", "many" and "few" led to the theory of generalized quantifiers within 'interpretational' frameworks and to the theory of restrictive quantification in the 'representational' tradition [2)]. On the latter account, we have logical representations of the following sort:

(16) [Most x: x is a porch] x has screens

Representations like (16) are interpreted in such a way, that the clause "x is a porch" has the function of restricting the domain of the quantifier "most". (16) is true in a world w, if and only if most values for x that satisfy "x is a porch" in w also satisfy "x has screens" in w.

For our material conditionals, worse was yet to come. In Adverbs of Quantification, David Lewis examines sentences like:

(17) Sometimes, if a man buys a horse, he pays cash for it.
(18) Always, if a man buys a horse, he pays cash for it.
(19) Most of the time, if a man buys a horse, he pays cash for it.

There is some discussion in Lewis' article as to what the type of entity is that adverbs like "sometimes", "always" and "most of the time" quantify over. For ease of exposition, let us assume that these entities are events (this is not Lewis' view, but nothing hinges on that [3)]). If adverbs of quantification are sentence adverbs and if indicative conditionals are formalized in the traditional way, we are led to the following logical representations of (17) to (19):

(20) There is an event e [if e is an event that involves a man buying a horse, then e is an event that involves that this man pays cash for it]
(21) For all events e [If...(e)......,then....(e)....]
(22) For most events e [If...(e).....,then...(e)...]

If some such analysis is the right analysis for adverbs of quantification, then the very same arguments that showed that material conditionals cannot be part of a formalization of sentences with nominal quantifiers will now show that material conditionals cannot be part of a formalization of sentences with adverbial quantifiers. And this is indeed the conclusion David Lewis draws. To see why, take sentence (19). If its logical form were as in (22) (with the conditional being interpreted as material implication), we would predict, for example, that it would be true in a world with the following properties: There are altogether a million events. Out of these, 2000 events involve a man buying a horse, and all of these 2000 events involve payment by check. Since all the events that are not horse buyings by a man at all, satisfy the conditional in (22), (22) and hence (19) are predicted to be true in this world.
Following Lewis (with the modifications mentioned above [4)]), we might propose the following improved logical form for (22):

(23) [Most events e: e involves a man buying a horse] e involves that this man pays cash for the horse

(23) has the same form as (16) above. And it is interpreted in the same way. (23) is true in a world w if and only if most events of w that involve a man who is buying a horse are events that involve cash payment. What originated as an "if"- clause in surface structure ended up as a restrictive clause in the corresponding logical form. On this proposal, there is no such thing as a two place "if...then" operator in any logical representation corresponding to English sentences like (17) to (19). The function of the "if"-clause is to restrict the domain of quantification of the adverb. If this is true, then we have to concede that there are indicative conditionals that cannot possibly be given a Gricean analysis.
Consider next an example due to Grice himself: Grice's paradox [5)].

Yog and Zog play chess according to normal rules, but with the special conditions that Yog has white 9 out of 10 times and that there are no draws. Up to now, there have been a hundred games. When Yog had white, he won 80 out of 90. And when he had black, he lost 10 out of 10. Suppose Yog and Zog played one of the hundred games last night and we don't yet know what its outcome was. In such a situation we might utter (24) or (25):

(24) If Yog had white, there is a probability of 8/9 that he won.
(25) If Yog lost, there is a probability of 1/2 that he had black.

Both utterances would be true in the situation described above. Let us now examine what the logical form of sentences like (24) and (25) may be. If we stick to a traditional analysis of conditionals, two candidates that come to mind are (26) and (27):

(26) If A, then x-probably B
(27) x-probably [if A, then B]

In (26) "x-probably" stays in its surface position; in (27) it has been raised out of its clause. Assuming a material implication interpretation of the conditional, both formalizations are absurd. A sentence of the form (26) is true whenever A is false. But Yog's having black would not be sufficient for making (24) true. Likewise, if Yog did indeed win, this wouldn't mean that for that reason alone, (25) would be true.
Taking (27) to be the formalization of (24) and (25) would give us the following:

(28) 8/9- probably [if Yog had white, then Yog won]
(29) 1/2- probably [if Yog lost, Yog had black]

And here is Grice's paradox: Given that in chess, not having black is having white, and that if there are no draws, not losing is winning, the two conditional sentences in (28) and (29) are contrapositions of each other. But then (assuming e.g. a material implication interpretation), they are equivalent. How come two equivalent sentences can have different probabilities? I think we have to examine very carefully how we arrived at our assignments of probabilities in the first place.

Reasoning for establishing the truth of (24): There was a total of 100 games. This is our universe of discourse. For the case at hand, we only have to consider the games in which Yog had white. This is what the "if"-clause tells us to do. We are now left with 90 games. Out of these, Yog won 80. These 80 games are 8/9 of the 90 games. Hence (24) is true.

Reasoning for establishing the truth of (25):
Our universe of discourse is as above. This time, the "if"-clause of the conditional tells us to consider only those games which Yog lost. There are 20 such games. Out of these, Yog had black in 10 cases. These 10 games are half of the 20 games considered. Hence (25) is true.

(Don't say: "That's just how we calculate conditional probabilities. So what?". We have to explain why conditionals and probability operators interact in such an 'exceptional' way [6].)

What was the function of the "if"-clause in these two pieces of reasoning? The answer must sound familiar by now: The role of the "if"-clause was to restrict the domain contextually provided for the operator "x-probably". We expect then, that the logical form of (24) and (25) should resemble those of the quantified sentences we have considered before. For the purposes at hand, we might think of representations like (30) and (31):

(30) [8/9-probably: g is a game and Yog had white in g] Yog won g
(31) [1/2-probably: g is a game and Yog lost g] Yog had black in g

Sentence (30) is true in a world w if and only if the proportion of things (in a general sense including events) satisfying "Yog won g" in w among all the things satisfying "g is a game and Yog had white in g" in w is 8/9. Quite generally, a sentence of the form (30) or (31) will be true in a world w if and only if the proportion of events satisfying the matrix clause in w relative to all the events satisfying the restrictive clause in w is as required by the probability operator. I am not saying that this is the correct interpretation of probability sentences. In previous work (Kratzer (1981)), I proposed a different account for (comparative) probability sentences not involving variables. In the present context, I don't want to choose between the two proposals. I rather want to stress what they have in common: On both accounts, we don't have a two-place conditional operator, and the only role of the "if"-clause is to restrict the universe of discourse necessary for the interpretation of some sort of 'quantifier'. Grice's paradox, then, is not only a counterexample to the material implication interpretation of conditionals. It is also an example that once more points to the role "if"-clauses seem to play quite generally: They are devices for restricting the domain of 'quantifiers'. As soon as this semantic function is reflected in the logical forms for (24) and (25), the paradox disappears. Formalizations of these two sentences in terms of restrictive quantification will not contain logically equivalent constituents anymore. In fact, they will not have "if....then"-clauses as constituents at all.

4. Indicative conditionals as modalized conditionals

The picture that emerged in the previous section was that "if"-clauses might quite generally serve as restrictive devices for certain operators, for adverbs of quantification or probability operators, for example. But what about cases where there is no operator the "if"-clause could restrict? What about 'bare' conditionals, the type of sentences we started out with? I argued in my dissertation (Kratzer 1978) and related work (Kratzer (1979) and (1981)) that we should consider simple indicative conditionals as implicitly modalized. On a slightly simplified view, we might think of modal operators as quantifiers over possible worlds. This set of possible worlds will then be the domain that an "if"-clause can restrict. On this account, the logical form of sentence (1) above might look as follows:

(32) [must: my hen has laid eggs] the Cologne Cathedral will collapse tomorrow

(32) is true in a world w if and only if the Cologne Cathedral will collapse tomorrow in all those worlds w' that are accessible from w and in which my hen has laid eggs. Which worlds are accessible

depends on the modality expressed by the modal "must" [7]. In different utterance situations, an utterance of "must" might express different sorts of relative necessities: Necessity in view of the evidence available in the utterance situation (epistemic necessity), necessity in view of what the law provides (deontic necessity), necessity in view of what we desire (bouletic necessity), and so on. The hidden modal in the logical representation of sentences like (1) (and of many other indicative conditionals without overt operators) seems to favor an epistemic interpretation [8]. If epistemic interpretations of modals are relativized to the evidence available in the utterance situation, different utterances of one and the same sentence involving such a modal might express different propositions (just like different utterances of sentences involving indexicals like "I" and "you" might express different propositions). Let us look at an example:

Suppose a man is approaching both of us. You are standing over there . I am further away. I can only see the bare outlines of this man. In view of my evidence, the person approaching may be Fred. You know better. In view of your evidence, it cannot possibly be Fred, it must be Martin. If this is so, my utterance of (33) and your utterance of (34) will both be true.

(33) The person approaching might be Fred.
(34) The person approaching cannot be Fred.

Had I uttered (34) and you (33), both our utterances would have been false. The modals in the two sentences , then, behave like true indexicals. Certain bare indicative conditionals show strikingly similar properties as shown by a famous example invented by Allan Gibbard (op. cit., p. 231):

"Sly Pete and Mr. Stone are playing poker on a Mississippi riverboat. It is now up to Pete to call or fold. My henchman Zack sees Stone's hand , which is quite good, and signals its content to Pete. My henchman Jack sees both hands, and sees that Pete's hand is rather low, so that Stone's is the winning hand. At this point, the room is cleared. A few minutes later, Zack slips me a note which says "If Pete called, he won," and Jack slips me a note which says "If Pete called, he lost." "

Zack's and Jack's utterances are both true. But again: If Zack had uttered "If Pete called, he lost" (given <u>his</u> evidence) and if Jack had uttered "If Pete called, he won" (given the situation <u>he</u> was in) the two men's utterances would have been both false. Like the epistemic modals above, these particular indicative conditionals behave like indexicals: They are interpreted with respect to the evidence available to their utterers . But this means that they are implicitly modalized .

5. <u>Gibbard's proof reconsidered</u>

I have now argued that certain bare indicative conditionals are implicitly modalized. But doesn't Gibbard's proof mentioned in section 2 show that that couldn't be the case in any interesting sense? This proof seems to demonstrate that conditional sentences endowed with obvious properties could not receive any other interpretation but material implication. Note however, that one property slipped in that by now is not obvious any longer: Gibbard's arrow is a two-place operator. We have seen above that in the logical forms for natural languages, conditional sentences have a very different structure: there simply is no two-place conditional connective, and conditional sentences as a whole don't even form constituents. Now what about stacked "if"-clauses? If one "if"-clause alone appears in the restrictive term of the quantifier, then two "if"-clauses in a row might very well both appear there. That is they may successively restrict the domain of one and the same quantifier like two adjectives or two relative clauses might successively restrict the extension of one and the same noun. That is a sentence like (7) above (here repeated as (35)) would have the logical form (36):

(35) If you are back before eight, then if the roast is ready, we will have dinner together.

(36) [Must: you are back before eight and the roast is ready] we will have dinner together.

(36) is obviously identical with the logical form of sentence (8) (here repeated as (37)):

(37) If you are back before eight and the roast is ready, we will have dinner together.

6. Grice reconsidered

On the present account, the reason that sometimes, there are nontruth-conditional grounds for accepting a bare indicative conditional sentence is its implicit modalization. In connection with Gibbard's riverboat example, we have seen that this implicit modalization might be an epistemic one that depends on the evidence available in the utterance situation. Suppose now that mathematicians and logicians behave like omniscient gods. For them, then, the modal appearing in a conditional is relativized to a total state of information [9]. This state of omniscience comprises everything that is the case in the world under consideration. Their conditionals will now reduce to material implication. As on Grice's account, material implication will come out as a special case. The difference is that mathematicians and logicians are not uncooperative anymore, they are just megalomanic. Sometimes, however, they go for the other extreme: The state of total ignorance. This will give them strict implication. Most of us prefer something in between [10].

7. Conclusion

The history of the conditional is the story of a syntactic mistake. There is no two-place "if...then" connective in the logical forms for natural languages. "If"-clauses are devices for restricting the domains of various operators. Whenever there is no explicit operator, we have to posit one. As shown above, epistemic modals are candidates for such hidden operators. Work by Farkas and Sugioka (1983) and by Wilkinson (1986) suggests that invisible generic operators may play a similar role.
Relying on Lewis' work on adverbs of quantification and on my work on conditional modality, Irene Heim argues in her dissertation (1982) that the operators that have to be posited in order to obtain a unified semantics for "if"-clauses might have an

additional use: They might act as unselective binders for variables. This assumption enabled Heim to develop a new solution for an old puzzle regarding the so-called "donkey-sentences". Heim's work, then, gives further support to the hidden operator approach to bare indicative conditionals.

"If"-clauses are not the only clauses that function as restrictive devices for operators. Stump (1981, 1985) showed that free adjuncts can play the same role. And Partee (1984) makes a similar point with respect to "when"-clauses. Obviously, all of these results impose certain requirements on the syntactic properties of that level of representation where semantic interpretation takes place. This level has to allow for the creation of restricted operator structures of various kinds. How precisely these structures are related to the corresponding surface structures of natural language sentences is largely unknown[11)].

Footnotes

1 This sentence is not quite Frege's sentence in Frege (1923).

2 See for example Barwise and Cooper (1981), Cushing (1976), McCawley (1981),

3 Arguments in Partee (1984) and in Baeuerle and Egli (1985) suggest that some such assumption might be more than just a convenient simplification.

4 For Lewis, adverbs of quantification are unselective binders. Unselective binders are operators that bind every free variable in their scope. A Lewis type logical form for (23) would look as follows:

(23') Mostly, x is a man and y is a horse and x buys y,
x pays cash for y

Syntactically, (23') is a tripartite construction, consisting of an unselective quantifier, a restrictive clause and a matrix clause. (23') is true in a world w, iff every pair of individuals that satisfies the restrictive clause in w also satisfies the matrix clause in w.

5 William James Lectures, lecture IV.

6 Lewis (1976) shows that there is no general way of interpreting a two-place conditional connective so that the probabilities of the resulting conditionals will equal the corresponding conditional probabilities.

7 See Kratzer (1977).

8 This interpretation is the backbone of data semantics, see Veltman (1984) and (1985) and Landman (1986). Data semantics captures one special use of conditional sentences. It emphasizes the important connection between epistemic modality and bare indicative conditionals.

9 See Peirce (1933).

10 Material implication and strict implication, then, can be seen as involving extreme cases of epistemic modality. In Kratzer (1981), I show that these two types of implication fall out as special cases from a more general account of conditional modality that provides a unified analysis for a wide range of conditionals including deontic conditionals and counterfactuals. Giving a common analysis to e.g. indicative conditionals and counterfactuals is not trivial: Such an analysis has to predict that the two types of constructions differ with respect to their inference properties, and also that counterfactuals are vague to an extent that by far exceeds the degree of vagueness found with indicative conditionals (see Lewis (1973)).

11 Unfortunately, I haven't seen Geis (1985) that I expect to contain relevant proposals, Geis p.c..

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