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Seminar on Vagueness
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Model-theoretic Vagueness vs. Epistemic Vagueness

1 Model-theoretic vagueness

The supervaluationist analyses of vagueness developed by Kamp and Fine (and others) can be described as 'model-theoretic' accounts of vagueness, because they rely crucially on the idea that the indeterminacy of vague predicates is a consequence of the structure of the models with respect to which linguistic expressions receive their interpretations: they may be partial (this derives borderline cases), but they can always be completed via a process of precisification, and crucial logical notions like (super-)truth and (super-)falsity are defined relative to (collections of) these complete models.

1.1 Advantages

The advantages of these accounts are that they:

- Preserve classical logic (sort of), and so retain important generalizations like the Law of the Excluded Middle.
- Provide a nice account of the relation between borderline cases and context (once we make them dynamic; Kamp's paper discusses this in most detail).
- Eliminate the Sorities Paradox, by rendering the second premise (super-)false.
- Provide the basis for a semantics of gradable predicates and comparison that does not require complicating the semantic ontology with abstract objects like 'degrees'

1.2 Disadvantages

There are some disadvantages of these accounts, too. First, it is not totally clear that they really explain the 'fuzziness' of borderline cases, since the boundary between the positive extension and extension gap in any particular incomplete model (or specification space, in Fine's terms) will be crisp. Likewise, it is not so clear that they explain why we fail to immediately notice the falsity of the second premise of a Sorites argument, if the semantics of universal quantification/generalization is stated in terms of super-truth.

On the linguistic side, while it's true that they support a neat account of gradability/comparison — importantly, one that we 'get for free' — it is not so clear that this is a good thing. By defining the semantics of the comparative in terms of the positive (i.e., in terms of possible precisifications/completions), they eliminate important distinctions between the two forms. Here is one example that illustrates their difference: explicit vs. implicit comparison.

(1) *Explicit comparison*

a. $[\![\text{This essay is longer than that one}]\!]^M = 1$ iff

b. $\{M' \mid [\![\textit{this essay is long}]\!]^{M'} = 1 \wedge M' \textit{ is a completion of } M\} \supset$
$\{M'' \mid [\![\textit{that essay is long}]\!]^{M''} = 1 \wedge M'' \textit{ is a completion of } M\}$

(2) *Implicit comparison*

a. $[\![\text{Compared to that essay, this one is long}]\!]^M = 1$ iff

b. same as (2b)??

c. $\exists M'[[\![\textit{this essay is long}]\!]^{M'} = 1 \wedge [\![\textit{that essay is long}]\!]^{M'} = 0 \wedge M' \textit{ is a completion of } M]$

There should be no difference in acceptability of explicit and implicit comparison in contexts involving CRISP JUDGMENTS: very slight differences between the compared objects. This is wrong:

(3) CONTEXT: A 600 word essay and a 200 word essay
  a. This essay is longer than that one.
  b. Compared to that essay, this one is long.

(4) CONTEXT: A 600 word essay and a 590 word essay
  a. This essay is longer than that one.
  b. ??Compared to that essay, this one is long.

The intuition about (4b): it's bad for the same reason that we get borderline cases and fail to recognize the problem with the second premise of the Sorities Paradox — small differences in a property can't make a difference for truth of the positive form. If the semantics of the comparative is defined in terms of the semantics of the positive, though, then we can't explain the difference between (4a-b).

A similar case: ABSOLUTE GRADABLE ADJECTIVES.

Rod A:  Rod B: 

(5) a. B is more bent than A.
  b. ??Compared to A, B is bent.

(6) a. A is straighter than B.
  b. ?Compared to B, A is straight.

(6b) is actually a bit better than (5b) (as long as A is not too far away from being straight). This probably relates to the fact that absolute adjectives with maximum standards are more likely to allow imprecise interpretations. The fact that (6b) works reasonably well suggests that we may want to model imprecision in terms of partial models; the contrast between (6b) and (4b) suggests that we don't want to handle vagueness in this way!

Absolute GAs may pose a larger problem for 'pure' model-theoretic accounts of vagueness and comparison (i.e., analyses that do everything without degrees); to see why, we need to go back to the handout from Week 2. [BREAK IN THE ACTION]

1.3 Synthesis?

Can we deal with these issues is to separate the semantics of gradability and comparison from the model-theoretic account of vagueness, by reintroducing a degree-based analysis of gradable predicates, and locating the 'partiality' in the meaning of the positive form.

(7) a. $[\![\textit{long}]\!]$ = a function from objects to their lengths
  b. $[\![\textit{more\_than\_d})]\!]$ = a function from GA meanings to (complete) functions from individuals to truth values, such that $[\![\textit{more\_than\_d}(\textit{long})]\!]$ is true of an object x if its $[\![\textit{long}]\!]$ applied to x returns a degree that exceeds d
  c. $[\![\textit{pos}]\!]$ = a function from GA meanings to partial functions from individuals to truth values, such that $[\![\textit{pos}(\textit{long})]\!]$ has exactly the sort of meaning that *long* has in a non-degree analysis like Klein's or Kamp's.

In particular, we can locate the partiality of *pos* in the 'standard function' that we posited a few weeks ago:

(8) $[\![pos]\!] = \lambda g \lambda x.\mathbf{s}(g(x))$

Assume that $\mathbf{s}$ is a partial function from degrees to truth values, which is true of a degree if it definitely exceeds the standard of comparison for the adjective in the context of utterance. Some do; some don't; and some are may or may not.

This approach allows us to maintain all the benefits of supervaluationist accounts of vagueness, and eliminates all the worries based on supervaluationist accounts of gradability and comparison, because it separates the latter from the former. (Note that it also leads to the expectation that there may be different kinds of vagueness: cases where the lexical meaning of a content term denotes a partial function vs. cases like this one, where the partiality is linked to a scalar term (*pos*). This is a distinction worth exploring.)

Any disadvantages would be more general disadvantages of a model-theoretic/supervaluationist account of vagueness in general. We've already talked about some; Timothy Williamson suggests some others.

2 Epistemic vagueness

2.1 Problems with supervaluations

Williamson (1992) presents the following *reductio* against accounts of vagueness that deny bivalence (that a proposition is either true or false), which includes supervaluationist analyses:

(9) Let $T(\text{`}p\text{'})$ represent ['p' is true] and $T(\text{`}\neg p\text{'})$ represent ['p' is false]. Then:

i. $\neg[T(\text{`}p\text{'}) \vee T(\text{`}\neg p\text{'})]$ (assumption: denial of bivalence)
ii. $T(\text{`}p\text{'}) \leftrightarrow p$ (Tarski's disquotational schema for truth)
iii. $T(\text{`}\neg p\text{'}) \leftrightarrow \neg p$ (ditto)
iv. $\neg[p \vee \neg p]$ (substitution)
v. $\neg p \wedge \neg\neg p$ (DeMorgan)

which is a contradiction, so the denial of bivalence must be wrong.

Williamson also considers an objection to this argument: if p is neither true nor false, then $T(\text{`}p\text{'})$ should be false, in which case at least one of (9ii-iii) is false, and we're not licensed to make the substitution in (9iv). This is what happens in e.g. cases of reference failure.

But Williamson argues that this is not what can be going on here, because then we wouldn't be able to make sense of the contrast between examples like those in (10), in a context in which the earth is borderline close to the sun and there is no thirteenth planet.

(10) a. ??If the atmosphere were clearer, we would be able to see the thirteenth planet with the naked eye.
b. If the earth's orbit were between those of Venus and Mercury, it would be close to the sun.

We can deny bivalence with reference failure because this has a consequence on use. According to Williamson, 'To deny bivalence for vague sentences while continuing to use them is to adopt an unstable position. The denial of bivalence amounts to a rejection of the practice of using them. One is rejecting the practice while continuing to engage in it. Rapid alternation between perspectives inside and outside the practice can disguise, but not avoid, this hypocrisy.'

One response: deny that sentences are *definitely* true or false. This is different from denying bivalence. But as TW points out, unless we know what *definitely* means, this doesn't really help. One analysis that is at least as good as any other is to say the following:

(11) 'p' is neither definitely true nor definitely false means that 'p' is *unknowably* true or false.

Which leads to an epistemic analysis of vagueness.

2.2 The core idea

The central hypothesis of the epistemic analysis of vagueness is that vague predicates (and in fact all predicates in language) sharply define a positive and negative extension (no extension gaps), but that the exact boundaries of these sets are *unknowable*. In other words: vague predicates are just those predicates whose extensions we do not fully know, and cannot fully know. The advantages of this account are pretty clear:

- It maintains classical logic and bivalence.
- It resolves the Sorites Paradox: the second premise is false.
- It derives borderline cases as instances of ignorance.
- It explains our willingness to accept the Sorites Paradox in terms of general constraints on knowledge acquisition.

Before we look at the challenges to the approach, let's see how it handles the core phenomena: borderline cases and the Sorites Paradox.

With respect to the former, the central question is: *why* are we ignorant about borderline cases? In other words, why can't we figure out what the sharp boundaries of a vague predicate are? Williamson's response:

- Assume that meaning supervenes on use (see below for more on this): a difference in meaning entails a difference in use, but not vice-versal.
- The meanings of some terms may be stabilized by natural divisions (H_2) vs. XYZ).
- The meanings of others (the vague ones) cannot be so stabilized: a slight shift in our disposition to say that the earth is close to the sun would slightly shift the meaning of *close to the sun. The boundary is sharp, but not fixed.*
- This means that an object around the borderline of a vague predicate P could easily have (not) been P had things been slightly different — different in ways that are too complex for us to even fully catalogue, let along compute. Given this instability, we can never know about a borderline case whether it is or is not P.

This last point leads to (12), which is another way of saying that vague knowledge requires a margin for error.

(12) *The Margin for Error Principle*
For a given way of measuring differences in measurements relevant to the application of property P, there will be a small but non-zero constant c such that if x and y differ in those measurements by less than c and x is known to be P, then y is known to be P.

Result: you cannot know that [x is P] is true and [y is P] is false when x and y differ by less than c. That's why we fail to reject the second premise of the Sorites, and also why 'big' changes make a difference. (Recall Aidan's observations last time.)

2.3 Challenges

1. To be a borderline case be a borderline case even in the presence of complete knowledge of the relevant facts.

Williamson's response: this begs the question against the epistemic analysis.

2. To be a borderline case is to be a borderline case in the knowledge of a set of facts on which the relevant facts *supervene*.

Williamson's response: supervenience generalizations cannot be known *a priori*. Metaphysically necessary generalizations of the form *anyone with properties x, y and z is P* can be unknowable as physically contingent ones like *x is P*.

3a. "Words mean what they do because we use them as we do; to postulate a fact of the matter in borderline cases is to suppose, incoherently, that the meanings of our words draw lines where our use of them does not."

Williamson's answer: We might use *that is water* to describe both H_20 and XYZ; that doesn't change the fact that it is false in the latter case.

3b. But here nature provides a sharp distinction in truth conditions ($H_20 \neq XYZ$). In the case of properties named by vague predicates, it does not. There is no line because "our use leaves not a line, but a smear."

Williamson's response: Not so fast. Consider a causal account of the relation between the property named by *thin* (precise) and the use of this term (vague): even if everything has or doesn't have this property (the epistemist position), "the reliability of our mechanism for recognizing it may depend on it giving neither a positive nor a negative response in marginal cases. The cost of having the mechanism answer in such cases would be many wrong answers." So vague use is consistent with precise meaning.

4a. If the epistemic view is right, can we ever know what we mean?

Williamson's answer:

(13) $[\![[_S \text{ Kim } [_{VP} \text{ is tall }]]]\!] = 1$ iff Kim is tall

Knowing truth conditions is not the same as verifying truth values!

4b. But we don't know the sense of a vague term, in addition to not knowing its reference. This means that the speech community as a whole has only partial understanding of the language its members speak. This is an untenable position!

Williamson's response: Knowing the sense of a term means being "completely inducted in a practice" that fully determines a sense. In the case of a vague term, if we take the causal line outlined above, this means knowing the verification conditions for a predicate P.

2.4 Semantics

How does the epistemic analysis of vagueness fare relative to the sorts of linguistic issues we have been considering?

The relative/absolute distinction If a property of a vague predicate is that it cannot be "stabilized by natural divisions", then we certainly expect all relative adjectives to be vague in the absence of linguistic material that introduces such divisions compositionally (e.g., comparative degree morphology, measure phrases, etc.).

On the other hand, absolute GAs provide the basis for natural divisions, via scale structure: having a property (not) completely; (not) having the property. Such predicates can (and should?) be associated with knowable boundaries, and so should resist borderline cases and the Sorites.

Gradability and comparison Williamson says nothing about these issues, but it appears to me that an epistemic account of vagueness is *forced* to assume something like a degree-based semantics of gradability and comparison, and therefore a distinctio between the former and the latter.

- If the meaning of (positive form) *tall* is a fixed property of individuals (with unknowable boundaries), then how could it possibly map onto a comparative form? On this view, (positive) *tall* is no different in terms of semantic type, etc. from *prime*.
- At best, we would have to do something like quantify over knowledge states (*Kim is taller than Lee* is true iff our knowledge that Kim is tall is better/stronger/... than our knowledge that Lee is tall). This seems wrong.

This is probably a good thing, because I don't see how else he's going to implement context dependence, except to say that we have a huge amount of polysemy.

(14) The Mars Pathfinder Mission was expensive.

So, the semantics we get on an epistemic account of vagueness is something like, the following, where **s** is a complete function from degrees and contexts to truth values (true of a degree if it exceeds the standard of comparison for that context).

(15) $[\![pos(A)]\!]^c = \lambda x.\mathbf{s}([\![A]\!](x))(c)$

In general, we cannot know is where exactly **s** sets the boundaries, for the reasons outlined above. However, in the special cases in which A uses a scale with minimal/maximal values, they provide a 'natural division' that supports stabilization of the predicate, and lets us know where the boundaries of the positive and negative extensions are. Essentially:

(16) a. $[\![pos(A_{max})]\!]^c = \lambda x.\mathbf{max}([\![A_{max}]\!](x))(c)$
b. $[\![pos(A_{min})]\!]^c = \lambda x.\mathbf{nonzero}([\![A_{max}]\!](x))(c)$

Here the conventional meanings of the adjectives override the role of context.

2.5 Questions

Is this satisfying? (From a linguistic perspective? From a philosophical perspective?) Does the contextual parameter eliminate Williamson's reductio against supervaluationist accounts? These questions and more next week!

References

Williamson, Timothy. 1992. Vagueness and ignorance. *Proceedings of the Aristotelian Society* 66:145–162.