Introduction to Semantics: Homework 4
Answer key

The meaining of *the*

One thing to notice about sentences like (1) is the presuppositions it brings about.

(1) The doctor smokes.

Such a sentence presupposes that there is a unique doctor and asserts that this doctor smokes. The presuppositions of existence and uniqueness can be shown by embedding sentences containing *the* under negation and observing that the relevant inferences are unchanged.

(2) a. There isn't any doctor. #The doctor smokes.
  b. There isn't any doctor. #The doctor doesn't smoke.

(3) a. There are two doctors. #The doctor smokes.
  b. There are two doctors. #The doctor doesn't smoke.

(2a) and (3a) are infelicitous, suggesting that sentences with *the* at least entail the existence and uniqueness of the individual satisfying the property denoted by the noun they combine with. Moreover, so are (2b) and (3b), suggesting that this entailment is also a presupposition.

Before moving on to an analysis of the definite determiner *the*, just a recap on the notation presented in class for representing both sets of simple meanings (e.g., individuals), assigned atomic types, and sets of complex meanings (e.g., functions from individuals to truth values), assigned complex types.

(4) a. $\mathrm{D_e}$ is the set of individuals (e is the type of individuals)
  b. $\mathrm{D_t}$ is the set of 1 and 0, i.e., TRUE and FALSE (t is the type of truth values)
  c. $\mathrm{D_s}$ is W, the set of worlds (s is the type of worlds)

(5) For any two sets D_α and D_β, $\mathrm{D}_{\langle\alpha,\, \beta\rangle}$ is the set of functions from (subsets of) D_α to D_β ($\langle\alpha, \beta\rangle$ is the type of such functions)

Now, given that a noun like *doctor* denotes as in (6),[1]

(6) $[\![doctor]\!] = f\colon \mathrm{D_e} \to \mathrm{D}_{\langle \mathrm{s,\, t}\rangle}$, such that for any $x \in \mathrm{D_e}$, $f(x)$ is the function mapping worlds in which x is a doctor to 1, and other worlds to 0

[1] See any remaining needed definitions of notation from the previous homework's answer key.

the meaning of *the* can be given as follows.[2]

(7) $[\![the]\!] = F$: $D_{\langle e, \langle s, t\rangle\rangle} \rightarrow D_{\langle s, e\rangle}$, such that for any $f \in D_{\langle e, \langle s, t\rangle\rangle}$, $F(f)$ is g: $W \rightarrow D_e$, such that, for any world w in which g is defined,
(a) $f(g(w))(w) = 1$ and,
(b) for any $y \in D_e$, either $f(y)(w) = 0$ or $y = g(w)$

Based on such denotations, a noun phrase like *the doctor* has the meaning in (9), given the principle in (8).

(8) **Function Application**: If A has meaning $[\![A]\!]$ and B has meaning $[\![B]\!]$, and $[\![A]\!]$ is a function with $[\![B]\!]$ in its domain, then $[\![A \circ B]\!] = [\![B \circ A]\!] = [\![A]\!]([\![B]\!])$, where $A \circ B$ and $B \circ A$ are expressions concatenating A and B.

(9) $[\![\textit{the doctor}]\!] =$
$[\![the]\!]([\![doctor]\!]) =$
g: $W \rightarrow D_e$, such that, for any world w in which g is defined
(a) w is in the set of worlds in which $g(w)$ is a doctor, and
(b) for any $y \in D_e$, either w is not in the set of worlds in which y is a doctor, or $y = g(w)$

Given any particular world w, $[\![\textit{the doctor}]\!](w)$ is an individual x, such that x is a doctor in w, and for any individual y, either y is not a doctor in w, or y and x are the same individual. The condition that x is a doctor is what guarantees the existence presupposition discussed above, and the condition that the only individual who is a doctor is x is what guarantees the uniqueness presupposition.

Given such a meaning for *the doctor*, *the doctor smokes* can be interpreted by assuming a denotation for *smokes* as in (10).

(10) $[\![smokes]\!] = f$: $D_{\langle s, e\rangle} \rightarrow D_{\langle s, t\rangle}$, such that, for any function g from W to D_e and any world w in which $f(g)$ is defined, $f(g)(w) = 1$ just in case $g(w)$ smokes in w

The interpretations of *the doctor* and *smokes* combine via Function Application to yield the following meaning.

(11) $[\![\textit{the doctor smokes}]\!] = h$: $\{w \in W \mid$ there is an x, such that (a) x is a doctor in w, and (b) for any y, either y is not a doctor in w, or $y = x\} \rightarrow \{1,0\}$, such that for any w on which h is defined, $h(w) = 1$ just in case the unique doctor in w smokes in w

[2]Reminder: '1' stands for TRUE and '0' stands for FALSE.

This function characterizes sets of worlds in which there is a unique doctor who smokes, and, moreover, is only defined on worlds in which there is a unique doctor. This semantic definedness condition is meant to explain the presuppositions of sentences containing *the*, as observed above, for example.