Counting and the Mass/Count Distinction

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Abstract
This article offers an account of the mass/count distinction and the semantics of count nouns, and argues that it is not based on an atomic/non-atomic nor on a homogeneous/non-homogeneous distinction. I propose that atomicity in the count domain is atomicity relative to a context k, where k is a set of entities that count as atoms (i.e. count as one) in a particular context. Assuming for simplicity Chierchia’s (1998a) and Rothstein’s (2004) theory of mass nouns, in which they denote atomic Boolean semi-lattices closed under the complete join operation, we define an operation \( \text{COUNT}_k \) that applies to the mass noun denotation \( N_{\text{mass}} \) and derives the count noun meaning: a set of ordered pairs \( <d,k> \) where \( d \) is a member of \( N \cap k \) and \( k \) is the context \( k \) relative to which the operation applied. So, there is a typal distinction between mass nouns, which are of type \( <d,t> \), and count nouns, which are of type \( <d \times k, t> \). The grammatical differences between count and mass nouns follow from this typal distinction. This allows us to encode grammatically the distinction between semantic atomicity, that is, atomicity relative to a context \( k \), and natural atomicity, that is, inherent individuability. We show a number of ways in which this distinction is grammatically relevant.

1 INTRODUCTION
This article offers an account of the semantics of the mass/count distinction, focusing on the semantics of count nouns. The mass/count distinction has interested linguists at least since Jespersen (1924) because it apparently provides a link between cognitive mechanisms of individuation and linguistic mechanisms for counting. In a language that has a mass/count distinction, some nouns can be directly modified by numeral expressions (e.g. three cats) and others cannot (e.g. *three furniture(s)). While the mass/count distinction apparently reflects the fundamental ontological distinction between ‘stuff’ or ‘substance’ and ‘objects’ or ‘things’, it is nonetheless an independent grammatical distinction that cannot be learned on the basis of the conceptual distinctions between stuff and objects. Count nouns naturally denote individuable entities or bounded entities with stable spatial properties across time (e.g. cat, boy, table, book), while mass nouns are associated
with substances that take their spatial dimensions from containers (e.g. water, mud), or whose physical boundedness varies over time or depends on the artefact constructed from it (e.g. wood, gold). Soja et al. (1991) have shown that infants are sensitive to the difference between individual objects and substance, and Prasada et al. (2002) have shown that children are aware that count nouns are the canonical form for denoting objects while mass nouns are the canonical form for denoting substances. Nonetheless, it is well known that despite the association of count terms with individuable objects and mass terms with substances, there are many mismatches between the grammatical form and the properties of the denotation. Nouns like furniture and jewellery denote individuable entities, and there is cross-linguistic variation in mass/count categorization (e.g. jewellery is mass in English but Hebrew taxšit/taxšitim is count). This indicates that the grammatical distinction is not a direct reflection of the conceptual distinction. Furthermore, as well as direct counting, grammatical operations like partitive construction and reciprocal resolution are sensitive to the distinction between mass and count nouns. Example (1) shows that numerical partitives distinguish between definite DPs with mass noun heads and definite DPs with count noun heads. The examples in (2) from Gillon (1992) show that reciprocal resolution is also sensitive to the mass/count distinction:

(1) a. Three of the books were damaged in transit.
   b. #Three of the furniture were damaged in transit.
   c. Three of the pieces of furniture were damaged in transit.

(2) a. The curtains and the carpets resemble each other.
   b. The curtaining and the carpeting resemble each other.

Example (2a) is ambiguous between the distributive reading where each curtain and carpet resembles the other curtains and carpets, and the collective reading where the carpets, as a plural entity, resemble the curtains as a plural entity and vice versa. In (2b), only the second reading is possible.

There are essentially three ways of trying to characterize a semantic distinction between mass and count nouns: the first, proposed by Link (1983), is to argue that mass nouns and count nouns have their denotations in different domains. The second is to propose that mass and count nouns denote entities of different types but that they are interpreted with respect to the same domain. This is the direction taken by Krifka (1989), who derives count nouns from mass meanings and suggests that count nouns denote extensive measure functions on entities in the mass domain. The third approach is represented by Chierchia (1998a), who argues that mass nouns and count nouns are not distinguished typally and
have their denotations in the same domain but that count nouns make a set of atoms lexically accessible.¹ In this article, I take the second approach and argue, like Krifka (1989), that count noun meanings are derived from mass noun meanings and are typically different from them but not in the way that Krifka suggests. I show that the assumption that (singular) count predicates make a set of atoms accessible is, by itself, insufficient to explain the behaviour of count nouns and that we need a theory of what atomicity is and how count nouns access the atoms grammatically. I start from the observation that the mismatch between form and denotation is two-way: as well as mass terms such as furniture and jewellery that denote sets of inherently individuable objects, there are also count terms that denote sets of entities that do not have spatial properties constant across time. Examples are fence, wall and bouquet. In order to treat these predicates as atomic, we need to know how the atomic elements in their denotation are determined. I shall propose that atomicity is context dependent and that part of specifying a context is specifying what counts as one entity in that context. Count nouns denote sets of entities indexed for the context in which they count as one. This results in a typal distinction between mass and count nouns that (unlike the typal distinction in Krifka’s account) is projected up to the DP, which is the extended projection of the nominal head. This allows us to explain the variety of grammatical differences between mass and count nouns, including the examples in (1) and (2).

More generally, I will argue that to explain the grammatical behaviour of mass and count nouns we need to distinguish between three kinds of atomicity: formal atomicity, that is, being an atom in a Boolean structure; natural atomicity, or being inherently individuable; and semantic atomicity, which I shall define in the course of the article, and which is the atomicity characterizing singular count predicates. I argue that this allows a theory of the mass/count distinction that is both cognitively and semantically plausible and that explains linguistic behaviour.

The article is structured as follows. In the next section, I review evidence that the mass/count distinction is a grammatical distinction independent of the structure of matter. In sections 3 and 4, I review previous accounts of the mass/count distinction and look briefly at the psycholinguistic evidence from Barner and Snedeker (2005). Sections 5–7 set up a semantics for count nouns based on the three-way

¹ There is also a syntactic approach that argues that there is no lexical or semantic difference between mass and count terms, and that all differences follow from the syntactic structure in which they are inserted. Borer (2005) is the most recent representative of this approach, which I discuss in section 10.
distinction between formal, natural and semantic atomicity. Section 8 compares this theory with Krifka (1989), and section 9 makes some preliminary extensions of the theory to classifier languages. In section 10, I argue against an approach that tries to derive the mass/count distinction from syntactic structure.

2 WHAT IS THE MASS/COUNT DISTINCTION?

The mass/count distinction, illustrated in (3), is the distinction between nouns that can and nouns that cannot be explicitly counted by using numeral modifiers:

(3) a. three girls,
    b. * three muds,
    c. three kilos of mud,
    d. three buckets of mud.

Girl is a count noun since it can be directly modified by the numeral modifier three, while mud, a mass noun, can only be counted via a classifier expression like kilo or bucket. Classifiers may be individuating or they may be non-individuating measure phrases. On the measure reading of bucket, three buckets of mud denotes a quantity of mud and is synonymous with three bucketfuls of mud. In the individuating use of (3d), it refers to three actual buckets filled with mud as in three buckets of mud were standing in a row. Doetjes (1997), Landman (2004) and Rothstein (2009a) discuss some of the different syntactic and semantic properties of these two uses of classifier expressions.

The distinction illustrated in (3) is a genuine grammatical distinction with linguistic implications. Count nouns are associated with a number of different syntactic and morphological properties, though not all grammatical differences appear in each language. The major differences that characterize the mass/count distinction are listed in (i) and (ii).

(i) properties of the noun
    (a) count nouns occur with numeral determiners, mass nouns do not:
        three chairs, *three furniture
    (b) count nouns take plural morphology, mass nouns do not:
        chair/chairs, furniture/furnitures
    (c) count nouns do not normally occur in the singular with classifiers, mass nouns do:
        *three pieces of chair/three pieces of furniture.

(ii) sensitivity of determiners to the mass/count distinction
    (a) some determiners select only count nouns:
each/every/a book, several/few/many books, *every/*several furniture(s)

(b) some determiners select only mass nouns:
   little/much water, *little/*much book(s)

(c) some determiners select mass and plural nouns:
   a lot of/plenty of wine, a lot of/plenty of books, *a lot of/*plenty of book,

(d) some determiners are unrestricted:
   the/some book(s), the/some water.

Not all of these properties show up in all languages that distinguish between mass and count nouns. In Turkish, numerals modify only count nouns, but these nouns are not marked for the singular/plural distinction. Similarly, the selectional properties of determiners show up differently in different languages. For example, Hebrew does not have the distinction between (how) much and (how) many, although there are other ways to determine which nouns are mass and which are count. In general, while identical diagnostics may not be available cross-linguistically, variations on the diagnostics show that the mass/count distinction is one that occurs in many languages.

The mass/count distinction is independent of the ‘structure of matter’. Chierchia (1998a) sums up many years of discussion by bringing four arguments in support of this claim:

(i) Entities that come in natural units of equal perceptual salience may differ in a single language as to whether they are mass or count, for example, rice is mass, while lentil/lentils is count.

(ii) Within a single language, there are pairs of synonyms, or near-synonyms, where one member of the pair is count and the other is mass.
   English: footwear/shoes, change/coins, carpeting/carpets, hair/hairs, rope/ropes, stone/stones.
   Dutch: het meubilair (the furniture)/het meubel (the piece of furniture).
   Hebrew: rihut (furniture)/rehit (piece of furniture).

Note that these synonyms are of three kinds:

(a) hair/hairs, a single lexical item can be realized as mass or count;
(b) carpeting/carpets, two lexical items based on the same root are related by a morphological operation. One has a mass use and the other has a count use; and
(c) footwear/shoes, there is no lexical relation between near-synonyms where one is mass and the other count.

(iii) Mass expressions in one language have count near-synonyms in another; for example, advice is mass in English but the Hebrew
equivalent *etza* has only a count use, as the contrast between (4a) and the ungrammatical (4b) shows.

(4) a. hi natna li śalos etzo
tshe gave me three-f advice-f-pl
‘She gave me three pieces of advice/*three advices’
b. hi natna li harbe etza
tshe gave me much advice-f-sg
intended reading: ‘She gave me much advice’

(iv) Some languages, such as Chinese, have only nouns that behave as mass expressions. Count usages require classifiers; for example, Chinese *xióng*, ‘bear’, is a mass expression, and counting requires a classifier (from Krifka 1995):

(5) a. sān zhī xióng
three classifier bear
‘three bears’ (objects)
b. sān qún xióng
three herd bear
‘three herds of bears’
c. sān zhōng xióng
three classifier bear
‘three bears’ (species)

These cross-linguistic differences show that while the mass/count distinction is clearly influenced by the structure of matter, it is not taken over from it. So, the question is: what is at the root of the mass/count distinction?

3 PREVIOUS ACCOUNTS OF THE MASS/COUNT DISTINCTION

3.1 Homogeneity and/or cumulativity is not at the root of the mass/count distinction

Much discussion of the mass/count distinction has focused on the downward and upward closure properties of the two kinds of nominals. Mass predicates are cumulative since their denotations are upwardly closed and homogeneous (divisible) since their denotations are downwardly closed: water + water forms a (possibly discontinuous) entity also in the denotation of *water*, while a quantity of water split into two gives two quantities of stuff both in the denotation of *water*. (Divisibility is usually said to work down to minimal parts, an issue which I will not discuss here.) This contrasts with singular count nouns that are characterized as neither cumulative nor homogeneous. The sum
of two entities in the denotation of *cup* cannot itself be in the denotation of *cup* (but only in the denotation of the plural *cups*), while splitting a cup into two more or less equal parts gives you two pieces, neither of which is in the denotation of *cup*. It is possible to split a cup unequally into, say, a chip and the rest of the cup, and the larger part will probably still count as a cup, but this contrasts with *water*, where splitting a quantity of water into two unequal divisions gives you two quantities, both of which count as water. As Link (1983) and Landman (1991) point out, the predicate that parallels the mass noun in upward and downward closure properties is not the singular count noun but the bare plural count noun, which is also divisible down to minimal parts, these minimal parts being of course the atomic individuals in the denotation of the singular count noun.

Link (1983) proposed a formal model to capture the difference between the mass and count domains. While explicitly not attempting to capture downward closure properties, he proposed a model that allowed linguists to represent the contrast between cumulative and non-cumulative predicates. Link proposed that both mass and count domains form lattices, with the essential difference that the count domain is atomic and the mass domain is non-atomic (Link leaves open whether it is or is not atomless or even Boolean). Nouns denote sets of entities that form Boolean sub-lattices of the respective domains. A singular mass noun denotes a sub-lattice of the non-atomic domain. A singular count noun denotes a set of atomic elements in the count domain. The corresponding plural count predicate denotes the closure of that atomic set under sum, and this plural set forms a Boolean sub-lattice of the count domain.

These two domains, the mass and the count, are separate but related by a ‘material part’ relation. For example, *gold* is a mass term denoting quantities of stuff, and *ring* is a count term denoting a set of atomic individuals. If a specific ring, *a*, is made of gold, there is some quantity *y* in the denotation of *gold*, such that *y* is a material part of *a*. Support for positing two domains comes from a consideration of examples such as (6):

(6) This ring is new, but the gold it is made out of is old.

Since the ring and the gold out of which the ring is made occupy the same spatiotemporal position and look like the same object, it seems as if (6) is predicating contradictory properties of a single object: the object that is the denotation of both *this ring* and *the gold* is simultaneously old and not-old. But if *this ring* denotes an entity *a* in the atomic domain, while *the gold* denotes an entity *b* in the non-atomic domain, related by the material part relation, then (4) is then no longer contradictory, but asserts that the old and the new
properties hold of two different objects \( a \) and \( b \). Link represents the meaning of (6) as in (7), where MatPart is the material part relation:

\[
(7) \quad \exists y \ [ \text{GOLD}(y) \land \text{RING}(a) \land \text{MatPart}(a,y) \land \text{OLD}(y) \land \neg\text{OLD}(a)].
\]

However, positing two different domains cannot be the solution to the paradox posed by (6) since the problem can be replicated with two nominals from the mass domain:\(^2\)

(8)\ a. This jewellery is new but the gold it is made of is old.
\b. The curtaining is new, but the fabric it is made of is old.\(^3\)

Much of the force of Link’s proposal has come from the naturalness with which it models the distinction between homogeneous and non-homogeneous predicates. Although Link explicitly refuses to discuss downward closure properties (because of the issues raised by the question ‘what are the minimal parts of water?’), his representation of the mass and count domains in terms of non-atomic and atomic lattice structures has often been made into the stronger distinction between atomless and atomic lattice structures, which directly expresses the distinction between mass and count predicates in terms of cumulativity and homogeneity. Upward homogeneity, or cumulativity, is defined as in (9) where \( \sqcup \) is the standard join or sum relation (the definition is based on Krifka 1998). Mass predicates are straightforwardly cumulative, while singular count predicates are not.

(9) Cumulativity:

\[ P \text{ is cumulative iff: } \forall x \forall y [x \in P \land y \in P \rightarrow x \sqcup y \in P] \]

‘\( P \) is a cumulative predicate if when \( x \) and \( y \) are in \( P \), then the sum of \( x \) and \( y \) is also in \( P \).’

\(^2\) A reviewer has noted that the paradox can be replicated in the count domain, as in The mosaic is new, but the stones it is built out of are old. Note that in order for the paradox to be replicated, the mosaic must consist of nothing other than the stones, that is, no cement, glue and so on.

\(^3\) The solution to this paradox has to be in a theory of intensional properties. The issue is discussed in Landman (1989b), Chierchia (1984) and in a different context in Heim (1998). Ascription of properties is not directly to entities but to entities presented under particular guises or perspectives. Thus, Landman (1989b) argues that even if the judges and the hangmen are the same individuals, they may have properties as judges, such as ‘being professional’, ‘being efficient’, or ‘being on strike’, which they do not have as hangmen, as in (i):

(i) John is efficient as a judge but inefficient as a hangman.

This seems to be what is going on with the examples in (8): the same entity may be old as gold but new when presented as a ring or as jewellery, without invoking a ‘constitutes’ or ‘material part’ relation.
Homogeneity or divisiveness is defined as in (10), where ‘\(\sqsubseteq\)’ is the part-of relation, \(\mathcal{O}\) is the overlap relation, and \(y\) and \(z\) are required to be non-empty (i.e. divisiveness cannot be satisfied trivially).

\[(10)\text{ Homogeneity (divisiveness):}\]

\[P \text{ is homogeneous iff } \forall x \in P: \exists y \exists z [y \sqsubseteq x \land z \sqsubseteq x \land \neg \mathcal{O}(y,z) \land y \in P \land z \in P]\]

‘\(P\) is a divisive (homogeneous) predicate if for every \(x\) in \(P\), there is a way of splitting \(x\) into two non-overlapping parts, both of which are also in \(P\).’

As discussed above, mass predicates are divisive or homogeneous since quantities of water or mud can be split into two portions of water or mud, but a count predicate like \(\text{cat}\) is not homogeneous or divisive since a cat cannot be split into two entities that both count as an instance of \(\text{cat}\). A different property of predicates discussed in Krifka (1992, 1998) is quantization. A predicate \(P\) is quantized if for all entities \(x\) in \(P\), no proper part of \(x\) is also in \(P\). Intuitively, this property is supposed to pick out predicates that have only minimal instantiations of the predicate in their denotation. Homogeneity or divisiveness as defined in (10) is stronger than the property of being non-quantized. Suppose a jacket has detachable sleeve parts, then \(\text{jacket}\) is non-quantized (since the jacket has a subpart—the part without the sleeves—which is also a jacket), but it is not homogeneous.

It is easy to see that an atomless Boolean algebra represents the denotation of a predicate that is both cumulative and non-trivially homogeneous. For any two elements in an atomless Boolean lattice \(L\), the join of those elements is also in \(L\) (cumulativity). Since the lattice is non-atomic, that is, it is not constructed from minimal elements, any element is non-trivially the join of its parts. In the count domain, a singular count noun \(C\) denotes a set of atoms, that is, a subset of the minimal elements in the domain. Thus, the sum of two elements in the denotation of \(C\) cannot, by definition, also be in the denotation of \(C\). Since atoms are minimal elements, a predicate of atoms cannot denote a non-trivially homogeneous property.

While the claim that homogeneity and cumulativity characterize mass nouns and not count nouns makes sense intuitively, it runs into problems as soon as we try to use these as criteria for classifying predicates as mass or count. First, predicates like \(\text{salt}\) or \(\text{rice}\) are not, when it comes down to it, homogeneous. These predicates have in their denotations entities that count as instances of rice or salt but that are too small to divide into two subparts that are both rice or salt, and at the most extreme end, both salt and rice are constructed out of salt and rice atoms that can be divided into parts that are not salt and rice, so \(\text{salt}\) and \(\text{rice}\) are not fully
homogeneous predicates. This is the problem with downward closure that led Link to remain agnostic as to whether the structures representing the mass domain were atomless or merely non-atomic. The issue has been discussed extensively in Gillon (1992), Chierchia (1998a) and Landman (2007). Second, not all mass nouns are even intuitively homogeneous. In particular, the group of mass nouns that are sometimes called ‘superordinates’ or quasi-kind terms, such are jewellery or furniture, are not homogeneous. But this is not a general property of quasi-kind terms, and there seems to be no predictable pattern: fruit is mass in British English, but has a count use in American English, while vegetable(s) is always count; furniture is mass but toy(s) is count and so on.

3.2 The mass domain is an atomic domain

One way to maintain the idea that homogeneity is involved in the mass/count distinction is to suggest that while mass nouns are indifferent as to the homogeneity/non-homogeneity of their denotations, count nouns are necessarily atomic. A theory that proposes this is Chierchia’s (1998a), who like Gillon (1992) argues that mass nouns as well as count nouns have their denotations in an atomic domain. Chierchia argues that mass predicates denote atomic Boolean semi-lattices, with the atomic entities in the denotation of a mass predicate being under-specified and vague. [Landman (2007) also argues that mass nouns have denotations in an atomic domain: he proposes that mass denotations are not Boolean but are composed out of substructures that are.4] Assume a Boolean domain with three individuals in it. It has the structure in (11):

(11)

\[
\begin{array}{c}
\text{b} \\
\text{a} \\
\text{c}
\end{array}
\]

The individuals on the bottom line are the singularities, the atoms of the model, and the entities on the higher lines are the plural entities. The Boolean semi-lattice models the domain partially ordered by \(\sqsubseteq\), the part-of relation, and closed under \(\sqcup\), the sum or join operation. Thus, (12) holds:

(12) a. \(a \sqsubseteq b \leftrightarrow a \sqcup b = b\).

b. Overlap: \(\forall a \forall b[a \circ b \leftrightarrow \exists c[c \sqsubseteq a \land c \sqsubseteq b]]\)

c. \(\forall a \forall b[a \sqsubseteq b \land \neg(a = b) \rightarrow \exists c[a \sqcup c = b]]\).

4 Problems with the assumption that the mass domain is a Boolean algebra and that mass nouns denote sub-algebras of this domain are discussed in Landman (2007).
In a standard account of the singular/plural distinction (Link 1983; Landman 1989a), semi-lattices like (11) are used to model denotations of count expressions, with the singular count noun denoting the set of atoms, or bottom elements of the semi-lattice, here \{a, b, c\}, and the plural denoting the set of atoms closed under sum, that is, the set of elements in the structure in (11) \{a, b, c, a \sqcup b, a \sqcup c, b \sqcup c, a \sqcup b \sqcup c\}. Chierchia (1998a) proposes that mass nouns too denote Boolean algebras, such as the one in (11). He argues that the crucial difference between mass and count nouns is that count nouns distinguish lexically between the set of atoms in the Boolean algebra and the set of plural elements, while mass terms are grammatically singular but lexically plural: ‘mass nouns come out of the lexicon with plurality already built in, and ... that is the only way they differ from count nouns’ (Chierchia 1998a: 53). So, a grammatically singular count noun denotes a set of atoms, and the plural of the count noun denotes that set closed under the sum operation, but a grammatically singular mass noun denotes the closure under sum of a set of atoms. If the predicate piece of furniture denotes the set in (13a), the plural of that predicate denotes the plural set in (13b). The mass term furniture as a lexical plural will have the denotation in (13c), although it is morphologically singular:

\[\text{(13)}\]

\[\begin{align*}
\text{a. piece of furniture} & \rightarrow \{\text{chair}_1, \text{chair}_2, \text{table}_1\} \\
\text{b. pieces of furniture} & \rightarrow \{\text{chair}_1, \text{chair}_2, \text{table}_1, \text{chair}_1 \sqcup \text{chair}_2, \\
& \phantom{\rightarrow} \text{chair}_1 \sqcup \text{table}_1, \text{chair}_2 \sqcup \text{table}_1, \text{chair}_1 \sqcup \text{chair}_2 \sqcup \text{table}_1, \} \\
\text{c. furniture} & \rightarrow \{\text{chair}_1, \text{chair}_2, \text{table}_1, \text{chair}_1 \sqcup \text{chair}_2, \\
& \phantom{\rightarrow} \text{chair}_1 \sqcup \text{table}_1, \text{chair}_2 \sqcup \text{table}_1, \text{chair}_1 \sqcup \text{chair}_2 \sqcup \text{table}_1, \}
\end{align*}\]

This means that Chierchia can explain why that is furniture and those are pieces of furniture apparently have the same truth conditions: the sum of entities in the denotation of furniture and pieces of furniture just are the same objects. The same holds for pairs such as carpeting and carpets, curtaining and curtains and so on.

The crucial difference between count nouns and mass nouns is that count nouns make a set of atoms grammatically accessible, while mass nouns do not. Count nouns do this since they ‘presuppose’ a set of atoms, and this presupposition makes the set of atoms salient in the

\[5\] In (13), the plural of the count noun and the mass noun denote the same set. This is not the case in Chierchia’s account since for him the plural count noun denotes the set of plural elements without the atoms. This is problematic since in normal discourse a bare plural includes atoms in its denotation. ‘Do you have children?’ allows the answer ‘Yes, one’, while ‘No, only one’ is infelicitous. Chierchia (1998a) proposes a mechanism for circumventing this, but it complicates the theory considerably. I shall adopt the more conventional account of plural predicates in (13) since in any case, I propose later in the article a structural distinction between the mass and the count domains that solves the problem of the apparent synonymy between (13b) and (13c).
discourse and available for the semantics to make use of. This replaces Link’s account of the contrasts in the examples in (3) and (4) above. Instead of count and mass nominals having their denotations in different domains, the fact that count nouns lexically access a set of atoms means that this atomic set is accessible to grammatical operations. Different grammatical operations exploit the mass/count distinction in different ways. Determiners that are sensitive to the mass/count distinction make use of functions that distinguish between mass and count predicates. For example, the function ‘SG’ applies to N and checks whether the denotation of N is either atomic or generated by pluralization from a lexically accessible atomic set. SG is the identity function when applied to singular or plural count nouns but is undefined for mass nouns. Since number modifiers like *three* are defined to apply to SG(N), *three* SG(N) will also only have a value when applied to count nouns. Other mechanisms account for the constraints on reciprocals and partitive constructions with mass nouns. Reciprocals require morphological plurals as antecedents and thus cannot take mass nouns such as *furniture* as an antecedent. Numerical partitives apply to definite plurals and not to definite mass nouns because *the* is sensitive to the distinction between plural and singular morphology and necessarily returns a singleton object when applying to a mass noun. Thus, *the furniture* denotes the singular group or collective entity formed out of the maximal plural object in the denotation of *furniture*. Since *three* requires a plural complement, it cannot apply to the singularity denoted by *the furniture*.

### 3.3 Homogeneous count nouns require an explicit theory of atomicity

Chierchia’s theory relies on a presupposition of atomicity to explain why and how count nouns access their set of atoms. This is problematic for two reasons. In the first place, some mass nouns like *furniture* and *footwear* denote sets of inherently individuable entities, and since *shoes* is a near-synonym of *footwear* but a count noun, there must be a great deal of lexical idiosyncrasy underlying whether a predicate of atomic individuals is or is not marked count. However, a much more serious problem is that there are count nouns that denote entities that our real-world knowledge tells us are not inherently atomic but homogeneous.

Homogeneous count nouns include nouns such as *fence, line, plane, sequence, twig* and *rope*. These have been noticed in the literature at various times over the past 20 years. Mittwoch (1988) shows that *line* and *plane* denote sets of entities that have proper parts that themselves are lines and planes; Krifka (1992) points out that entities in the denotation of *sequence* and *twig* have proper parts that count as a sequence or a twig; Gillon (1992)
makes the same point for ‘flexible’ nouns that have both a mass and a count form, such as rope or stone: one rope can be cut into many ropes, one stone can be broken into stones and so on. Rothstein (1999, 2004) shows that the phenomenon is even more general and includes a wide variety of nouns such as fence, wall, hedge and bouquet. Other nouns with the same property denote organisms: a bacterium reproduces by dividing and a flatworm can be divided into two flatworms. These nouns are divisive in the sense of (10): there is a way of dividing a fence or a bouquet or a bacterium into two parts, both of which count as fences or bouquets or bacteria. There have been only a few attempts to explain why these nouns are in fact count.\footnote{Zucchi and White (2001) discuss why non-quantized nominals in direct object position induce telic readings of accomplishment-headed VPs, but they do not discuss why homogeneous head nouns are count.}

Furthermore, these facts show that homogeneity is a real-world property and not a semantic property, and so a characterization in terms of homogeneity does not capture the independence of the mass/count distinction from the structure of matter. The reason why a flatworm can be divided into two flatworms, while a cat cannot, is because of differences in the systemic make up of flatworms and cats. Similarly, the reason why furniture is not homogeneous and linoleum is has to do with the different real-world properties of the two types of stuff, even though they are both mass predicates. Other predicates are sometimes homogeneous, depending on what entities are in their denotation in a particular model. For example, a part of a notepad may or may not be a notepad. If the original is one of those blocks with 500 pages joined by glue where you can tear off clumps of pages to make smaller notepads, then there is a bipartition of the notepad into two smaller notepads. If the notepad is bound and has a cover, then this cannot be done. Crucially, it depends on whether the original item has a structure that makes such a bipartition available. But in a model in which only the first kind of notepad exists, then notepad is a homogeneous predicate. Fence is more strongly homogeneous since the same piece of fencing may be analysable as one or several non-overlapping fences in the same situation under different criteria of individuation. The example given in Rothstein (1999, 2004) is as follows. Suppose four farmers, A, B, C and D build a fence each, as in Figure 1:

Figure 1

\begin{figure}[h]
\centering
\begin{tikzpicture}
  \node[draw] (A) {A};
  \node[draw, below of=A] (B) {B};
  \node[draw, right of=B] (C) {C};
  \node[draw, below of=C] (D) {D};
  \node[draw, below of=B, anchor=north] {The Field};
\end{tikzpicture}
\end{figure}
Then, either each farmer built a fence, and there are four fences, one on each side of the field, or the field is enclosed by a fence, in which case there is only one fence around the field. Here is a different example: suppose that I have a bouquet of flowers that I split, giving half to my daughter and half to her friend. Then, either there is a single bouquet that has been split so that each girl has half a bouquet, or each girl has a bouquet of flowers (albeit smaller than the original one). Similar examples can be constructed for count nouns such as wall, twig, quantity of milk and so on.

Cumulativity does not fare any better as a defining property of the mass/count distinction. Mass nouns are indeed cumulative since water + water gives an entity in the denotation of water. But while cat + cat is not in the denotation of cat, there may well be situations in which the sum of two non-overlapping fences could be in the denotation of fence, given a context in which this sum of fence parts can be treated as a singularity, or two bouquets can be put together to make a single bouquet.

What this shows is that inherent, or natural, atomicity is neither a necessary nor a sufficient criterion for count noun predicates, and homogeneity v. non-homogeneity cannot be at the root of the mass/count distinction. Furniture is mass but naturally atomic and non-homogeneous since it denotes sets of individual units and fence is count but homogeneous and not naturally atomic. This means that a theory of count nouns cannot rely on presuppositions of atomicity. Instead, we need a theory of what atomicity is and in what way count nouns like fence are atomic, while mass nouns like furniture are not. In what follows, I shall propose that Chierchia’s ‘association with a salient set of atoms’ is the result of a semantic operation deriving count nouns that is at the heart of grammatical countability. This operation, which derives semantically atomic predicates, as opposed to naturally atomic predicates, results in a typal difference between mass and count nouns that allows us to give a unified explanation of the grammatical differences between them.

4 EVIDENCE FROM BARNER AND SNEDEKER (2005)

In this section, we look briefly at the results of Barner and Snedeker (2005), which support the claim that natural atomicity and semantic or count atomicity are distinct phenomena. Barner and Snedeker investigate experimentally the basis of quantity judgments by asking adults and children the question, ‘Who has more X?’ in three different situations:
(a) where $X$ is a mass substance term, such as *mud*
(b) where $X$ is a mass superordinate term, such as *furniture*
(c) where $X$ is a flexible noun, such as *stone/stones, brick/bricks* etc.

We are interested only in the adult data. Adults were presented with pictures of quantities of the relevant entity and asked, ‘Who has more $X$’, and the results were as follows: (details of the experiments and the statistical analysis are given in Barner & Snedeker 2005).

Result I: In situation (a), where the question was asked using mass substance nouns, for example, ‘Who has more mud?’, quantity judgments depended on overall quantity of stuff. One big heap of mud was consistently judged to be ‘more mud’ than three small heaps of mud.

Result II: In situation (b), when the question was asked using mass superordinate terms, for example, ‘Who has more furniture/silverware?’, quantity judgments rely on number. Several small pieces of silverware were judged to be more ‘silverware’ than one big piece, whose overall volume was bigger than the smaller pieces. Several small chairs were judged to be more ‘furniture’ than one big chair, even when the volume/mass of the single big chair was greater than the combined mass of the small chairs. No context was given for the quantity judgment: the stimulus presented two sets of entities in a context-independent way.

Result III: In situation (c), which tested nouns having both mass and count forms such as *stone/stones*, quantity judgments depended on the syntax of the noun. The stimulus was a picture of several small stones and one big stone, where the volume of the big stone was greater than the combined volumes of the small stones. When the question was asked using a count noun, three small stones were judged to be more than one big stone, and when the question was asked using a mass noun, one big stone was judged to be more than three small stones.

7 It is plausible that varying the situational context would affect what is understood as the individual entities relevant for quantity judgments. For example, suppose the relevant issue is who can seat more people in her office. I have three chairs and a colleague has two sofas. I might then say, ‘You have more furniture: we should have the meeting in your office’.
Barner and Snedeker conclude from results I and II that some mass nouns denote sets of individuals, while others do not. A more precise formulation is that some mass nouns allow individuals to be salient for quantity judgments, and in these cases, the quantity judgments are based on comparing numbers of individuals and not overall mass. Result III shows that mass/count syntax influences the basis of quantity judgments where both options are available. Given a flexible noun with both mass and count forms, then when the noun is mass, even where individual lumps of stone are salient, quantity judgments are by comparison of total mass and not by comparison of number. So, although some mass nouns denoting sets of individuals make the individuals salient as a basis for quantity judgments, other mass nouns do not. Furthermore, result III indicates that count nouns individuate and that count stones required a comparison in terms of numbers of individuals and not in terms of overall volume.

Barner and Snedeker suggest that there are two kinds of mass terms, ‘substance mass’ terms such as mud, salt and stone (on its mass reading), and those that pattern like furniture in result II, which they call ‘object mass’ and which tend to denote heterogeneous classes of objects. These object mass terms are those that rely on the salience of inherently individuable entities for quantity judgments. They therefore suggest that these terms appear in the lexicon marked [+individual], although they are not count, and that this feature marking represents that fact that ‘the conceptual apparatus associated with individuation is distinct from the linguistic feature which licenses its direct expression in the language’ (Barner & Snedeker 2005: 59). Without accepting the necessity for using a lexical feature [+individual], we see from their results that even when the salience of the individuals in the denotation of a mass term allows quantity judgments in terms of implicit counting, you still cannot count grammatically. If A has one big sofa and B has two small chairs and a small table, and you think that B has more furniture based on a comparison of numbers of pieces of furniture, you still cannot say ‘B has three furnitures’. Similarly, you can say ‘B has more furniture than A’ but not ‘B has two more furnitures than A’. This means that what is relevant for quantity judgments is not relevant for linguistic expressions of counting. In other words, the conceptual apparatus of individuation and the grammatical mechanisms that allow direct counting of individuals are distinct. This means that the grammar of count nouns is not directly dependent on or derived from the cognitive or perceptual salience of individuals.
5 TOWARDS AN ACCOUNT OF THE SEMANTICS OF COUNT NOUNS

5.1 Two observations

We now summarize the two observations made in the previous sections:

(i) Observation 1: Even when you can count the objects in the denotation of mass nouns implicitly because the units are salient, you cannot count grammatically. While Barner and Snedeker (2005) show that the answer to ‘Who has more furniture?’ is determined by comparing the cardinality of two sets of individuals, English still does not allow (14):

\[
\begin{align*}
(14) & \\
    a. & \#John \text{ has three furnitures.} \\
    b. & \#John \text{ has three furnitures more than Bill.}
\end{align*}
\]

A second piece of data showing that individual units may be salient even when nominals cannot be directly counted comes from adjectival modification. In Mandarin Chinese, bare nouns can never be directly counted but always require a classifier, as demonstrated in (15).

\[
\begin{align*}
(15) & \\
    a. & \text{liǎng ge píngguǒ} \\
         & \text{two Cl-} \text{apple(s)} \\
         & \text{‘two apples’} \\
    b. & \text{*liǎng píngguǒ} \\
         & \text{two Cl-apple(s)}
\end{align*}
\]

Nonetheless, the individual unit is salient and can be directly modified. So while the modifier ‘big’ cannot modify nouns such as shuǐ ‘water’, as in (16a), the same modifier can directly modify the noun when the noun is of the ‘count type’ as in (16b).

\[
\begin{align*}
(16) & \\
    a. & \text{*liǎng bēi dā shuǐ} \\
         & \text{two Cl-cup} \text{big water} \\
    b. & \text{liǎng ge dà píngguǒ} \\
         & \text{two big apples} \\
    c. & \text{wò mǎi le dà píngguǒ} \\
         & \text{I buy} \text{perfective big apple(s)} \\
         & \text{‘I bought big apples.’}
\end{align*}
\]
So even though pingguo, ‘apple(s)’, cannot be directly counted in Mandarin, the nominal denotes entities whose unit structure can be modified.\(^8\) The same is true of mass nouns like furniture in English, as (17) shows. This is noted independently in Schwarzschild (forthcoming), who calls predicates like big ‘stubbornly distributive predicates’. [Note that the verbal predicates distribute over the individual pieces of big furniture, particularly in the contexts relevant for (17c/d), although it might be argued that big furniture is a kind term in (17b)].

(17) a. The furniture in our house is big.
   b. In a department store: ‘The big furniture is on the third floor.’
   c. To movers who are emptying the house: ‘Please take the big furniture down first.’
   d. ‘Don’t buy big furniture, the stairs are too narrow to carry it up.’

Doetjes (1997: 37) brings another example of linguistic sensitivity to perceptual salience. In Dutch, the classifier stuk ‘piece’ is used anaphorically in answering a question. It has two forms: stuks and stukken. In answering questions involving a count noun, stuks is used, and if the count noun is replaced by a mass noun, stukken must be used.

   ‘How many books do you take? Two.’
   b. Hoeveel kaas heb je gegeten? Twee stukken/*stuks.
   ‘How much cheese have you eaten? Two pieces.’

Doetjes notes that when the question involves a mass noun denoting a set of perceptually salient individuals, there is a strong tendency to use stuks and not stukken.

(19) Hoeveel meubilair neem je mee?
   ‘How many/much furniture mass take you with? Two pieces.’
   Twee stuks/*stukken.
   Two pieces.

(ii) Observation 2: Things that you can count grammatically do not necessarily come in individuated units and are not inherently atomic (Mittwoch 1988; Krifka 1992; Rothstein 1999, 2004; Zucchi & White 2001). As we saw, twig, sequence, line, fence, wall and quantity of milk

\(^8\) Even if dà pingguo is understood as a complex kind, that is, the kind ‘big apples’, the kind exists as a generalization over the set of individual apples with the ‘big’ property.
may rely on context to determine what one counts as one entity in the denotation of the singular predicate. So, when N is a count noun, the entities in its denotation can be counted even when what counts as a single unit is not uniquely determined.

These two observations lead to the following conclusions:

Conclusion I: We cannot define the mass/count distinction in terms of properties of the denotations of the nouns themselves, or via a presupposition of atomicity (or lack of it).

Conclusion II: Although there is a clear tendency for naturally atomic objects to be denoted by count nouns and ‘stuff’ to be denoted by mass nouns, being a naturally atomic predicate is neither a necessary nor a sufficient condition for being a count noun. Therefore,

Conclusion III: the mass/count distinction can only be explained in terms of how the expressions refer, and not in terms of the things they refer to. This means it is a grammatical and not an ontological distinction.

We will argue that count nouns are a mechanism for grammatical counting. They allow grammatical counting because they keep track of their atomic members via a semantic operation, and not presuppositionally or because of any ‘real-world’ properties such as inherent individuability/atomicity. So, while agreeing with Chierchia that count nouns make accessible a set of atoms, we need to now show what the semantic mechanism is which does this.

5.2 Modelling the Mass/Count distinction

We assume that nominals are interpreted with respect to a complete atomic Boolean algebra M. $\sqcup_{M}$, the sum operation on M is the complete Boolean join operation (i.e. for every $X \subseteq M$: $\sqcup_{M}X \in M$). With Chierchia, I assume that the set of atoms $A$ of M is not fully specified, vague. The denotation $N_{\text{root}}$ of a root noun is the Boolean algebra generated under $\sqcup_{M}$ from a set of atoms $A_{N} \subseteq A$ [so root noun denotation $N_{\text{root}}$ has the same 0 as M, its atoms are $A_{N}$ and its 1 is $\sqcup_{M}(A_{N})$]. I assume that mass nouns have the denotations of root nouns, so $N_{\text{mass}} = N_{\text{root}}$.\(^{9}\) (Note that we assume this particular theory of mass nouns for simplicity but the choice is not essential to

\(^{9}\) Arguably, mass nouns should denote kinds, as proposed in Chierchia (1998a,b). For a version of this theory in which the meaning of the mass noun is the kind associated with $N_{\text{root}}$, see Rothstein (2009b).
what follows. In particular, the mass domain may be only partially atomic.)

For a noun like *furniture* that naturally denotes a set of inherently individuable units that are pieces of furniture, the atoms in the denotation of the nominal will usually be these single pieces of furniture. These are the individuals that Barner and Snedeker have shown are relevant for quantity judgments. But the choice of the atoms in the denotation of a mass predicate is vague and underdetermined, and in different contexts, the set may be constructed on the basis of different salient minimal elements, for example, the set of parts of modular, build-it-yourself furniture or the set of sitting places made available by the furniture (see footnote 7). Whatever the atomic elements are, they are not lexically accessible, and there is no lexical item that denotes the set of atomic elements.

For mass substance nouns like *mud*, we assume, like Chierchia, that the atoms of the set are the minimal relevant quantities of mud. Chierchia argues that the minimal elements here are specified by context or may be left vague and unspecified, and thus, the information as to what counts as a minimal element is usually neither explicitly nor implicitly specified, nor recoverable from context nor identifiable via perceptual salience. This explains why, in these cases, quantity evaluation judgments in Barner and Snedeker’s experiment are evaluated in terms of overall quantity and not in terms of number of minimal elements. As with *furniture*, the set of minimal elements is not lexically accessible and is not countable.

Count nouns differ from mass nouns because they allow direct grammatical counting. Counting is putting entities in one-to-one correspondence with the natural numbers and requires a contextually determined choice to what counts as one entity. Grammatical counting is direct modification of a nominal by a number word, expressing the results of this one-to-one matching operation. As our discussion of nouns like *fence* and *wall* showed, count nouns do not necessarily presuppose a specific set of salient atomic entities; instead, the model needs to specify a context-dependent choice of atomic elements relative to which the count noun is derived, and count noun denotations must specify the context in relation to which they are to be interpreted. In general, along with specifying contextual parameters such as those that provide values for indexicals, time and location, part of specifying a context is specifying what are the set of ‘things which count as one for the purposes of counting’. The choice of this counting parameter is determined by discourse considerations and is updated *ceteris paribus* in the standard way (see,
e.g. discussion in Partee et al. 1990/1993: chapter 15). Since in this article the only relevant contextual parameter is the counting parameter under discussion, I will for ease of notation identify contextual counting parameters and contexts. If you think this is too short you may read counting context or counting perspective wherever it says context.

(20) A context k is a set of objects from M, k ⊆ M, K is the set of all contexts.

(21) The set of count atoms determined by context k is the set

\[ A_k = \{ <d,k> : d \in k \} \]

\( A_k \) is going to be the set of atoms of the count structure \( B_k \) to be determined below. The objects in k are not mutually disjoint with respect to the order in M since we may want, in a single context, say, my hands and each of my fingers to count as atoms, that is, to be members of the same contextual set of atoms. Thus, it may be the case that for two entities lt and lh (my left thumb and my left hand), \( lt \subseteq M lh \), but nevertheless \( lt, lh \in k \). In that case \( <lt,k>, <lh,k> \in A_k \). So both my left thumb and my left hand are atoms to be counted in context k. Given this, we cannot lift the order on the count Boolean domain from the mass domain. We want the count domain \( B_k \) to be a complete atomic Boolean algebra generated by the set of atoms \( A_k \). Up to isomorphism, there is only one such structure:

(22) \( B_k \) is the unique complete atomic Boolean algebra (up to isomorphism) with set of atoms \( A_k \). We let \( \sqcup_k \) stand for the corresponding complete join operation on \( B_k \).

However, we would like to lift this order from the mass domain as much as we can. The idea is: if \( k' \subseteq k \) and \( k' \) is a set of mutually non-overlapping objects in M, there is no problem in lifting part-of relations of the sums of \( k' \)-objects from the mass domain. (\( k' \) is a set of mutually non-overlapping objects in M iff for all \( d, d' \in k' \): \( d \cap M d' = 0 \)).

Thus, we impose the following constraint:

(23) For any set \( k' \subseteq k \) such that the elements of \( k' \) are mutually M-disjoint, the Boolean substructure \( B_{k'} \) of \( B_k \) is given by: \( B_{k'} = \{ <\sqcup_M X,k> : X \subseteq k' \} \) with the order lifted from \( \sqcup_M \).

The plurality order is not lifted from the mass domain for objects that overlap, that is, the sum of my hands and my fingers is a sum of 12
atoms, hence not lifted from the mass domain (atom here is a metalanguage predicate).

(Singular) count predicates, in particular count nouns, denote subsets of $A_k$. We propose that they are derived as follows.

All lexical nouns $N$ are associated with a root noun meaning $N_{\text{root}}$. As I stated above, the root noun meaning is a Boolean algebra generated under $\bigcup_M$ from a set of $M$-atoms. The mass noun denotation, $N_{\text{mass}}$, is identical to the associated $N_{\text{root}}$, that is, $N_{\text{mass}} = N_{\text{root}} \subseteq M$.

Count noun meanings are derived from root noun meanings by an operation $\text{COUNT}_k$, which applies to the root noun meaning $N_{\text{root}}$ and gives the set of ordered pairs $\{<d, k>: d \in N \cap k\}$. These are the entities that in the given context $k$ count as atoms and thus can be counted. The parameter $k$ is a parameter manipulated in context. Thus, in the course of discourse we have as many relevant $k$’s around as is contextually plausible. We can think of these contexts as contextually defined perspectives on a situation or model, and the set of contextually relevant contexts is rich enough so that there may be different numbers of $N$ entities in a situation depending on the choice of $k$, that is, the choice of counting perspective that is chosen.

In sum:

1. For any $X \subseteq M$: $\text{COUNT}_k(X) = \{<d, k>: d \in X \cap k\}$
2. The interpretation of a count noun $N_{\text{count}}$ in context $k$ is: $[[N_{\text{count}}]] = \text{COUNT}_k(N_{\text{root}})$. We will use $N_k$ for the interpretation of $N_{\text{count}}$ in $k$.

The denotation of a singular count noun in context $k$ is thus $N_k$, an ordered pair whose first projection is a set of entities $N_{\text{root}} \cap k$ and whose second projection is context $k$. We call such sets semantically atomic sets since the criterion for what counts as an atom is semantically encoded by the specification of the context. The set $N_{\text{root}} \cap k$, or $N_{\text{root},k}$ is the set of atomic $N$-entities used to evaluate the truth of an assertion involving $N_{\text{count}}$ in a particular context $k$.

The atoms in $k$ are not constrained by a non-overlap condition since we want to allow examples like (25), which make reference to atomic elements and their atomic parts:

(25) a. I can move my hand and my five fingers.
   b. It took 2500 bricks and a lot of cement to build this wall.
But since the entities in \( k \) are strictly atomic, the relation between the wall and the bricks that it is made of will not be expressed visibly in \( B_k \).

10 As two reviewers have pointed out, a difficulty still remains when we want to be able to count singular entities in \( k \), which themselves consist of a sum of a set of atoms in \( N_{root} \cap k \), for some \( N_{root} \). One example of this is the predicate deck of cards. A deck of cards consists of 52 cards. If we treat the count noun deck as an ordinary count predicate, then it will denote a set of entities in \( M \times \{k\} \), each of which is a sum of 52 cards. This means that when in context \( k \) we construct the count noun meaning of the singular count noun card, \( CARD_k \), \( CARD_{root} \cap k \) will include not only the individual atomic cards but also the sums of 52 cards in the denotation of deck. This is obviously not desirable.

Landman (1989a,b) analyses nouns like group (of boys), class (of children) and deck (of cards) as ‘group nouns’, singular count nouns that refer to atomic collections of count entities. These group nouns do not have ordinary count noun denotations. I shall analyse them as a form of count noun classifier, as follows: We assume that alongside \( B_k \) there is a set \( GROUP_k \), the set of (abstract) singular entities that are derived from sums of count entities via ‘group-formation’ or parcelling, which uses the group function \( \uparrow \) defined as follows (see also Chierchia 1998a, who defines essentially the same function):

(i) For any plural entity \( y \) in \( B_k \), \( \uparrow \pi(y) \) is the group entity whose members are the atomic parts of \( \pi_1(y) \), that is, the members of the group are those parts of \( \pi_1(y) \) that are in \( k \).

(ii) \( \downarrow \uparrow \pi_1(y) = \pi_2(y) \), that is, \( \downarrow \) applied to a group gives the original plurality back again. We now define the set \( GROUP_k \) of group entities, and the set \( k^* \), a superset of \( k \), which includes members of \( k \) and the singular groups constructed from them via \( \uparrow \). We also define the set of atomic groups \( A_{GROUP_k} \).

(iii) \( GROUP_k = \{ x: \exists y \in B_k: x = \uparrow \pi_1(y) \} \)

(iv) \( k^* = k \cup GROUP_k \)

(v) \( A_{GROUP} \) is the set of ordered pairs \( \{ <x,k^*>: x \in GROUP_k \} \)

\( GROUP_k \) is the unique atomic Boolean algebra generated by \( GROUP_k \) with the corresponding join operation \( \sqcup_{GROUP_k} \) and \( GRP_k \) is the unique atomic Boolean algebra generated by \( A_{GROUP_k} \) and the corresponding join operation \( \sqcup_{GRP} \) with the order lifted from \( GROUP_k \). Nouns such as deck denote functions from \( k \) into \( k^* \), that is, functions from plural entities into the atomic collections formed from them via the \( \uparrow \) operation. Plural group predicates such as decks denote sub-algebras of \( GRP_k \). This means that deck does not have a denotation in \( k \), and, thus \( CARD_{root} \cap k \) is just the set of atomic individual cards in \( k \) and does not include any sums or decks. I discuss these group classifiers further as part of study of the semantics of classifiers in work in progress.

A more difficult version of the problem arises with pairs of singular count nouns such as brick and wall, not in the general case illustrated in (25b) but in the specific case where [unlike the case in (25b)] a wall entity consists only of a sum of bricks and has no other parts, such as cement. This would occur if the wall was a dry-stone wall (see footnote 2). If such a wall entity is in \( k \), but is represented merely as a sum of bricks, then the wall entities will be in \( BRICK_{root} \cap k \) and thus in the denotation of the singular count noun brick. One possible solution is to treat wall analogously to deck, justifying this by the plausible assumption that walls are greater than the sums of bricks that compose them. However, against this is the intuition that while deck is defined as a set of cards, wall denotes a set of entities that are objects in their own right, rather than being an expression that classifies bricks, and thus, wall should have a denotation in \( k \). However, this problem is a version of the problem that occurs in the mass domain too, as we saw in the examples in (8) above. This jewellery is new, but the gold it is made of is old. The mass entity in jewellery cannot be equated with the mass entity in gold since they have different properties, even though they are apparently identical. This implies that generally ‘artefact’ predicates like jewellery involve a packaging or perspective function as part of their lexical meaning, so that \( d \in GOLD_{root} \) and \( d' \in JEWELLERY_{root} \) can be identified as the same spatiotemporal entity but presented under different perspectives or guises and with different properties. But if this kind of lexical packaging is needed anyway in the mass domain, then the problem of the wall and the sum of bricks that makes it up can be solved at the level of \( WALL_{root} \) and \( BRICK_{root} \), in which case \( BRICK_{root} \cap k \) will not include the sum of bricks presented as a wall.
Non-overlap is not irrelevant though. I assume it comes in as a constraint on default contextual interpretations:

(26) Constraint on count predicates:
In a default context k, the interpretation of singular count predicate P is a set of mutually non-overlapping atoms in k (where \(<a,k>\) and \(<a',k>\) do not overlap iff \(\bigcap_M a' = 0\)).

This guarantees that when we count entities in the denotation of \(N_k\), we will be counting contextually discrete non-overlapping entities.

*Plural count nouns* are derived from singular count noun meanings, using the standard plural operation, defined for these count structures. The plural operation gives the closure of \(N_{\text{root},k}\) under the sum operation, while keeping track of the context. Link’s (1983) plural operation is given in (27):

(27) \(*A = \{d: \exists Y \subseteq A: d = \bigcup Y\}\)

For a two-place relation \(N_k\), the \(n\)-th projection of \(N_k\) (where \(n = 1, 2\)) is given by:

(28) \(\pi_1(N_k) = \{d: <d,k> \in N_k\}\)
\(\pi_2(N_k) = k\)

For convenience, we also define \(\pi_n\) directly for pairs:

(29) \(\pi_1(<d,k>) = d\)
\(\pi_2(<d,k>) = k\)

Note that for any \(<d,k> \in N_k\), \(\pi_2(<d,k>) = \pi_2(N_k) = k\).

With this, we lift the \(*\)-operation to the present count structures:

(30) In default context k:
\(\text{PL}(\llbracket N_{\text{count}} \rrbracket) = *N_k = \{<d,k>: d \in *\pi_1(N_k)\}\)
(In non-default contexts, we do not lift plurality from the mass domain:
In non-default context k: \(*\pi_1(N_k) = \{d: \exists Y \subseteq A_k: d = \bigcup_k Y\}\)

Some crucial points: first, the non-overlap condition in (26) guarantees that in default contexts, the order of the plural count noun denotation is lifted directly from M. The denotation of the plural count noun depends on the contextually determined denotation of the singular \(N_k\). It denotes a set of ordered pairs where the first element is in the closure of \(N_{\text{root},k}\) under sum and the second element is the context k. \(N_{\text{root},k}\) may vary depending on choice of k, and the denotation of the plural set will similarly vary. Crucially, the information about the context determining the set of atoms is preserved in the plural denotation. Note
that, even with a flexible predicate like *hair*, there is no guarantee that HAIR\textsubscript{root,k} and the set of atoms in HAIR\textsubscript{root} are the same set. So though *the hair\textsubscript{mass}* and *the hairs* may well refer to the same real-world entity, this is not necessarily the case.

The second point is that since k is not constrained by a non-overlap condition, the plural domain may contain elements not lifted from M. These plural entities will not be in the denotations of lexical predicates, but they will be in the denotations of other expressions built up in the grammar like the conjunctive definite *my hand and its five fingers* in (25a). In a normal context, *my hand* will count as an atom, and *its five fingers* as a sum of five fingers, and consequently, in B\textsubscript{k} *my hand and its five fingers* will denote a sum of six atoms. Nevertheless, when I say in a context I moved *my hand and its five fingers*, it is not necessarily the case that it would be felicitous in that context to conclude, Hence I moved six body parts. The reason is that counting is a grammatical operation introduced by numerical modifiers that applies at the N-level to lexical predicates. The predicate *body part* in a default context will be interpreted as denoting a set of non-overlapping objects, and thus, though there are six atoms in *my hand and its five fingers*, we are unlikely in a normal context to allow the overlapping entities, hands and fingers, into the denotation of body parts. There are specific contexts in which this default assumption may be overruled, for example, a medical examination of a paralytic, but then the predicates will usually be reinterpreted so the entities in its denotation are not overlapping. *He can move five limbs: one hand and four fingers* is naturally interpreted as a statement about movement at the hand joint and the finger joints rather than movement of overlapping entities.  

Similarly, How many

\footnote{There seems to be a constraint even at the N-level that N conjunctions of predicates can be modified by numerals only if the conjunction can be reinterpreted as a quasi-lexical item or natural predicate. (i) and (ii) are felicitous but note that the conjunction is interpreted differently in each case:  
(i) Ten boys and girls came to the party.  
(ii) There are six cups and saucers in the cupboard.  
In (i), boys and girls is naturally interpreted as a synonym for the ‘superset’ children and then sentence (i) asserts that ten children came. In (ii), we count cup + saucer pairs and (ii) is not equivalent to There are six pieces of crockery in the cupboard, that is, it cannot be used to describe a situation where the cupboard contains three cups and three saucers (or four cups and two saucers). However, these strategies are highly constrained in ways that are not generally understood. Thus, (iii) and (iv) are not felicitous [although (iv) is minimally different from the felicitous a cat has four paws]:  
(iii) #?John has twenty fingers and toes.  
(iv) #?A human has four hands and feet.  
(A cursory search on Google produced about 60 000 references to ‘walking on hands and feet’ and no references at all to ‘walking on four hands and feet.’) Similarly, he received 10 letters and parcels is more felicitous than he received 10 letters and magazines presumably because in the first, and not the second, the conjoined N predicate can be reinterpreted as pieces of mail.}
things are there on the table? in the default case usually assumes a contextual decision as to what non-overlapping entities count as \( \text{THING}_k \). There are a few cases where we do count genuinely overlapping entities without reinterpreting the predicate or making contextual decisions about what the non-overlapping atomic entities are. These are of two kinds. First, there are a few lexical predicates that are defined as having overlapping entities in their denotations. The most obvious example (pointed out by a reviewer) is subset, as in *I wrote down the 16 subsets of the four member set*. Here, the meaning of the predicate (which, as a technical term, has a formal definition of what counts as a countable entity) overrules the default assumption in (26). The second case in which we count overlapping entities is in contexts that are explicitly or implicitly intensional, where we are instructed to count in alternative \( k \)-contexts and sum the results. An example is a situation where you are given a picture of embedded cubes and asked *How may cubes do you see in this picture?* Here, we are asked to consider alternative choices of sets of non-overlapping atoms, where the union of these sets is a set of overlapping entities. *How many cubes do you see in this picture* is as an instruction to count the number of atoms in each relevant context \( k_1, \ldots, k_n \) and take the sum of the results.

The variety of nominal denotations is thus summed up as follows:

- **Root nouns**: \( \text{N}_{\text{root}} \subseteq \text{M} \): Root nouns denote a Boolean algebra of mass entities, the closure of a set of atoms in \( \text{M} \) under the sum operation \( \sqcup_{\text{M}} \).

- **Mass nouns**: \( \text{N}_{\text{mass}} = \text{N}_{\text{root}} \): Mass nouns just are root nouns.

- **Singular count nouns**: \( \text{N}_k \subseteq \text{M} \times \{k\} \): A singular count noun denotes a set of ordered pairs of which the first projection is \( \text{N}_{\text{root}} \cap k \), a subset of \( \text{N}_{\text{root}} \) whose members do not (generally) overlap, and the second projection is the context \( k \).

- **Plural count nouns**: In a default context \( k \), \( \text{PL}(\text{N}_k) \subseteq \text{M} \times \{k\} \), where the first projection is the closure of \( \text{N}_{\text{root}} \cap k \) under sum, and the second projection is \( k \).

The definition of context that I gave in (20) is purely formal. While I have not given any constraints on the construction of contexts other than formal ones, we can assume that the set of contexts, although rich enough to give us what we need, is not unconstrained. Intuitively, \( \text{N}_{\text{root}} \cap k \), the set of \( \text{N} \)-entities that count as one in a particular context, are \( \text{N} \)-entities that count as one by a single criterion of measurement.
As we saw in the discussion about fences above, how we count fences depends on what we choose to consider one fence, how we pick out the atomic fence units. So a context k can be thought of as pragmatically constrained by a set of ‘atomicity conditions’, which (partly) specify what the criteria are for an N-entity to count as an atomic N in context k. This is what allows contexts to be distinguished pragmatically and governs the appropriate choice of context in a particular interpretation. It is also what underlies the constraint on the derivation of lexical count nouns in (26).

Grammatical counting and related grammatical operations apply to N_k and PL(N_k) since what counts as one is semantically encoded in these noun denotations. The two kinds of nominal expressions, mass nouns and count nouns, are of different types: mass nouns are of type <d,t>, while count expressions are of type <d×k, t>, and this explains why some semantic operations will distinguish between them, as we will see below. Some operations do not distinguish between them: many adjectives apply equally well to mass or count expressions, as in expensive chairs, expensive furniture. For these cases, we use the π_n function defined above in (28)–(29). We use P as a variable of predicates of type <d,t>, P for count predicates of type <d×k, t> and x and x for variables of type d and type d×k, respectively. Expensive, when it applies to mass nominal expressions of type <d,t>, denotes the function λPλx.P(x) ∧ EXPENSIVE(x). When it applies to count expressions of type <d×k, t>, it denotes the derived function λPλx.P(x) ∧ EXPENSIVE(π_1(x)).

The π_n function is also used in deriving conjunctions of mass and count DPs, as in (31):

(31) Tables and other furniture were standing around the room.

Conjunctions of this kind must be at the type of mass noun, as shown by the contrast in partitive constructions in (32a/b).¹²

¹² A referee suggested that if tables and other furniture are conjoined at the mass type, the conjunction should induce singular agreement, as mass nouns usually do, rather than plural agreement as in (31). But conjunctions of mass nouns above the N-level generally allow and usually indeed prefer plural agreement as in (i) and (ii), and when definite mass terms are conjoined as in (iii), plural agreement is obligatory.

(i) Furniture and curtaining were/was on sale at much reduced prices last weekend.
(ii) Bread and milk are/is being delivered between 2 and 4 this afternoon.
(iii) The bread and the milk are/is being delivered between 2 and 4 this afternoon.

Given these facts it is expected that tables and other furniture can induce plural agreement too. Note that some mass/count conjunctions may also induce singular agreement as in (iv), and plural count phrases may induce singular agreement when understood as denoting quantities as in (v):

(iv) Carpets and curtaining is on the 5th floor of the store.
(v) Ten tables and twenty chairs is not enough.
    b. Some/Much of the tables and the furniture arrived damaged.

    (As we will see in section 8, conjunction at the mass type also gives
the right results for reciprocal resolution.) Furthermore, as example
(33) shows, a numeral cannot have scope over a conjunction of a mass
and a count term. Example (33) refers to a sum consisting of furniture
that includes at least 10 tables, as well as other furniture.

(33) The movers delivered ten tables and furniture.

    Given that and conjoins arguments at the same type, in cases of type
mismatch, the count nouns lower to a mass reading via the $\pi_n$ function,
which, as we have just seen, is available for adjectival modification
anyway:

(34) $\llbracket$tables and (other) furniture$\rrbracket$
    = AND($\pi_1(\llbracket$tables$\rrbracket)$, $\llbracket$furniture$\rrbracket$)
    = AND($\ast$TABLE$_{root,k}$, FURNITURE)

    Turning to coordinations of definite expressions, I assume, following
Link (1983) and references cited there, that the is defined using the join
operation: $\llbracket$the$\rrbracket$(X) = $\sigma$(X) = $\sqcup$X if $\sqcup$X $\in$ X, otherwise undefined.
In our case, this gives the following interpretations, with $\sqcup$M and $\sqcup_k$
being the complete join operations on M and on B$_k$, respectively.

For mass nouns: $\sigma$N = $\sqcup$MN, the (unique) maximal entity in N, if
defined.
For count nouns: $\sigma$N$_k$ = $\sqcup_k$(N$_k$) = $<\sqcup_M\pi_1(N_k), k>$, if defined.

For mass nouns, $\sqcup_M$N is a plural individual, while for plural count
nouns $\sqcup_k$N is the ordered pair consisting of the maximal entity in the
pluralization of N$_{root,k}$ and its context k. (For singular count nouns, it
gives the relevant ordered pair if and only if N denotes a singleton set,
otherwise it is undefined.) The interpretation of coordination of
definite count and mass nouns follows naturally:

(35) $\llbracket$the tables and the other furniture$\rrbracket$
    = AND($\pi_1(\llbracket$the tables$\llbracket_k$), $\llbracket$furniture$\rrbracket$)
    = AND($\pi_1(<\sigma\pi_1$(TABLE$_k$), k$>$), $\sigma$furniture$\rrbracket$)
    = AND($\sigma\pi_1$(TABLE$_k$), $\sigma$furniture$\rrbracket$)
    = AND($\sigma*$(TABLE$_{root,k}$), $\sigma$furniture$\rrbracket$)

    Despite the difference in types between mass nouns and count
nouns, there is no ontological distinction between the entities with
respect to which they are interpreted, and these entities are thus
recoverable from the count noun meaning. This captures both Chierchia’s intuition that the carpeting and the carpets can be used to refer to the same pile of entities and also fact that they need not do so (see discussion in section 6).

To sum up so far, we have argued against Chierchia’s (1998a) proposal that the grammatical accessibility of atomic sets in the denotation of singular count nouns is determined by the notion of salience, derived presuppositionally. Instead, we have proposed a grammatical definition of countability, namely that we count semantic atoms, entities that count as one relative to a particular context $k$ and that are indexed for that context. Count nouns are of a different type from mass nouns and are derived from mass nouns via the operation $\text{COUNT}_k$. We can now look more closely at the lexical derivation of mass and count nouns (in section 6) and the differences in their grammatical behaviour (in section 7).

6 MASS NOUNS V. COUNT NOUNS

Mass nouns are root nouns. Like plural nouns, they denote Boolean algebras, which is why Chierchia (1998a) calls them lexically plural. A mass noun has the same denotation as a root noun, that is, a mass noun denotes $\text{N}_{\text{mass}}$ the closure under sum of a vague set of minimal parts in $\text{N}_{\text{mass}}$. These minimal parts may be perceptually salient in a particular context but need not be so. Predicates that consistently denote sets of salient individuals such as furniture or jewellery are naturally atomic, and the minimal entities in their denotations are available as the basis of quantity judgments (Barner & Snedeker 2005). Crucially, even when the contextually relevant minimal elements of $\text{N}$ are perceptually salient, this is not encoded semantically, and they are not grammatically accessible for counting and related operations. This is because they are not of the right type: mass nouns are of type $<d,t>$, that is, functions from individuals to truth values, while counting operations apply to count nouns of type $<d\times k, t>$, that is, functions from ordered pairs to truth values. In order for the minimal elements of $\text{N}_{\text{mass}}$ to be counted, a classifier must be used. The most neutral classifiers are unit of and piece of and can be thought of as an explicit expression of the $\text{COUNT}_k$ operation as in (36):

(36) I bought a unit of furniture/one piece of furniture.

Without going seriously into the semantics of classifiers here, we can hypothesize that unit of is analysed as a function from $\text{M}$ into
M × K, which applies to a mass noun and which individuates entities relative to a particular context. This is illustrated for unit in (37a,b).

Some classifiers add more lexical information than unit, for example, strands of hair, cups of coffee and so on. We assume that these add properties conjunctively to the semantic atoms as in (37c,d), though we will not discuss this further here.

\[(37)\]
\[\begin{align*}
\text{a. } & \text{unit}_k \text{ of } = \lambda x. \pi_1(x) \in (P \cap k) \land \pi_2(x) = k \\
\text{b. } & \text{unit}_k \text{ of furniture} = \text{unit}_k \text{ of } ([\text{furniture}])
\quad = \lambda x. \pi_1(x) \in (\text{FURNITURE} \cap k) \land \pi_2(x) = k \\
\text{c. } & \text{strand}_k \text{ of } = \lambda P \lambda x. \pi_1(x) \in (P \cap k) \land \\
\ & \text{LONG-AND-THIN}(\pi_1(x)) \land \pi_2(x) = k \\
\text{d. } & \text{strand}_k \text{ of hair} = \text{strand}_k \text{ of } ([\text{hair}])
\quad = \lambda x. \pi_1(x) \in (\text{HAIR} \cap k) \land \\
\ & \text{LONG-AND-THIN}(\pi_1(x)) \land \pi_2(x) = k
\end{align*}\]

Singular count noun meanings are derived from root noun meanings via the COUNT_k function. Intuitively, there are two sorts of count nouns, those that denote things that are inherently individuable and those that do not. The first kind are naturally atomic predicates.

Case 1: naturally atomic count nouns: boy, pencil, cat, etc.

These are the predicates that first come to mind when we think of count predicates. They are naturally atomic because what counts as one entity is not determined by context but by the naturally atomic structure of the stuff. What counts as one P is part of our knowledge of what a P is, whether P is cat, boy, pencil and so on. So the atomic entities in N_k are not determined by the choice of k but by our knowledge of the world.

It would be nice to say that naturally atomic predicates are never context dependent and that COUNT_k(N_root) always yielded the same set on a particular domain for these predicates, no matter what the choice of context, but this is an oversimplification. Child is a naturally atomic predicate since children come in inherently individuated units. What counts as the denotation of child, though, may vary depending on aspects of the situation that are independent of the choice of atomic entities. What child means with respect to paying bus fares (under 12 in the Netherlands and under 14 in the UK) is different from what it means when filling out tax declarations. But this kind of context dependence is different from the context dependence shown by fence: child is a vague predicate because of borderline cases, but there is never any doubt as to what counts as ‘one’. The kind of context dependence shown by child is not
dependent on the choice of contextual counting parameter \( k \) since a 13-year-old girl will count as ‘one entity’ and thus be a member of \( k \) for any choice of \( k \), whether or not she counts as a child in \( k \). Vagueness shows up in the mass domain too; childhood is vague in the same way that child is, as can be seen from the vagueness of expressions like during childhood. Vagueness of this kind may be treated by the theory of supervaluations proposed by Kamp (1975) or by other treatments in which precisifications of vague predicates are dependent on contextual factors.

Given the existence of borderline vague naturally atomic predicates, we cannot say that if \( N \) is naturally atomic, \( \text{COUNT}_k(N) \) is a constant function independent of choice of \( k \). Rather, when \( N \) is naturally atomic, \( \text{COUNT}_k(N) \) is constrained so that for any two contexts \( k \) and \( k' \), if \( d \) is a semantic atom in context \( k \), then it will be a semantic atom in \( k' \) as well. So a thirteen-year-old may count as a child in some contexts and not in others, but if she does count as a child in context \( k \), then she will necessarily count as a semantic atom in context \( k' \) too. This means we can give a definition of natural atomicity as follows:

(38) Natural atomicity:

If \( N \) is a naturally atomic predicate then:

\[
\forall x \forall k \forall k' [x \in \pi_1(N_k) \land x \in \pi_1(*N_{k'}) \rightarrow \pi_1(N_{k'})]
\]

‘If \( N \) is naturally atomic, then for any two contexts \( k \) and \( k' \), if \( x \) is an atom of \( N_k \), and \( x \) is in the denotation of \( N_{k'} \), \( x \) is also an atom in \( N_{k'} \).’

Note that what counts as a natural atom here is usually not dependent on size but on some systemic property that defines what counts as one \( N \). For example, what counts as one boy is dependent on ‘boy-ness’. A giant preteenager and a small premature male baby each count as one instance of boy and together they make a plurality of boys with the cardinality two. [Note also that (38) can be used to define naturally atomic mass as well as count predicates.]

Case 2: homogeneous nouns: fence, wall, sequence, quantity, bouquet.

These predicates are not naturally atomic since the entities do not come in inherently individuated units. The predicate itself does not uniquely determine an individuating function, and thus, the set of atomic entities will vary from context to context. Assume a root noun denotation \( \{a, b, c, a \sqcup b, a \sqcup c, b \sqcup c, a \sqcup b \sqcup c\} \). \( \text{COUNT}_k(N) \) applied to this set could choose as a set of atoms \( \{a, b, c\} \) or \( \{a, b \sqcup c\} \) or \( \{a \sqcup b, c\} \) or \( \{a \sqcup c, b\} \) or \( \{a \sqcup b \sqcup c\} \) depending on the choice of \( k \). The set of atomic entities in \( \text{fence}_k \) need not be identical to the set of
contextually salient minimal elements in $fence_{\text{root}}$ and need not even cover it. For example, suppose the contextually salient minimal elements in the denotation of the root noun are the pieces of fencing which different individuals possess, but that not all of these fence pieces are big enough to count as actual fences in a given context $k$. Some of these small fence pieces in $\text{FENCE}_{\text{root}}$ may be construed as parts of other atomic fences, while other pieces may be so small as to be irrelevant and may not be included at all. Which set we get in $\text{FENCE}_k$ depends on what the context $k$ chooses to count as ‘one’ fence.

As mentioned above, the denotation of the plural of a count noun depends directly on the choice of $k$. So while Chierchia is correct in arguing that the same pile of objects may be the denotation of the furniture and the pieces of furniture or the carpeting and the carpets, this need not be the case. Suppose that the denotation of carpeting is generated in $M$ from a set of atoms that includes all pieces of carpeting, for example, $\{a, b, c, d\}$, but that $\text{CARPET}_k = \text{COUNT}_k(\text{CARPETING})$ is the intersection of that set with $k$ and includes only those atoms in carpeting that are in the context ‘big enough’ to count in $k$, for example, $\{a, b\}$. Then, the maximal entity in $\pi_1(*\text{CARPET}_k)$ will not be the same entity as the maximal entity in carpeting. This seems right. If I say to the movers, ‘Please take all the carpets out of the attic’, they can say they have complied if they leave a few odds and ends lying there. But if I say, ‘Please take all the carpeting out of the attic’, that requires them to take all the small pieces away too. [Landman (2007) suggests other cases where substituting apparently synonymous mass and count predicates for each other leads to different truth-values.]

Once we start thinking about it, we can find more cases in which the atomic individuals in the denotation of a count noun are context dependent:

(i) If I and my neighbour build adjoining walls, we may say either ‘Together we built a wall in front of both our houses’ or ‘We each built a wall in front of our respective houses’.
(ii) If I have a bunch of flowers and I divide it in two and give a part of each to my daughter and her friend, then either each has a bunch of flowers or each has half a bunch of flowers.
(iii) If a restaurant owner puts together tables $a$ and $b$ to make a bigger table $a \sqcup b$ and tables $c$ and $d$ to make another bigger table $c \sqcup d$, then $a$, $b$, $c$ and $d$ no longer count as semantic atoms in that context. There are either two tables or four tables in the restaurant,
but no other possibility (i.e. not six and not three). This makes clear the contrast with the mass domain, where if I put sand together with sand, the original sand parts and the sum of the parts fall under the denotation of *sand* simultaneously.

However, even with non-naturally atomic predicates, there is frequently a default measure unit, usually something like ‘perceptually salient, spatially distinct unit’, although it can be overridden by an appropriate context. Context and convention determine how far the denotation can vary with choice of context. In the UK, final school examinations (A-level exams) are taken in three or four subjects, and for each subject, the exam may consist of a number of sessions. One can refer to ‘the geography exam I took this morning’ or say that ‘There are four different geography exams’, using *exam* to mean ‘examination session’. Or one can say, ‘The geography exam has four parts’ or ‘This year’s geography exam was very difficult and the grades for the exam will be low’, using the singular term to denote the subject examination as a whole. Still, it would be infelicitous to refer to the whole set of school finals across subjects as ‘the A-level exam’ in the singular rather than the plural.

It seems then that natural atomicity is a property that comes in degrees, and the range of contextual variation that is possible with different choices of k depends on the meaning of the predicate. Tables seem to come in inherently distinguishable units, and for most contexts k, COUNT$_k$(TABLE) will yield a set of inherently individuable tables with little variation, but nonetheless, as we just saw above, COUNT$_k$(TABLE) may still vary for some choices of k. While we can define a formal notion of natural atomicity as in (38), there is a sense in which natural atomicity is a gradable property. Example (38) defines 'highly naturally atomic predicates', and the degree of natural atomicity can perhaps be defined in terms of the proportion of relevant contexts for which (38) is violated. Note though that when a predicate P is highly naturally atomic, COUNT$_k$, the function deriving count nouns, cannot choose a context that overrides natural atomicity and pick entities other than the natural atoms as the atoms of the context. Overriding natural atomicity in count nouns is done through group-forming classifier, as in *a class of boys, a deck of cards* and so on (see footnote 10).
With these distinctions in place, we can explain the grammatical differences between count and mass nouns. We have already seen why grammatical pluralization is restricted to count nouns: it is an operation on semantic atoms and thus restricted to expressions of type \(<<d,k>, t>\). It is irrelevant in the mass domain because the mass domain is already Boolean.

As we saw in section 2, there are three other central issues: (i) modification of N by numerals and sensitivity of determiners in general to the mass/count distinction, (ii) sensitivity of the numeral phrase in partitive constructions to the mass/count status of the head of the complement DP, and (iii) sensitivity of each other to the mass/count status of the head noun of its antecedent. I shall show that in each of these cases, the relevant grammatical operation is sensitive to the typal distinction between mass and count nouns.

(i) Modification by numerals: what stops \textit{three} from modifying furniture in (39)?

(39) *three furnitures/three chairs/three pieces of furniture

Krifka (2008b) suggests that the difference is presuppositional, that is, that number words presuppose the atomicity, or discreteness, of the set that they modify, and thus can modify count nouns and not mass nouns. But as we have seen, some mass nouns such as \textit{furniture} do denote sets of discrete entities, while some count nouns such as \textit{fence} do not, and a presuppositional account is not able to make the necessary distinctions.

In the analysis of the mass/count distinction presented here, numerals cannot modify mass nouns because there is a type mismatch: mass and count nouns denote different entities and numerals are sensitive to this distinction. Determiners may be sensitive to the distinction, although some determiners such as \textit{some} and \textit{the} apply to both types.

We treat numerals as adjectival modifiers and assume that a numeral such as \textit{three} in its modifier meaning denotes a function from \(M \times \{k\}\) into \(M \times \{k\}\), that is, from count predicate denotations into count predicate denotations. It applies to a plural count expression and gives a set of ordered pairs where the first element has three atomic parts and the second element preserves the information about the context in which these parts count as atoms. The typal sensitivity of the numeral is justified theoretically by the fact that while the cardinality function \(\|\) is a function mapping entities onto the number of its atomic parts, numeral modifiers can only count relative to
a fixed context $k$. So *three*, and other numeral modifiers, make use of the parameterized cardinality function $|\cdot|_k$ that assigns a value to a plural entity depending on the number of its atomic parts in $k$. Numerical modifiers thus apply to noun denotations in which a context $k$ is grammatically encoded.

The numeral expression *three* thus denotes a function from count noun denotations into count noun denotations and is of type $\langle\langle d \times k, t \rangle, \langle d \times k, t \rangle\rangle$. It applies to a set of ordered pairs $N_k$ and gives the subset of $N_k$, such that all members of $\pi_1(N_k)$ are plural entities with three parts each of which is an (atomic) entity in $k$. [As above, $P$ is a variable over count predicates of type $\langle d \times k, t \rangle$ and $x$ a variable of type $d \times k$. $\pi_2(P)$ is the context parameter on the parameterized cardinality function, which is dependent on the context relative to which the count predicate has been derived.]

$$
\begin{align*}
\text{(40) } [\text{Three}\langle\langle d \times k, t \rangle, \langle d \times k, t \rangle\rangle] &= \lambda P \lambda x. P(x) \land |\pi_1(x)|_{\pi_2(P)} = 3 \\
&= 3
\end{align*}
$$

*Three* denotes a function which applies to a count predicate of type $\langle d \times k, t \rangle$ and gives the subset of the count predicate i.e. a set of ordered pairs where the first projection of each ordered pair has three parts which count as atoms in $k$.

Since *three* must apply to a count predicate with a denotation in $M \times \{k\}$, the infelicity of *three furniture(s)* is due to a type mismatch. If *three* is treated as a determiner denoting a function from predicates into generalized quantifiers, then a similar explanation can be given in terms of the selectional properties of the determiners. This can be extended to other determiners: *every*, which does not occur with mass nouns, denotes a function from singular count predicates into sets of count predicates or generalized quantifier denotations; *many* denotes a function from plural count predicates into generalized quantifier denotations; and *much* from type $\langle d, t \rangle$ into generalized quantifier denotations.

(ii) Numerical partitives take only count DPs as complements.

Numerical partitives take only count DPs as complements while expressions such as *some of the* take either mass or count DPs as complements as in (41):

$$
\begin{align*}
\text{(41) } &\text{a. three of the pieces of furniture/*three of the furniture} \\
&\text{b. much of the furniture/*much of the pieces of furniture} \\
&\text{c. some of the pieces of furniture/some of the furniture}
\end{align*}
$$

These examples are crucial support for this theory of count nouns, as they show that the distinction between mass and count
nominals must be accessible outside the DP itself, and that the structural difference between mass and count nouns must be preserved at all levels of compositional structure that they project. Three modifies plural count nouns, and thus, three of applies to DPs lexically headed by plural count nouns; much (of) applies to mass nouns and definite DPs headed by mass nouns, while some (of) applies both to count and mass nouns, at the N and the DP level. (I assume that D is the functional head of DP and N its lexical head.)

The embedded DP in partitive constructions is always definite. In section 6, we gave the interpretation of the following Link (1983) and references cited there in terms of the $\sigma$ operation:

$$\sigma(X) = \sqcup X \text{ if } \sqcup X \in X, \text{ otherwise undefined.}$$

For mass nouns: $\sigma(N) = \sqcup_M N$, the (unique) maximal entity in $N$, if defined.

For count nouns: $\sigma(N_k) = <\sqcup_M \pi_1(N_k), k>$, if defined.

We need to recover the denotation of the predicate head from the DP. We define an operation PARTITIVE on definite DPs that gives the set of parts of $\sqcup N$, $N$ the lexical head of the DP.

We lift the part-of relation on ordered pairs in $M \times \{k\}$ from $M$:

$$<x_1, k> \sqsubseteq_k <x_2, k> \text{ iff } x_1 \sqsubseteq_M x_2$$

The partitive operation follows the following definition schema, operating on a definite complement and giving the set of its parts (we use $x$ as a generalization over $x$ and $\bar{x}$):

$$\text{PARTITIVE} (\sigma N) = \{x : x \sqsubseteq (\sigma N)\}$$

For a mass predicate, $\text{PARTITIVE} (\sigma(N_{\text{mass}})) = \{x : x \sqsubseteq_M \sigma(N_{\text{mass}})\}$, which is $N_{\text{mass}}$ itself.

For a count predicate, in a default context $k$, $\text{PARTITIVE} (\sigma(N_k))$ is again lifted from $M$:

$$\text{PARTITIVE} (\sigma N_k) = \{<x,k> : <x,k> \sqsubseteq_k <\sigma(\pi_1(N_k)), k>\}$$

Crucially, since we kept track of the context $k$ during all the operations involving the composition of the embedded DP, the operation giving the set of parts of $\sigma N_k$ will still have access to the original context $k$.

The partitive determiners three and some can now have exactly the same semantics as they have as N modifiers. Since three (of) makes use of the parameterized cardinality function that makes reference to $k$, it can apply to $\text{PARTITIVE} ([\text{the chairs}])$ or $\text{PARTITIVE} ([\text{the pieces of furniture}])$ that are sets of type $<d \times k, t>$, but not to $\text{PARTITIVE} ([\text{the}}$
furniture], which is a set of type <d,t>. Much(of) can take only PARTITIVEnmass and some(of) can take any definite complement.\textsuperscript{13,14}

(iii) A reciprocal cannot take a mass noun as antecedent although it is ‘lexically plural’.

This is illustrated in (42) and in Gillon’s examples, in (43) repeated from (4).

(42) a. (The) pieces of furniture are piled on top of each other.
    b. #The furniture is piled on top of each other.
    c. #Furniture was piled on top of each other.

(43) a. The curtains and the carpets resemble each other.
    b. The curtaining and the carpeting resemble each other.

The interpretation of reciprocals is too complicated to discuss in detail here, involving as it does a variety of interpretations including the so-called intermediate interpretations. I offer only a suggestion of how aspects of the mass/count distinction may be involved in the process.

\textsuperscript{13} I assume that the PARTITIVEn operation also applies to group predicates as in three of the decks of cards. For group nouns: $\sigma_{NGROUP} = \bigcup_{GRP_{k}}$, if defined.

$\text{PARTITIVEn}(\sigma_{NGROUP}) = \{<x,k*>: <x,k*> \in \text{GRP}_{k}, \sigma_{NGROUP}\}$

$\text{PARTITIVEn}([\text{the decks of cards}])$ is the set of group parts of the maximal entity in decks of card (since the join relation is not lifted from $B_{k}$ but from $\text{GROUP}_{k}$), that is, the set of deck entities. Thus, three of the decks of cards denotes pluralities of decks (of cards) and not pluralities of cards.

\textsuperscript{14} A reviewer suggests that a syntactic copy account of partitives makes the problem of how to project the typal distinction between mass and count nouns beyond the DP level irrelevant. In such an account (e.g. Sauerland & Yatsuhiro 2004 and references cited there), three of the boys contains a copy of the sortal boys outside the definite. At LF, the expression to be interpreted is three boys of the boys. The typal distinction between mass nouns and count nouns is thus directly available outside the definite DP. But the semantic interpretation of the syntactic copy theory has only been worked out for the simple cases and not for more complicated cases where the embedded nominal is a relational noun. Three of the mothers of children in this class is not equivalent to three mothers of the mothers of children in this class just as three of the successors of 10 is not equivalent to three successors of the successors of 10. If of is constrained to be interpreted as a partitive relation, as in Sauerland and Yatsuhiro (2004; see also Barker 1998), then the higher sortal cannot be a copy of the denotation of the embedded noun but needs to shift into the appropriate non-relational meaning. Doing this is non-trivial (see, e.g. the discussion in Partee and Borschev 2003 about shifts of this kind in genitive constructions), especially since non-relational uses of nouns like mother are highly restricted. The attractiveness of the copy theory as an alternative to the theory that I propose here is thus considerably reduced. Sauerland and Yatsuhiro (2004) do not discuss how the copy theory would handle these cases. In a different version of the copy theory, Barker (1998) assumes a null nominal head without suggesting that the content is copied from the lower noun, thus avoiding the problem of relational nouns. However, he does not discuss the mass noun/count noun contrast in partitives and gives a theory of partitivity that does not explain the ungrammaticality of *three of the furniture. In fact, his theory cannot explain it without adopting a typal difference between mass and count nouns.
As has long been known, whatever interpretations (intermediate, non-intermediate) are acceptable, the antecedent of a reciprocal must be a plural noun phrase, as illustrated in (44):

\[(44)\]
\[
a. \text{The boys helped each other.}
\]
\[
b. \text{The chairs stood on top of each other.}
\]
\[
c. \text{The boy and the girl helped each other.}
\]
\[
d. \text{John and Mary helped each other.}
\]
\[
e. *\text{The boy helped each other.}
\]
\[
f. *\text{The furniture stood on top of each other.}
\]

Plural noun phrases can be definite plurals on a distributive reading as in (44a/b), conjunctions of singular definites or proper names as in (44c/d), but not singular definites or mass nouns as in (44e/f). As (43) shows, conjunctions of definites and conjunctions of mass nouns are also possible. Conjunctions of mass and count nouns are also possible, as in (45):

\[(45)\] The curtaining and the carpets resemble each other.

Chierchia suggests that the impossibility of (44e/f) should be put down to a requirement that antecedents of reciprocals must be morphologically plural, but this is too weak for two reasons: first, this makes appeal to morphological agreement when other argumentation (both in this and in his article) has been at the level of semantic structure, and second, there are examples with singular antecedents that are much better than (44e/f), namely examples where the singular is an inherently group term. While the examples in (46) are not perfect, they are better than (44e/f).

\[(46)\]
\[
a. \text{The family stood next to each other and consoled each other at the funeral.}
\]
\[
b. \text{The committee argued with each other for some time before coming to a unanimous conclusion.}
\]

We assume that the semantic effect of a reciprocal is twofold: it identifies two thematic arguments as satisfied by the same plural DP, and it constrains semantically how the atomic parts of the plural DP participate in the event denoted by V. We show how this works explicitly, using the account of plural arguments from Landman (1997, 2001).

Landman (1997, 2000) argues that verbs can be either semantically singular, denoting sets of singular events or semantically plural, denoting sets of pluralities of events. He assumes that singular verbs assign singular thematic roles to singular DPs, while plural verbs assign plural roles to plural DPs. As plural events are sums of singular events, so plural roles are
sums of thematic roles. The intuition behind this is that only singular entities participate directly in events. *John helped Mary* is true if the individuals denoted by *John* and *Mary* participated directly in the helping event. Plural individuals do not directly participate in events: rather, event participation distributes down to the atomic parts of the entities denoted by the plural noun phrase. In *The boys helped the farmer,* the boys satisfies the plural agent role of helped syntactically, and the sentence is true iff the atomic parts of the plural individual, that is, the individual boys denoted by *the boys,* each participated in helping the farmer. Intuitively, if *the boys* is the antecedent of a reciprocal, as in (42a), the thematic participation in the event must distribute down to the atomic parts of the plural DP in such a way that for any two atomic parts of DP, the relevant relation between them is reciprocal.

Collective readings of DPs such as *the boys* are derived as follows. The plural entity denoted by *the boys* is $\sigma^{\text{BOYS}},$ the maximal sum of individual boys. This is lifted to the group reading $\uparrow(\sigma^{\text{BOYS}})$ and is treated as a higher order singular entity that can be assigned a thematic singular role. On the collective reading, *The boys helped the farmer* is true if the group collectivity denoted by *the boys* directly participated as a singularity in the event of helping the farmer. The individual parts of the group entity are not accessible to the interpretation and so the collective cannot be an antecedent for a reciprocal, as (47) shows:

(47) The boys helped each other carry the piano upstairs (#together).

This account applies straightforwardly to reciprocal resolution with mass and count antecedents. Reciprocal antecedents must be plural count nouns such as *the boys* or conjunctions of singularities like *John and Mary* or *the boy and the girl* but not mass nouns or collectives. Assume that proper names always denote semantic atoms, that is, are inherently of type $d \times k.$ This is plausible since, as constants, they are contextually rigid and denote the same individual cross-contextually relative to a model. (If proper names are rigid designators, this follows automatically.) Then, antecedents for reciprocals are constrained to be pluralities of semantic atoms, as in (48):

(48) Constraint on reciprocal interpretation:

An antecedent for a reciprocal must be a plural entity in $M \times \{k\}.$

In (44a), *The boys helped each other,* the reciprocal associates the value of the theme relation with the value of the agent. *Helped* denotes the set of plural events $^n\text{HELP},$ and $^n\text{Ag}$ and $^n\text{Th}$ are the plural roles

---

15 The antecedent can be also be a plural of a group predicate. So more correctly, the antecedent of a reciprocal must be a plural entity in Boolean algebra generated by $A_k^*$ (see footnote 10).
assigned by *HELP to its arguments. \( x \) remains a variable of either type \(<d,t>\) or \(<d\times k, t>\) since help does not restrict its arguments to either mass or count DPs.

Help each other is interpreted as in (49a). The reciprocal requires that the variable must be of type \(<d\times k, t>\) and introduces the constraint on interpretation in (49b):

\[
(49) \quad \text{a. } \left[ \text{Help each other } \right] = \lambda x.\ast\text{HELP}(e) \land \ast\text{Ag}(e) = x \land \ast\text{Th}(e) = x \\
\text{b. Interpretation of the reciprocal:} \\
\forall y,z \quad [y \subseteq \text{ATOM} x \land z \subseteq \text{ATOM} x \land \neg(y = z) \rightarrow \\
\exists e' \quad [e' \subseteq e \land \text{Ag}(e') = y \land \text{Th}(e') = z]]
\]

Plural DPs with count noun heads can be antecedents of reciprocals since they are of type \(d\times k\). Conjunctions of singular definites and proper names can be antecedents of reciprocals, under the assumption (Link 1983) that conjunction of entities is a summing operation that results in pluralities. We add to Link’s summing operation the constraint that both conjuncts within a conjoined DP must be interpreted relative to the same context. Conjunctions of proper names then denote pluralities in \(M \times \{k\}\).

Singular definites and proper names cannot be antecedents for reciprocals since they do not denote pluralities. We can either stipulate that \( x \) in (49b) ranges over plural objects or assume that (49b) cannot be satisfied trivially. Mass nouns cannot be antecedents for reciprocals since they are of the wrong type. With respect to examples like (46), we assume that some (but very few) of the count nouns in English that denote groups of individuals (i.e. group nouns) are marked as inherently plural and under certain circumstances allow access to the atomic entities that they group together. This is why they can marginally be antecedents for reciprocals, and also why they allow plural agreement as in The family are arriving this evening or The committee are arguing about it right now.\(^{16}\)

We now go back to Gillon’s examples in (43), repeated here:

\[
(43) \quad \text{a. The curtains and the carpets resemble each other.} \\
\text{b. The curtaining and the carpeting resemble each other.}
\]

\(^{16}\) This appears to be possible only if the noun denotes a group of animate individuals, that is, family, class of boys and herd of cattle appear to take plural agreement but not deck of cards and set of silverware.
Example (43a) is ambiguous. The first reading, the distributive reading, is that all the curtains and carpets resemble each other. On this reading, the conjoined DP denotes the sum of the maximal plurality of curtains and the maximal plurality of carpets, and the interpretation of the reciprocal requires every two atomic entities (i.e. atomic parts of $\sigma$CURTAINS \& $\sigma$CARPETS) to resemble each other. On the second reading, the curtains as a group, or singular collection, resemble the carpets as a group, or singular collection, and vice versa. On this reading, $\sigma$CURTAINS and $\sigma$CARPETS are treated as collections and are raised to the group atoms $\uparrow(\sigma$CURTAINS) and $\uparrow(\sigma$CARPETS) (see Landman 1989a,b).

In (43b), where the antecedent of the reciprocal is the curtaining and the carpeting, the distributive reading is not available. Curtaining and carpeting are nouns of type <d,t> and the definites denote maximal sums of entities in M, so condition (48) is not satisfied. The group reading is available since $\sigma$CURTAINING and $\sigma$CARPETING can be raised to atomic collections $\uparrow(\sigma$CURTAINING) and $\uparrow(\sigma$CARPETING), respectively. They then satisfy (48) in exactly the same way that the collections based on count nouns do. Finally, note that in (45), repeated here, we get only the group reading. Example (45) means the same as (43b).

(45) The curtaining and the carpets resemble each other.

This is predicted since, as we saw above in section 5.2, coordination of the mass and count nominals is at the mass type, and thus, within the coordination the carpets will denote the same type of entity as the carpeting.

To conclude this section, we have shown that the characteristic grammatical distinctions between mass and count nouns are due to the sensitivity of some grammatical operations to the typal distinction between them. This sensitivity reflects the fact that the grammatical operations that are essentially counting operations (numerical modification and partitives), as well as reciprocals, are constrained to apply to semantic or counting atoms, atoms that count as one relative to a particular context. This is quite a natural restriction for counting operations, but perhaps less natural for reciprocals. However, we do not want the connection between reciprocals and semantic atomicity to be too direct because in fact there are languages where it does not hold and where reciprocal resolution is sensitive to natural as well as semantic atomicity. As Pires de Oliveira and Rothstein (in preparation) show, reciprocal resolution in Brazilian Portuguese is one such case. Although (50a) is ungrammatical in English, (50b) is perfectly acceptable.
in Brazilian Portuguese, and also in European Portuguese, although, predictably, (50c) is not.

(50) a.*Furniture (of this brand) fits into each other
    b. Mobília (desssa marca) encaixa uma na outra.
    Furniture (of this brand) fits one in+the other
    ‘Furniture (of this brand) fits into each other.’
    c.*Ouro cai um atrás do outro.
    Gold falls one behind of-the other.

This indicates that there is a cross-linguistic parametric difference as to whether reciprocal resolution is constrained to have plural semantic atoms as antecedents and further supports the claim that the distinction between semantic and natural atomicity is a grammatical one.

8 KRIFKA (1989)

The relevance of natural atomicity to the semantics of count nouns has been pointed out in the literature before, notably in Krifka (1989, 1995). He also pursues the idea that count nouns involve an implicit measure function and are derived from abstract nominal predicates and that they thus differ in type from mass nouns. However, the theory proposed here differs from Krifka’s work in a number of important respects. Most crucial is the fact that Krifka’s typal distinction between mass and count nouns is a lexical distinction that is neutralized as soon as a noun is inserted into a nominal phrase. The typal difference is not projected higher up into the NP nor into the DP and is not accessible to operations such as partitivity and reciprocal resolution.

Krifka (1989) analyses count nouns as two-place relations between numbers and entities, taking as a model expressions in English such as five head of cattle. He analyses the classifier head of as a measure function. In five head of cattle, the noun mass predicate cattle is interpreted as \( \lambda x. \text{CATTLE}(x) \), a function applying to plural entities. The function is associated with a lattice structure \( L \) representing the set of individual cattle closed under sum. Head of introduces a measure function represented by NU (for natural unit), and five head thus denotes the measure function \( \lambda P. \lambda x. P(x) \land \text{NU}(P)(x) = 5 \). This measure function, when applied to the denotation of cattle, yields the function compatible with the lattice \( L \), namely \( \lambda x. \text{CATTLE}(x) \land \text{NU}(\text{CATTLE})(x) = 5 \), equivalently, the set of plural entities in the denotation of cattle that are sums of five individuals.

Krifka proposes that in count nouns, the reference to a natural unit is built into the meaning of the head noun. The count noun cow
(as opposed to cattle) denotes the two-place relation COW’ between numbers and entities given in (51a), where the relation between COW’ and an abstract predicate COW is given in (51b):

\[(51) \quad \begin{align*}
    a. & \quad \lambda n \lambda x. COW'(x,n) \\
    b. & \quad COW'(x,n) \leftrightarrow COW(x) \land \text{NATURAL UNIT}(COW)(x) = n.
\end{align*}\]

COW is in effect what we have called N_root, and COW’ the denotation of the count noun. This noun either combines directly with a numeral, for example, five, to give the predicate \(\lambda x. COW'(x,5)\), or there is existential quantification over the expression in (51a) to give the expression \(\lambda x \exists n. COW'(x,n)\). Thus, the count noun as used in the syntax is of the same type as the mass noun. Count nouns differ structurally from mass nouns since they are born at type \(<n, <d,t>>\), while mass nouns are born as predicates at type \(<d,t>\), but the typal difference is not accessible above the N-level. Singular count nouns do not have a special status; they are simply functions from the number 1 to sets of singleton cows and are just one instance of the relation denoted by COW’. One cow or a cow denotes the predicate \(\lambda x. COW'(x,1)\), while five cows denotes \(\lambda x. COW'(x,5)\) and the bare plural cows denotes \(\lambda x. \exists n[COW'(x,n)]\). Plural marking, that is, the contrast between cow and cows, is a matter of agreement: one or a induces singular morphology on the head noun, while five induces plural morphology on the head noun. Krifka’s account makes countability the result of an individuation operation and treats count nouns as measure operations from a number \(n\) to sets of plural individuals with cardinality \(n\). But the content of the NU function is presupposed and is as general as possible: ‘... we can assume that NU yields the same measure function for entities of a similar kind ... NU(CATTLE) and NU(GAME) should denote the same measure function’ (Krifka 1989: 84).\(^{17}\)

The theory proposed here differs crucially, both grammatically and conceptually. In both theories, the grammatical distinction is expressed in the typal difference between mass and count nouns, but for Krifka, the difference is in basic nominal meanings, and the typal resolution is designed so as to be neutralized as low as possible in the syntactic projection of the N. When count nouns combine with a number, they get back immediately to type \(<d,t>\), and if they do not combine with a number, existential quantification over the \(n\) argument at the N-level gets the noun

\(^{17}\) Krifka (2008a) has accepted that the problems raised by nouns like fence and sequence require the NU function to be at least partially context dependent, though he has not been explicit about how this should be implemented.
back to type \(<d,t>\). Thus, operations that apply at the NP or DP level cannot distinguish between count and mass nouns. In the account presented in this article, the typal distinction between mass and count nouns is not neutralized but persists up to the level of the DP and is used to explain a series of syntactic and semantic combinatorial differences (number modification, partitive construction, plurality and reciprocal resolution). The difference in meaning between mass and count nouns thus determines the combinatorial possibilities all the way up the tree. Since Krifka (1989) is concerned with aspectual issues rather than with explaining the grammatical differences between mass and count nouns, he does not discuss how to account for partitives and reciprocals, but there is no natural way in which a typal difference between count and mass nouns neutralized at the N-level could explain them.

The different ways in which the two theories exploit typal distinctions corresponds with the conceptual difference between the two theories. For Krifka, count nouns are extensive measure functions that measure in terms of (presupposed) natural units. Count nouns are analogous to expressions such as kilo and litre, which measure in terms of kilo and litre units. Units that have measure value 1 have no special status beyond entities whose value is 5 or 2500. In this article, however, we do not treat singular count nouns as measure functions but as expressions that denote sets of countable units, where a countable unit is a pair consisting of an entity and a context. These are what we have called semantically atomic sets. The idea underlying this is that counting and measuring are two very different operations. Counting puts entities (which already count as ‘one’) in correspondence with the natural numbers, while measuring assigns an (plural) individual a value on a dimensional scale. So while Krifka treats count nouns as extensive measure functions assigning pluralities of entities a value \(n\), we treat them as expressions grammatically encoding countability, indexing individuals for the contexts in which they count as ‘one’. And this is essential since, as we saw in sections 3 and 4, countability is not necessarily a property of inherently individuable entities, and natural atomicity (being a natural unit) is neither a necessary nor a sufficient condition of countability. Furthermore, counting pluralities is a modification operation on the plural of the count noun, indicating how many atomic (countable) parts the plurality has and not a measure function on the denotation of the root nominal. This distinction between counting and measuring is an important one that shows up in other places in language, in particular in the interpretation of classifier constructions (see discussion in Rothstein 2009a).
To sum up, we have distinguished between three different kinds of atomicity: formal atomicity, semantic atomicity and natural atomicity, all of which are relevant in different ways. Formal atomicity is a property of Boolean algebras generated by a set of atoms. By hypothesis (Chierchia 1998a, b), mass nouns have denotations in the formally atomic domain. A count predicate makes accessible a set of semantic atoms, derived via the $\text{COUNT}_k$ operation, and mechanisms such as grammatical counting and reciprocals are sensitive to semantic atomicity. Naturally atomic predicates denote a set of entities that are inherently individuable and that are cognitively salient as individuals. Natural atomicity is not the basis of grammatical countability but is a phenomenon that grammatical systems are sensitive to. Grammatical counting operations and semantic atomicity may hitch a ride on the back of natural atomicity but do not always do so. As Barner and Snedeker have shown, quantity judgments are sensitive to natural atomicity. Other grammatical phenomena that are sensitive to natural atomicity include modification by predicates such as $\text{big}/\text{small}$, as well as the alternation between $\text{stuks}$ and $\text{stukken}$ in Dutch. Furthermore, there appears to be cross-linguistic variation as to whether reciprocal resolution is sensitive only to semantic atomicity (e.g. English) or may also be sensitive to natural atomicity (e.g. Brazilian and European Portuguese).

By hypothesis, the dependence of grammatical counting on semantic atomicity is presumed to be universal, although semantic atomicity may not be expressed via count nouns. In section 9, we look briefly at how the distinction between semantic and natural atomicity shows up in classifier languages, where semantic atomicity is expressed only through classifier phrases.

9 EXTENSIONS TO CLASSIFIER LANGUAGES: MANDARIN CHINESE

An obvious question for any theory of the mass/count distinction is how it extends to classifier languages. A detailed discussion is impossible in the framework of this article, but we look briefly in this section at how this theory extends naturally to Mandarin Chinese. Classifier languages are those that do not grammaticalize the mass count distinction as a distinction between nominal heads and require classifiers to intervene between all numerals (and demonstratives) and the nominals they modify. Classifier languages exhibit variation in the use they make of classifiers (Sybesma 2008), but nonetheless some preliminary generalizations are possible (some of which apply also to English classifier + mass noun/plural count noun constructions).
that in English (Chierchia 1998a; Landman 2004; Rothstein 2009a), classifiers may denote either individuating/counting functions or measure functions. We assume, following Li (in progress), that classifiers in Mandarin show the same variation and restrict our discussion here to counting or individuating classifiers.

Since classifiers are needed to make all nouns countable, there is no grammatical difference between bare ‘count’ or bare ‘mass’ nouns. Thus, pingguo, ‘apple(s)’, in Mandarin Chinese cannot be explicitly counted without a classifier, as we saw above.

(52) liany * (ge) pingguo
two Cl apple(s)

We assume (as does Krikifa 1995) that the count operation that derives count nouns in the lexicon in languages with a lexical mass/count distinction is expressed by the classifier in languages such as Mandarin Chinese. In our theory, this means that Mandarin Chinese does not allow the COUNT\_k operation as a lexical operation, but requires it to be introduced explicitly by a syntactic element, the classifier. As well as being explicit expressions of the COUNT\_k operation, classifiers may add information about the criterion used for individuation in a particular context k. Thus cloud can be the complement of at least four different classifiers (Li, in progress)

(53) a. yi duoshu yun
one Cl\_blossom cloud
b. yi pian yun
one Cl\_piece cloud
c. yi tuan yun
one Cl\_ball cloud
d. yi lu yun
one Cl\_stream cloud

This is parallel to the way that, in English, we can say pieces of cotton or strands of cotton, but strands gives additional information, namely that the pieces are long and thin.

In general, for individuating classifiers (or classifiers on their individuating uses), \[[\text{Classifier}(N)] = \text{COUNT}_k(N_{\text{root}} \cap Q)\] where Q is a (possibly empty) expression \(<d,t>\), with the classifier performing the individuation function and Q possibly adding more information about the properties of the individual unit. Frequently, these properties are topological, as in piece of v. strand of or the examples in (53) but they need not be; zhi, from example (5) applies to naturally atomic predicates and includes the information that the individual d is a unit of animal, while ke used in liang ke shu ‘two units of tree’ applies to naturally atomic predicates and includes the information that d is a plant. In (52), ge is the neutral classifier, also used as a default classifier
(Erbaugh 2002), which applies to naturally atomic predicates adding no additional information (i.e. Q is empty).

10 GRINDING, NATURAL ATOMICITY AND THE LEXICAL DERIVATION OF COUNT NOUNS

In this article, I have argued that count nouns and mass nouns are of different types and denote different kinds of entities. I have argued further that while mass nouns are root nouns, count noun meanings are derived from root noun meanings by a lexical operation. In principle, there is a second way to think about count nouns: we might assume that all nouns come out of the lexicon as root nouns and that count nouns are derived in the syntax, with the $\text{COUNT}_k$ operation triggered by count syntax. On this approach, *hair* is a unique unambiguous lexical item, whose default interpretation is mass. When it occurs in the context of count syntax, it shifts to a count interpretation, $\text{COUNT}_k(\text{hair})$. So the syntactic context forces count semantics on the nominal, and learning English requires learning that in the presence of indicators of count syntax, the $\text{COUNT}_k$ operation must apply. A general discussion of the pros and cons of these two approaches is given in Pelletier and Schubert (1989).

Sharvy (1978) suggests a slightly different version of this approach, that is, that count nouns in English are in fact mass nouns that appear as the complement of null classifiers in contexts in which count interpretation is obligatory. Borer (2005) has proposed a syntactic account that is very similar to Sharvy’s: she suggests that classifiers and number fill the same syntactic node and have the semantic function of introducing a ‘dividing’ operation, $\text{DIV}$, on the denotation of the nominal predicate. This dividing operation is obligatory if counting is to take place. When number is present, $\text{DIV}(\text{N})$ derives a count noun that can then be counted. When a classifier is present, a classifier + mass noun concatenation is derived, which can also be counted. If neither a classifier nor a number fills the node, then the noun is interpreted as mass. The complementary distribution between classifiers and plurality in a DP is thus explained.

There is one very strong argument against these approaches, which is based on the semantic distinction between natural and semantic atomicity, and the fact that *furniture* and *boy* are naturally atomic, although only the latter is semantically atomic. If count nominals are derived in the syntax, there should be no reason why a noun like *boy* should not be usable without count syntactic indicators, in which case it would receive a mass interpretation. Earlier discussions of the data
(e.g. Borer 2005) have implicitly assumed that a mass interpretation of a noun like boy should have only the universal grinder interpretation discussed in Pelletier and Schubert (1989), where, following suggestions by David Lewis, it is suggested that the mass interpretation of boy/bicycle is derived by grinding atomic entities into stuff:

(54) a. After he had finished the job, there was bicycle all over the floor.
    b. After the accident, there was boy all over the ground.

But, as predicates like furniture show, mass interpretation does not require a non-atomic interpretation of the predicate. On a mass interpretation, boy would denote BOY<sub>root</sub> and display the same kind of behaviour as furniture: it would be syntactically a mass noun but would denote a set of naturally individuable entities. These expressions could not be directly counted, but when counting is not relevant, this should not be a problem for interpretation. This is of course the situation in Mandarin Chinese, where a bare naturally atomic noun like píngguǒ, ‘apple’, has mass syntax and a classifier is inserted only if individuation is grammatically necessary, for example, in counting contexts.

So, if count semantics is induced by count syntax, English should allow a bare ‘count’ noun such as boy to have the same kind of interpretation as a naturally atomic nominal without a classifier in Mandarin Chinese. This means that (55b/d) should be allowed on a par with (55a/c). However, as these examples show, this is not the case.

(55) a. There is now furniture in my house.
    b. *There is now boy in my class.
    c. There is a lot of furniture in my house now that the four chairs and two tables have been delivered.
    d. *There is a lot of boy in my class now that John, Bill and Peter have enrolled.

Furthermore, one might expect nouns like boy to be usable with classifier expressions if there is no expression of number. (Note that to say that with some nouns, expression of number is obligatory is just to say that some nouns are marked as explicitly count, which is exactly what the syntactic account of count nouns is trying to avoid.)

(56) *There are four units/pieces of boy in my class.

But (56) is ungrammatical just like (55b) and (55d), indicating that boy really is lexically a count expression, independent of the syntactic context. It is no good arguing that real-world knowledge leads us to prefer the count form for some nominals: exactly what we have seen is
that knowledge of natural atomicity is neither a necessary nor a sufficient condition for imposing count syntax (though other linguistic mechanisms might be sensitive to it).

Notice also that the assumption that count nouns are marked as such in the lexicon explains why different languages can mark different nouns as count since lexical derivational process are known to be idiosyncratic,\(^{18}\) while type-shifting operations are not. If count nouns are derived via a syntactic type-shifting operation, we would need an explanation for why different languages allow the operation to apply to different nouns in parallel syntactic contexts.

Examples such as (54), derived via the ‘universal grinder’, are genuine mass nouns, although some people feel that the examples are odd and have only an ironic interpretation. There are related examples using what Lewis called the ‘universal packager’ function, illustrated in (57), which shifts noun meanings from mass to count interpretations. Note that many people consider these much more natural than the examples in (54).

(57) They ordered two orange juices, two beers and a single malt scotch.

I suggest (54) illustrates a genuine operation of syntactic type shifting. In examples like there was bicycle/boy all over the floor, the noun is not being used in its root/mass form since a naturally atomic mass predicate is not a predicate of ground stuff. Instead, the lexically count noun is being shifted into a mass interpretation in the syntax, triggered by the syntactic context that allows only mass nouns. The semantic effect of type shifting cannot be to shift the noun back to its original interpretation: since boy and bicycle are naturally atomic predicates, the original root/mass meaning is not significantly different from the count meaning, and a plausible extension of the blocking principle of Chierchia (1998a), allows us to assume that type shifting will be blocked unless the resulting interpretation is sufficiently different from interpretations that are otherwise available. Type shifting from count to mass is thus associated with a new interpretation, the ‘ground’ interpretation. [Blutner (2000) gives a very similar account of grinding in terms of optimality theory.]

In the ground interpretation, the atomic elements in the denotation of the predicate are not the naturally atomic entities but parts of the

\(^{18}\) Although the division between mass and count nouns differs from language to language, there may be both cross-linguistic generalizations as to what is count and what is mass, and patterns that are specific to a particular language. Wierzbicka (1988) discusses English v. Slavic patterns in mass/count classification, and Smith-Stark (1974) argues that animacy is a linguistic feature associated with count nouns cross-linguistically.
naturally atomic entities: $\text{SHIFT}_{\text{mass}}$ applied to a count noun is the operation given in (58), which applies to a singular count predicate $P$ of type $<d\times k, t>$ and gives an expression of type $<d, t>$, denoting the set of proper parts of some semantic atoms of $P$.

(58) $\lambda P \forall x. \exists y [y \in \pi_1(P) \land x \subseteq y \land \neg x = y]$

‘Grinding’ the denotation of boy will give (59), the set of proper parts of some semantically atomic entities in BOY$_k$. This is a set of contextually determined minimal boy-parts and their sums, but not whole atomic boys.

(59) $\lambda x. \exists y [y \in \pi_1(\text{BOY}_k) \land x \subseteq y \land \neg x = y]$

So if a mass denotation is imposed via type shifting because of a syntactic mismatch, the minimal parts are not the individual boys and the individual bicycles but the set generated by a set of smaller-than-atomic boy-parts and bicycle-parts, hence the interpretation of (54).

This explains several points. First, grinding, or using count nouns in mass contexts, is odder than packaging, or using mass nouns in count contexts. This is because grinding involves reanalysing a naturally atomic predicate so as to override its naturally atomic structure, while packaging involves imposing an individuating structure on a mass domain. But then packaging is an operation that the grammar uses naturally to interpret classifiers. So (57) can be straightforwardly analysed as involving an implicit classifier and it fits naturally into the range of syntactic structures and semantic operations available.

Second, we have an explanation for the observation in Cheng et al. (2008) for why Mandarin Chinese does not allow ground interpretations of bare nouns. Cheng et al. point out that (60a) only has a plural reading, and the ground interpretation requires (60b). ‘Substance’ nouns such as shuí behave as they do in English.

(60) a. qiáng-shang dōu shì gǒu
   wall-top all COP dog
   ‘There are dogs all over the wall’ (NOT ‘There is dog all over the wall.’)

b. qiáng-shang dōu shì gǒu-ròu
   wall-top all COP dog-flesh/meat
   ‘There is dog all over the wall.’

c. dì-shang dōu shì shuǐ
   floor-top all COP water
   ‘There is water all over the floor.’
On the account given here this is expected. The ground reading of *dog/boy* is obtained only as a result of type shifting to resolve a syntactic mismatch occurring when a count noun appears in a mass syntactic context. But in (60a), there is no such mismatch since a naturally atomic mass noun appears in a perfectly appropriate context. Since the denotation of a naturally atomic mass noun is the set of individual dogs closed under sum, the appropriate interpretation of (60a) is that a plurality of (individual) dogs is all over the wall. The ground reading requires an explicit operation deriving a set of dog parts from the naturally atomic set. I discuss this further in Rothstein (2009b).

A third issue relating to type shifting is raised by Barner and Snedeker’s (2005) results. They show that with mass nouns like *furniture*, quantity judgments involve comparing individuals, whereas with flexible terms like *stone*, the basis for comparison depends on whether the noun is count or mass. Mass nouns such as *stone* never allow quantity judgments based on a comparison of number of individuals but only on the basis of overall volume. They suggest that this is because *furniture* but not *stone* is marked as [+individual], and thus, three small chairs can be judged as ‘more furniture’ than one big chair. However, we can now explain this without recourse to a [+individual] (or [+naturally atomic]) feature.

Assume that in general, when a mass predicate is naturally atomic, quantity judgments compare quantities of perceptually salient natural atoms, even when the syntax does not allow grammatical counting. This explains the results that Barner and Snedeker got for predicates like *furniture*. We now need to explain why in expressions like *stone*, the basis of quantity judgments is determined by the syntax of the noun and not by whether in context the predicate denotes a set of perceptually salient individuals. Put differently, if in context, *stone* denotes a set of salient entities, why does *who has more stone?* require you to ignore the natural atoms and base quantity judgments on overall volume? There is an obvious pragmatic explanation. We have been assuming that when COUNT<sub>k</sub> applies to a nominal root predicate, the root nominal is no longer available in the active lexicon, and thus, for example, *boy* and *fence* do not have mass forms lexically available. Flexible nouns such as *stone*, *rope* and *brick*, which have mass and count forms, are an exception to this in English since the root nominal is still available in the active lexicon as a mass noun even after the count predicate has been derived.

Assume, then, that it is part of our knowledge of English that *stone* has a mass form and a count form. Since we have the possibility of choosing which form to use, the question *who has more stones?* is an explicit request to form a quantity judgment on the basis of number. As a consequence, the question *who has more stone?* using the mass noun
will naturally be taken as a request to make an evaluation based on quantity and to ignore the number of the atomic stones. So, the availability of both mass and count nouns forces us to interpret the choice of one or another noun as a request for a particular kind of quantity judgment.

Notice finally that even with predicates like *furniture* that are naturally atomic to a relatively high degree, we are not always forced to take the individual pieces of furniture as the entities relevant for making quantity judgments. Landman (2007) points out that in a context in which furniture is modular, the minimal parts are not the individual pieces of furniture but the modular parts. Thus, if I have three chairs and you have a couch made of the identical parts, I can naturally say:

(61) We have the same furniture.

But the elements relevant for making the quantity judgments do need to stay constant through a single context. So, if you and I have the same modular parts, but I have three chairs and you have a couch, I can either say (61) or I can say ‘I have more furniture than you’, but I cannot say (62), although each conjunct separately may be true relative to a different analysis of minimal parts.

(62) #We have the same furniture but I have more than you.

The big question that we still have not answered is of course why some nouns have mass forms, some have count forms and some have both. Some generalizations can be made. For example, Smith-Starke (1974) pointed out that if a language has count nouns, then animate entities will naturally be denoted by count nouns, and Wierzbicka (1988) has shown that other cross-linguistic patterns can be observed. But the point of this article has not been to analyse patterns of mass/count distribution cross-linguistically. It has been to show that natural atomicity in the denotation of a predicate is neither a necessary nor a sufficient condition for count syntax, that countability is grammatically encoded and that it is dependent on semantically atomic structure, which is grammatically derived and contextually dependent.

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