Introduction to Semantics: Homework 1
Answer key

Since

(1) the basic expressions of our language are D = {‘0’, ‘1’, ‘2’, . . . , ‘9’}, whose meanings are 0, 1, 2, . . . , 9 ∈ ℕ,

and our grammar is

(2)
a. If x is a D with meaning $[\![x]\!]$, then x is an N with meaning $[\![x]\!]$.
b. If x is an N with meaning $[\![x]\!]$, and y is a D with meaning $[\![y]\!]$, then xy is an N with meaning $(10 * [\![x]\!]) + [\![y]\!]$.
c. If x and y are Ns with meanings $[\![x]\!]$ and $[\![y]\!]$, respectively, then
   (i) x ‘+’ y is an N with meaning $[\![x]\!] + [\![y]\!]$.
   (ii) x ‘–’ y is an N with meaning $[\![x]\!] - [\![y]\!]$.
   (iii) x ‘*’ y is an N with meaning $[\![x]\!] * [\![y]\!]$.
   (iv) x ‘÷’ y is an N with meaning $[\![x]\!] / [\![y]\!]$.
   (v) x ‘=’ y is an S with meaning TRUE if $[\![x]\!] = [\![y]\!]$ and FALSE otherwise.

the expression ‘2 * 3 + 10 = 26’ is ambiguous. In particular, it is assigned TRUE by a derivation using (2) in which ‘3 + 10’ is derived in a sub-derivation, and it is assigned FALSE by a derivation using (2) in which ‘2 * 3’ is derived in a sub-derivation. This is shown by deriving them as follows, where strings (and statements of their category membership) and meanings generated by the grammar are given as pairs. Look at the second lines of each derivation.

L1 ⟨‘10’ ∈ N, 10⟩	(1), (2-a), (2-b)
L2 ⟨‘2 * 3’ ∈ N, 6⟩	(1), (2-a), (2-c-iii)
L3 ⟨‘2 * 3 + 10’ ∈ N, 16⟩	L1, L2, (2-c-i)
L4 ⟨‘26’ ∈ N, 26⟩	(1), (2-a), (2-b)
L5 ⟨‘2 * 3 + 10 = 26’ ∈ S, FALSE⟩	L3, L4, (2-c-v)

L1 ⟨‘10’ ∈ N, 10⟩	(1), (2-a), (2-b)
L2 ⟨‘3 + 10’ ∈ N, 13⟩	L1, (1), (2-a), (2-c-i)
L3 ⟨‘2 * 3 + 10’ ∈ N, 26⟩	L2, (1), (2-a), (2-c-iii)
L4 ⟨‘26’ ∈ N, 26⟩	(1), (2-a), (2-b)
L5 ⟨‘2 * 3 + 10 = 26’ ∈ S, TRUE⟩	L3, L4, (2-c-v)