

## First steps towards a semantics of measurement

### 1 The starting point

The focus of this class is the semantic analysis of expressions whose meanings (appear to) involve measurement. So the first question is: what expressions are we talking about? Here are some potential answers:

- (1) Expressions that can be associated with measure phrases.
  - a. Kim is six feet *tall*.
  - b. The ocean is 10 degrees *colder* than the river.
  - c. I need two pounds of *bluefish*.
  - d. I need a two pound *bluefish*.
  - e. The balloon *ascended* two kilometers.
  - f. I live two blocks *from* Danny.
  
- (2) Expressions that can appear in partitive or classifier constructions:
  - a. Pat ate most of *the pie*.
  - b. Almost all of *the children* have brown eyes.
  - c. I wrote two chapters of *my thesis* in one day.
  - d. There are several boxes of *books* in the hallway.
  - e. Pat ate two helpings of *pie*.
  - f. I need two thousand words of *text* by tomorrow morning.
  
- (3) Expressions that can appear in comparative constructions.
  - a. Kim is *taller* than Lee.
  - b. Lee isn't as *intelligent* as Kim.
  - c. Felix bought *fewer onions* than we needed.
  - d. Felix bought more *rice* than we needed.
  - e. I *love* my dog as *much* as (I love) my baby.
  
- (4) Expressions that can be modified by 'degree terms' like *very, how, much, so, too* etc.
  - a. Kim is *very tall*.
  - b. How *intelligent* is Lee?
  - c. Felix bought *too many onions*.
  - d. I *love* my dog *very much*.
  - e. Joan ran way *up* the hill.

We might also throw in the following sorts of cases:

- (5) Expressions that show general properties of vagueness: they may be true or false of the same object in different contexts and they may have borderline cases.
  - a. The Mars Pathfinder mission was *expensive*.
  - b. My coffee is neither *cold* nor *hot*.
  - c. I stepped in a *puddle*.
  - d. Minneapolis is *near* Chicago.
  - e. CK *likes* mushrooms.

What all of these examples have in common is that their meanings can be paraphrased in terms of the amount to which an object possesses some property. This property is introduced by the italicized word in each example.

- (6) The cost of the Mars Pathfinder mission was greater than *\$n*.
- (7)
  - a. Kim's height exceeds *n* by a large amount.
  - b. The degree to which I love my dog is very high.
- (8)
  - a. Kim's height is greater than Lee's height.
  - b. The amount of onions Felix bought is less than the amount of onions we needed.
- (9)
  - a. The amount of pie that Pat ate is greater than half the total amount of pie.
  - b. The amount of books in the hallway is enough to fill several boxes.
- (10)
  - a. The amount of bluefish that I need is two pounds.
  - b. The distance from Danny's house to where I live is two blocks.

One of the crucial questions that we will address is whether these paraphrases are indicative of the actual meanings of the italicized expressions (and the various modifiers and measure terms), or whether they are 'mere paraphrases'. That is, do we really need to introduce amounts, measures, or whatever (and various operations on these things) into our semantic toolbox, or can we just get by with ordinary objects and operations on them? Part of the goal of today's class is to give a 'yes' answer to this question, but the class as a whole (I think) will hopefully end up as a big 'yes'.

In addition, we will be interested in the following questions:

- What sorts of linguistic phenomena crucially involve a semantics of measurement?
- What do these phenomena (and their analysis) tell us about the way that language encodes measurement, amount, etc.?
- What can we learn about other aspects of the grammar (e.g., lexical representation, quantification, aspect, etc.) from a close investigation of the semantics of measurement?
- Finally, is there reason to believe that a single, general semantics of measurement is at work across the different linguistic expressions that seem to involve this feature, or are there different types of measurement systems for different classes of linguistic expressions?

For the first class, I want to look specifically at the case of measurement in adjectival constructions, since this is in some sense 'where it all started'.

- Establish that we need a semantics of measurement.
- Get a basic picture of what such a semantics might look like.

The rest of the class will focus more on measurement in other grammatical contexts — in NPs, VPs, and PPs. At the end, we will hopefully have some idea of whether there are some shared semantic features unifying these different kinds of expressions.

## 2 Two semantic analyses of gradable adjectives

### 2.1 Degree-based analyses

#### 2.1.1 Basic assumptions

The following assumptions are shared by most approaches to the semantic analysis of gradable adjectives (see e.g. Seuren 1973; Bartsch and Vennemann 1973; Cresswell 1977; Hellan 1981; von Stechow 1984a; Heim 1985, 2000; Bierwisch 1989; Klein 1991; Kennedy 1999 and others):

1. Gradable adjectives map their arguments onto abstract representations of measurement, or DEGREES.
2. Degrees are formalized as points or intervals totally ordered along some DIMENSION (e.g., height, cost, etc.; the set of ordered degrees corresponds to a SCALE).
3. Propositions constructed out of gradable adjectives express relations between degrees on a scale.

The most common implementation of this general view assumes that gradable adjectives denote relations between individuals and degrees.

Specifically, GAs contain as part of their meanings a measure function and a partial ordering relation, as specified in (11). (See Bartsch and Vennemann 1973; Kennedy 1999 for approaches in which the adjective just denotes a measure function.)

- (11) a.  $\llbracket[_A \text{ expensive}]\rrbracket = \lambda d \lambda x. \mathbf{expensive}(x) \succeq d$   
b. **expensive** = a function from objects to (positive) degrees of cost

#### 2.1.2 Comparatives and degree modifiers

This approach provides a straightforward analysis of GA+degree morpheme constructions like comparatives. Specifically, we can assume that degree morphemes have interpretations along the lines of (12): they impose restrictions on the semantic value of the degree argument of the adjective.

- (12)  $\llbracket \text{Deg}(P) \rrbracket = \lambda G \lambda x. \exists d [\mathbf{R}(d) \wedge G(d)(x)]$

Different degree morphemes assign different values to the restrictive clause **R**. For example, the comparative morphemes restrict the degree argument as shown in (13), where  $d_c$  is the semantic value of the comparative clause.

- (13) a. more:  $\mathbf{R} = \lambda d. d \succ d_c$   
b. less:  $\mathbf{R} = \lambda d. d \prec d_c$   
c. as:  $\mathbf{R} = \lambda d. d \succeq d_c$

The details of the compositional interpretation of comparatives will differ a bit depending on assumptions about syntax and the syntax-semantics interface (see e.g. Heim 1985, 2000; von Stechow 1984a; Bierwisch 1989; Gawron 1995 for different implementations). One way of handling a sentence like (14a) is illustrated below.

- (14) a. The Viking Mission was more expensive than the Pathfinder Mission (was).

- b. The Viking Mission was  $[_{AP} \text{ } [_{DegP} \text{ -er [than } wh \text{ the Pathfinder Mission was } t \text{-expensive]} \text{ expensive}]]$
- (15) a.  $\llbracket [_A \text{ expensive}] \rrbracket = \lambda d \lambda x. \mathbf{expensive}(s) \succeq d$   
 b.  $\llbracket [\text{than } wh \text{ the Pathfinder Mission was } t \text{-expensive}] \rrbracket = \max\{d' \mid \mathbf{expensive}(\text{pathfinder}) \succeq d'\}$   
 c.  $\llbracket [_{DegP} \text{ -er [than } wh \text{ the Pathfinder Mission was } t \text{-expensive]}] \rrbracket = \lambda G \lambda x. \exists d [d \succ \max\{d' \mid \mathbf{expensive}(\text{pathfinder}) \succeq d'\} \wedge G(d)(x)]$   
 d.  $\llbracket [_{AP} \text{ } [_{DegP} \text{ -er than } wh \text{ the Pathfinder Mission was } t \text{-expensive}] \text{ expensive}] \rrbracket = \lambda x. \exists d [d \succ \max\{d' \mid \mathbf{expensive}(\text{pathfinder}) \succeq d'\} \wedge \mathbf{expensive}(x) \succeq d]$   
 e.  $\llbracket (14a) \rrbracket = 1$  iff  $\exists d [d \succ \max\{d' \mid \mathbf{expensive}(\text{pathfinder}) \succeq d'\} \wedge \mathbf{expensive}(\text{viking}) \succeq d]$

### 2.1.3 Measure phrases

This general approach can handle to measure phrases as well, either by analyzing them as properties of degrees or as expressions that denote degrees.

- (16) Spartacus is 13 years old.  
 a.  $\mathbf{old}(\text{spartacus}) \succeq 13\text{-years}$   
 b.  $\exists d [13\text{-years}(d) \wedge \mathbf{old}(\text{spartacus}) \succeq d]$

Schwarzschild (2002): Measure phrases are predicates of scalar intervals (= degrees). We're going to look into his proposals in a lot more detail in a couple of weeks, but here's a quick overview.

- (17) a. \$2 more expensive, 2 lbs. too heavy, 2 feet away, 2 degrees below zero  
 b. that much faster, much too spicy, that much above the house
- (18) a. Rod A is 2 inches longer than rod B (is).  
 b. Rod B is 2 inches shorter than rod A (is).
- (19) a. LENGTH: 0 ● ———  $\mathbf{long}(A)$  ——— ● ———  $\mathbf{short}(A)$  —————  $\rightarrow \infty$   
 b. LENGTH: 0 ● ———  $\mathbf{long}(B)$  ——— ● ———  $\mathbf{short}(B)$  —————  $\rightarrow \infty$   
 c.  $\mathbf{long}(A) - \mathbf{long}(B) =$  ● —●  
 d.  $\mathbf{short}(B) - \mathbf{short}(A) =$  ● —●
- (20) a.  $\llbracket [2 \text{ inches}] (\mathbf{long}(A) - \mathbf{long}(B)) \rrbracket = 1$   
 b.  $\llbracket [2 \text{ inches}] (\mathbf{short}(B) - \mathbf{short}(A)) \rrbracket = 1$

This analysis of measure phrases suggests a meaning along the lines of (21) for the comparative morpheme, possibly with existential closure over the difference function when a differential degree is not expressed:

$$(21) \quad \llbracket \text{more} \rrbracket = \lambda d_1 \lambda d_2 \lambda f_{(d,t)}. f(d_1 - d_2)$$

This is more or less the way things are done in Schwarzschild and Wilkinson 2002.

#### 2.1.4 The positive form

Paradoxically, the analysis of ‘simple’ predicates involving GAs — henceforth the POSITIVE FORM — turns out to be the most problematic within this set of assumptions.

At first, things seem straightforward. To keep things fully compositional, we may assume that a null degree morpheme *pos* introduces an existential quantifier with an implicit restriction variable to bind the degree argument of the adjective, as in (22) (Cresswell 1977; von Stechow 1984a).

$$(22) \quad \llbracket [\text{Deg } \text{pos}] \rrbracket = \lambda G_{\langle d,et \rangle} \lambda x. \exists d [\mathbf{C}(d) \wedge G(d)(x)]$$

This gives us the truth conditions for e.g. (5a) shown in (23).

$$(23) \quad \llbracket \text{The Mars Pathfinder Mission was expensive} \rrbracket = 1 \text{ iff} \\ \exists d [\mathbf{C}(d) \wedge \mathbf{expensive}(\text{pathfinder}) \succeq d]$$

$\mathbf{C}$  here determines the STANDARD OF COMPARISON: the degree to which an object must possess some gradable property in order for a predicate like ‘(is) expensive’ to be true of it.

Crucially, since the restriction on the degree argument in (23) is a variable, we allow for the possibility of different standards of comparison in different contexts:

- If the value of  $\mathbf{C}$  is fixed to pick out degrees above the cost of the Mars Pathfinder Mission, then (23) will be false.
- If it picks out degrees below the cost of the mission, then (23) will be true.

The truth conditions of (5a) are therefore predicted to vary in just the way we want. Moreover, this analysis, plus (something like) the analysis of comparatives presented above, will capture an important generalization: there is typically no entailment relation between the comparative and positive forms.

- (24)
- a. This bottle of wine is more expensive than that one, but neither is expensive.
  - b. Both the Viking Mission and the Pathfinder Mission were expensive, but the former was more expensive than the latter.

This fact provides an important argument for separating out the calculation of the standard from the lexical meaning of GAs.

Unfortunately, when we take a closer look at GAs in the ‘positive’ form, we see that this basic approach runs into problems. This is a topic for another class, however. (See Kennedy 2003 for detailed discussion.)

#### 2.1.5 Summary

One thing is clear: if this is the right way to handle the semantics of gradable adjectives, measure phrases, etc., then we ought to find independent linguistic evidence that measures (formalised in terms of scales and degrees) have linguistic significance. That is, if representations of measurement are part of the semantic ontology, then we should find constructions that are sensitive to properties of these objects.

### 2.1.6 Some additional questions

For those who want to think more about some of the issues outlined here:

- (25)
- a. How exactly do linguistic vs. contextual factors influence the value of the standard of comparison?
  - b. What about GAs in prenominal (attributive) position?
  - c. What about cases where we do get entailments from e.g. comparatives to the unmarked form?
  - d. What is the right logical form for comparatives?

## 2.2 Vague predicate analyses

### 2.2.1 The basics

The degree-based analysis we discussed above in effect takes the clear examples of measurement — comparatives, measure phrases, degree modifiers — as indicative of the underlying semantics. Because of this, GAs are assigned a different semantic type from other predicative expressions (relations between objects and degrees.)

A very different sort of approach to the semantic analysis of gradable adjectives is developed by Klein (1980) (see also McConnell-Ginet 1973; Kamp 1975; Fine 1975). Klein’s analysis, in contrast, takes the positive form as ‘basic’, starting from the assumption that GAs have the same semantic type as other predicates — they are functions from objects to truth values. They differ in two crucial ways, however:

1. They are partial functions from  $U$  to  $\{0,1\}$ . That is, a gradable adjective  $G$  partitions its domain into three sets:
  - (a) the positive extension of  $G$ :  $\{x \mid \llbracket G(x) \rrbracket = 1\}$
  - (b) the negative extension of  $G$ :  $\{x \mid \llbracket G(x) \rrbracket = 0\}$
  - (c) the extension gap of  $G$ :  $\{x \mid \llbracket G(x) \rrbracket \text{ is undefined}\}$
2. Their interpretations are context-dependent, such that the exact compositions of the positive and negative extensions and extension gap are a function of the context of utterance.

Specifically, Klein assumes that every context  $c$  determines a *comparison class* of objects that provides the domain of the adjective. This allows him to maintain the assumption that e.g., *expensive* has a single core meaning along the lines of (26):

- (26) For any context  $c$  and comparison class  $\mathbf{C}(c)$ ,
- a.  $\llbracket expensive(x) \rrbracket_c = 1$  iff  $x$  is definitely expensive,
  - b.  $\llbracket expensive(x) \rrbracket_c = 0$  if  $x$  is definitely not expensive, and
  - c.  $\llbracket expensive(x) \rrbracket_c$  is undefined otherwise.

The variability comes from the fact that the composition of the comparison class will affect what does and does not ‘count as’ definitely expensive.

- (27) The Mars Pathfinder mission was expensive.
- a.  $\mathbf{C}(c_1) = \{x \mid x \text{ is named ‘Pathfinder’}\}$

- b.  $\llbracket expensive(p) \rrbracket_{c1} = 1$
- c.  $\mathbf{C}(c_2) = \{x \mid x \text{ is a mission to outer space}\}$
- d.  $\llbracket expensive(p) \rrbracket_{c2} = 0$

But what decides whether something is ‘definitely expensive’ or not? Note that there is nothing in Klein’s representations that corresponds to the standard of comparison — this is evidently outside the linguistic system. (This is an important difference from the degree analysis, where the answer to this question is ‘greater than the standard of comparison’.)

### 2.2.2 Comparatives and degree modifiers

Turning to comparatives, Klein’s analysis starts from his analysis of degree modifiers in which the function of a degree modifier is to provide a new comparison class for the adjective it modifies. For example, *very expensive* has the meaning in (28).

$$(28) \quad \llbracket very(expensive) \rrbracket_c = \llbracket expensive \rrbracket_{c[X]}, \text{ where } X \text{ is the positive extension of } expensive \text{ at } c.$$

In other words, *very expensive* is a vague predicate just like *expensive*, except that the comparison class for the former is just the positive extension of the latter.

- The analysis of degree modifiers suggests a straightforward explanation of examples like *Julian is a short tall boy*, *Julian is a tall tall boy*, etc., or stacked modifiers *Julian is very very tall*.

Comparatives involve existential quantification over degree modifier meanings:

$$(29) \quad \begin{array}{l} \text{a. The Viking mission was more expensive than the Pathfinder mission.} \\ \text{b. } \exists D[\llbracket D(expensive)(v) \rrbracket_c = 1 \wedge \llbracket D(expensive)(p) \rrbracket_c = 0] \end{array}$$

In other words, (29a) is true in a context  $c$  just in case there is some way of fixing the comparison class for *expensive* such that *The Viking mission was expensive* is true wrt that  $c$ -class and *The Pathfinder mission was expensive* is false wrt that  $c$ -class.

Since the derived  $c$ -class need not be the same as the one determined by the context of utterance, we will not get entailments from the comparative form to the unmarked form.

$$(30) \quad \begin{array}{l} \text{A couple questions:} \\ \text{a. This analysis of } very \text{ seems to predict that } very \text{ expensive presupposes that its} \\ \quad \text{argument is expensive. Is this correct?} \\ \text{b. What does the analysis predict about things like } ??Julian \text{ is more } very \text{ tall} \\ \quad \text{than Nigel?} \end{array}$$

### 2.2.3 Measure phrases

Klein actually doesn’t give a very good analysis of measure phrases, and it’s difficult to see exactly how this would work. See von Stechow 1984a for detailed criticism along these lines.

#### 2.2.4 Summary

Crucially, since this analysis crucially does not introduce scales and degrees into the linguistic system — for Klein, *there is no ‘semantics of degree’!* — we should not expect to find phenomena that are crucially sensitive to properties of these objects. We should however expect to find phenomena that are crucially sensitive to (properties of) the comparison class (though we probably also expect this on a degree analysis).

### 3 Arguments for degrees

Are scales and degrees merely convenient formal tools for representing the meanings of gradable adjectives, or do their properties have linguistic significance? How can we tell?

Formally, a scale can be defined as a set of objects  $S$  asymmetrically ordered along some dimension  $\delta$ .

$$(31) \quad \langle S, \succeq_{\delta} \rangle$$

To see whether scales are linguistically significant, we should look for phenomena that are sensitive either to properties of the objects ordered (degrees) or to properties of the ordering relation.

#### 3.1 Incommensurability

Evidence that the nature of the ordering relation (in particular, the dimensional parameter) is linguistically significant: *incommensurability*. The examples in (32) show that we can compare objects according to what look like different gradable properties:

- (32) a. They call him ‘The Bus’ because he’s kind of as wide as he is tall. (National Public Radio broadcast, 1/26/02)  
b. [This comparison] is unfair both to him and the quarterbacks like Dan Marino and John Elway who excelled for almost as long as [Peyton] Manning is old. (*Chicago Tribune*, 11/2/00)

These properties must be commensurable, however:

- (33) a. ??They call him ‘The Bus’ because he’s kind of as wide as he is punctual.  
b. ??These quarterbacks excelled for almost as long as Peyton Manning is talented.

What’s the difference between (33) and (32)?

Assuming that orderings along different dimensions entail different scales, and that comparative morphemes presuppose that the degrees they order come from the same scale (see Kennedy 2001), the examples in (33) are correctly predicted to be anomalous.

In contrast, the pairs of adjectives in (32) make use of the same scale. However, they still differ with respect to the measure functions they incorporate, which provide different perspectives on the property they measure. For example, *wide* corresponds to a horizontal perspective on linear extent, and *tall* to a vertical one, with the result that the two adjectives impose different orderings on the same domains.

### 3.2 Scale structure

Turning to scale structure, several different properties of the scale could in principle be linguistically significant:

- whether the set of degrees is finite or infinite
- whether it is dense or discrete
- whether it contains minimal or maximal elements
- ...

Let's focus on the OPEN (no minimal/maximal elements) vs. CLOSED (minimal/maximal elements) distinction.

Intuitively, this looks like the right way to characterize the difference between the adjectives in (34a) and those in (34b).

- (34) a. *Open scale adjectives?*  
long, short, fast, slow, interesting, inexpensive, ...  
b. *Closed scale adjectives?*  
empty, full, dry, clean, open, closed, ...

This intuition is supported by linguistic data involving PROPORTIONAL MODIFIERS like *completely*, *partially*, and *half*, which are acceptable with some gradable adjectives and unacceptable with others:

- (35) a. completely {empty, full, open, closed}  
b. partially {empty, full, open, closed}  
c. half {empty, full, open, closed}
- (36) a. ??completely {long, short, interesting, inexpensive}  
b. ??partially {long, short, interesting, inexpensive}  
c. ??half {long, short, interesting, inexpensive}

**An interesting possibility:** Might these expressions be actually modifying the argument of the adjective, and so don't actually tell us anything about scale structure?

- (37) a. The shirt is completely/half/partially dry.  
b. All/half/part of the shirt is dry.

This may be possible (it's worth exploring at least when we talk about partitives), but I think that these modifiers also need to be able to modify the adjectives.

- (38) a. Eight glasses are half full.  $\neq$  Half of eight glasses are full.  
b. Few rooms were completely empty.  $\neq$  All of few rooms were empty.
- (39) a. ??The books were completely inexpensive.  
b. All of the books were inexpensive.
- (40) a. ??The boys were half tall.  
b. Half of the boys were tall.

- (41) a. ??The shirt is half drier than the pants.  
 b. Half of the shirt is drier than the pants.

If the modifiers *completely*, *half* and *partially* have interpretations along the lines of those in (42), where  $S(G)$  returns the scale associated with a gradable adjective  $G$ , they should be compatible only with adjectives that map their arguments onto scales with maximal or minimal elements. (The DIFF function returns the difference between two degrees; see Kennedy 2001.)

- (42) a.  $\llbracket \text{completely} \rrbracket = \lambda G \lambda x. \exists d [d = \max(S(G)) \wedge G(d)(x)]$   
 b.  $\llbracket \text{half} \rrbracket = \lambda G \lambda x. \exists d [\text{DIFF}(\max(S(G)), d) = \text{DIFF}(d, \min(S(G))) \wedge G(d)(x)]$   
 c.  $\llbracket \text{partially} \rrbracket = \lambda G \lambda x. \exists d [d \succ \min(S(G)) \wedge G(d)(x)]$

See Kennedy and McNally 2002 and Kennedy 2003 for additional arguments from the distribution of degree modifiers and the interpretation of the positive form that scale structure is a linguistically significant semantic feature.

### 3.3 Positive and negative degrees

Most GAs come in pairs: a ‘positive’ form and a ‘negative’ form:

- (43) a. tall/short, fast/slow, wide/narrow, old/young  
 b. smart/stupid, good/bad, easy/difficult  
 c. interesting/uninteresting, likely/unlikely

Antonymous pairs of adjectives provide the same kind of information about an object (both ‘tall’ and ‘short’ characterize an object’s height, for example), but they provide different perspectives on the projection of an object on a scale.

Seuren 1978; von Stechow 1984b; Kennedy 2001: We should capture this intuition in terms of a structural distinction between two sorts of degrees: *positive* and *negative degrees*.

Roughly speaking, positive degrees are intervals that range from the lower end of a scale to some point, and negative degrees are intervals that range from some point to the upper end of a scale.

Specifically, the set of positive and negative degrees for any scale  $S$  ( $POS(S)$  and  $NEG(S)$ , respectively), can be defined as in (44).

- (44) a.  $POS(S) = \{d \subseteq S \mid \exists p_1 \in d \forall p_2 \in S [p_2 \preceq p_1 \rightarrow p_2 \in d]\}$   
 b.  $NEG(S) = \{d \subseteq S \mid \exists p_1 \in d \forall p_2 \in S [p_1 \preceq p_2 \rightarrow p_2 \in d]\}$

Finally, assume that for any object  $x$ , the positive and negative projection of  $x$  on a scale  $S$  ( $pos_S(x)$  and  $neg_S(x)$ , respectively) are related as in (45), where MAX and MIN return the maximal and minimal elements of an ordered set.

- (45)  $\text{MAX}(pos_S(x)) = \text{MIN}(neg_S(x))$

The positive and negative projections of an object  $x$  on a scale  $S$  are (join) complementary intervals on the scale, as illustrated by the diagram in (46).

- (46)  $S: 0 \text{ ————— } pos_S(x) \text{ ————— } \bullet \text{ ————— } neg_S(x) \text{ ————— } \rightarrow \infty$

A result of this analysis is that the set of positive degrees on a scale  $S$  and the set of negative degrees on  $S$  are disjoint.

Adjectival polarity can thus be characterized as a difference in the ranges of the measure functions expressed by positive and negative adjectives:

- Positive adjectives express functions from individuals to positive degrees.
- Negative adjectives express functions from individuals to negative degrees.

Antonymy holds when two adjectives have the same domains but different ranges, and they map identical arguments onto (join) complementary regions of the same scale.

In addition to requiring actual degrees in the first place, this sort of analysis makes a broader claim: degrees (and representations of measurement in general?) must be formalized as intervals, not points.

There are two empirical arguments for this sort of approach.

### 3.3.1 *Measure phrases*

Measure phrases are acceptable only with positive GAs, though not all positive adjectives accept them (even when a standard of measurement is defined):

- (47) a. Julian is two feet tall.  
b. ??Julian is two feet short.
- (48) a. ??Hillary was driving 160 mph fast.  
b. ??Hillary was driving 20 mph slow.

‘Differential’ measure phrases are acceptable with both positive and negative adjectives in comparatives; even with adjectives that don’t accept them otherwise:

- (49) a. Julian is six inches taller/shorter than his cousin.  
b. Hillary was driving 10 mph faster/slower than she should have been.

This seems to be true cross-linguistically: examples like (47a) are not possible in Japanese and Korean, for example (instead we get something like *Julian has two feet of height*), but comparatives like (49a) are OK.

We can explain these facts in terms of Schwarzschild’s proposal that MPs are predicates of scalar intervals, by assuming that the functions denoted by measure phrases are defined only for degrees on particular scales, determined by the head of the MP, and for degrees of particular sorts — closed intervals on a scale (cf. von Stechow 1984b).

- (50)  $\llbracket \text{MP} \rrbracket = \lambda d_d: d$  is a degree on scale  $S$  and  $d$  has minimal and maximal values. $d$  measures MP-much

The first condition will handle cases like *?8 years tall*. The second condition will handle cases like (51b), since negative degrees will typically not satisfy this condition.

- (51) a. The plant is 17 inches high.  
b. ??The plant is 17 inches low.

Maybe some positive adjectives do not include the minimal value of the corresponding scale in their range, even if the scale is a closed one?

- (52) a. ??This rock is 30 ounces heavy/light.  
b. This rock is 30 ounces heavier/lighter than that one.  
c. This rock has a weight of 30 ounces.

Differential degrees in comparatives will always satisfy the second condition in (50), so we're all set there.

### 3.3.2 Cross-polar anomaly

Kennedy 2001 shows that comparatives constructed out of antonymous adjectives are anomalous:

- (53) a. ??Alice is taller than Carmen is short.  
b. ??*The Brothers Karamazov* is longer than *The Idiot* is short.  
c. ??The Mars Pathfinder mission was cheaper than the Viking mission was expensive.

These facts actually can't be explained either in a vague predicate analysis or in a degree analysis that represents degrees as points. It follows quite naturally in the system outlined above, however:

1. The arguments of an ordering relation must be elements of the same ordered set. We may assume in particular that this requirement is part of the meaning of expressions of ordering in natural language: the comparative morphemes 'more', 'less' and 'as' presuppose that their degree arguments are elements of the same ordered set.
2. If this requirement is not met, the relations denoted by comparative morphology are undefined, and a truth value cannot be computed.
3. Positive and negative degrees are elements of disjoint sets.
4. Therefore, a comparative that defines an ordering between positive and negative degrees violates this requirement.

For example, the (simplified) logical representation of (53a) is (54).

$$(54) \quad \mathbf{short}(a) \succ \mathbf{tall}(c)$$

**short**(*a*) and **tall**(*c*) denote degrees in different ordered sets (*NEG*(*height*) and *POS*(*height*), respectively). The ordering relation introduced by the comparative morpheme is undefined for its two arguments, rendering the sentence anomalous. (Examples in which the adjectives are reversed are explained in exactly the same way.)

## 3.4 Summary

There is strong linguistic evidence from the domain of gradable adjectives that we need a semantics of measurement. Whether the sort of framework that has been shown to be useful for explaining properties of gradable adjectives will extend to other cases of measurement is a question that we will hopefully answer before the quarter is over!

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