ANGELIKA KRATZER

WHAT 'MUST' AND 'CAN'
MUST AND CAN MEAN[1]

ABSTRACT. In this paper I offer an account of the meaning of 'must' and 'can' within the framework of possible worlds semantics. The paper consists of two parts: the first argues for a relative concept of modality underlying modal words like 'must' and 'can' in natural language. I give preliminary definitions of the meaning of these words which are formulated in terms of logical consequence and compatibility, respectively. The second part discusses one kind of insufficiency in the meaning definitions given in the first part, which arise from the 'ex falso quodlibet' paradox of logical consequence. In stepwise fashion, I make an attempt to avoid most of the consequences of this paradox for the meaning definitions of 'must' and 'can'.

1.1. 'must' and 'can'

Most utterances of words, phrases, sentences do mean something in certain conversations and situations. It is the task of semantics to describe all those features of the meaning of utterances of linguistic expressions which stay invariable in whatever context these expressions may be used. This invariable element we may call the meaning proper of a linguistic expression. This is of course a simplification which neglects many things. But it is a simplification which will help us to approach our problem. I shall try in the following to keep very close to the task I have proposed for a semantic description of linguistic expressions. But I am sure that I shall meet difficulties everywhere. And here is the first one: Nobody would claim that a semantic description of the words 'must' and 'can' should try to capture whatever is common to the meaning of the two respective occurrences of these words in the sentence I am going to utter right now:

(1)   You must and you can store must in a can.

In cases like this it is generally said that the two occurrences of 'must' are not occurrences of the same word, but occurrences of two different words which just happen to look the same. The must which you can store in a can has nothing to do with necessity, and the can you can store your must in has nothing to do with possibility.

So the word 'must' in English has at least two different meanings and so has the word 'can'. I said "at least two different meanings", and this is very

moderate. So far, we have seen that there is a noun 'must' and a verb 'must', and a noun 'can' and a verb 'can'. I think everyone will accept this. But most people claim that even if we take just the verbs 'must' and 'can', they are ambiguous too; there are really many verbs 'must' and many verbs 'can'. In order to justify these claims, sentences like the following four are proposed:

(2)   All Maori children *must* learn the names of their ancestors.

(3)   The ancestors of the Maoris *must* have arrived from Tahiti.

(4)   If you *must* sneeze, at least use your handkerchief.

(5)   When Kahukura-nui died, the people of Kahungunu said: Rakaipaka *must* be our chief.

The 'must' in sentence (2) is often called a deontic 'must': it refers to a duty. The 'must' in sentence (3) is usually called an epistemic 'must': it refers to a piece of knowledge or information.

The kind of 'must' in sentence (4) has been called a dispositional 'must':[2] it refers to dispositions people have, which are for example such that they cannot help sneezing. The 'must' in sentence (5) is sometimes called a preferential 'must': it refers to preferences and wishes. These distinctions among four different kinds of 'must' are the ones which are usually made.[3] Perhaps I have forgotten some. Perhaps the classification should be refined. Perhaps we will have to consider some more kinds of 'must'. Some? Look at the following four bits of conversation:

"The Maori children must learn the names of their ancestors."

"Do they really? Is there a law in New Zealand which provides that the Maori children learn the names of their ancestors?"

"No, of course there is no such law in New Zealand. At least no official law. But the Maoris have their tribal laws, and it was these laws I had in mind when I said 'All Maori children must learn the names of their ancestors'."

"The ancestors of the Maoris must have arrived from Tahiti."

"No, they could have arrived from somewhere else. We know that their technical means permitted them much longer trips. They could have even arrived from Peru."

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[2] See Grabski (1974).
[3] See Grabski (1974).

"But we know that they did not arrive from Peru. We know it from their tribal history. We know it from Polynesian mythology. We simply know it. They must have arrived from Tahiti."

"I must sneeze."

"Don't be silly. You must not. Everyone knows how to prevent sneezing. You feel that something fuzzy is going on in your nose. You feel it a good time in advance. And you can suppress it. That's all."

"But once I have missed the right moment, I cannot help sneezing any more. It just comes out. It is too late to suppress it. I simply must sneeze."

"Rakaipaka must be our chief."

"No, he must not. The Queen does not like him particularly. She does not dislike him particularly, either. He could be our chief, but there are others who could be just as well."

"I do not care whether the Queen likes Rakaipaka. I only care about our tribe. I only consider what is good for our tribe. That is why Rakaipaka must be our chief."

How many kinds of 'must' do we have to distinguish? How many deontic ones? How many epistemic ones? How many dispositional ones? And how many preferential ones?

Obviously many, many of each group. We do not just refer to duties. We refer to duties of different kinds. To different duties different persons have towards different persons at different times.

We do not simply refer to a bit of knowledge or information – once and for ever the same. We refer to different kinds of knowledge or information in different situations.

We do not simply consider dispositions. Dispositions change. My dispositions now are not the same as my dispositions two minutes ago.

We do not always refer to the same wishes or preferences when we use a preferential 'must'. Sometimes it is the wish of the Queen, sometimes it is the wish of our tribe, sometimes we consider even our own wish.

All this leaves us with many different 'must's and 'can's. What can we do with them? We could give them different names.[4] Numbers have been

4 See Grabski (1974).

proposed. Let's have

'must$_1$', 'must$_2$', 'must$_3$', ...
'can$_1$', 'can$_2$', 'can$_3$', ...

But we might not have enough numbers. How many bits of knowledge are there, to which we can refer? How often does the Queen change her mind? How often do I change my dispositions? How many kinds of duties can we take into consideration? And even if we had enough numbers, it would not be very sensible to use them here. In everyday conversation we do not use subscripts when we use the words 'must' and 'can'. Somehow we do without them. And even quite easily. There must be another way by means of which we make ourselves understood using these words.

If I look at the four different occurrences of the word 'must' in sentences (2) to (4), I do have the feeling that there is something in the meaning they have there which stays invariable. I feel that the connection between these four occurrences is much stronger than the connection between any of these occurrences and an occurrence of the word 'must' referring to the must which we can store in a can. Let us try to find out what this connection is. Let us try to paraphrase what we might mean if we utter these four sentences and try to make explicit what we might refer to when we use the word 'must' in sentences like (2), (3), (4) or (5).

Consider for this purpose the following sentences (2*) to (4*):

(2*)   *In view of what their tribal duties are*, The Maori children *must* learn the names of their ancestors.

(3*)   *In view of what is known*, the ancestors of the Maoris *must* have arrived from Tahiti.

(4*)   If – *in view of what your dispositions are* – you *must* sneeze, at least use your handkerchief.

(5*)   When Kahukura-nui died, the people of Kahungunu said: *In view of what is good for us*, Rakaipaka *must* be our chief.

Let us suppose that sentences (2*) to (5*) do express roughly what I wanted to say when I uttered sentences (2) to (5). Then what happened to the four occurrences of the word 'must' in these paraphrases? They seem to have shifted some of their meaning they had in the utterances of sentences (2) to (5) to different 'in view of' phrases. And what we find now in these four paraphrases are four different occurrences of the word 'must' which all seem to have the same meaning. And this meaning seems to be the common kernel of meaning whose presence we felt somehow in the four occurrences of the word 'must' in the sentences (2) to (5).

The examples (2) to (5) and (2*) to (5*) suggest that a semantic description of the word 'must' should capture that common kernel of meaning which we have isolated above. It is this kernel of meaning which seems to stay invariable whenever the verb 'must' is used. In such a semantic description there will be only one verb 'must'. Whoever wants to keep billions of different 'must's would be forced by examples (2) to (5) to accept still another verb 'must': a neutral 'must'. Sentences (2*) to (5*) are English sentences too, and an appropriate description of the meaning of 'must' has to account for this 'neutral' meaning which the verb 'must' has here as well.

Let us now have a closer look at this common kernel of meaning which we have peeled out of all the occurrences of the verb 'must', we have considered so far. In sentences (2*) to (5*) the verb 'must' is used relationally. We might say that what we have in these sentences is not an absolute 'must' but a relative 'must in view of'. This relative modal phrase 'must in view of' has two arguments: a phrase like 'what is known' or 'what is good for us' etc. and a sentence. Picture 1 would then be a rough representation of the semantic structure of sentence (3*).
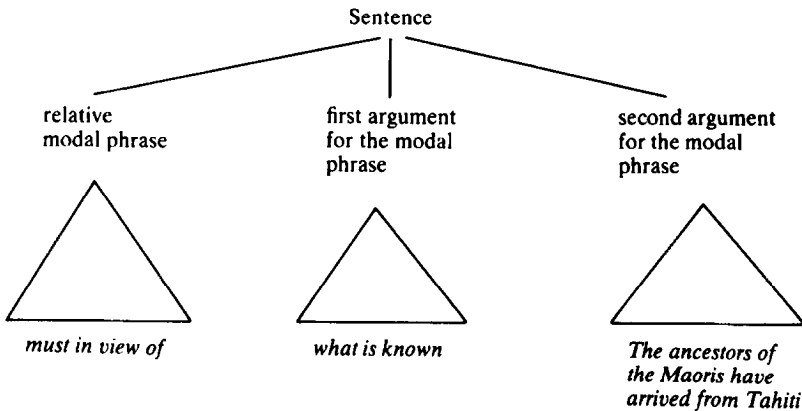
Put into purely semantic terms, we may say that the neutral meaning of all the occurrences of the word 'must' in sentences (2*) to (5*) is something which requires two arguments. The first argument is provided by the meaning of a phrase like 'what is known', and the second argument is provided by the meaning of a sentence like 'the ancestors of the Maoris have arrived from Tahiti'. According to our considerations above, the neutral meaning of the occurrences of the verb 'must' in sentences (2*) to (5*) is identical with the kernel of meaning common to all occurrences of this verb in sentences (2) to (5). If the neutral meaning of the occurrences of the verb 'must' in sentences (2*) to (5*) is something which requires two arguments of a certain kind, then the kernel of meaning common to the occurrences of the

word 'must' in sentences (2) to (5) should be something which requires equally two arguments of that kind.

Sentences (2) to (5), however, provide only one such argument explicitly. It is that kind of argument which is provided by a sentence like 'the ancestors of the Maoris have arrived from Tahiti'. The other argument, which is of the kind that would be provided by a phrase like 'what is known', is missing – at least it is not explicit in the sentence. Where must we look for it? The context of utterance provided it, of course. The context seemed to provide a deontic argument when I uttered sentence (2), an epistemic argument when I uttered sentence (3), a dispositional argument when I uttered sentence (4), and a preferential argument when I uttered sentence (5). I tried to make this a bit more explicit when I gave a kind of paraphrase for what I said when I uttered each of those four sentences.

I want to claim now that the impression that the occurrences of the word 'must' were deontic, epistemic, dispositional, and preferential, in sentences, (2) through (5), respectively, was due to the fact that when I uttered these sentences, one of the missing arguments joined with that very kernel of meaning whose presence we felt in all occurrences of the verb 'must' in these sentences; that is, one of these missing arguments joined with the meaning proper of the verb 'must'. After this fusion we had the impression that there were different kinds of 'must' present. Similar considerations hold for the word 'can'.

We may draw the following conclusion: relative modal phrases like 'must in view of' and 'can in view of' should be considered as the foundation of the modals 'must' and 'can', respectively.

The meanings of these modal phrases require two arguments – one argument which could be provided by a phrase like 'what is known', and another argument which is provided by a sentence. The first argument may be delivered explicitly or else it may be provided by the context of utterance. I do not want to deal with context dependence here. This would make everything much more complicated. So I shall consider in the following only examples where both arguments of the modal phrase are explicitly present in the sentence. Of course, in real life this is very seldom the case. But sometimes it would help us to avoid or to settle misunderstandings if we made use of the opportunity to be a bit more explicit about this first argument.

Consider the following case of a misunderstanding. Last year I attended a lecture in ethics given by a man called 'Professor Schielrecht'. Professor Schielrecht is a third-generation offspring of the Vienna Circle, so his main concern in philosophy is to show that most of what most people say most of the time does not make sense.

Suppose a judge asks himself whether a murderer could have acted otherwise then he eventually did. Professor Schielrecht said that the judge asks himself a question which does not make sense. Why not? Professor Schielrecht's answer was: Given the whole situation of the crime, which includes of course all the dispositions of the murderer, this man could not have acted otherwise than as he did. If he could have acted otherwise than he eventually did, he would have acted otherwise than he eventually did. So the answer to the question is trivial; there is no need to spend a single second on the problem. There is really no problem. And it does not make sense if one spends any time on a problem which only *seems* to be a problem but is not really one.

But there *is* a problem. The answer to the question of the judge is *not* trivial. The judge asked himself: Could this murderer have acted otherwise than he eventually did? Professor Schielrecht claimed that the judge asked himself whether – given the whole situation of the crime – the murderer could have acted otherwise than he eventually did. The judge did not make explicit the first argument required by the word 'could' which he used in his question. Professor Schielrecht provided such an argument by the phrase 'given the whole situation', but he provided an argument which does not match what the judge meant. He misunderstood the judge: what the judge probably meant was: Given such and such aspects of the situation, could the murderer have acted otherwise than he eventually did?

The misunderstanding could have been avoided if Professor Schielrecht had asked the judge: In view of *what* could the murderer have acted otherwise than he did? Perhaps the judge would have been embarrassed by the question, but this does not concern us here.

We shall be explicit about the first argument of the modal word in the following. But I do not thereby claim that we must or even that we can be explicit enough to avoid misunderstandings of the kind which I have just described. I only want to make things easier to describe: I do not want to get into the mess of context theories.

1.2. Considerations for an Analysis of the Meaning of 'must' and 'can' within the Framework of Possible World Semantics: Step 1

I next want to show how a description of the meaning of 'must' and 'can' can be given within the framework of possible worlds semantics. What do we require of such a description?

There are several things which puzzle linguists today. If they work in syntax, they are puzzled by the fact that someone produces or recognizes well-formed sentences which he has never heard before. If they work in

semantics, they are puzzled by the fact that someone understands these
sentences which he has never met before. And after they have been puzzled
by these facts for a while, they try to explain them by designing machinery
which does more or less the same job: generating an infinite set of sentences
from a finite set of words (or smaller entities) and assigning a meaning to
each of these sentences beginning with an assignment of meaning to each
word (or smaller syntactic entity).

Let us suppose here that we have already designed syntactic machinery
which generates the set of all English sentences starting from a finite
vocabulary. I want to sketch what the corresponding semantic machinery
would do with some of these sentences which contain our two words 'must'
and 'can'. It would, of course, assign a meaning to them. What kind of entity
is a meaning of a sentence? A proposition. And what is a proposition? For
the purposes of this paper, the only characteristic property of a proposition
which I am interested in is that it is either true or false in a possible world.

Given this, we can go so far as to identify propositions with the set of
possible worlds in which they are true. A proposition is a set of possible
worlds.[5] Starting from the set W of all possible worlds, we have got almost
everything we need for our purposes here. The set of all propositions will
then be the power-set of W. On the basis of these two sets we can formulate
the following definitions which are common in possible-worlds semantics:

DEFINITION 1. A proposition p is true in a world w of W if and only if w is
a member of p. Otherwise p is false in w.

DEFINITION 2. If A is a set of propositions and p is a proposition, then p
follows (logically) from A if and only if there is no possible world where all
members of A are true but p is false.

DEFINITION 3. A set of propositions is consistent if and only if there is a
possible world where all its members are true. Otherwise it is inconsistent.

DEFINITION 4. A proposition p is compatible with a set of propositions A
if and only if $A \cup \{p\}$ is consistent.

With these tools, we can go back to picture 1. This picture gives us a certain
structure for the sentence 'In view of what is known, the ancestors of the
Maoris must have arrived from Tahiti'. It is not a very refined structure, but

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[5] This way of reconstructing the notion of a proposition has been shown to be extremely useful
in a number of recent publications. See for example Cresswell (1973) and Lewis (1973).

it is fine enough for our purposes. I want to sketch now what the semantic machinery would do with this sentence.

It assigns propositions to sentences, so it would assign a proposition to the constituent sentence 'The ancestors of the Maoris have arrived from Tahiti'. It would be that proposition, call it p, which is true in exactly those possible worlds where the ancestors of the Maoris have arrived from Tahiti. Let us suppose that p is a proposition which we have already at our disposal. We do not ask how this proposition is obtained by the meaning of the constituents of the sentence 'The ancestors of the Maoris have arrived from Tahiti'.

Now, what is the meaning of the phrase 'what is known'? What is known changes from world to world. If Lord Rutherford had not existed, we would not know many things we do in fact know. If Darwin had never travelled with Captain Fitzroy, it would not be known that we all descend from the monkeys. We can imagine worlds where people know more than we do. There are possible worlds where it is known who made the statues on Easter Island. And what is it that is known in a world? In our world it is known, for example, that Lord Rutherford was a physicist, that Darwin visited New Zealand, that 1 plus 1 equals 2, etc., so what is known in a possible world is a set of propositions. We can now say what the meaning of the phrase 'what is known' is within the framework of possible worlds semantics: the meaning of the phrase 'what is known' is a function from possible worlds into sets of propositions. To be more specific, the meaning of the phrase 'what is known' is that function from the set of possible worlds into the set of all propositions which assigns to each possible world the set of propositions which are known in that world. Let us call this function 'f'.

We should now be able to say what the meaning of the modal phrase 'must in view of' is. In picture 1, this phrase is syntactically something which has two arguments. It makes a sentence out of the expression 'what is known' and the sentence 'the ancestors of the Maoris have arrived from Tahiti'. Its meaning behaves in pretty much the same way: The meaning of 'must in view of' is something which assigns a sentence meaning to a pair consisting of a function like f and another sentence meaning. The meaning of 'must in view of' is a function which assigns a proposition to a pair consisting of a function like f and another proposition. This is the sort of meaning the phrase 'must in view of' has. What is its specific meaning, that is, which function ζ is the meaning of the phrase 'must in view of'? Let us look at our example.

How do we get the meaning of the whole sentence of picture 1 from the meaning of its parts? That is, which proposition does ζ assign to the pair consisting of f and p? The answer is: ζ assigns to the pair consisting of f and p that proposition which is true in exactly those possible worlds w where p

follows logically from the set of propositions which f assigns to w. In other words, ζ assigns to the pair consisting of f and p that proposition which is true in exactly those possible worlds where it follows logically from what is known in these worlds, that the ancestors of the Maoris arrived from Tahiti.

If we replace the word 'must' in picture 1 by the word 'can' we get another picture – picture 2.
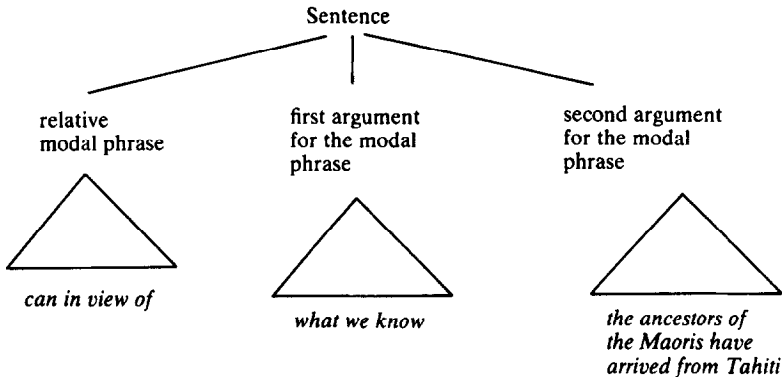
The question is now: how is the meaning of the sentence of picture 2 determined by the meaning of f, p and the meaning of the phrase 'can in view of'? Of course the meaning of the phrase 'can in view of' is again a function which assigns propositions to pairs consisting of a function like f and another proposition. But which function χ is it? Or, for our particular case, which proposition does this function χ assign to the pair consisting of f and p? The answer is easy to guess by now. χ assigns to the pair consisting of f and p that proposition which is true in exactly those possible worlds w which are such that the set of propositions which f assigns to w is compatible with p. That is, the meaning of the sentence of picture 2 is that proposition which is true in exactly those possible worlds where it is compatible with everything which is known in these worlds that the ancestors of the Maoris arrived from Tahiti. All these considerations lead to the following definitions for the meaning of 'must in view of' and 'can in view of'.

DEFINITION 5. The meaning of 'must in view of' is that function ζ, which fulfils the following conditions:
(i)   If p is a proposition and f a function which assigns a set of propositions to every $w \in W$, then (f, p) is in the domain of ζ.
(ii)   For any f and p such that (f, p) is in the domain of ζ, $\zeta(f, p)$ is that proposition which is true in exactly those $w \in W$ for which the following holds: p follows (logically) from $f(w)$.

DEFINITION 6. The meaning of 'can in view of' is that function χ which fulfils the following conditions:
(i)   As in Definition 5 with χ for ζ.
(ii) For any f and p such that (f, p) is in the domain of χ, $\chi(f, p)$ is that proposition which is true in exactly those $w \in W$ for which the following holds: p is (logically) compatible with $f(w)$.

The general idea behind these definitions is very simple. The semantics of 'must in view of' and 'can in view of' is given by means of a function f which assigns sets of propositions to every possible world. A proposition is necessary in a possible world w in view of such a function f, if it follows logically from the set of propositions which f assigns to w. A proposition is possible in a possible world w in view of such a function f, if it is logically compatible with the set of propositions which f assigns to w.

2. INCONSISTENT SETS OF PROPOSITIONS: TOWARDS A MEANING DEFINITION FOR 'MUST' AND 'CAN': STEP 2

2.1. *The Problem*

I have given an account of the meaning of 'must' and 'can' in terms of logical consequence and compatibility. In doing so I must be prepared to face all the old paradoxes which are connected with these notions. For example, 'ex falso quodlibet' says that any proposition at all follows from an inconsistent set of propositions. I want to show in this section that we need not accept this paradox in our case. We do have quite clear intuitions about what follows or does not follow from an inconsistent set of propositions, and we do have the technical tools to match these intuitions, more or less.[6]

Let us imagine a country where the only source of law is the judgements which are handed down. There are no hierarchies of judges, and all judgements have equal weight. There are no majorities to be considered. It does not matter whether one judgement has a hundred judgements against it; it does not have less importance for all that. Let New Zealand be such a country.

There is one judgement in New Zealand which provides that murder is a crime. Never in the whole history of the country has anyone dared to attack this judgement. No judgement in the whole history of New Zealand suggests that murder is not a crime. There are other judgements, however. Some

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[6] This was demonstrated by Rescher (1973). The tools which I am going to use are more general than the ones developed by Rescher.

judges did not quite agree, and there were even judges who disagreed so much that they did not talk to each other any more.

Here is an example of such a disagreement. In Wellington a judgement was handed down which provided that deer are not personally responsible for damage they inflict on young trees. In Auckland a judgement was handed down which provided that deer *are* personally responsible for damage they inflict on young trees. This means that the set of propositions which the New Zealand judgements provide is an inconsistent set of propositions.

The situation which I have just presented is not a very unusual one. It may happen every day that two judges hand down judgements which contradict each other. But the definitions I have given for the meaning of 'must' and 'can' cannot cope with such a simple situation. According to them, each of the propositions expressed by the following two sentences (6) and (7) should be true in the described situation:

(6)   In view of what the New Zealand judgements provide, murder must be a crime.

(7)   In view of what the New Zealand judgements provide, it must be that murder is not a crime.

(I have cheated a bit by putting 'must be that' in place of 'must' in order to get the right reading of sentence (7).)

We do certainly agree with (6), but (7) is not really what we want. And having both propositions expressed by these sentences turn out true is the last thing we want. But there is no help as long as we accept the definitions I have given. As the set of propositions which form the content of all the New Zealand judgements in our world is an inconsistent set of propositions, it follows logically from this set that murder is a crime and that murder is not a crime. Although no New Zealand judge has ever doubted that murder is a crime, our semantic analysis forces us to suppose nevertheless that murder must be a crime and not a crime in view of the New Zealand judgements.

The situation is just as bad if we consider the personal responsibility of deer. As the set of all propositions which form the content of the New Zealand judgements is inconsistent, we can add any proposition whatsoever to that set, and it does not become consistent. This means that no proposition is compatible with this set, and according to our definitions, both propositions expressed by the following sentences are therefore false in our world:

(8)   In view of what the New Zealand judgements provide, deer can be personally responsible for damage they inflict on young trees.

(9)   In view of what the New Zealand judgements provide, it can be

that deer are not personally responsible for damage they inflict on young trees.

What can we do in a situation like this? What should we think about the personal responsibility of deer? One thing is certain – whatever decision we make, it will not be one which keeps to New Zealand law. We really have run into an odd situation. Our semantic analysis forces us to assume that murder must be both a crime and not a crime in view of the New Zealand judgements, and it prevents us from making any decision about the personal responsibility of deer without offending the law. All this trouble arose only because once upon a time there were two judges who disagreed.

2.2. *Improved Definitions*

Let us have another look at the problem. First we shall simplify it a bit further. Let us suppose that the *whole* content of what a New Zealand judgement has ever provided in our world is expressed by the following sentences:

(10)   Murder is a crime.
(11)   Deer are personally responsible for damage they inflict on young trees.
(12)   Deer are not personally responsible for damage they inflict on young trees.

What do we want in a situation like this? We do want, for example, that the proposition expressed by sentence (9) be true given this situation in our world. And we do want that the proposition expressed by sentence (7) be false in our world under the supposed circumstances. Furthermore, we want certainly that the proposition.expressed by the following sentence be false in our world:

(13)   In view of what the New Zealand judgements provide, it can be that murder is not a crime.

Now, how should we decide according to New Zealand Common Law when the personal responsibility of deer is concerned? There is a judgement which provides that deer *are* personally responsible for damage they inflict on young trees, and there is a judgement which provides the contrary. These are the two possibilities, and we may choose either one, so we want both the proposition expressed by (8) and that expressed by (9) to be true in our world if things are there as we have assumed. Let us try to make some corrections in our preliminary definitions of the meaning of 'must' and 'can' which keep closer to our intuitions.

Some abbreviations will be useful for what follows. Let A be the set of propositions provided by all the New Zealand judgements in our world. In our example, this set contains only three members: p, the proposition expressed by (10); q, the proposition expressed by (11); and $-q$, the proposition expressed by (12).

Why didn't the old Definition 5 work in this case? It was based on a relation between a set of propositions and a proposition, and this relation was simply logical consequence. Since A is inconsistent, the relation on which Definition 5 is based holds between A and any proposition whatever. In particular, it holds between A and p and between A and the negation of p. What we are looking for is a definition for the meaning of 'must' based on a stricter relation, a relation which should hold, for example, between A and p but not between A and $-p$. Let us try to find such a relation, which, of course, will have to cope with more than just the special case we are considering here.

We must find a way of removing the inconsistency of A while staying as close as possible to what the propositions in A provide. One way of overlooking the inconsistency of A is to consider the set of all consistent subsets of A. Let us call this set 'X'. Perhaps we can establish the relation we are looking for by means of X.

The following proposal might come to our minds: the relation we want holds between a proposition and A if and only if this proposition follows logically from every consistent subset of A, that is, if and only if this proposition follows logically from each member in X. Let us see whether this relation does the required job. We have:

$$A = \{p, q, -q\}$$

$$X = \left\{ \begin{matrix} \{p\}, \{q\}, \{-q\}, \varnothing \\ \{p, q\}, \{p, -q\} \end{matrix} \right\}$$

Certainly, $-p$ does not follow from every set in X. It does not follow from the set which contains q as its only member, for example. This is pleasant. Unfortunately, p does not follow from this set, either. This shows that our relation is too rigorous. We cannot go as far as that.

Let us try the following: the relation we want holds between A and a proposition p if and only if for every consistent subset of A there is a consistent extension in A from which p follows logically, that is, if and only if for every set in X there is a superset in X from which p follows logically. This relation does the job in our case. For every set in X, there is a superset in X from which p follows logically. But there is not for every set in X a superset in X from which the negation of p follows logically. There is, for example, no

superset in X of the set containing p and q as its only members from which the negation of p follows logically.

This result is quite pleasant. We have found a method to clean up an inconsistent set of propositions while still staying as close as possible to all that these propositions tell us. Of course, the example which helped us to find this method was very simple, and our cleaning machinery is designed to cope with cases which are much more complicated. For example, an inconsistency may arise in a set of propositions in a great variety of ways., In our example, there were only two propositions involved, which were con-tradictions of each other. Furthermore, the proposition that murder is a crime is certainly not the only proposition which is necessary in view of what the New Zealand Law provides. There are others which have never been expressed explicitly in a New Zealand judgement, for example, the proposi-tion that Bully Hayes commits a crime when he murders the barber in his shop. These complicated cases lurk behind the following Definition 7, although I chose a simple example to motivate it.

DEFINITION 7. The meaning of 'must in view of' is that function ζ which fulfils the following conditions:
(i)   As in Definition 5.
(ii) For any f and p such that (f, p) is in the domain of ζ, $\zeta(f, p)$ is that proposition which is true in exactly those $w \in W$ for which the following holds: if X is the set of all consistent subsets of $f(w)$, then there is for every set in X a superset in X from which p follows (logically).

Similar considerations lead to an improvement of Definition 6. Why didn't this definition work in our case? Because it is based simply on the relation of logical compatibility between a set of propositions and a proposi-tion. Since A is an inconsistent set of propositions, this relation does not hold between A and any proposition at all, and in particular, it does not hold between A and q nor between A and $-q$. We need a suitable relation between a set of propositions and a proposition which does hold between A and q and between A and $-q$, for example, but not between A and $-p$. Again we overlook the inconsistencies of A by considering the set X of all consistent subsets of A, and this time, we try to get the right relation straight away. The relation we want holds between A and a proposition p if and only if there is a consistent subset in A such that p is compatible with all its consistent extensions in A, that is, if and only if there is a set in X such that p is compatible with all its supersets in X.

Let us see how this relation works in our case. There is in X a set such that q is compatible with all its supersets in X. Take for example the sets containing q itself. And the same holds for $-q$. But there is no set in X such

that $-p$ is compatible with its supersets in X. Again, it was a simple example which led to an improved definition, and again, this Definition 8 has to be seen against a background of more complicated cases.

DEFINITION 8. The meaning of 'can in view of' is that function which fulfils the following conditions:
(i)   As in Definition 6. .
(ii) For any f and p such that (f, p) is in the domain of χ, $\chi(f, p)$ is that proposition which is true in exactly those $w \in W$ for which the following holds: If X is the set of all consistent subsets of $f(w)$, then there is a set in X such that p is (logically) compatible with all its supersets in X.

2.3 Objections to Step 2?

Our hopes that these two improved definitions might be general enough to cope with the problem of making the best out of inconsistent sets of propositions seem to vanish as soon as we look at the following situation.[7]

The pupils of a Whare Wananga, which was a kind of University in the Maori Society, have to be educated according to the recommendations of the former principals of the school. Of course these principals had different opinions about what is good for a pupil to learn. There was, for example, Te Miti, who recommended that his pupils practise striding and flying. And there was Te Kini, who recommended that his pupils do not practise striding under any circumstances. In his opinion the practice of striding overstrained his pupils' legs. But he had no objections to the practice of flying.

Let us suppose that Te Miti and Te Kini were the only principals of this particular Whare Wananga. And let us further assume that the two recommendations mentioned are the only ones these two principals ever gave. What the recommendation of Te Miti provides can be expressed by sentence (14):

(14)   The pupils practise striding and flying.

Let us refer to the proposition expressed by (14) by '$p \cap q$'. What the recommendation of Te Kini provides can be expressed by sentence (15):

(15)   The pupils do not practise striding.

Let us refer to the proposition expressed by this sentence by '$-p$'. Given this situation in our world, we certainly want the propositions expressed by (16) to be true in our world:

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[7] An objection based on a similar situation was brought to my attention by Irene Heim.

(16)   In view of what the former principals of the Whare Wananga
         have recommended, the pupils must practise flying.

Does Definition 7 match these intuitions? It does not. Let A be the set of
propositions which form the content of all the recommendations which the
former principals of the Whare Wananga have ever given in our world. In
our example, A contains only two members, $p \cap q$ and $-p$. The set X of all
consistent subsets of A is:

$$X = \{\{p \cap q\}, \{-p\}\}, \varnothing\}$$

Now, there is not for every set in X a superset in X from which q follows
logically. The set which contains $-p$ as its only member does not have such a
superset in X. What can we do? Is there any possibility of escaping this
unpleasant situation?

There are at least two possibilities. One consists in trying to improve our
definitions once more. The other consists in checking our intuitions once
more. The definitions do not match our intuitions; either the definitions are
wrong or our intuitions are misguided. Let us investigate the second
possibility, going step by step through the argument which made us feel that
we are in an unpleasant situation. Te Miti recommended that his pupils
practise striding and flying. Te Kini recommended that his pupils do not
practise striding under any circumstances. In such a situation, we argued, we
wanted certainly to save Te Miti's recommendation concerning the pupils'
flying. This recommendation has obviously not been contradicted by what
Te Kini recommended. "Certainly," I said, and "obviously", but I think we
cannot be certain about this at all, and our claim is far from being obvious.
Look at the following situation. Te Miti recommended that his pupils
practise both striding and flying. He recommended that they do both
together. These were his considerations: striding stretches the legs and flying
stretches the arms. If you do both, that's a good combination. But if you
practise striding without flying or flying without striding, the proportions of
your body somehow get funny. You get only your legs stretched and your
arms remain very short, or else you get your arms stretched and your legs are
left behind. Neither is good. But practising both arts together will yield a
good shape for your body.

If Te Kini thinks that striding is bad in any case, that its effects on your
body are so disastrous that you should not practise it at all, then this
challenges Te Miti's recommendation as a whole, not just the part of it
concerning striding. Once the pupils do not stride any more, Te Miti would
not want them to practise flying any more.

Let us look at the problem from another angle. How did we proceed in
showing that our definition did not match our purported intuitions. We said

that the proposition $p \cap q$ formed the whole content of Te Miti's recommendations. If Te Miti recommended the proposition $p \cap q$, which worlds did he therefore recommend? The worlds where $p \cap q$ is true, of course. These are all the worlds were p and q are both true together, and this is exactly the case where we should say that Te Miti recommended that both striding and flying should be practised together. And that is exactly the case where we should say that Te Kini's recommendations contradict Te Miti's recommendations as a whole. And our definitions match this intuition: they do not save the pupils' flying.

In what case would we want to save the pupils' flying? When would we want to say that Te Kini's recommendation does not contradict Te Miti's recommendation as a whole, but only the part of it concerned with striding? This would be in the case where Te Miti recommends that his pupils stride and recommends that his pupils fly. In this case, Te Miti really gives two recommendations. He does not recommend the proposition $p \cap q$ but recommends the proposition p and recommends the proposition q. And this is something different. Here Te Miti recommends all the worlds where p is true and all the worlds where q is true; he does not insist on both being true together. If one of his recommendations is challenged, we would still want to keep the other one.

Our definitions match this intuition as well. If the set of propositions which form the whole content of what the former principals of our Whare Wananga have ever recommended is the following:

$$A = \{p, q, -p\}$$

then there is indeed for every consistent subset of A a consistent extension in A from which q follows logically. So far our definitions seem to be right and our intuitions were false. We made a simple mistake in not realizing that recommending the practice of striding and the practice of flying is not the same as recommending the practice of striding and recommending the practice of flying. There is a distinction, although we do not usually make this distinction explicitly. 'Te Miti recommends his pupils practise striding and flying' may mean both.

So far we have talked only about recommending, but the phenomenon discussed is rather general. If Te Miti says that the pupils practise striding and practise flying, it is not the same as if Te Miti says that the pupils practise striding and says that the pupils practise flying. We hardly feel the difference here. The difference gets a bit clearer, however, as soon as we consider inconsistencies. If Te Kini says that the pupils do not practise striding, then what he says contradicts the whole of what Te Miti says in the first case, but only part of it in the second case. And we can go on. Believing that the pupils

practise striding and practise flying is not the same as believing that the pupils practise striding and believing that the pupils practise striding. Commanding that the pupils practise striding and practise flying is not the same as commanding that the pupils practise striding and commanding that the pupils practise flying etc.

One lesson to draw from this is as follows: In recommending, saying, believing, or commanding a proposition we do not automatically recommend, say, believe, or command all the logical consequences of this proposition. So what is recommended, said, believed, commanded, etc. in a world need not be a set which is closed under logical consequence.

Sonderforschungsbereich 99 *"Linguistik"*,
Universität Konstanz

REFERENCES

Cresswell, M. J.: 1973, *Logics and Languages*. London: Methuen.
Grabski, M.: 1974, Syntax und Semantik der Modalverben in Aussagesätzen des Deutschen. Dissertation, Universität Stuttgart.
Kratzer, A.: 1976, 'Was "können" und "müssen" bedeuten können müssen'. *Linguistische Berichte* **42**.
Lewis, D. K.: 1973, *Counterfactuals*. Oxford: Basil Blackwell.
Rescher, N.: 1973, *The Coherence Theory of Truth*. Oxford: Clarendon Press.