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The Notional Category of Modality

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"It would be considered naive today to attempt, as did Wegener (1885), to describe the semiotic stratification of human language with examples restricted to German, Greek and Latin. But it is remarkable how well Wegener's theory stands up now that the range of our evidence has been vastly broadened. It takes only a slightly more flexible calculus, I believe, to accommodate all the varieties of semiotic structure evident in ordinary discourse."

Uriel Weinreich

Introduction

In this study, I want to explore the notional category of modality as reflected in certain expressions of German. I chose German since this is the language I know best. There is a number of very detailed investigations of the German modal system. I profited from all of them.

In dealing with the semantics of modals, the main danger one is facing is to get utterly lost in the variety of interpretations one and the same expression can receive in different utterance situations. As a result, one may be tempted to develop sophisticated classifications and to study the characteristics of major types like alethic, epistemic or deontic uses of a modal expression. I am not primarily interested in such classifications. My main concern is to answer three questions:

1. What is the logical nature of these interpretations?
2. What is their variety due to?
3. How is this variety restricted by the vocabulary of German itself?

These questions are very much in the spirit of Gunnar Bech. I think, however, that I am in a better position today than he was: In modal logic, a semantic framework has been developed, which is more suitable for describing semantic relations between modal expressions than the tools available thirty years ago.

Traditionally, investigations of modality have concentrated on expressions like necessarily, possibly, must, can, should or may. Little attention has been paid to the fact that natural languages have means of grading and comparing possibilities. Furthermore, conditionals are usually not considered in connection with modality. Yet, if-clauses very often serve to restrict modals in an explicit or implicit way.

In what follows, I am trying to present a unified analysis of modality, which incorporates these facts.

Many insights gained in separate examinations of some of these phenomena will then come out as special cases of a few very general principles.

1. Expressing Modality in German

Modality has to do with necessity and possibility. In German, as in other languages, there are many ways of expressing these notions. Here is a selection:

1.1 Inherent Modality

(1) Niemand läuft in zehn Minuten von Andechs nach Aufhausen. Nobody runs in ten minutes from Andechs to Aufhausen.

(2) Dieses Auto fährt zwanzig Meilen pro Stunde. This car goes twenty miles per hour.

(1) and (2) have a modalized reading:

(1') Nobody is able to run from Andechs to Aufhausen in ten minutes.

(2') This car can go twenty miles an hour.

Sentences (1') and (2') make explicit the modal element which seems to be inherent in the verb in the two original sentences.

1.2. Suffixes on Adjectives

There are two suffixes in German which often have a modal meaning: -lich and -bar.

Consider the following lists, parts of which I borrowed from Hermann Paul.
The English translations are very rough approximations. The exact meaning of most of these auxiliaries will be discussed in detail as we go along. I included wärde, as I was convinced by the arguments Heinz Vater gives in his article “Werden als Modalverb”.

1.4. Sentence Adverbs and Impersonal Constructions

 möglichweise possibly
notwendigerweise necessarily
wahrscheinlich probably

Phrases like:
es ist möglich daß it is possible that
es ist notwendig daß it is necessary that
es ist wahrscheinlich daß it is probable that

are used in a similar function.

1.5. Adjectival Phrases

imstande sein to be able
in der Lage sein to be in the position

What becomes obvious from this selection is that there is no syntactic category corresponding to the notional category of modality.

What then is modality?
The following sections are meant to shed some light on this question.

2. Basic Notions

Most of what I have to say in this section is found in more detail in my dissertation or in related articles listed in the bibliography. Anyone who is already familiar with my previous work on modals can skip whatever does not sound new to him or her.

In order to see what is involved in modality, let us look at the following example:

The Murder:
Much-Girl has been murdered on his way home. The police start investigations. Certain conclusions may be drawn from what is known about the circumstances of the crime. Utterances of the following sentences are likely to have occurred in such a situation:

2 Vater (30).
Logical Consequence:
A proposition $p$ follows from a set of propositions $A$ if, and only if, $p$ is true in all worlds of $W$ where all propositions of $A$ are true.

Consistency:
A set of propositions $A$ is consistent if, and only if, there is a world in $W$ where all propositions of $A$ are true.

Logical Compatibility:
A proposition $p$ is compatible with a set of propositions $A$ if, and only if, $A \cup \{p\}$ is a consistent set of propositions.

Conversational Backgrounds:
We know already that a conversational background is the kind of entity which might be referred to by the utterance of a phrase like what is known (we might ignore the in view of bit). What is known is different from one possible world to another. And what is known in a possible world is a set of propositions. In our semantics, a conversational background will therefore be construed as a function which assigns sets of propositions to possible worlds. In particular, the meaning of what is known will be that function from $W$ into the power set of the power set of $W$, which assigns to any world $w$ of $W$ the set of all those propositions which are known in $w$. This is an example of an epistemic conversational background. We will consider other kinds of conversational backgrounds later. First, I want to say something about modal relations.

The most familiar of these relations are simple necessity and possibility. Assume for the following that $f$ is an arbitrary conversational background, that is a function from possible worlds into sets of propositions.

Simple Necessity:
A proposition is a simple necessity in a world $w$ with respect to the conversational background $f$ if, and only if, it follows from $f(w)$.

Simple Possibility:
A proposition is a simple possibility in a world $w$ with respect to the conversational background $f$ if, and only if, it is compatible with $f(w)$.

The obvious thing to do now, is to link the meaning of the German modals corresponding to must, necessarily, it is necessary that, can, possibly or it is possible that to the notions I have defined above. We might want to say — for example — that a certain modal expresses simple necessity. I am going to spell out for one example what this would mean.
The Meaning of Notwendigerweise:
Consider an utterance of a sentence \( \alpha \) of the form \( \text{notwendigerweise} \ \beta \)
such that the proposition \( q \) is expressed by the utterance of the constituent sentence \( \beta \).

We have then:

(i) A proposition is expressed by the utterance of \( \alpha \) only if there is one, and

(ii) If a proposition \( p \) is expressed by the utterance of \( \alpha \), and if \( f \) is the conv-

versational background for this utterance, then \( p \) is that proposition

which is true in exactly those worlds \( w \) of \( W \), such that \( q \) is a simple

necessity in \( w \) with respect to \( f \).

Let us take this as a first approximation for a meaning rule for modals related to necessity.

One may wonder why there should be a unique conversational background for a modalized sentence to express a proposition. We’d better assume that in the case of several conversational backgrounds, there are several propositions expressed, one relative to each background. It would then be part of the vagueness of modal expressions that sometimes, it remains unclear which proposition was intended. These considerations lead directly to the work Manfred Pinkal has done about definite descriptions.

There is also a problem if the constituent sentence contains further modals, each requiring a conversational background of its own. To account for this, we would have to split up the utterance situation of \( \alpha \) further and consider those parts where each modal is uttered. I elaborated this in (19) and I don’t want to spend any more time on these kinds of refinements.

The analysis as it is, allows for one parameter to be fixed by the context of use. It implies that it is this parameter which is responsible for the variety of interpretations many modals can receive. In the murderer example, we had an epistemic conversational background. An epistemic conversational background leads to an epistemic interpretation of modal expressions. Other kinds of conversational backgrounds could lead to different interpretations. For further reference, I would like to draw attention to the following kinds of conversational backgrounds:

Realistic Conversational Backgrounds: In view of facts of such and such kind ...
A realistic conversational background is a function \( f \) which assigns sets of propositions to members of \( W \), such that for any \( w \in W: w \in \cap f(w) \).
That is, \( f \) assigns to every possible world a set of propositions which are true in it.

3 Strictly speaking, rules like this would have to apply on a level of logical form, where all modal operators are sentential operators.

4 See Pinkal (25) and (26).

Totally Realistic Conversational Backgrounds: In view of what is the case ...
A totally realistic conversational background is a function \( f \) which assigns sets of propositions to members of \( W \) such that for all \( w \in W: w \in \cap f(w) \).
That is, \( f \) assigns to any world a set of propositions which characterize it in a unique way. For each world, there are many ways of characterizing it uniquely. This is the source of the vagueness of counterfactuals as we’ll see in a later section.

Epistemic Conversational Backgrounds: In view of what is known ...
An epistemic conversational background is a function \( f \) which assigns sets of propositions to members of \( W \) such that for all \( w \in W: f(w) \) contains all those propositions which are established knowledge in \( w \) – for a group of people, a community etc.
Since only true propositions can be known, epistemic conversational backgrounds are special cases of realistic ones.
Of particular interest are:

Stereotypical Conversational Backgrounds: In view of the normal course of events ...
A stereotypical conversational background is a function \( f \) which assigns sets of propositions to members of \( W \) such that for any \( w \in W: f(w) \) contains all those propositions \( p \) such that it is the normal course of events in \( w \) that \( p \) – for someone, for a community etc.

Deontic Conversational Backgrounds: In view of what is commanded
A deontic conversational background is a function \( f \) which assigns sets of propositions to members of \( W \) such that for any \( w \in W: f(w) \) contains all those propositions \( p \) such that it is commanded in \( w \) that \( p \) – by someone, by the Law etc.

Teleological conversational backgrounds are related to aims and bulletic conversational backgrounds have to do with wishes. An extreme case is the empty conversational background:

The Empty Conversational Background:
The empty conversational background is that function which assigns to any \( w \in W \) the empty set.

We might think now that the ‘semantic field’ of modal expressions could be described along two axes: One specifying a modal relation and the other one specifying restrictions for admissible conversational backgrounds.

For example:

\( mub \)
Modal relation: Simple necessity
Conversational backgrounds: No restrictions
3. Grades of Possibility

I would like to take up the murderer example again. Instead of (4) or (5), the police inspector might have uttered one or several of the following sentences:

(7) Es kann gut sein, daß der Gauzner-Michl der Mörder war. It can well be that the Gauzner-Michl the murderer was.
There is a good possibility, that Gauzner-Michl was the murderer.

(8) Es besteht aber immer noch eine geringe Möglichkeit, There is however still a slight possibility
 daß der Kastenjakl der Mörder war. that the Kastenjakl the murderer was.
That is, however, still a slight possibility that Kastenjakl was the murderer.

(9) Der Gauzner-Michl kann eher der Mörder sein als der Kastenjakl. The Gauzner-Michl can rather the murderer be than the Kastenjakl.
Kastenjakl. Gauzner-Michl is more likely to be the murderer than Kastenjakl.

(10) Es ist wahrscheinlich, daß der Gauzner-Michl der Mörder war. It is probable that the Gauzner-Michl the murderer was.
It is probable that Gauzner-Michl was the murderer.

The police inspector does not know what the real world is like. But he can draw conclusions from the growing evidence available to him.
At any time, this evidence is compatible with a set of worlds which ‘could’ be the real world. These are the epistemically accessible worlds.

There is a straightforward connection between conversational backgrounds and accessibility relations as used in modal logic: If τ is a conversational background, then the set of worlds which are accessible in a world w with respect to τ is simply τ (w). That is, the set of worlds where all propositions of τ (w) are true.

There are certain worlds among the accessible worlds which are more far-fetched than others. A world where Kastenjakl is the murderer is more far-fetched than a world where Gauzner-Michl has killed Girgl. Gauzner-Michl couldn’t stand Girgl, but Kastenjakl got along very well with him. Even more far-fetched are worlds where someone from the other end of the world committed the crime. Far-fetched in respect to what? In respect to what is the case in the real world? This can’t be true, since it seems quite natural to say that something which was almost impossible, turned out to be the case. Actually, it is things like this which usually happen in detective stories. The most unlikely candidate is the murderer. What is far-fetched about someone from the other end of the world having killed Girgl, is that things like that do not correspond to the normal course of events. Normally, you don’t meet people from the antipodes in that village. And should someone show up who does not actually live in the neighbourhood, he wouldn’t just go and kill Girgl. Normally people need a motive for killing someone. It couldn’t have been for money since Girgl wasn’t robbed: all his money was found on him. In view of the normal course of events, it is far-fetched that someone from the other end of the world has killed Girgl. And in view of the normal course of events it is more far-fetched for Kastenjakl to be the murderer than for Gauzner-Michl.

Worlds in which the normal course of events is realized are a complete bore, there are no adventures or surprises. The concept of a normal course of events is analogous to the concept of ‘frame’ which plays an important role in psychology and artificial intelligence.

In our example, the epistemic conversational background determines for every world the set of worlds which are accessible from it. It forms the modal base.

There is a second conversational background involved in the above uses of modals, a stereotypical background. It induces an ordering on the set of accessible worlds, thereby functioning as ordering source.5

Quite generally, a set of propositions A can induce an ordering \( \leq_A \) on the set of all possible worlds in the following way: (The idea is taken from David Lewis’ work on ordering semantics, personal communication.)

The Ordering \( \leq_A \):

For all worlds \( w \) and \( z \in W \):

\( w \leq_A z \) if and only if \( \{ p : p \in A \text{ and } z \in p \} \subseteq \{ p : p \in A \text{ and } w \in p \} \)

The intuitive idea is this: A world \( w \) is at least as close to the ideal A as a world \( z \) if, and only if, all propositions of A which are true in \( z \), are true in \( w \) as well.

It can be shown that the relation \( \leq_A \) is reflexive and transitive.

We are now in the position to define some additional modal relations:

Human Necessity:

A proposition \( p \) is a human necessity in a world \( w \) with respect to a modal base \( f \) and an ordering source \( g \) if, and only if, the following condition is fulfilled:

5 The term is inspired by what Franziska Raynaud calls “source” in French.
Simple and Human Possibility:
A proposition is a simple possibility in a world \( w \) with respect to \( f \) if, and only if, it is a human possibility in \( w \) with respect to \( f \) and \( g \).

As a new start, we may try now to describe the semantic field of modal expressions along three axes specifying:
(i) a modal relation
(ii) conditions for the modal base
(iii) conditions for the ordering source.

In the following section I will begin with a discussion of the two major types of modal bases which are realized in German.

4. Two Basic Kinds of Modal Reasoning

We have seen that in modal reasoning, a conversational background may play the role of a modal base or an ordering source. The modal base determines the set of accessible worlds and the ordering source determines an ordering on it.

In this chapter, I want to investigate the two major types of modal bases which are relevant for German.

Some examples will be useful:

Root or Circumstantial Modal Bases:

(19) Sie wollte schreiben und konnte nicht, gewann aber, indem endlich die Herrschaft über ihre paralytischen Glieder.  
finally the control over her paralyzed limbs.  
Genovev was so terrified that she was unable to move.

(20) Der Jani-Hans schimpfte nie, fluchen konnte er gar nicht.  
The Jani-Hans scolded never, curse could be at all not.

Jani-Hans had such a mild character that he just wasn’t capable of getting angry.

(21) Hier können die Tomaten gedeihen.  
Here can the tomatoes prosper.

(22) Wer nichts hat, dem kann man auch nichts nehmen.  
Who nothing has, from whom can one also nothing take away.

Epistemic Modal Bases:

(23) Es kann nur einer gewesen sein, der sich im Haus  
It can only someone been have, sebo (refl.) in the house  
acknowledged been.

been at home has.
The Heimarth's have been burgled and Girrl tries to find out who might have been the thief. It must have been someone who was familiar with the house.

(24) Sie hatten den Befehl, den jungen König zu suchen, der sich
They had order the young king to look for, who (refl.)
in einer seiner Jagdhütten aufhalten mußte.
in one of his hunting huts stay must (past).

The young king has disappeared and in view of what is known, he must be hiding in one of his hunting huts.

Unlike the English must, the German muß has a past tense form mußte.

(25) Soweit wir wissen, muß es für sie nie etwas
As far as we know, must there for them never anything
anderes gegeben haben als Geborenwerden, Aufwachsen,
else been have but being born, growing up,
ermüdliche Arbeit und Sterben.
tired work and dying.

Oskar Maria Graf draws this conclusion from the historical sources about the life of the Heimarth family some centuries ago.

The term "epistemic modality" is familiar in linguistics and philosophy. The term "root modality" is usual in the tradition of generative grammar. "Circumstantial modality" is in the spirit of Terence Horgan (15).

There is a clear intuitive difference between the two kinds of occurrences of modals which I grouped under the two headings. It is a difference in the kind of premises from which we reason. If we use an epistemic modal, we are interested in what else may or must be the case in our world, given everything we know already. And if we use a circumstantial modal, we are interested in what can or must happen, given circumstances of a certain kind. Circumstances of a certain kind are facts of a certain kind. Facts concerning the outside world, our bodies or our mind, for example. Usually, circumstances permit or exclude that certain things happen. Only sometimes do they necessitate an event or an action: We have to die, to cough, to vomit, to laugh, to cry or to realize that we are lost.

Epistemic modality and circumstantial modality involve a different categorization of the facts. The problem is now to find out some more details about this partition.

I shall present a few observations towards this goal. Consider the following pair of sentences:

(26) (a) Aus dieser Kanne Milch kann die Katlb ein Pfund
From this can of milk can the Kathlb one pound of
Quark machen.
cottage cheese make.

(26) (b) Es kann sein, daß die Katlb aus dieser Kanne Milch
It may be that the Kathlb from this can of milk
ein Pfund Quark macht.
one pound cottage cheese makes.

(27) (a) In dieser Gegend können Zweietschenbäume wachsen.
In this area can plum trees grow.

(27) (b) Es kann sein, daß in dieser Gegend Zweietschenbäume wachsen.
It may be that in this area plum trees grow.

Sentences (26) (a) and (27) (a) have a circumstantial reading besides an epistemic one. For sentences (26) (b) and (27) (b), the epistemic reading is prominent.

Given a circumstantial reading for the (a)-sentences and an epistemic reading for the (b)-sentences, we can imagine situations where I say something true in uttering an (a)-sentence, but something false in uttering the corresponding (b)-sentence. Take the first two sentences: In view of quite general conditions concerning the production of cottage cheese, it is possible that Kathl is going to produce a pound of cottage cheese from the milk in the can. We know, however, that Kathl never uses the whole can of milk for the production of cheese. She uses a bit for her coffee, a bit for her porridge, a bit for the cat and the rest for her cheese. This means, that in view of everything we know, it is not possible that Kathl is going to produce a pound of cottage cheese from the milk in the can.

In using a circumstantial modal, we neglect certain kinds of facts. In our case, it is facts about what Kathl always actually does.

The situation is similar with the sentences (27) (a) and (27) (b). Suppose I am travelling in an exotic country and discover that soil and climate are very much like that in my own country, where plum trees prosper everywhere. In such a situation, an utterance of (27) (a) in its circumstantial sense would probably be true. But (27) (b) could very well be false, given that this country had no contacts whatsoever with western civilization and the vegetation is altogether different from ours. Since we know this, it is impossible in view of what we know that plum trees grow in this area.

Again, we have to neglect certain facts for (27) (a), although we might be aware of them.

The kind of facts we take into account for circumstantial modality are a rather slippery matter. This may give rise to misunderstandings and jokes. I once heard a philosopher say that one of the defining properties of a cup is, that you can pour things like coffee in it. A student objected to this in pointing out that if this were true — a cup which has coffee in it already, would not be a cup anymore.

When we talk to each other, we hardly ever make explicit in view of which circumstances something should be necessary or possible. We may give hints.

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* See Horgan (15), Kratzer (18) or Lewis (23) for a further illustration of this point.
Usually people understand. And they all understand in pretty much the same way.
Consider the following sentence:

(28) Ich kann nicht Posaune spielen.
I can not trombone play.

Depending on the situation in which I utter this sentence, I may say quite different things. I may mean that I don’t know how to play the trombone. I am sure that there is something in a person’s mind which becomes different when he or she starts learning how to play the trombone. A programme is filled in. And it is in view of this programme that it may be possible that I play the trombone.

Or suppose that I suffer from asthma. I can hardly breathe. In view of my physical condition I am not able to play the trombone, although I know how to do it. I may express this by uttering (28). Or else imagine that I am travelling by sea. The ship sinks and so does my trombone. I manage to get to a lonely island and sadly mumble (28). I could play the trombone in view of my head and my lungs, but the trombone is out of reach.

There are more conceivable interpretations for an utterance of (28), but most of them involve other conditions in addition to the facts. That is, most of them involve a non-empty ordering source. I’ll discuss such cases in the following chapters.

A distinction between circumstances concerning mainly the outside world, the body or the mind of a person, plays a role in the semantic development of können. According to Gustav Deggau, a student of Otto Behaghel’s, the Old High German equivalent of this modal was first used for intellectual capacities. Then, it could express possibilities in view of the outside situation. Only considerably later was it used for talking about physical abilities.

Ferenc Kiefer (17) has shown that similar distinctions are made in Hungarian. In Hungarian, the verbal suffix -bet expresses possibility. In its circumstantial reading, it can only be used for possibilities in view of the outside situation. In Kiefer’s own terms: “Modal sentences with -bet can only express outer dispositions”.

Taking up some of Kiefer’s further observations, I would like to present some analogous facts about modern German.

Consider a phrase like imstande sein (to be able).

I could say

(29) Ich bin nicht imstande, Posaune zu spielen.
I am not able trombone to play.

if I have asthma or weak nerves or if I am just too stupid. I doubt whether I would say it in a situation where I haven’t learnt how to play the trombone.

* Gustav Deggau (9).

And I could never say it on the island with my trombone lost at sea. The prominent circumstances for imstande sein are concerned with the strength of our body, character or intellect.

For kann, there is a further type of restrictions.
Consider:

(30) Dieses Messer kann nicht schneiden.
This knife can not cut.

(31) Dieser Hut kann den Kopf warmhalten.
This hat can the head keep warm.

This stove can not properly heat.

These sentences sound funny. They suggest that the knife, the hat or the stove are agents which take an active part in the cutting, the warming of the head or the heating. To avoid this effect, we would have to say:

(33) Dieses Messer schneidet nicht.
This knife cuts not.

(34) Dieser Hut hält den Kopf warm.
This hat keeps the head warm.

(35) Dieser Ofen heizt nicht richtig.
This stove heats not properly.

I think that sentences (30) to (32) have some features in common whose cooccurrence might be responsible for the fact that they sound bizarre.

One of these properties is concerned with agency: The knife is not an agent, but an instrument for cutting something. The hat is not an agent, but an instrument for warming the head. And the stove is not an agent, but an instrument for heating a room. After all, it’s you who cuts the bread, keeps the head warm and heats the house. Some machines, like music boxes, can do things all by themselves, thus functioning as true agents. I can’t find anything peculiar about (36):

(36) Diese Spieluhr kann “La Paloma” spielen.
This music box “La Paloma” play.

Here, the music box is an agent and the use of kann is appropriate.

Another feature is concerned with the kinds of actions which are said to be possible or impossible for a knife, hat or stove to be involved in. That a knife cuts, a hat keeps the head warm or a stove heats a room, is fairly well compatible with our stereotypical notions about knives, hats or stoves. Consider in contrast:

(37) Dieses Messer kann einen Felsen zerschneiden.
This knife can a rock cut into pieces.

(38) Dieser Hut kann epileptische Anfälle verhindern.
This hat can epileptic attacks prevent.
(39) Dieser Ofen kann wahlweise mit Kohle oder Öl beizen.
This stove can at choice with coal or oil heat.

Knives which cut rocks into pieces, huts which prevent epileptic attacks and stoves which work with coal or oil at choice come as a surprise. I think this is the reason why sentences (37) to (39) sound all right although the knife, the hat and the stove remain instruments for the actions under consideration. Further research has to be done in this area.

What these examples (as well as Kiefer's examples) show, however, is that it is still a simplification to describe the meaning of modal expressions by specifying nothing more but a modal relation and some restrictions for possible modal bases or ordering sources. Some constraints seem to involve agency or stereotypes associated with natural kind terms. I shall nevertheless stick to this simplification. I think it is still rewarding to examine the modal system of a language with respect to these three parameters, even if this is not the whole story.

In this chapter, I have examined the two major kinds of modal bases which are relevant for German (and all other languages I know): Circumstantial and epistemic modality are both based on realistic conversational backgrounds, but involve a different categorization of the facts.

The distinction is clearly marked in the vocabulary. Verbs with inherent modality, modal adjectives on -lich and -bar and phrases like imstande sein or in der Lage sein never express epistemic modality.

Sentence adverbs like wahrscheinlich or möglicherweise and auxiliaries like wird always express epistemic modality - if they express modality at all. Some of these expressions involve a grading. In the examples discussed in this chapter, I avoided grading as far as possible.

In the following sections, I will show how different modal bases interact with different kinds of ordering sources to yield the variety of the German modal system.

5. The Quest for Certainty

In section three, I gave an example of the grading of an epistemic modal base. As a result, we obtained some new modal relations which were linked to expressions like there is a good possibility that or it is probable that. In this section, I want to discuss some further issues concerning the grading of epistemic modal bases.

It has often been observed that I make stronger claim in uttering (40) than in uttering (41): 12

10 Ewalt Lang proposed an explanation along these lines (personal communication).
11 See Putnam (27).
12 See for example Karttunen (16) or Brunner and Redder (4).

(40) Das ist die Bürgermeister-Weiβ-Straße.
This is the Bürgermeister Weiss Street.

These utterances present a problem if we assume that muß gets a 'pure' epistemic interpretation. In this case, the proposition expressed by the utterance of (40) would follow from the proposition expressed by the utterance of (41) but not vice versa. Thus, uttering (41) should lead to a stronger claim than uttering (40). Since this is not the way things are, we have good reasons to assume that the utterance of muß in (41) does not express 'pure' epistemic necessity. In our framework, this means that the ordering source is not empty.

In uttering (41) instead of (40), I signalize that I don't reason from established facts alone. I use other sources of information which may be more or less reliable. Take for example the route description of a friend, a tourist guide or my own vague memories from years ago. These other sources of information may form ordering sources for epistemic modal bases.

A set of facts is always consistent. Other sources of information may themselves be inconsistent or else be inconsistent with the established facts. If these other sources function as ordering sources and are not part of the modal base, it can be explained why they can still be useful, even if there are inconsistencies. And why they never override the facts: In the case of a conflict, established facts have priority over route descriptions, tourist guides and memories. I shall give an illustration of the treatment of inconsistencies in section seven. So I needn't go into details here.

The next point I want to discuss, was brought up by John Lyons (24):

"In principle, two kinds of epistemic modality can be distinguished, objective and subjective. This is not a distinction that can be drawn sharply in the everyday use of language; and its epistemological justification, is, to say the least, uncertain . . . . It is nonetheless of some theoretical interest to draw the distinction between objective and subjective epistemic modality."

The distinction is manifest in the vocabulary of German. Imagine that Lenz, who often has bad luck, is going to leave the Old World by boat, today, on Friday thirteenth. On hearing about this, someone might utter one of the following sentences: 13

(42) Wahrscheinlich sinkt das Schiff.
Probably sinks the boat.

(43) Es ist wahrscheinlich, daβ das Schiff sinkt.
It is probable that the boat sinks.

(44) Das Schiff wird (bestimmt) sinken.
The boat will (certainly) sink.

13 The inspiration for these examples came from Gerald Gazdar.
Das Schiff dürfte sinken.
The boat is likely to sink.

It is probable that the boat will sink.

In German, the auxiliary *dürfte* has a temporal and a modal use. I intended the modal reading for (44). I couldn't find an appropriate gloss for *dürfte*, so I left a gap. In uttering (42) or (44), I make a more subjective claim than in uttering (43) or (45). I may be rather superstitious. I couldn't defend my claim on objective grounds. But I would have to do so if I uttered (43) or (45). There are established facts about the boat, the technical equipment nowadays or the weather. And there are commonly held conceptions about the normal course of events. In a world reigned by science and technology, these conceptions don't include superstitions. *Es ist wahrscheinlich, daß* and *dürfte* seem to require an 'objective' stereotypical background as their ordering source. *Wahrscheinlich* und *dürfte* prefer 'subjective' stereotypical backgrounds.

John Lyons believes that in its subjective reading, an epistemic modal doesn't contribute to the propositional content of an utterance at all. This is a very debated issue on the border of semantics and pragmatics. I don't want to go into it, as I won't be able to examine the different positions here.

In the following section, I want to discuss ways of grading circumstantial modal bases.

6. Approaching Ideals

In this section, I am going to examine how different ordering sources interact with a circumstantial modal base and how this is reflected in German.

Circumstantial conversational backgrounds are special kinds of realistic ones. They involve the sort of categorization of facts which we have discussed in section four. We can include the empty conversational background as a special case of a circumstantial one.

Circumstances create possibilities. The set of possible worlds which are compatible with them. These worlds, which are accessible in the circumstances under consideration, may be closer or further away from

The Law,
What my father provided in his last will,
What is good,
What you think is good,
Our plans,
Our aims,
Our hopes,
Our wishes.

To all of these ideals correspond conversational backgrounds. In the terms of possible worlds semantics, these would be functions "g from possible worlds into sets of propositions, such that for every world w, g(w) is the set of all those propositions p such that"

The Law provides that p in w,
My father provided that p in his last will,
p is good in w
In w, you think that p is good,
Our plans in w provide that p,
It is our aim in w that p,
We hope in w that p,
We wish in w that p,
It is in w our conception of a good life that p,
Ferdl recommends p to his wife in w,
In w, is rational that p.

All of these 'normative' conversational backgrounds could be proper ordering sources for a circumstantial modal base. Just as in section two, they would induce an ordering on the set of accessible worlds. From this, we get corresponding notions of human necessity, human possibility, slight possibility and comparative possibility.

Some modal expressions of German tolerate a wide range of ordering sources. Others have to obey more restrictions. Let us look at some examples:

Können and dürfen:

(46) Du kannst doch nicht nur Häuser bauen oder Semmeln backen
You can not only houses build or rolls bake
und wenn du dann gestorben bist, ist alles aus,
and when you then dead are is everything finished,
alles weggeschichtet.
everything wiped out.

Shortly before his death, the old Graf realizes that in view of some conception of an ideal life, you should do more than just care for your property or do your daily work.

(47) Sagst kannst gewiß nicht, daß ich dir einmal schlecht
Say can you certainly not that I you once bad
geraten hab'.
advice given have.
Jani Hans always advised the Heimrath widow well. Given this fact, it is impossible in view of an ideal of truthfulness and trust, that she says anything to the contrary.

(48) **Dieses Brot kann man ja direkt seiner Majestät empfehlen.**
This bread can one indeed straight away to his Majesty recommend.

This bread is good. If you recommend him something good, his Majesty will be pleased. If you recommend him something bad, however, his Majesty will hate you. Given these facts, it is possible in view of an ideal where his Majesty loves you, that you recommend this bread to his Majesty.

(49) **Kann ich jetzt gehen?**
Can I now leave?

Imagine a pupil who says (49) to his teacher. The teacher is the source of law and order for him. What she wants is commanded and nothing is commanded unless she wants it. The boy wants to know whether it is possible in view of what is commanded that he leaves. In this case, the **kann** in (49) is deontic.

Welke (31) and Buscha-Heinrich-Zoch (6) think that this purely deontic use of **kann** is colloquial. Klaus Welke quotes from “Muttersprache” (“Mother Tongue”), where teachers of German are advised to correct pupils who use **kann** for expressing permission. They should say **darf** (may). For me, **kann** may express permission and I don’t feel that there is anything colloquial about it.

For **darf**, a deontic ordering source is common but not obligatory. Suppose two burglars are trying to enter a farm house and whisper to each other:

(50) **Jetzt dürfen wir keinen Lärm machen.**
Now may we no noise make.

It is not that they are not allowed to make a noise. They can’t make a noise in view of their aim to burgle the farmers without getting caught.

**Kann** and **darf** have similar meanings. Both express human possibility. But there are differences. **Darf** requires an ideal in view of which possibilities are assessed. **Kann** is more neutral in this respect. Here, possibilities may depend on brute facts only, that is, the ordering source may be empty. On the other hand, **darf** doesn’t admit any ‘normative’ conversational background as ordering source.

Suppose I have a horrible headache and say with a deep sigh:

(51) **Ich kann das nicht aushalten.**
I can this not bear.

This use of **kann** involves standards concerning normal tolerance thresholds for pain. I couldn’t express the same thing in uttering:

(52) **Ich darf das nicht aushalten.**
I may this not bear.

**Darf** does not tolerate a ‘normal standards’ — ordering source. On the other hand, **kann** may have difficulties with buletic ordering sources: Tomorrow is the coronation of the King and I utter:

(53) **Morgen darf es nicht regnen.**
Tomorrow may it not rain.

What I say here is roughly, that in view of what we all want, it shouldn’t rain tomorrow. I couldn’t get this interpretation in uttering:

(54) **Morgen kann es nicht regnen.**
Tomorrow can it not rain.

We can conclude that there are certain restrictions for **kann** and **darf** which concern the admissible ordering sources. Again, more detailed investigations have to reveal the exact nature of these restrictions.

That an expression requires a complement of a certain kind to be provided by the context of use, has important consequences for the way we understand these expressions. These rules of use can influence certain features of the utterance context itself by means of what David Lewis has called “rules of accommodation”. In our case, a rule of accommodation in the style of David Lewis would look as follows:

**Rule of Accommodation:**
If the utterance of an expression requires a complement of a certain kind to be correct, and the context just before the utterance does not provide it, then ceteris paribus and within certain limits, a complement of the required kind comes into existence.

This is black magic, but it works in many cases. Suppose, I have a broken leg and say:

(55) **Ich darf nicht laufen.**
I may not walk.

So far, I have been talking about how I fell down the ladder, how they plastered my leg . . . just facts and nothing else. With the utterance of (55), suddenly ideals start entering the picture: ideals where people don’t have crooked legs, where they don’t feel pain or where they just listen to their physician. As David Lewis shows, rules of accommodation play an important role in our conversations. So this is an example of how the way we understand a particular
occurrence of a modal can be at least partly explained by an interaction of independently motivated semantic and pragmatic principles.

Müssen and Sollen

56. Wegen der Lola Montez hat er dem Thron entsagen müssen.
   Because of Lola Montez has the throne abdicate must (inf.).

Ludwig I of Bavaria loved Lola Montez. People became angry. Revolution broke out. In view of the public interest, it was necessary in this situation that he resigned. (Note the use of the infinitive müssen here. You would expect the participle perfect passive gemuβt. This peculiarity of German is discussed by Edmonds (10.).)

57. Es muβ mir gehörig, es muβ.
   It must to me belong, it must.

Kastenjäckl is desperate to buy a piece of land from the Heimrath’s. In view of what he wants, it must belong to him.

58. Lump muβ man sein, nur als Lump zwingt man die lumpige Welt.
   Crook must one be, only as crook conquers one the crookier world.

Zenzen presents his aim in the second part of the sentence. Given our world as it is, it is necessary in view of the aim to conquer the world, to be a crook.

59. Arbeiten haben wir bis jetzt müssen, arbeiten werden
   Work have we up to now must (inf.), work will
   wir auch weiter müssen.
   we also in future must (inf.).

The Heimrath’s are peasants. Given their social status, they have to work in view of an ideal of a decent and honest life. They don’t want to be beggars or burglars.

Like kann, muβ accepts a wide range of ordering sources. The ordering source may be empty too. This is suggested by sentences like:

60. Er muβte husten.
   He must (past) cough.

Like darf, soll requires a non-empty ordering source. Let us consider some examples:

   A Richard Wagner festival hall shall (past) after the designs of the architect Semper built be.

In view of the plans of King Ludwig II of Bavaria, a Richard Wagner festival hall was to be built after the designs of the architect Semper.

(62) Ich bitt euch gar schön, der hochwürdige Herr Pfarrer soll kommen.
   I ask you very nicely, the reverend Sir curate shall come.

Gauzner Michl is dying. In view of what he wants, a priest must come.

In Luther’s translation, God uses sollen a lot when he talks to Moses.

(63) Sechs Tage soltn erbeiten und alle deine Werke tun.
   Six days shalt thou labour and all thy work do.

In view of what God wants, it is necessary that you work six days a week. In some societies, what God wants is commanded. In other societies, what God wants is good and recommended, but not commanded. If I lived in a society of the first kind, I would most naturally say:

(64) Ich muß sechs Tage arbeiten und alle meine Werke tun.
   I must six days work and all my work do.

If I lived in a society of the second kind, however, I would prefer to say:

(65) Ich soll sechs Tage arbeiten und alle meine Werke tun.
   I shall six days work and all my work do.

Sollen expresses necessity. It requires an ordering source which is created by what is good, planned or recommended, or by what a particular someone wants, plans or recommends. Actually, it is not just what anyone wants, plans or recommends. The one who does so cannot be identical with the individual referred to by the subject of the sentence in which sollen occurs. I can’t say

(66) Ich soll ein Bäcker werden.
   I shall a baker become.

I am supposed to become a baker.

if it is mine but no-one else’s wish that I become a baker. Compare this with Gunnar Bech’s characterization in (2): “sollen . . . bezeichnet einen nicht dem Subjekt innewohnenden Willen”, “sollen refers to a will which is not inherent in the subject”. If we assume that in a passive sentence like (67), er is not the logical subject, (67) is not a counterexample to this principle:

(67) Er soll in Rühe gelassen werden.
   He shall in peace left be.

I think that I could use (67) for expressing that it is in view of what he wants himself that he shouldn’t be bothered.

Muß is neutral with respect to who wants me to become a baker.

(68) Ich muß ein Bäcker werden.
   I must a baker become.

may be used if I want to say that it is in view of my own wishes that I have to become a baker.
The suffixes -bar and -lich allow all kinds of ordering sources, depending on the adjective they are attached to.

- **bar**
- **lich**

Consider:

(69) **Dieses Eintrittsbillet ist nicht übertragbar.**

*This admission ticket is not transferable.*

In view of the regulations, it is not possible to give this ticket to someone else.

(70) **Diese Tasse ist zerbrechlich.**

*This cup is fragile.*

I think that this is a case of 'pure' circumstantial modality. It is in view of certain properties inherent in the cup, that it is possible that it breaks. The ordering source seems to be empty.

(71) **Dieser Vorschlag ist annehmbar.**

*This proposal is acceptable.*

In view of our common aims, it is possible to accept this proposal.

(72) **Diese Lage ist unerträglich.**

*This situation is intolerable.*

Every night, Marie-Louise's living room becomes the meeting place for all the cats in the neighbourhood. This is intolerable in view of quite normal standards concerning property, noise and smell. We may add a phrase like *for Marie-Louise* to indicate that the standards involved are more subjective.

(73) **für Marie-Louise ist diese Lage unerträglich.**

*For Marie-Louise is this situation intolerable.*

Ordering sources permit the grading of possibilities:

(74) **Ich kann eher Bäcker als Stellmacher werden.**

*I can rather be a baker than a cartwright.*

*Ich würde eher ein Bäcker als Stellmacher werden.*

Maxl was wounded during the war against the Prussians. Given this, he comes closer to an ideal where everyone is good in whatever his craft may be, if he becomes a baker and not a cartwright.

**Kann eher . . . als** expresses comparative possibility. In section two, the main motivation for introducing a clear-cut distinction between conversational backgrounds functioning as modal bases or as ordering sources, was the necessity to obtain notions of graded possibility.

In the following section, I want to discuss further arguments in favour of this bipartition.

7. **Practical Inference**

There is an obvious connection between my way of analyzing modals and what has been called "practical inferences". A practical inference may have the following form:

\[
\begin{align*}
&I \text{ want to become mayor.} \\
&I \text{ will become mayor only if I go to the pub regularly.}
\end{align*}
\]

Therefore:

\[
\begin{align*}
&I \text{ must go to the pub regularly.}
\end{align*}
\]

Let us adapt this inference to the present framework. If \( w \) is any possible world, we would have:

\[
\begin{align*}
&\text{In } w, \text{ all I want is to become mayor.} \\
&\text{In } w, \text{ the relevant circumstances are such that I will become mayor only if I go to the pub regularly.}
\end{align*}
\]

Therefore:

\[
\begin{align*}
&\text{Considering the relevant circumstances and what I want, it is necessary in } w \text{ that I go to the pub regularly.}
\end{align*}
\]

The reader can easily check that this inference should be valid. To do this, we have to interpret some expressions in a certain way, namely: *Necessary* expresses human necessity. The phrase the relevant circumstances contributes a modal base \( f \). \( f \) is that function from possible worlds into sets of propositions which assigns to any world the set of propositions which constitute the relevant circumstances in it. The phrase what I want contributes the ordering source \( g \). \( g \) is that function from possible worlds into sets of propositions which assigns to any possible world the set of those propositions which constitute what I want in it. For the particular world \( w \) mentioned in the inference, \( f(w) \) contains just one proposition, namely that I will become mayor only if I go to the pub regularly. And \( g(w) \) contains nothing but the proposition that I will become mayor. The union of \( f(w) \) and \( g(w) \) is a consistent set of propositions. It can be proved that if this is so, then it is a human necessity in \( w \) with respect to \( f \) and \( g \) that I go to the pub regularly if, and only if, it follows from the union of \( f(w) \) and \( g(w) \) that I do so. It does indeed follow. Thus the inference is valid according to our definitions.

I should like to look at a more intricate example:

\[
\begin{align*}
&\text{In } w, \text{ all I want is two things, namely to become mayor and} \\
&\text{not to go to the pub regularly.}
\end{align*}
\]

\[17\] See Anscombe (1), Brünn (3) or von Wright (32) and (33).
In \( w \) the relevant circumstances are such that I will become mayor only if I go to the pub regularly.

Therefore: Considering the relevant circumstances and what I want,

Conclusion one: it is necessary in \( w \) that I go to the pub regularly.

Conclusion two: it is necessary in \( w \) that I don't go to the pub regularly.

Conclusion three: it is possible in \( w \) that I don't go to the pub regularly and still become mayor.

Conclusion four: it is possible in \( w \) that I go to the pub regularly.

Conclusion five: it is possible in \( w \) that I don't go to the pub regularly.

This is the horrible story of someone who wants something but rejects the necessary means leading to the fulfillment of her desires. Which conclusion can we draw in such a case? I think that the first three conclusions are faulty, but the last two are correct. The above analysis predicts this. Let us see why. The expressions necessary, the circumstances and what I want are interpreted as above. Possible expresses human possibility. This time, \( g(w) \) contains exactly two propositions: That I will become mayor and that I don't go to the pub regularly. We may now reason as follows:

\( \cap f(w) \) is the set of worlds which are accessible from \( w \).

(a) For all worlds \( v \in \cap f(w) \), we have:

If I don't go to the pub regularly in \( v \), I won't become mayor in \( v \).

Given the definition of human possibility, it follows immediately that conclusion three is false. Let us now consider the set \( g(w) \). It induces a tripartition of the set \( \cap f(w) \) of accessible worlds as follows:

A is the set of all those possible worlds \( v \) of \( \cap f(w) \) such that I will become mayor in \( v \).

B is the set of all those possible worlds \( v \) of \( \cap f(w) \) such that I don't go to the pub regularly in \( v \).

C is the set of all those possible worlds \( v \) of \( \cap f(w) \) such that I won't become mayor but yet do go to the pub regularly in \( v \).

The reader may verify that:

(b) A, B and C are not empty, they are pairwise disjoint and

\[ A \cup B \cup C = \cap f(w). \]

It is easy to check now that all of the following statements concerning the ordering relation \( \leq_{g(w)} \) are true:

(c) For all \( v \) and \( z \in \cap f(w) \):

If \( v \in A \) and \( z \in B \), then neither \( v \leq_{g(w)} z \) nor \( z \leq_{g(w)} v \).

(d) For all \( v \) and \( z \in A \): \( v \leq_{g(w)} z \).

(e) For all \( v \) and \( z \in B \): \( v \leq_{g(w)} z \).

(f) For all \( v \) and \( z \in \cap f(w) \): If \( z \in C \) and \( v \in A \cup B \), then not \( z \leq_{g(w)} v \).

It follows from (b), (c), (d) and (f), that there is a world \( v \in \cap f(w) \) such that for any world \( z \in \cap f(w) \) such that \( z \leq_{g(w)} v \), I will become mayor in \( z \).

Given (a), it follows that there is a world \( v \in \cap f(w) \) such that for any world \( z \in \cap f(w) \) such that \( z \leq_{g(w)} v \), I go to the pub regularly in \( z \).

This means that it is a human possibility in \( w \) with respect to \( f \) and \( g \) that I go to the pub regularly.

Thus, conclusion two is false and conclusion four is correct. An analogous argument would show that conclusion one is false and conclusion five is correct.

In a practical inference, facts have priorities over ideals. You can't give up facts in favour of an ideal. That's why conclusion three is false.

The analysis I proposed in (18) and (19), cannot cope with these more complicated examples in a straightforward way. I did not distinguish facts and ideals. For the second example, there would be false predictions since we would proceed as follows: We would not have two conversational backgrounds \( f \) and \( g \), but just one, \( h \). For any world \( w \), \( h(w) = f(w) \cup g(w) \).

\( h(w) \) is an inconsistent set of propositions. We would try to make the best out of this inconsistent set by looking at all its maximal consistent subsets. If a proposition follows from all of them, it would be necessary in \( w \) with respect to \( h \). If it is compatible with one of them, it would be possible in \( w \) with respect to \( h \). Unfortunately, there is a maximal consistent subset of \( h(w) \) which contains all I want in \( w \), namely that I will become mayor and that I don't go to the pub regularly. Thus, conclusion three should be correct under this interpretation of possibility. As it isn't, we have good reasons to prefer my new analysis to the old one. There is another reason. The new analysis offers a very natural way for treating certain kinds of conditionals. In (19) and (20), I was not able to say what happens, if an \( if \)-clause modifies an arbitrary modal. I had to give meaning rules for each modal separately. Doing this, I missed an obvious generalization.

In the following section, I will sketch how conditional modalities fit into the present framework.

8. Conditionals

I argued in (19) and (20) that many conditionals seem to involve modals in an explicit or implicit way. I want to talk about these conditionals in this section. They may have the following form:

(If \( \ldots \ldots \), then necessarily \( \ldots \ldots \))

(If \( \ldots \ldots \), then possibly \( \ldots \ldots \))

(If \( \ldots \ldots \), then probably \( \ldots \ldots \))

etc.

Franziska Raynaud raised an objection of this kind, personal communication.
The second part of these constructions is a normal modalized sentence of the kind we have discussed so far. (Let us forget about the *then* in what follows). The first part is an *if*-clause. Its job is very easy: It makes sure that a hypothesis is added to the modal base required by the modal expression to follow.

I would like to make this more precise:

**Conditional modality**

Consider an utterance of a sentence of the following form:

\[(\alpha), \ (\text{then} \ \alpha \ldots )\]

This utterance has two parts: the first part consists of the utterance of the *if*-clause, and the second part consists of the utterance of the *then*-clause.

Suppose that the proposition $p$ is expressed by the utterance of $\alpha$.

The rule is now:

(i) The first part of the utterance requires one, and only one, modal base and one, and only one, ordering source to be correct.\(^{19}\)

(ii) If $f$ is the modal base and $g$ the ordering source for the first part of the utterance, then $f^+$ is the modal base and $g$ the ordering source for the second part of the utterance. $f^+$ is that function from possible worlds to sets of propositions, such that for any world $w$, $f^+(w) = f(w) \cup \{p\}$.

We obtain different kinds of conditionals by fixing the parameters $f$ and $g$ in different ways. I want to demonstrate this with a few examples.

For the following, consider utterances of sentences which have the following form:

\[(\alpha), \ (\text{then necessarily } \beta)\]

Suppose that $p$ and $q$ are the propositions expressed by $\alpha$ and $\beta$ respectively, and that *necessarily* expresses human necessity. As our first example, let us look at material implication:

**Material Implication:**

A material implication is characterized by a *totally realistic modal base* $f$ and an *empty ordering source* $g$. We have to prove that these requirements for $f$ and $g$ indeed give us material implication.

**Sketch of a Proof:**

Let $w$ be any possible world.

We must show that $q$ is a human necessity in $w$ with respect to $f^+$ and $g$, if and only if, $q$ is true or $p$ is false in $w$.

\(^{19}\) Instead of the uniqueness condition, a Pinkal solution would be preferable here as well. There is quite a bit of vagueness around conditionals.

**Case one:** Suppose that $p$ is true in $w$.

Then $f^+(w) \cup \{p\} = f^+(w)$ is a consistent set of propositions. Since $\cap f^+(w) = \{w\}$ and $f^+(w)$ is a consistent superset of $f(w)$, $\cap f^+(w) = \{w\}$ as well. It follows immediately, that in this case, $q$ is a human necessity in $w$ with respect to $f^+$ and $g$ if, and only if, $q$ is true in $w$.

**Case two:** Suppose that $p$ is false in $w$. Then $f(w) \cup \{p\} = f^+(w)$ is an inconsistent set of propositions and $\cap f^+(w)$ is the empty set. Then it is vacuously true that $q$ is a human necessity in $w$ with respect to $f^+$ and $g$.

Our next example is strict implication:

**Strict implication:**

A strict implication is characterized by an empty modal base $f$ and an empty ordering source $g$. Again, we have to prove that these requirements for $f$ and $g$ yield strict implication.

**Sketch of a Proof:**

Let $w$ be any possible world.

We must show that $q$ is a human necessity in $w$ with respect to $f^+$ and $g$ if, and only if, $q$ is true in all worlds in which $p$ is true. Since $g(w)$ is the empty set, we have:

For all worlds $u$ and $v \in \cap f^+(w): u \leq_R (w) v$.

Since $f^+(w) = f(w) \cup \{p\} = \{p\}$, this means that $q$ is a human necessity in $w$ with respect to $f^+$ and $g$ if, and only if, $q$ is true in all worlds of $\cap \{p\} = p$.

The most interesting kinds of conditionals are counterfactuals. They are the exact mirror images of material implications.

**Counterfactuals:**

A counterfactual is characterized by an empty modal base $f$ and a totally realistic ordering source $g$.

It follows from David Lewis' work mentioned above, that this analysis of counterfactuals is equivalent to the one I give in (21). I don't want to discuss counterfactuals in detail here. I do this in (21). The idea is this: All possible worlds in which the antecedent $p$ is true, are ordered with respect to their being more or less near to what is actually the case in the world under consideration. 'What is actually the case' is a vague concept. There are many ways of uniquely characterizing a world.

In formal terms: There are many functions $g$ from $W$ which assign to any world $w$ of $W$ a subset of the power set of $W$ such that $\cap g(w) = \{w\}$.
Let us consider an example:

Two totally realistic conversational backgrounds $g_1$ and $g_2$ may differ in the following way: for some world $w$

$g_1(w) = \{r, s\}$

$g_2(w) = \{r \cap s\}$

$g_1$ assigns to $w$ a set which contains two propositions, the propositions $r$ and $s$. $g_2$ assigns to $w$ a set which contains one proposition, the conjunction (that is the intersection) of $r$ and $s$.

If $g_1$ and $g_2$ function as ordering sources, such a difference may become important. $g_1(w)$ and $g_2(w)$ induce different orderings on the set of all possible worlds. Consider two worlds $u$ and $v$ such that $r$ is true and $s$ is false in $u$, and $r$ and $s$ are both false in $v$.

We have now: $v \leq g_1(w)$, but not $v \leq g_2(w)$.

I think that this vagueness about ‘what is the case’ is responsible for the vagueness of counterfactuals. It is worth noticing that no such vagueness can arise for material implications where totally realistic conversational backgrounds function as modal bases.

As a last example, I would like to discuss a kind of conditional which has led to paradoxes in the past. 20

Deontic Paradoxes:

Consider utterances of the following sentences:

(75) Jedein Menschen muß Gerechtigkeit widerfahren.
    To every person must justice be given.

(76) Wenn jemand ungerecht behandelt wurde, muß das Unrecht
    If someone unjustly treated was, must the injustice
    wieder gutgemacht werden.
    amended for be.

(77) Wenn jemand ungerecht behandelt wurde, muß er muntot gemacht
    If someone unjustly treated was, must be reduced to silence
    werden.
    be.

In traditional modal logic, sentences like this lead to problems. I think that these problems arise because of two reasons: On the one hand, conditional sentences like (76) or (77) are analyzed as modalised material implications. They would have the following logical form:

Necessarily ($\alpha \rightarrow \beta$)

On the other hand, the interpretation of the modal is based on nothing else but a simple accessibility relation.

20 Hansson (14), van Fraassen (11), Lewis (22) give a detailed discussion of the problem.

In our case, the traditional analysis would look as follows: The proposition I express by my utterance of (75) would be true in a world $w$, if, and only if, it is true in all worlds which are morally accessible from $w$, that justice is given to everyone. A world is morally accessible from a world $w$, if, and only if, the moral ideals prevailing in $w$ are all realized in it. The proposition I express by my utterance of (76) would be true in a world $w$, if, and only if, for all world $w^+$ which are morally accessible from $w$, the following holds: If someone has been treated unjustly in $w^+$, the injustice is amended for in $w^+$. And the proposition I express by my utterance of (77) would be true in a world $w$, if, and only if, for all worlds $w^+$ which are morally accessible from $w$, the following is true: If someone has been treated unjustly in $w^+$, he is reduced to silence in $w^+$.

What is paradoxical about all this is that, supposing that the proposition expressed in uttering (75) is true in a world, the propositions I expressed in uttering (76) and (77) would both be vacuously true in this world. If there is no injustice in any morally accessible world, anything you like is true in a those morally accessible worlds where someone has been treated unjustly.

The analysis of conditionals which I proposed above, avoids this paradox. Assume that for my utterance of (75) and the first part of (76) and (77), the modal base $f$ was empty 21 and the ordering source $g$ was determined by what is morally commanded. If $f^+$ is the modal base for the second part of (76) and (77), then for any world $w$, $f^+(w)$ contains nothing but the proposition that someone has been treated unjustly. Roughly speaking, the three proposition which I expressed in uttering (75), (76) and (77) would be true under the following conditions: The first proposition would be true in a world $w$, if and only if, justice is given to everyone in all those possible worlds which are closest to what is morally commanded in $w$. The second proposition would be true in a world $w$, if, and only if, the injustice is amended for in all those possible worlds of $\cap f^+(w)$ which are closest to what is morally commanded in $w$. And the third proposition would be true in a world $w$, if, and only if, the one who has been treated unjustly is reduced to silence in all those worlds of $\cap f^+(w)$ which are closest to what is morally commanded in $w$.

Under this analysis, it is not excluded, for example, that the first two propositions are true, but the third is false in a world. For us, a world where unjust is amended for, is not ideal, since there is no injustice in an ide world. But it may still be closer to what is ideal than any world where people who suffered injustice are reduced to silence.

Whether an analysis of conditionals is appropriate is usually assessed by examining their predicted behaviour in certain kinds of inferences like ‘transitivity’, ‘strengthening the antecedent’ or ‘contraposition’. 22 The analysis I am proposing here predicts that these three inference patterns can't be expected.

21 This assumption is not essential.
22 See for example Lewis (22), Kratzer (20).
Let us consider an example:

Two totally realistic conversational backgrounds \( g_1 \) and \( g_2 \) may differ in the following way: for some world \( w \)

\[
g_1(w) = \{r, s\} \\
g_2(w) = \{r \cap s\}
\]

\( g_1 \) assigns to \( w \) a set which contains two propositions, the propositions \( r \) and \( s \). \( g_2 \) assigns to \( w \) a set which contains one proposition, the conjunction (that is the intersection) of \( r \) and \( s \).

If \( g_1 \) and \( g_2 \) function as ordering sources, such a difference may become important. \( g_1(w) \) and \( g_2(w) \) induce different orderings on the set of all possible worlds. Consider two worlds \( u \) and \( v \) such that \( r \) is true and \( s \) is false in \( u \), and \( r \) and \( s \) are both false in \( v \).

We have now: \( u \subseteq g_1(w) \), but not \( v \subseteq g_1(w) \).

I think that this vagueness about ‘what is the case’ is responsible for the vagueness of counterfactuals. It is worth noticing that no such vagueness can arise for material implications where totally realistic conversational backgrounds function as modal bases.

As a last example, I would like to discuss a kind of conditional which has led to paradoxes in the past.\(^{20}\)

**Deontic Paradoxes:**

Consider utterances of the following sentences:

\[
(75) \text{Jedem Menschen muß Gerechtigkeit widerfahren.} \\
\text{To every person must justice be given.}
\]

\[
(76) \text{Wenn jemand ungerecht behandelt wurde, muß das Unrecht } \\
\text{If someone unjustly treated was, must the injustice } \\
\text{wieder gutgemacht werden.} \\
\text{amended for be.}
\]

\[
(77) \text{Wenn jemand ungerecht behandelt wurde, muß er mundtot gemacht } \\
\text{If someone unjustly treated was, must be reduced to silence } \\
\text{werden.} \\
\text{be.}
\]

In traditional modal logic, sentences like this lead to problems. I think that these problems arise because of two reasons: On the one hand, conditional sentences like (76) or (77) are analyzed as modalized material implications. They would have the following logical form:

\[
\text{Necessarily } (\alpha \rightarrow \beta)
\]

On the other hand, the interpretation of the modal is based on nothing else but a simple accessibility relation.

In our case, the traditional analysis would look as follows: The proposition I express by my utterance of (75) would be true in a world \( w \) if, and only if, it is true in all worlds which are morally accessible from \( w \), that justice is given to everyone. A world is morally accessible from a world \( w \) if, and only if, all moral ideals prevailing in \( w \) are all realized in it. The proposition I express by my utterance of (76) would be true in a world \( w \) if, and only if, for all world \( w^* \) which are morally accessible from \( w \), the following holds: If someone has been treated unjustly in \( w^* \), the injustice is amended for in \( w^* \). And the proposition I express by my utterance of (77) would be true in a world \( w \) if, and only if, for all worlds \( w^* \) which are morally accessible from \( w \), the following is true: If someone has been treated unjustly in \( w^* \), he is reduced to silence in \( w^* \).

What is paradoxical about all this is that, supposing that the proposition expressed in uttering (75) is true in a world, the propositions I expressed in uttering (76) and (77) would both be vacuously true in this world. If there is no injustice in any morally accessible world, anything you like is true in a those morally accessible worlds where someone has been treated unjustly.

The analysis of conditionals which I proposed above, avoids this paradox. Assume that for my utterance of (75) and the first part of (76) and (77), the modal base \( f \) is empty\(^{21}\) and the ordering source \( g \) was determined by \( w \) and is morally commanded. If \( f^* \) is the modal base for the second part of (76) and (77), then for any world \( w \), \( f^*(w) \) contains nothing but the proposition that someone has been treated unjustly. Roughly speaking, the three proposition which I expressed in uttering (75), (76) and (77) would be true under the following conditions: The first proposition would be true in a world \( w \) if, and only if, justice is given to everyone in all those possible worlds which are closest to what is morally commanded in \( w \). The second proposition would be true in a world \( w \) if, and only if, the injustice is amended for in all those possible worlds of \( f^*(w) \) which are closest to what is morally commanded in \( w \). And the third proposition would be true in a world \( w \) if, and only if, the one who has been treated unjustly is reduced to silence in all those possible worlds of \( f^*(w) \) which are closest to what is morally commanded in \( w \).

Under this analysis, it is not excluded, for example, that the first two propositions are true, but the third is false in a world. For us, a world where injustice is amended for, is not ideal, since there is no injustice in an ideal world. But it may still be closer to what is ideal than any world where people who suffered injustice are reduced to silence.

Whether an analysis of conditionals is appropriate is usually assessed by examining their predicted behavior in certain kinds of inferences like ‘transitivity’, ‘strengthening the antecedent’ or ‘contraposition’.\(^{22}\) The analysis I am proposing here predicts that these three inference patterns cannot be expected.

\(^{20}\) Hansson (14), van Fraassen (11), Lewis (22) give a detailed discussion of the problem.

\(^{21}\) This assumption is not essential.

\(^{22}\) See for example Lewis (22), Kratzer (25).
to be valid for all those types of conditionals which involve a non-empty ordering source.

In the literature, the failure of these inference patterns is often discussed in connection with deontic conditionals, probability conditionals and counterfactuals. If we analyse these conditionals in the way outlined above, their specific behaviour in inferences is an automatic consequence of the analysis.

Conclusion

A person who has a complete grasp of the modal system of German has certain abilities. It was the aim of this paper to say exactly what these abilities are. As a result we have

(i) The ability of categorizing conversational backgrounds according to the requirements imposed by the vocabulary.
(ii) The ability of drawing inferences of various strength involving two conversational backgrounds: a modal base and an ordering source.

Actually, it is a simplification to assume that there is never more than one ordering source involved in modal reasoning. Suppose I draw conclusions which involve established facts, the Encyclopaedia Britannica, the local newspaper and the gossip I picked up at the corner. And suppose further that the established facts have priority over the Encyclopaedia Britannica, the Encyclopaedia Britannica has priority over the local newspaper and the local newspaper has priority over the gossip I picked up at the corner. How do we reason in such a case?

I think that the semantics of modals which I have presented so far can be extended in a straightforward way to handle these cases. The interpretation of a modal expression would have to depend on a modal base $f$ and a finite sequence of ordering sources $g_1, \ldots, g_n$. For any world $w$, $g_i(w)$ would induce an ordering on $f(w)$ in the usual way, $g_2(w)$ would refine this ordering in undoing the ‘ties’ left by its predecessor and so on for every successive member in the sequence.

Probably, we can’t assume that the different ordering sources form a natural sequence with respect to having priority over each other. There may be ordering sources which have equal priority. This all sounds as if it were the beginning of my next paper.

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Bibliography

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Words and Worlds

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1. Aims and First Questions

Sentence semantics is tied to word semantics by the principle of meaning composition. So, most of the factors which influence the meaning of sentence, apart from factors which can be contributed to syntax or intonation, do this because they already influence the meaning of its constituent parts. In other words, if the meaning of some sentences is, for instance, dependent on conceptions which we may call 'possible worlds', then the meaning of the words of these sentences is at least formally dependent on these conceptions.

* Some of my ideas originated 7 years ago when I was working on my Ph. D. (1974). At the time my main concern was the contribution of a reasonable notion of possible world to a context-free grammar, whereas now I am concentrating on the relation between words and particular worlds. Since then many more people have been working in that area and I have especially profited from discussions with Lennart Åqvist, Michael Grabski, Hans Kamp, John Mackie, Frank Rella, Christian Rohrer and Dana Scott about these matters. I must not forget to draw the reader's attention to Moisil (1979). He takes up quite a few of my ideas from my Ph. D. (1974) and extends them in a very interesting way. Unfortunately, it was on published after the completion of this article in January, 1979.

I am especially grateful to my wife Elizabeth C. Lutzeier for checking my English.

There is a less fortunate name around since Margalit (1978): The 'platitude' principle.