ABSTRACT. This paper uses the distribution and interpretation of antonymous adjectives in comparative constructions as an empirical basis to argue that abstract representations of measurement, or ‘degrees’, must be modeled as intervals on a scale, rather than as points, as commonly assumed. I begin by demonstrating that the facts in this domain must be accounted for in terms of the interaction of the semantics of adjectival polarity and the semantics of the comparative, rather than principles governing the (overt) expression of particular types of adjectives in comparatives. I then show that a principled account of the full range of data under consideration can be constructed within a model in which degrees are formalized as intervals on a scale and adjectival polarity is characterized in terms of two structurally distinct and complementary sorts of ‘positive’ and ‘negative’ degrees.

KEY WORDS: antonymy, comparatives, degrees, gradable adjectives, polarity

1. INTRODUCTION

A fundamental problem for the semantic analysis of gradable adjectives like tall, long, and expensive is that many of the sentences in which they occur are vague. (1), for example, could be true in one context and false in another.

(1) The Mars Pathfinder mission was expensive.

In a context in which the discussion includes all objects that have some cost value associated with them, (1) would most likely be judged true, since the cost of sending a spacecraft to Mars is far greater than the cost of most things (e.g., nails, dog food, a used Volvo, etc.). In a context in which

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only missions involving interplanetary exploration are salient, however, (1) would probably be judged false, since a unique characteristic of the Mars Pathfinder mission was its low cost compared to other projects involving the exploration of outer space (see Sapir (1944), McConnell-Ginet (1973), Kamp (1975), Klein (1980), Ludlow (1989), Kennedy (1999a) and others for relevant discussion).

What this example shows is that the criteria for deciding whether an utterance of ‘x is φ’ is true, where φ is a gradable adjective, may vary according to factors external to the adjective, such as the meaning of x and features of the context of utterance. Determining the truth of ‘x is φ’ in a particular context requires figuring out what these criteria are, and then making a judgment of whether x ‘counts as’ φ in that context. A basic requirement of a semantic analysis of gradable adjectives, then, is to both provide a means of making this judgment, ensuring that sentences like (1) have definite interpretations, and to allow for variability of interpretation across contexts.

One approach to this problem, first formalized in Seuren (1973) and Cresswell (1976) but since adopted in some form by many semantic analyses of gradable adjectives, meets this requirement by constructing an abstract representation of measurement and defining the interpretation of gradable adjectives in terms of this representation (see e.g., Hellan (1981), Hoeksema (1983), von Stechow (1984a,b), Heim (1985), Bierwisch (1989), Pinkal (1989), Moltmann (1992), Gawron (1995), Rullmann (1995), Hendriks (1995), and Kennedy (1999a); see Klein (1991) for general discussion, and see Faller (1998, to appear) for a related approach in terms of vector space semantics (Zwarts (1995), Zwarts and Winter (1997))). This abstract representation, or scale, can be construed as a set of objects under a total ordering, where each object represents a measure, or degree, of φ-ness.1 The introduction of scales and degrees into the ontology makes it possible to analyze predications involving gradable adjectives in terms of relations between objects and degrees on a scale.

This analysis in turn provides the basis for an account of vagueness. A sentence of the form ‘x is φ’ is taken to mean that the degree to which x is φ is at least as great as some other degree d(φ) on the scale associated with φ that identifies a standard for φ. The semantic function of the standard-denoting degree is to provide a means of separating those objects

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1 In Cresswell’s analysis, scales are actually constructed out of equivalence classes of objects partially ordered according to some gradable property. For the purposes of the following discussion, we can make the simplifying assumption that degrees are abstract objects that stand in a one-to-one relation with the (totally ordered) set of equivalence classes derived in a Cresswell-style approach (see Klein (1991)).
for which the statement ‘x is φ’ is true from those objects for which ‘x is φ’ is false (see Bartsch and Vennemann (1973), Cresswell (1976), Siegel (1979), Klein (1980), von Stechow (1984a), Ludlow (1989), Bierwisch (1989), and Kennedy (1999a)). The problem of vagueness is thus recast as the problem of determining the value of the standard degree in a particular context.

For example, a sentence like (1), on this view, is true just in case the degree that represents the cost of the Mars Pathfinder mission is at least as great as the standard value for ‘expensive’ in the context of utterance.\(^2\) In a context in which all objects in the domain of expensive are relevant, the standard value would fall below the degree that represents the cost of the Mars Pathfinder mission, and (1) would be false. In a context in which only interplanetary expeditions are considered, however, the standard value would exceed the cost of the Pathfinder mission, and (1) would work out to be true.

While the many semantic analyses of gradable adjectives that make use of scales and degrees differ in non-trivial ways, one important question applies equally to all of them, which concerns the appropriate formal characterization of such abstract representations of measurement as scales and degrees. What kinds of objects are they, and how does their structure affect or possibly determine aspects of the interpretation of gradable adjectives?\(^3\)

The goal of this paper is to address this question by investigating a set of facts that provide unusual insight into the empirical consequences of different assumptions about the nature and structure of scales and degrees. Specifically, I will focus on the distribution and interpretation of antonymous pairs of adjectives in comparatives, and I will show that the

\(^2\) The standard assumption is that the standard value is set indexically, with respect to some contextually relevant set of objects (a ‘comparison class’ in Klein’s (1980) terms) that provides the basis for identification of a ‘norm’, although there are a number of problems with this view (see Klein (1980), Ludlow (1989), and Kennedy (1999a)).

\(^3\) A second question concerns a broader ontological issue: is the introduction of scales and degrees into the semantics really necessary, or can the semantic properties of gradable adjectives be explained in terms of some alternative mechanisms that do not make reference to abstract objects? This question is of particular importance, because such alternative analyses have been developed (see for example McConnell-Ginet (1973), Kamp (1975), Klein (1980, 1982), Van Benthem (1983), Larson (1988), and Sánchez-Valencia (1994)). These approaches build vagueness directly into the meaning of a gradable adjective, by associating with it a family of functions from individuals to truth values, each with a different extension. In any context of use, a particular function from this family must be selected in order to achieve a definite interpretation, but no reference to degrees or other abstract objects is required. In Kennedy (1999b), I show that such accounts cannot be maintained, as they do not provide a principled explanation of the facts discussed in this paper.
facts in this domain lead to the following conclusions. First, degrees must be formalized as *intervals* on a scale (as in Seuren (1978, 1984) and von Stechow (1984b); cf. Bierwisch (1989) and Löbner (1990)), rather than as points on a scale (as is more commonly assumed). Second, a structural distinction must be made between two sorts of degrees: *positive degrees*, which range from the lower end of a scale to some point, and *negative degrees*, which range from some point to the upper end of a scale.

The structure of the paper is as follows. I begin in Section 2 by introducing the range of data that forms the empirical basis for the argument. I show that comparatives constructed out of antonymous adjectives do not have the interpretations that we would expect if degrees were formalized as points on a scale, and that the explanation of the facts lies in the interaction of adjectival polarity and the semantics of the comparative. In Section 3, I introduce a model in which degrees are formalized as (positive or negative) intervals and adjectival polarity is represented as a sortal distinction between adjectives, and I show that this model supports a principled explanation of the empirical phenomena presented in Section 2.

2. **Comparison and Polar Opposition**

2.1. **Cross-Polar Anomaly**

The empirical starting point for this paper is a phenomenon that I will refer to as *cross-polar anomaly* (CPA), which is exemplified by the sentences in (2) (see Hale (1970), Bierwisch (1989), and Kennedy (1997a)).

(2)a. ?Alice is shorter than Carmen is tall.

b. ?*The Brothers Karamazov* is longer than *The Idiot* is short.

c. ?The Mars Pathfinder mission was cheaper than the Viking mission was expensive.

d. ?New York is dirtier than Chicago is clean.

e. ?A Volvo is safer than a Fiat is dangerous.

These sentences demonstrate that comparatives formed out of so-called ‘positive’ and ‘negative’ pairs of adjectives are semantically anomalous. This anomaly cannot be accounted for in terms of syntactic ill-formedness: structurally identical examples of ‘comparative subdeletion’ in which both
adjectives have the same polarity, such as those in (3), are perfectly well-formed.4

(3)a. The space telescope is longer than it is wide.

b. After she swallowed the drink, Alice discovered that she was shorter than the doorway was low.

Given the acceptability of examples like these, we can tentatively conclude that the factors underlying cross-polar anomaly involve the interaction of the semantics of positive and negative adjectives and the semantics of the comparative construction, a conclusion that will be further refined and strengthened in Section 2.3.

The classification of gradable adjectives as positive or negative can be made based on a number of empirical characteristics (see Seuren (1978) for general discussion of this issue). For example, negative adjectives license downward entailments and negative polarity items in clausal complements, but positive adjectives do not (see Seuren (1978), Ladusaw (1979), Linebarger (1980), Sánchez-Valencia (1994), Kennedy to appear a); and positive but not negative adjectives can appear with measure phrases (compare ‘2 meters long’ with ‘?2 meters short’; see Section 3.4 below).

Of particular relevance to the current discussion is the fact that a large class of antonymous positive and negative pairs make (4) valid, as shown by (5).5

(4)  \( x \) is more \( \phi_{pos} \) than \( y \) if and only if \( y \) is more \( \phi_{neg} \) than \( x \).

(5)a. Carmen is taller than Alice if and only if Alice is shorter than Carmen.

b. A Volvo is safer than a Fiat if and only if a Fiat is more dangerous than a Volvo.

4 ‘Subdeletion’ is the term adopted by Bresnan (1975) to refer to comparative constructions with the form in (i).

(i) \( [S \ NP_x \ BE \ more \ A_1 \ than \ NP_y \ BE \ A_2] \).

5 It should be acknowledged that not all adjectives that are intuitively antonymous make (4) valid. ‘Endpoint oriented’ antonyms like tiny and huge, for example, do not, because the comparative forms entail the non-comparative forms (see Lehrer (1985) and Sánchez-Valencia (1994) for relevant discussion). For the purposes of this paper, I will focus on the class of ‘complementary’ antonyms which do fit into the pattern in (4), but the proposals I make here should eventually be folded into a larger theory that encompasses different types of antonymy.
These facts can be straightforwardly explained by adopting three very natural assumptions. First, as discussed in Section 1, gradable adjectives are characterized as expressions that map objects to abstract representations of measurement (scales), which are sets of points (degrees) that are totally ordered along a dimension determined by the adjective (e.g., height, weight, temporal precedence, etc.; see Cresswell (1976), Hellan (1981), and especially Pinkal (1989)). Second, comparatives define ordering relations between degrees, so that an expression of the form in (6a) has the interpretation in (6b) (see e.g., Russell (1905), Postal (1974), Williams (1977), Hoeksema (1983), Heim (1985), and Kennedy (1999a) for examples of this type of approach).

(6)a.  $x$ is more $\phi$ than $y$

b.  the degree to which $x$ is $\phi$ $>_{\phi}^\circ$ the degree to which $y$ is $\phi$

Finally, antonymous pairs of adjectives such as ‘tall’ and ‘short’ map identical arguments onto the same degrees (and are therefore associated with the same scales), but they introduce the opposite ordering relations (Rullmann (1995)). In other words, such pairs are duals: for all antonymous adjectives $\phi_{pos}, \phi_{neg}$ that map their arguments onto a shared scale $S$, and for all $d_1, d_2 \in S$, the relation in (7) holds.

(7)  $d_1 >_{\phi_{pos}} d_2 \iff d_2 >_{\phi_{neg}} d_1$

Given these assumptions the truth conditions of e.g., (5a) can be paraphrased as in (8).

(8)  the degree to which Carmen is tall $>_{tall}$ the degree to which Alice is tall  $\iff$  the degree to which Alice is short $>_{short}$ the degree to which Carmen is short

Since the degree of Alice’s tallness is the same point on the scale of height as her degree of shortness (and likewise for Carmen), the two conjuncts in (8) are correctly predicted to be logically equivalent.

The very same reasoning that provides an elegant account of the validity of (4) makes exactly the wrong predictions about cross-polar anomaly, however. As noted above, examples of cross-polar anomaly are subdeletion constructions. Assuming that the comparative clause (the complement of ‘than’) in such constructions is transparently interpreted as a definite description of a degree (Heim (1985), Izvorski (1995)), an example like (2a) should have the interpretation in (9).

(9)  the degree to which Alice is short $>_{short}$ the degree to which Carmen is tall
The difference between the logical representation in (9) and that of the first conjunct in (8), for example, is that in the former, only one of the arguments of the ordering relation is a degree whose polarity agrees with the adjective that determines the type of ordering relation. (I assume here that the type of ordering relation is determined by the morphologically comparative adjective, though the argument would be the same if the situation were reversed.) This suggests a plausible explanation of CPA. If it were the case that the ordering relation in comparatives were defined only for degrees of the same polarity, then the anomaly of CPA constructions would follow: such constructions could not be assigned a truth value.

Although this explanation of CPA is exactly the one that I will present in Section 3.2, within the set of assumptions outlined so far, it is unavailable. Specifically, if degrees correspond to points in an ordered set, and if positive and negative adjectives map their arguments onto the same degrees – an assumption that is necessary to account for the validity of constructions with the form in (4) – then (9) is equivalent to (10).

\[ (10) \quad \text{the degree to which Alice is short} \succ_{\text{short}} \text{the degree to which Carmen is short} \]

The result is that (2a) should not only not be anomalous, it should be logically equivalent to ‘Alice is shorter than Carmen’ (and likewise ‘Carmen is taller than Alice’).

The conclusion to be drawn from this discussion is that within a model in which positive and negative degrees are the same objects there is no means of distinguishing between them in a way necessary to achieve the desired result that the ordering relation in (9) is undefined – this result can only be achieved by stipulation. Cross-polar anomaly thus provides a serious challenge to such a model.\(^6\)

\[ (i) \quad \exists d \text{ such that Alice is at least as short as } d, \text{ and } \forall d' \text{ such that Carmen is at least as tall as } d', d \succ_{\text{short}} d'. \]

If the set of degrees that satisfy the restriction on the existential quantifier are the degrees ordered above Alice’s degree of height, while the ones that satisfy the restriction on the universal quantifier are the degrees ordered below Carmen’s degree of height, then given the assumptions about how \(\succ_{\text{short}}\) is satisfied, (i) works out to be a contradiction. However, even if we are willing to explain cross-polar anomaly in terms of contradiction (and see Ladusaw (1986) for reasons why we might not want to do this), there are independent reasons for rejecting such an analysis. As shown in Rullmann (1995, p. 109) and

\(^6\) An alternative semantics for comparatives within this system that appears at first glance to have more success at dealing with cross-polar anomaly assigns the truth conditions paraphrased in (i) to (2a) (see Atlas (1984), Lerner and Pinkal (1992, 1995), Moltmann (1992), Gawron (1995) for examples of this type of approach).
2.2. *Comparisons of Divergence and Deviation*

The picture presented in the previous section is complicated by two sets of facts that appear at first glance to counterexemplify the claim that comparatives constructed out of positive and negative pairs of adjectives are anomalous. The first set of facts, brought to my attention by Chris Barker (personal communication), is illustrated by the sentences in (11).

(11)a. The A-string on your guitar is sharper than the D-string is flat.

b. My watch is faster than your watch is slow.

c. She was earlier than I was late.

These sentences, which I will refer to as *comparisons of divergence* (for reasons that will become clear below), appear to involve comparatives constructed out of adjectives of opposite polarity, but they are clearly not anomalous. There is good reason to believe that the adjectives in the comparatives in (11) are not antonymous in the same way as the adjectives involved in cross-polar anomaly, however. While the pairs in (11) are clearly opposites in some sense, there are three compelling pieces of evidence that this opposition is not one of polarity.

First, all of the adjectives in (11) can take measure phrases; as observed above, this is a characteristic of positive adjectives, but not negative adjectives:

(12)a. Your A-string is 30 Hz flat/sharp.

b. My watch is 10 minutes fast/slow.

c. She was an hour early/late.

Second, statements using the adjectives in (11) that appear to be substitution instances of (4) are not valid. For example, the first conjunct of (13), would be true, but the second conjunct would be false, in a context in which the A-string is farther above its proper pitch than the D-string is, but the D-string is higher in absolute pitch than the A-string (the normal situation for a properly tuned guitar).

(13) The A-string is sharper than the D-string if and only if the D-string is flatter than the A-string.

Kennedy (1997b, pp. 74–75), this approach fails to accurately capture the truth conditions of comparatives with ‘less’.
Finally, the sentences in (11) are non-anomalous only on a very specific interpretation: one in which the adjectives measure divergence from some common point of reference, rather than the ‘absolute’ degree to which an object has some gradable property. For example, (11a) compares the degrees to which two notes differ from their respective pure tones, and (11b) compares the amounts to which our watches deviate from ‘on time’, whatever that may be. When absolute degrees are compared, the adjectives in (11) trigger cross-polar anomaly. This is shown by the minimal pair (11b) and (14).

(14) ?My car is faster than your car is slow.

These facts suggest that both members of the pairs of adjectives in (11) have the same polarity (and are in fact positive), in which case they do not represent counterexamples to the generalization established in Section 2.1. They do, however, raise larger questions about the nature of antonymy and polar opposition. The adjectives ‘fast’ and ‘slow’ are opposites in both (11b) and (14), for example, yet the nature of the opposition is different in the two cases. At an intuitive level, the difference is that in the former, the two adjectives measure divergence from a common point in different directions; in the latter, they provide opposite perspectives on the same value (the speed of the cars). The fact that (14) is anomalous but (11b) is not indicates that this difference has empirical consequences; the question that must be answered is how this difference should be characterized. I will return to this question in Section 3.5.

A second set of apparent counterexamples to the generalization that comparatives constructed out of antonymous adjectives are anomalous is illustrated by the naturally occurring examples in (15).

(15)a. [The Red Sox] will be scrutinized as closely as the Orioles to see whether they are any more legitimate than the Orioles are fraudulent. [New York Times, Summer 1998 (exact date unknown)]

b. Grace especially had a forgettable playoff series that won’t soon be forgotten. Grace was as cold as he was hot in the 1989 playoffs. [Chicago Tribune, October 4, 1998, Section 3, p. 6]

c. I can still remember the sound it made, a lovely special sound, as light and thin as the clothes were solid and heavy. [Dibdin, M.: 1989, Ratking, Bantam Crime, New York, p. 178]
These examples, which I will refer to as comparison of deviation (COD) constructions, have two characteristics that distinguish them from standard comparatives.

First, COD constructions compare the relative extents to which two objects deviate from some standard value associated with the adjective (cf. Bierwisch (1989, p. 220). The meaning of the comparative in (15a), for example, can be paraphrased as in (16).

(16) The degree to which the Red Sox exceed a standard of legitimacy is greater than the degree to which the Orioles exceed a standard of fraudulence.

In contrast, standard comparatives and equatives compare the absolute projections of two objects on a scale. The most natural paraphrase of the equative construction in (17), for example, is (18). (17) also has a comparison of deviation-like interpretation, a point that I will return to below.


(18) The depth of the hole is at least as great as the height of a ten-story building.

Second, unlike typical comparatives, COD constructions entail that the properties predicated of the compared objects are true in the absolute sense. (19a), for example, is contradictory, but (19b) is not.

(19)a. The Red Sox are more legitimate than the Orioles are fraudulent, but they’re not legitimate.

b. The hole is deeper than a two-year old is tall, but it’s not deep.

This property is clearly related to the interpretation of COD. Since the truth of an expression of the form ‘\( x \) is \( \phi \)’ is determined by checking whether the degree to which \( x \) is \( \phi \) exceeds an appropriate standard value (see the discussion of this point in Section 1), the fact that comparison of

\footnote{In the discussion that follows, I will follow Seuren (1984, p. 116) in treating the equative as a partial ordering relation, rather than one of equality, although nothing hinges on this decision.}
deviation constructions compare the degrees to which two objects exceed their respective standard values derives the observed entailment patterns.

Two observations should be made about comparison of deviation. The first is that interpretations of this type are not restricted to comparatives involving positive and negative pairs of adjectives. This is most clearly illustrated by (20), which can have either the ‘standard’ interpretation paraphrased in (21a), which is false, or the COD interpretation paraphrased in (21b), which is (arguably) true.

(20) The Sears Tower is as tall as the San Francisco Bay Bridge is long.

(21)a. The height of the Sears Tower is at least as great as the length of the San Francisco Bay Bridge.

b. The degree to which the Sears Tower exceeds a standard of tallness (for buildings) is at least as great as the degree to which the San Francisco Bay Bridge exceeds a standard of length (for bridges).

Given these facts, it must be the case that comparison of deviation interpretations represent a freely available option in comparatives (cf. Bierwisch (1989, pp. 220–221), where the availability of such interpretations is claimed to be a marked option).

The second point to make about COD is that in comparatives that are in fact constructed out of positive and negative pairs of adjectives, the COD interpretation is the only interpretation available. This is most clearly illustrated by equative constructions such as (15b)–(15c) and (22).

(22) The Cubs are as old as the White Sox are young.

This sentence cannot mean that the (average) age of the players on the Cubs is the same as the (average) age of the players on the White Sox, which is what a standard equative interpretation would give us (cf. (17)–(18) above). It can only mean that the degree to which the average age of the Cubs exceeds a standard of oldness (for baseball teams) is the same as the degree to which the average age of the White Sox exceeds a standard of youngness (for baseball teams). This fact is important because it shows that comparison of deviation constructions, like the comparisons of divergence in (11), are not counterexamples to the descriptive generalization originally made on the basis of cross-polar anomaly. Instead, (22) highlights the fact that comparatives constructed out of antonymous pairs of adjectives do not
have the type of interpretations that we would expect them to have given the assumptions about the nature of degrees and adjectival polarity laid out in Section 1. COD thus reinforces the conclusion that the standard model cannot be maintained.

Although the facts discussed in this section are not counterexamples to the generalization that comparatives constructed out of antonymous pairs of adjectives are anomalous, they do indicate that this generalization needs to be refined. What is unique about comparisons of divergence and deviation is that both constructions compare the degrees to which two objects deviate from some reference point (which need not be the same for the two objects) — a conventionalized value in the former case and a contextually determined standard value in the latter. In contrast, standard comparatives compare the absolute measures of two objects on a scale. The importance of cross-polar anomaly is that it shows that the standard interpretation is impossible when the compared measures are constructed on the basis of adjectives of opposite polarity: only the comparison of deviation/comparison of divergence interpretations are available.

These observations are summarized in the descriptive generalization in (23), which is stated not in terms of the distribution of (types of) lexical items, but rather in terms of the ‘sorts’ of the compared degrees.

(23) Comparatives are semantically well-formed only if they define ordering relations between the same sorts of degrees: between positive degrees, between negative degrees, or between degrees that measure divergence from a reference point.

One way to derive (23) is with an analysis along the lines of the one sketched in Section 1: the ordering relation introduced by the comparative morphology is defined only for degrees of the same sort. In order to implement such an analysis, however, it must be the case that degrees can be sortally distinguished in the appropriate way, something that we have already seen to be impossible in a model in which degrees are formalized as points on a scale. In Section 3, I will introduce an alternative model in which degrees are formalized as intervals on a scale, and I will show that it supports a sortal distinction between degrees that derives the generalization in (23) in exactly the way sketched here.

2.3. Polar Opposition and the Syntax-Semantics Interface

Before moving to a semantic analysis of cross-polar anomaly, however, a different type of approach to the problem should be considered. In Section 2.1, I showed that the standard assumptions about the nature of
degrees and adjectival polarity do not support a principled semantic explanation of CPA. I also pointed out that the well-formedness of examples of comparative subdeletion involving adjectives of the same polarity, such as (24), indicates that CPA cannot be explained in terms of syntactic ill-formedness.

(24)a. The space telescope is longer than it is wide.

b. After she swallowed the drink, Alice discovered that she was shorter than the doorway was low.

There is another possibility, however, that would allow us to maintain the assumptions about scales and degrees outlined in Section 2.1. It could be the case that the explanation of cross-polar anomaly lies neither in syntax nor in semantics, but rather in aspects of the syntax-semantics interface, in particular, the principles governing when a particular type of adjective may appear in the comparative clause.

Precisely this type of explanation is discussed in Bierwisch (1989, 1998) and Faller (1998). Bierwisch (1989, 1998), for example, develops an analysis in which negative adjectives are ruled out in the comparative clause across the board. The starting point of this analysis is the observation that both of the German sentences in (25a) and (25b) are equally unacceptable (as are their English counterparts; see Bierwisch (1989, pp. 104–105)).

(25)a. ?Der Tisch ist 10cm höher als er schmal ist.
   ‘The table is 10cm taller than it is narrow.’

b. ?Der Tisch ist 10 cm niedriger als er schmal ist.
   ‘The table is 10 cm lower than it is narrow.’

The use of a measure phrase in these examples is important, as it rules out the possibility of a comparison of deviation interpretation: as Bierwisch observes, a COD interpretation is possible in both the German and English sentences in (26). (I will return to a discussion of measure phrases in comparatives in Section 3.3.)

(26)a. Der Tisch ist niedriger als er schmal ist.

b. The table is lower than it is narrow.

Bierwisch uses facts like these to argue for a semantic analysis of gradable adjectives and comparatives in which a negative adjective in the
comparative clause can only give rise to a COD interpretation, and standard comparative interpretations cannot be computed in such constructions. A corollary of this analysis is that all standard comparative interpretations are derived from representations in which the adjective in the comparative clause is positive. In other words, in Bierwisch’s analysis, both (27a) and (27b) are derived from an underlying representation along the lines of (28), where the struck-through material is deleted or unpronounced in the surface form.

(27)a. The table is longer than the rug is.

b. The table is shorter than the rug is.

(28) The table is longer/shorter than the rug is long.

While Bierwisch’s assessment of sentences like (25a)–(25b) appears to be correct, examples like (24b) above contradict the claim that negative adjectives cannot appear in the comparative clause when the entire construction has a standard interpretation, since there is strong evidence that it is not a comparison of deviation construction.

First, as shown by the passage in (29), it is possible to insert a measure phrase into comparatives like (24b) without anomaly in certain contexts.

(29) Alice came upon a very small, very low doorway, with a bottle of some kind of potion next to it marked DRINK ME. Alice swallowed the potion, and began to shrink. When she had stopped shrinking, Alice discovered that she was exactly two inches shorter than the doorway was low.

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8 It follows from these assumptions that examples of CPA in which the matrix adjective is negative and the adjective in the comparative clause is positive (such as (2a)) should be perfectly well-formed, contrary to fact. The explanation of the unacceptability of such constructions must therefore come from somewhere other than their semantics. I will address this point below when I discuss Faller’s (1998) analysis of CPA constructions, which builds on ideas in Bierwisch (1989).

9 The naturally occurring example in (i) also constitutes a potential counterexample to Bierwisch’s claim, as it clearly contains negative adjectives in the comparative clause.

(i) Your rhetoric is as low and hypocritical as your mind is narrow and mean. [Sterling, B.: 1997, The Artificial Kid, Hardwired, San Francisco, p. 37 (originally published in 1980 by Harper and Row).]

However, the adjectives in (i) are used in an ‘evaluative’ sense. Evaluative adjectives have a number of properties that distinguish them from the dimensional adjectives discussed elsewhere in this paper, and are in fact given a different analysis in Bierwisch (1989) that may be extendible to (i).
Second, the entailments of (30) show that comparatives with negative adjectives in the comparative clause can have standard interpretations.

(30) After she swallowed the potion, Alice became as thin as the doorway was narrow.

(30) clearly has a comparison of deviation interpretation: the degree to which Alice exceeds a standard of thinness (for girls) is at least as great as the degree to which the doorway exceeds a standard of narrowness (for doorways). However, (30) also has a standard comparative interpretation, which asserts that Alice’s thinness is at least as great as the doorway’s narrowness. The crucial difference between the two interpretations is that only the latter entails that Alice’s width is as small as that of the doorway, permitting the inference that Alice can fit through the door. The COD interpretation carries no such entailment, since the truth conditions for this reading do not necessarily require it to be the case that Alice and the doorway have the same width. The fact that (30) can be used to make exactly this assertion (in fact, this is the most natural reading of this sentence) indicates that this sentence can have a standard interpretation, contrary to Bierwisch’s claims.

Given these observations, I conclude that the anomaly of Bierwisch’s examples do not provide evidence that negative adjectives in the comparative clause can never have a standard interpretation. Instead, I will suggest that their anomaly reflects a general pragmatic restriction on the use of negative adjectives. A well-known difference between pairs of positive and negative adjectives is that the latter are marked with respect to the former; this markedness has semantic consequences (see Lehrer (1985) for an overview). For example, how-questions, which ask for the degree to which some object possesses a gradable property, typically do not assert that the property actually holds of the object to which the property applies. Negative adjectives in how-questions, however, introduce the presupposition that the property they describe does hold of the target of predication. This is illustrated by the contrast between (31a), which presupposes that the table is narrow, and (31b), which does not presuppose that the table is wide.

(31)a. How narrow is the table?
   b. How wide is the table?

Like how-questions, comparatives typically make no claims about whether the gradable property that forms the basis of the comparison
actually holds of the compared objects, because the basic goal of a comparison is only to establish an ordering relation between two objects along some dimension. It follows that in the default case, the unmarked (positive) form(s) of the adjective(s) should be used, and the marked (negative) form(s) should be reserved for contexts that are consistent with their presuppositions. The crucial difference between e.g., (29) and (25b) is that the former provides just such a context. (29) describes a state of affairs that is part of a scenario in which Alice is reduced to a small size as a result of drinking a potion, a context that is perfectly compatible with (and arguably demands) the use of the negative forms.

A related but slightly different approach to CPA is taken by Faller (1998), who argues that cross-polar anomaly arises not when the two adjectives in the comparative construction are of opposite polarity, but rather only in the specific context in which the two adjectives are antonyms. Faller (1998, p. 38) points to the fact that (32a) seems less acceptable than (32b) on a standard comparative interpretation to make this point.

(32)a. The ski poles are shorter than the box is long.

b. ?The ski poles are shorter than the box is wide.

To account for the difference in acceptability between these two sentences, Faller proposes an ‘anti-redundancy’ condition that prohibits the overt realization of an adjective in the comparative clause when it maps its argument onto the same scale as the adjective that heads the comparative construction (cf. Bierwisch (1989, p. 104)).

The intuition underlying this proposal – redundant information must be deleted from the comparative clause – clearly reflects a very deep property of comparatives.\(^\text{10}\) However, there are a number of reasons to doubt that this is the correct explanation of CPA. First, although (32a) is more acceptable than (32b), similar examples involving measure phrases (which force a standard interpretation; see the discussion of (25) above) are judged to be anomalous:

(33)a. ?After she swallowed the potion, Alice discovered that she was two inches shorter than the doorway was high.

b. ?The hole is a foot shallower than the gravedigger is tall.

\(^{10}\) Indeed, the obligatoriness of deletion in virtually all contexts (but see (35) below) sets comparatives apart from other constructions that involve some kind of structural or semantic identity, where deletion is optional (in particular, ellipsis constructions). This point is used by Kennedy (to appear b) to argue that the principles governing deletion in comparatives are essentially syntactic in nature (see also Bresnan (1975) for relevant discussion).
Second, the comparison of divergence and deviation constructions discussed in Section 2.2 show that comparatives can be constructed out of antonymous adjectives as long as they obey the generalization in (23), i.e., as long as their interpretations involve comparisons of similar sorts of measures (degrees of difference). Faller’s proposal would require us to assume that e.g., fast and slow in (34) map their arguments onto different scales, an assumption that not only is at odds with the clear intuition that both adjectives locate their arguments on the same scale (the timeline), but also makes it accidental that they measure divergence from the same point (the ‘correct time’).

(34) My watch is faster than your watch is slow.

Finally, it is not only possible for antonymous adjectives associated with the same scale to appear together in comparisons of divergence and deviation, it is also possible for the adjective in the comparative clause to be identical to the adjective that heads the comparative when it has contrastive stress, as in (35) (where capitalization indicates emphasis; see Chomsky (1977)).

(35) The ski poles are not only longer than the box is wide, they’re also longer than the box is LONG.

(35) has a standard comparative interpretation, in which it is asserted that the length of the ski poles exceeds the length of the box (this sentence does not entail that the ski poles (or the box) are long, so it is not a COD construction). While it is the case that the second occurrence of ‘long’ must be prosodically contrastive with the first in order for the sentence to be well-formed, it is clearly not the case that the two occurrences of ‘long’ must map their arguments onto different scales.

11 The naturally occurring example in (i) provides an additional illustration of this fact, and further shows that what is contrasted in such examples are the adjective phrases, not just the adjectival heads.

(i) Watching the Cubs on his satellite dish has been almost as difficult for Beck as watching Beck close games has been difficult for the CUBS. [Chicago Tribune, June 6, 1999, section 4, p. 3 (emphasis added to indicate the required pronunciation)]

This example also supports the claim made above that negative adjectives can appear in the comparative clause under certain circumstances: ‘difficult’ is a canonical example of a negative adjective (see Kennedy (to appear a)).
Bierwisch (1989) proposes a redundancy condition that is slightly different from Faller’s, and more compatible with facts like (35), which requires deletion of a lexically identical adjective from the comparative clause when it is not prosodically contrastive with the adjective in the matrix. Bierwisch (1998) extends this observation to claim that prosodic contrast induces comparison of deviation interpretations, pointing to the contrasts in (36) and (37) to make this point (where capitalized lexical items are focused and items marked with a ‘‘↓’’ are deaccented).

\[(36)\]
\[
\begin{align*}
&\text{a. } \text{Kim is TALLer than Lee is SHORT.} \\
&\text{b. } ?\text{Kim is taller than Lee is ↓short.}
\end{align*}
\]

\[(37)\]
\[
\begin{align*}
&\text{a. } \text{Lee is SHORTer than Kim is TALL.} \\
&\text{b. } ?\text{Lee is shorter than Kim is ↓tall.}
\end{align*}
\]

However, while it is certainly true that COD interpretations require prosodic contrast, it is not the case that this is a unique property of such constructions. Rather, all instances of subdeletion require prosodic contrast, a point that has not been adequately recognized in the literature on these constructions. This is illustrated by the contrast between (38a) and (38b), which is just as clear as the contrasts in (36) and (37).

\[(38)\]
\[
\begin{align*}
&\text{a. } \text{The table is LONGer than it/the carpet is WIDE.} \\
&\text{b. } ?\text{The table is longer than it/the carpet is ↓wide.}
\end{align*}
\]

This point is crucial, because it shows that the distribution and interpretation of antonymous adjectives in comparatives cannot be explained in terms of requirements on prosodic contrast any more than they can be explained in terms of requirements on ‘scale contrast’. If all subdeletion constructions require prosodic contrast, then the impossibility of a standard comparative interpretation in CPA environments must be due to some other factor.

To summarize, we have seen that two plausible and coherent attempts to state the restrictions on the distribution of polar adjectives in comparatives in terms of the syntax-semantics interface result either in undergeneration (Bierwisch’s proposal) or in overgeneration (Faller’s proposal).\(^{12}\) These results, together with the observations in Section 2.2 summarized in the

\(^{12}\) Although the arguments against Faller’s analysis are sound, it must be acknowledged that the example she presents in (32b) is surprisingly acceptable, and is moreover prob-
descriptive generalization (23), reinforce the conclusion originally stated at the beginning of Section 2.1: the explanation for the distribution and interpretation of antonymous adjectives in comparatives must be sought in the interaction of the semantics of adjectival polarity and the semantics of the comparative construction. Assuming that the basic analysis of the comparative that I have adopted is essentially correct (comparatives denote ordering relations between degrees; see Kennedy (1999a, 1999b) for justification of this approach over alternatives that do not make reference to degrees), it must be the case that the theory of adjectival polarity outlined in Section 2.1 needs to be revised.

3. INTERVALS AND ADJECTIVAL POLARITY

As pointed out at the end of Section 2.2, a potential explanation for the descriptive generalization about the distribution and interpretation of antonymous adjectives in comparatives, repeated below in (39), is that the ordering relation introduced by the comparative morphology is undefined when its degree arguments are of different sorts.

(39) Comparatives are semantically well-formed only if they define ordering relations between the same sorts of degrees: between positive degrees, between negative degrees, or between degrees that measure divergence from a reference point.

The problem for the traditional model, in which degrees are formalized as points on a scale, is that the (independently necessary) assumption that positive and negative degrees are the same objects rules out the possibility of making the necessary sortal distinction between them. In order to achieve this result, it is necessary to give up this assumption; but at the same time, any model in which positive and negative degrees are analyzed as different sorts of objects must also support a principled account of the semantic properties of polar adjectives and comparatives. In particular, it must explain the validity of constructions like (4), the empirical phenomenon that most strongly argues for the traditional analysis. A model that meets these requirements can be found in the work of Seuren (1978, 1984) and von Stechow (1984b) (see also Lübner (1990)), in which degrees are analyzed as intervals on a scale. In this section, I will show that this
characterization of degrees not only satisfies the requirements of descriptive adequacy in the domain of the semantics of gradable adjectives and comparatives, but also satisfies the larger goal of explanatory adequacy by supporting a principled analysis of the distribution and interpretation of polar adjectives in comparatives.

3.1. Capturing Adjectival Polarity

A clear intuition about antonymous pairs of adjectives is that they provide the same kind of information about an object (both ‘tall’ and ‘short’ characterize an object’s height, for example), but they provide different perspectives on the projection of an object on a scale. This observation provides the basis for the analysis of degrees originally presented in Seuren (1978) and elaborated in Seuren (1984) and in particular von Stechow (1984b), which I will adopt and further develop here. In this approach, degrees are represented as intervals on a scale, and a structural distinction is made between two sorts of degrees: positive and negative degrees. Roughly speaking, positive degrees are intervals that range from the lower end of a scale to some point, and negative degrees are intervals that range from some point to the upper end of a scale.13

More formally, I will define a scale $S$ as a linearly ordered, infinite set of points, associated with a dimension that indicates the type of measurement that the scale represents (e.g., height, length, weight, brightness and so forth).14 A degree $d$ can then be defined as a convex, nonempty subset of a scale, i.e., a subset of the scale with the following property: $\forall p_1, p_2 \in d \forall p_3 \in S [p_1 < p_3 < p_2 \rightarrow p_3 \in d]$ (cf. Landman (1991, p. 110); this is simply the definition of an interval for a linearly ordered set of points). The set of positive and negative degrees for any scale $S$ ($POS(S)$ and $NEG(S)$, respectively), can then be defined as in (40).

---

13 Bierwisch (1989) also develops an interval-based formalization of degrees, but his model differs from the one I will adopt here in that it characterizes the semantics of negative adjectives in terms of degree subtraction, rather than in terms of negative degrees. As was argued in Section 2.3, Bierwisch’s approach is too restrictive to account for the full range of facts under consideration in this paper.

14 The dimensional properties of scales are important for accounting for the anomaly of cross-scalar comparisons such as (i), sometimes referred to as ‘incommensurability’ (see Klein (1991) and Kennedy (1999a)).

(i) ?Alice is taller than she is clever.

Since the dimensional properties of scales are not crucial to the explanation of the facts under discussion here, I will not address this issue in the following discussion.
POLAR OPPOSITION AND THE ONTOLOGY OF ‘DEGREES’ 53

\[(40)\]  
\[a. \quad POS(S) = \{d \subseteq S \mid \exists p_1 \in d \forall p_2 \in S[p_2 \leq p_1 \rightarrow p_2 \in d]\}\]  
\[b. \quad NEG(S) = \{d \subseteq S \mid \exists p_1 \in d \forall p_2 \in S[p_1 \leq p_2 \rightarrow p_2 \in d]\}\]

Finally, let us assume that for any object \(x\), the positive and negative projection of \(x\) on a scale \(S\) (\(pos_S(x)\) and \(neg_S(x)\), respectively) are related as in (41), where \(\text{MAX}\) and \(\text{MIN}\) return the maximal and minimal elements of an ordered set.

\[(41) \quad \text{MAX}(pos_S(x)) = \text{MIN}(neg_S(x))\]

The result of these assumptions is that the positive and negative projections of an objects \(x\) on a scale \(S\) are (join) complementary intervals on the scale, as illustrated by the diagram in (42).

\[(42) \quad S: 0 \quad pos_S(x) \quad \bullet \quad neg_S(x) \quad \rightarrow \infty\]

The intuition that the structural distinction between positive and negative degrees is designed to capture is exactly the one I mentioned above: that antonymous pairs of adjectives provide complementary perspectives on the projection of an object onto the scale. This structural distinction provides the basis for an account of adjectival polarity.

Following Kennedy (1999a) (see also Bartsch and Vennemann (1973)), I assume that gradable adjectives denote ‘measure functions’ – functions from objects to degrees.\(^{15}\) A consequence of the definitions in (40) is that the set of positive degrees on a scale \(S\) and the set of negative degrees on \(S\) are disjoint. Given this, adjectival polarity can be characterized as a difference in the ranges of the functions denoted by positive and negative adjectives: positive adjectives denote functions from individuals to positive degrees; negative adjectives denote functions from individuals to negative degrees. Antonymy, in this view, holds when two adjectives have the same domains but different ranges, and they map identical arguments onto (join) complementary regions of the same scale.

This approach to adjectival polarity supports a comprehensive and general account of the semantic properties of gradable adjectives and comparatives. (See Kennedy (to appear a) for arguments that the approach also

\(^{15}\text{In contrast, standard degree-based analyses treat gradable adjectives as relations between individuals and degrees. Since relational analyses must also postulate a measure function as part of an adjective’s meaning, the basic account of adjectival polarity proposed here extends to such analyses as well. One of the reasons for adopting the measure function analysis is that it supports a simpler semantics for comparatives, which does not require quantification over the degree argument of the adjective. See Kennedy (1999a) for arguments that comparatives do not involve quantification over degrees.}\)
supports an explanation of the monotonicity properties of polar adjectives.)

Starting with comparatives, let us assume for reasons of perspicuity that all comparatives are derived from representations that have the syntax of subdeletion constructions; i.e., all comparatives have the shape in (43a) at the level of logical representation (abstracting away from the difference between incorporated and nonincorporated comparative morphology), and that such structures are mapped onto interpretations of the form in (43b), which defines a total ordering between degrees.\(^{16}\) (The analysis of comparatives with ‘less’ and ‘as’ is parallel, modulo the appropriate change in ordering relations.)

\[
(43) \quad a. \ x \text{ is more } \phi_1 \text{ than } y \text{ is } \phi_2 \\
     b. \ \phi_1(x) > \phi_2(y)
\]

Since degrees qua intervals are formalized set-theoretically, we can formulate the truth conditions of comparatives with ‘more’, ‘less’, and ‘as’, in terms of the Boolean definitions of ordering relations in (44a)–(44c), respectively, where \(d_1\) and \(d_2\) are elements of some set of degrees \(D\) on a scale \(S\) (i.e., \(POS(S)\) or \(NEG(S)\)).

\[
(44) \quad \forall d_1, d_2 \in D:\n
a. \ d_1 > d_2 \iff d_1 \cap d_2 = d_2 \land d_1 \neq d_2 \\
b. \ d_1 < d_2 \iff d_1 \cap d_2 = d_1 \land d_1 \neq d_2 \\
c. \ d_1 \geq d_2 \iff d_1 \cap d_2 = d_2
\]

Given these assumptions, a simple comparative such as (45a) has the interpretation in (45b), which is true just in case the degree to which Alice is tall exceeds the degree to which Carmen is tall; i.e., just in case the positive projection of Alice on a scale of height stands in the relation defined

\(^{16}\) In Kennedy (1999a), I show how interpretations of the type in (43b) can be compositionally derived from syntactic representations in which adjectives project extended functional structure headed by degree morphology (as in Abney (1987), Grimshaw (1991), and Corver (1990, 1997)). While it is not actually necessary to assume that all comparatives are derived from clausal forms (though see Lechner (1999) for arguments that they are), I will adopt this position here as a simplifying assumption since all of the crucial facts under investigation involve comparatives in which the complement of ‘than’ is a clause.
in (44a) to the positive projection of Carmen on the height scale, as in the
diagram in (46).\(^\text{17}\)

(45)a. Alice is taller than Carmen (is tall).

b. \(\text{tall}(a) \succ \text{tall}(c)\)

(46)
\[
\begin{array}{l}
\text{height:} & 0 \quad \text{tall}(a) \quad \bullet \\
\text{height:} & 0 \quad \text{tall}(c) \quad \bullet \\
\end{array}
\]

The analysis of comparatives with negative adjectives is exactly the
same. (47a) has the interpretation in (47b), which is true just in case the
negative projection of Carmen on the height scale exceeds the negative
projection of Alice. According to (44a), this will hold just in case \(\text{short}(c)\)
extends further down the scale (i.e., has a lower minimal element) than
\(\text{short}(a)\), as indicated in (48).\(^\text{18}\)

(47)a. Carmen is shorter than Alice (is short).

b. \(\text{short}(c) \succ \text{short}(a)\)

(48)
\[
\begin{array}{l}
\text{height:} & 0 \quad \bullet \quad \text{short}(a) \quad \rightarrow \infty \\
\text{height:} & 0 \quad \bullet \quad \text{short}(c) \quad \rightarrow \infty \\
\end{array}
\]

This discussion illustrates an important aspect of the analysis, namely
that it explains the validity of (49).

(49) Alice is taller than Carmen if and only if Carmen is shorter than
Alice.

\(^{17}\) Overt subdeletion constructions such as (i) receive exactly the same analysis, assum-
ing either that ‘deep’ and ‘high’ denote different functions from objects to degrees on a
shared scale of e.g., physical extent (see Kennedy (1999a) for arguments in favor of this
position), or that a mapping can be established between degrees on the dimension-specific
scales of depth and height to a more general scale of physical extent.

(i) The hole is as deep as a ten story building is high.

\(^{18}\) The assumption that the overt expression ‘Carmen is shorter than Alice (is)’ is de-

erived from ‘Carmen is shorter than Alice is short’ does not make any claims about the
(un)acceptability of the latter, which I take to be a matter of syntax; note that ‘Alice is taller
than Carmen is tall’ is also unacceptable without contrastive focus. See the discussion of
this point in Section 2.3 and Note 10.
According to the semantic analyses of (45a) and (47a) outlined above, the interpretation of (49) is a substitution instance of (50), where \( \text{pos}_S \) and \( \text{neg}_S \) are functions from objects to positive and negative degrees on a scale \( S \), respectively, i.e., antonymous positive and negative gradable adjectives.

\[
\text{(50)} \quad \text{pos}_S(x) > \text{pos}_S(y) \iff \text{neg}_S(y) > \text{neg}_S(x)
\]

The validity of the schema in (50) follows from the relation in (41), which asserts that the positive and negative projections of an object on a scale are join complementary. Using (41) as a starting point, the complements of positive and negative degrees can be defined as follows:

\[
\text{(51) a. } -\text{neg}_S(x) = \text{pos}_S(x) - \text{MAX}(\text{pos}_S(x)) \\
\text{ b. } -\text{pos}_S(x) = \text{neg}_S(x) - \text{MIN}(\text{neg}_S(x))
\]

If \( \text{pos}_S(x) > \text{pos}_S(y) \), then \( \text{pos}_S(x) - \text{MAX}(\text{pos}_S(x)) > \text{pos}_S(y) - \text{MAX}(\text{pos}_S(y)) \), since \( \text{MAX}(\text{pos}_S(x)) \) and \( \text{MAX}(\text{pos}_S(y)) \) are the maximal elements of \( \text{pos}_S(x) \) and \( \text{pos}_S(y) \), respectively. It follows that \( -\text{neg}_S(x) > -\text{neg}_S(y) \), by substitution, and finally that \( \text{neg}_S(y) > \text{neg}_S(x) \), by contraposition. The other direction of the biconditional can be proved in exactly the same way.

Non-comparative predications can also be easily accommodated within the analysis. The basic account presented in Section 1 can be implemented by analyzing the truth conditions of the examples in (52) as in (53), where \( d_s(\text{tall}) \) and \( d_s(\text{short}) \) are free variables over standard-denoting degrees, whose values are determined contextually.

\[
\text{(52) a. } \text{Alice is tall.} \\
\text{ b. } \text{Alice is short.}
\]

\[
\text{(53) a. } \text{tall}(a) \geq d_s(\text{tall}) \\
\text{ b. } \text{short}(a) \geq d_s(\text{short})
\]

In a context such as the one represented by the diagram in (54), ‘Alice is tall’ would be true, and ‘Alice is short’ would be false, since the requisite
ordering relation between the two degrees in (53a) is satisfied, but the one
in (53b) is not.\textsuperscript{19}

\begin{equation}
\text{height: } 0 \quad \text{tall(}a\text{)} \quad \bullet \quad \text{short(}a\text{)} \quad \rightarrow \infty
\end{equation}

An important aspect of the proposal is that all that is required to eval-
uate the truth conditions of an adjectival predication, for both positive and
negative adjectives, is that a relation between two degrees can be estab-
lished (the projections of two objects on a scale for comparatives, and the
projection of one object on a scale plus the relevant standard-denoting
degree in non-comparatives). The actual values of the relevant degrees
are of secondary importance, and are only relevant insofar as they interact
with the relevant ordering relation. This point is crucial, as it responds to a
common criticism of the approach advocated here, which runs as follows:
since all negative degrees are infinite (at least for scales without a maximal
element, see below for discussion of this point), they are all equal, and the
analysis makes the obviously wrong prediction that all objects have equal
shortness, narrowness, slowness, etc.

In fact, this is not the case. While it is true that all negative degrees
are infinite (a point that can be exploited to account for the distribution
of measure phrases, as will be shown in Section 3.3), it is not the case
that they are all equal. The basic assumptions about scale structure (scales
are totally ordered, infinite sets of points) allow negative degrees to differ
in their minimal values, with the consequence that orderings of the sort in
(44) can be evaluated. These orderings in turn impose a relational structure
on the domain of a particular adjective, effectively capturing differences in
shortness, narrowness, slowness, and so forth.\textsuperscript{20}

\textsuperscript{19} That the standards for e.g., tallness and shortness need not stand in a strict com-
plementation relation is shown by the fact that the sentence ‘Alice is neither tall nor
short’ could (in some context) be true, in which case Alice’s height would fall between
the maximal and minimal points of the positive and negative standards, respectively. (In
Sapir’s (1944) terminology, ‘Alice’ would fall in the \textit{zone of indifference}; cf. Klein’s (1980)
discussion of the \textit{extension gap} of the (positive) predicate.)

\textsuperscript{20} A second objection to negative degrees is one of ‘conceptual implausibility’ (see
e.g. Bierwisch (1989), p. 247ff). Since the strength of such an argument can only be evalu-
ated in the context of psycholinguistic evidence, which has up to now not been presented,
I will not address it here. However, it should be observed that psycholinguistic studies
on adjectival polarity have demonstrated asymmetries in positive and negative adjectives.
(For example, the latter take longer to process; see Clark (1970) for an overview.) Whether
the inherent asymmetry between positive and negative degrees can be exploited to provide
some account for the observed psycholinguistic facts is a question that should be a focus
of future work.
3.2. Cross-Polar Anomaly Explained

We are now in a position to see how the interval-based formalization of degrees and corresponding analysis of adjectival polarity supports an explanation of cross-polar anomaly. The basic form of the explanation is the same as the one I rejected in my earlier discussion of the degree-as-point approach in Section 2: comparatives constructed out of antonymous pairs of adjectives are anomalous because they involve comparison of different sorts of degrees. Within the system proposed here, the fact that ‘degree sort mismatch’ leads to anomaly can be explained in terms of very general principles of ordering relations. A fundamental property of an ordering relation is that its arguments must be elements of the same ordered set. Let us assume that this requirement is part of the meaning of expressions of ordering in natural language; specifically, that the comparative morphemes ‘more’, ‘less’ and ‘as’ presuppose that their degree arguments are elements of the same ordered set. If this requirement is not met, the relations denoted by comparative morphology are undefined, and a truth value cannot be computed. It is precisely this type of breakdown that is at the root of examples of cross-polar anomaly.

The basic claim of the analysis of adjectival polarity presented in the previous section is that positive and negative adjectives denote functions with different, in fact disjoint, ranges: positive adjectives denote functions from objects to positive degrees, negative adjectives denote functions from objects to negative degrees, and the structural distinction between the two sorts of degrees has the consequence that these two sets are disjoint. Since the truth conditions for comparatives are formulated in terms of the ordering relations in (44), which require their arguments to be degrees in the same ordered set, it follows that any comparative constructed out of adjectives of opposite polarity should fail to have a truth value.

For illustration of the analysis, consider the following examples. The logical representation of (55a), in which the adjective in the main clause is negative and the adjective in the comparative clause is positive, is (55b).

(55)a. ?Alice is shorter than Carmen is tall.

b. short(a) $>$ tall(c)

The problem is that short(a) and tall(c) denote degrees in different ordered sets ($NEG(height)$ and $POS(height)$, respectively). As a result, the ordering relation introduced by the comparative morpheme is undefined for its two arguments, rendering the sentence anomalous. (Examples in which the adjectives are reversed are explained in exactly the same way.)
This analysis extends beyond examples involving pure antonyms like (55a) to any comparative in which the adjective in the main clause is of different polarity from the adjective in the comparative clause. (56) illustrates an example of this type.

(56)a. ?Alice is shorter than the doorway is high.

b. \textit{shorter}(a) < \textit{high}(d)

Even if we assume that ‘short’ and ‘high’ map their arguments onto a shared scale – call it \textit{physical extent} (see Note 17) – the same problem that arises in examples like (55a) shows up here. The negative degree denoted by \textit{short}(a) and the positive degree denoted by \textit{high}(d) are objects in disjoint sets ($\text{NEG}($\textit{physical extent}$)$ and $\text{POS}($\textit{physical extent}$)$), so the comparative relation is undefined for its arguments, and the sentence is correctly predicted to be anomalous.

The most important point to take away from this discussion is that an analysis of the semantics of gradable adjectives and adjectival polarity in which degrees are formalized as (positive or negative) intervals supports an explanation of cross-polar anomaly precisely because it is built on a fundamental structural distinction between positive and negative degrees. This distinction provides the basis for a sortal theory of adjectival polarity, which in turn supports an analysis of cross-polar anomaly in terms of general properties of ordering relations. Since we have already seen that other approaches fail to fully account for the data under consideration, we can take the success of the interval-based analysis in this regard as strong evidence for its correctness.

3.3. \textit{Measure Phrases and Differential Comparatives}

An interesting consequence of the analysis of cross-polar anomaly presented in the previous section is that CPA constructions are explained in the same way as another well known class of anomalous adjectival predications, namely sentences in which a negative adjective is associated with a measure phrase, as in (57a) (cf. (57b); see Hale (1970) for early discussion of this phenomenon).

(57)a. ?‘The Dream of a Ridiculous Man’ is 21 pages short.

b. ‘The Dream of a Ridiculous Man’ is 21 pages long.

The basic explanation for the anomaly of (57a) within a model that assumes positive and negative degrees is presented in von Stechow (1984b).
For any scale with a minimal element, positive degrees correspond to intervals that range from the minimal element of the scale (the zero point) to some positive value. Assuming that scales typically have no maximal element (see von Stechow (1984a) and Rullmann (1995), but see also Kennedy and McNally (1999) for arguments that some scales do have maximal values), it follows that positive degrees correspond to finite, closed intervals, but negative degrees correspond to infinite, open intervals. If we further assume that nominals like ‘meter’, ‘centimeter’, ‘pound’, and ‘kilometer per hour’ refer to (conventionally determined) degrees that begin at the zero point of the scale (cf. Bierwisch (1989)), and that numerals induce concatenation of such measures (see Krantz et al. (1971) for discussion of concatenation of closed scale segments), then it follows that measure phrases introduce only positive degrees.

Building on the semantics for the non-comparative form adopted in Section 3.1 (or some equivalent formulation), (57b) can be assigned the logical representation in (58), in which the measure phrase provides the standard value.

(58) \[ \text{long}(r) \geq 21 \text{ pages} \]

Since \text{long}(r) and the degree denoted by ‘21 pages’ are both positive degrees, the partial ordering in (58) can be evaluated. Note also that (58) says nothing about the relation between the length of \textit{The Dream of a Ridiculous Man} and the standard for ‘long’. As a result, it does not imply that ‘\textit{The Dream of a Ridiculous Man} is long’ is true, which is exactly what we want.

By the same reasoning, the logical representation of (57a) should be (59).

(59) \[ \text{short}(r) \geq 21 \text{ pages} \]

(59) imposes a partial ordering relation on the negative degree \text{short}(r) and the degree denoted by ‘21 pages’. The measure phrase cannot denote a negative degree, however, so the ordering relation cannot be evaluated. The end result is that examples like (57a) are correctly predicted to be anomalous because the ordering relations they impose are undefined, just like examples of cross-polar anomaly.

Although measure phrases are incompatible with negative adjectives in their non-comparative forms, this is not true of comparatives. As shown by the ‘differential comparatives’ in (60), measure phrases are felicitous with
both positive and negative adjectives (see Hellan (1981) and von Stechow (1984a) for detailed discussion of differential comparatives).

(60)a. Alice is 12 cm shorter than Carmen (is).

b. *The Brothers Karamazov* is 122 pages longer than *The Idiot* (is).

What is interesting about these constructions is that the measure phrases play a different role from the one they play in examples like (57b). In (57b), the measure phrase provides information about the positive projection of the argument of the adjective on the scale (its length). In (60a) and (60b), however, the phrases ‘12 cm’ and ‘122 pages’ denote degrees that measure the difference between the compared (positive or negative) degrees.

This difference in interpretation can be exploited to account for the acceptability of examples like (60a). Since positive and negative degrees are defined set-theoretically, they are subject to operations on sets. In particular, for any two degrees, it is possible to identify their difference as the interval that is contained in one but not the other.\(^{21}\) Crucially, such ‘differential degrees’, like positive degrees, correspond to finite, closed intervals. Given this, all that is necessary to interpret differential comparatives is the introduction of a function that maps differential degrees onto degrees with minimal elements that correspond to the zero point of the scale in a structure-preserving way. A function \(\text{ZERO}\) that achieves this result is defined in (61).

\[
\text{ZERO} = \{(d_1, d_2) | \text{MIN}(d_1) = \text{MIN}(d_2)\}
\]

The interpretation of differential comparatives can then be formulated as an extension of the basic semantics for comparatives introduced in Section 3.1. Specifically, a differential comparative with the form in (62a)

\[
\forall d_1, d_2 \subseteq S : d_1 - d_2 = \{ p \in S | p \in d_1 \land p \notin d_2 \}
\]

Specifically, the difference operator for degrees can be defined as in (i).
(where ‘MP’ is a measure phrase) can be assigned the interpretation in (62b). 22

(62)a. \( x \text{ is MP more } \phi_1 \text{ than } y \text{ is } \phi_2 \)

b. \( \phi_1(x) > \phi_2(y) \land \text{ZERO}(\phi_1(x) - \phi_2(y)) \geq MP \)

The logical representation of (60a), for example, is (63), which is true just in case the difference between short\(a\) and short\(c\) can be mapped onto a degree that is at least as great as the degree denoted by ‘12 cm’.

(63) \( \text{short}(a) > \text{short}(c) \land \text{ZERO}(\text{short}(a) - \text{short}(c)) \geq 12 \text{ cm} \)

Crucially, the fact that this example involves a negative adjective is irrelevant. Since the difference between two negative (or two positive) degrees is a closed interval on the scale, a mapping to a degree in the set of degrees named by measure phrases can be established by the \text{ZERO} function. Since this degree and the degree denoted by ‘12 cm’ are elements of the same set, the ordering relation between them is defined, and the entire expression in (62b) can be assigned a truth value. 23


Hellan and von Stechow also argue that the differential interpretation is basic, and that the simpler comparatives involve existential quantification over a measure phrase variable. Since the purpose of this section is only to show that the model supports a semantic analysis of differential comparatives, I will not take a stand on this question here.

23 This section has established that the characterization of adjectival polarity argued for in this paper supports a semantics for differential comparatives, but the story is far from complete. Of particular interest here are pairs like ‘warm/cold’, which accept measure phrases in differential comparatives, but not in the positive (or negative) non-comparative forms, as shown by the examples in (i).

(i)a. Today is 45 degrees warmer than yesterday.

b. Yesterday was 45 degrees colder than today.

c. ?Today is 70 degrees warm.

d. ?Yesterday was 25 degrees cold.

One plausible explanation for these facts is that for some adjectives, the set of degrees that comprises the range of the positive form may be disjoint from the set of degrees that can be referred to by measure phrases, e.g. because the former includes intervals that approach,
3.4. Comparisons of Divergence and Deviation Revisited

The remaining question to be addressed is how the well-formed examples of comparatives constructed out of antonymous adjectives – comparisons of divergence and deviation – are to be explained. Building on the analysis of differential comparatives discussed above, I will show that these constructions can be analyzed as comparisons of differential degrees. Before going into the details, though, we can observe that if this hypothesis is correct, it explains why comparisons of divergence and deviation do not trigger cross-polar anomaly. As the discussion in Section 3.3 demonstrated, differential degrees can be mapped to degrees in the set of intervals that begin at the zero point of a scale, regardless of whether they correspond to differences between positive or negative degrees. It follows that comparisons between such derived degrees should be semantically well-formed, since orderings between them can be evaluated.

Consider first the case of comparison of deviation, an example of which is repeated in (64) (cf. (15a)).

(64) The Red Sox are more legitimate than the Orioles are fraudulent.

The hypothesis that this sentence involves comparison of differential degrees can be implemented by assigning it the interpretation in (65), in which the ZERO function is invoked to compare two differential degrees: the intervals that correspond to the differences between the (positive and negative) projections of the compared objects on the scale and the corresponding standard values for the antonymous adjectives.

\[
\text{ZERO}(\text{legitimate}(r) - d_{s(\text{legit})}) \succ \text{ZERO}(\text{fraudulent}(o) - d_{s(\text{fraud})})
\]

According to this analysis, (64) is true just in case the degree to which the Red Sox deviate from a standard of legitimacy exceeds the degree but do not include, the zero point of the scale. If this were the case, then the anomaly of examples like (ic) could be explained in exactly the same way as the anomaly of (57a) (and (id)).

Independent empirical evidence that adjectives may differ in terms of whether or not their ranges consist of (positive or negative) degrees that include endpoints comes from a number of sources. For example, Kennedy and McNally (1999) argue that this factor affects the distribution of degree modifiers and the identification of an adjective’s standard value, and Hay, Kennedy and Levin (1999) claim that it affects the aspectual properties of so-called ‘degree achievements’ derived from gradable adjectives. A full discussion of these issues is presented in Kennedy and McNally (in preparation).
to which the Orioles deviate from a standard of fraudulence. This logical representation accurately captures the meaning of this sentence, and more importantly, since the degrees derived by applying the zero function to the differences in (65) are comparable, it correctly predicts (64) to be acceptable.

The representation in (65) also accounts for the entailment patterns observed in COD constructions. In order for the difference operator to return a degree, it must be the case that its first argument exceeds its second argument (recall from the definitions in Section 3.1 that degrees must be nonempty convex subsets of a scale). In the case of (65), this means that the degree to which the Red Sox are legitimate must exceed the standard value for ‘legitimate’ in the context of utterance. Since the truth conditions for the non-comparative state that a sentence of the form ‘x is $\phi$’ is true just in case $\phi(x)$ is at least as great as the standard for $\phi$, the truth conditions for the non-comparative are satisfied whenever the truth conditions for the comparison of deviation interpretation are satisfied.24

Comparisons of divergence such as (66) can be analyzed in much the same way as comparisons of deviation, with one important difference: the properties of the former suggest that the adjectives in these constructions map their arguments directly onto differential degrees.

(66) My watch is faster than your watch is slow.

Recall from the discussion in Section 2.2 that one of the properties of the adjectives in these constructions is that both members of the ‘antonymous’ pair accept measure phrases, as illustrated by the examples in (67).

(67)a. My watch is 15 minutes fast.

b. Your watch is 10 minutes slow.

24 This discussion demonstrates that the general account of degrees and adjectival polarity presented here can accommodate the semantics of COD, though a full-scale compositional analysis of these constructions remains to be developed. One plausible hypothesis, given the fact that COD interpretations are available even to examples that do not involve adjectives of opposite polarity (see (20) in Section 2.2), is that the comparative morphemes are ambiguous between readings that map onto standard comparative interpretations and readings that map onto COD interpretations like (65). In examples involving adjectives of the same polarity, both interpretations are possible. In examples involving adjectives of opposite polarity, however, the standard interpretation is ruled out by the principles underlying cross-polar anomaly, leaving the COD reading as the only possible interpretation.
A second property that characterizes the adjectives in these constructions is that their interpretations differ from those associated with their standard uses. In (66) and (67), for example, ‘fast’ and ‘slow’ measure the degrees to which their arguments diverge from some arbitrary point (whatever counts as ‘on time’ in the context of utterance). In contrast, when ‘fast’ and ‘slow’ measure the absolute degrees to which their arguments possess some gradable property, they give rise to cross-polar anomaly in comparatives, as illustrated by (68).

(68) ?My car is faster than your car is slow.

These facts can be explained by assuming that the difference between the ‘measuring-from-a-reference point’ interpretations of these adjectives and their standard interpretations is that in the former case, the adjectives map their arguments onto intervals that extend in different directions (depending on the polarity of the adjective) from some arbitrary point (‘on time’ in (66) and (67)). For the positive member of the pair, this point provides the lower bound of the interval; for the negative member, this point provides its upper bound. The basic idea is illustrated in (69), where the subscript $\delta$ indicates the differential interpretation.

(69) \[ \text{time: } 0 \rightarrow \text{[on time]} \rightarrow \infty \]

\[ \cdot \text{slow}_\delta(w_\gamma) \rightarrow \text{fast}_\delta(w_\mu) \cdot \]

The acceptability of (67a) and (67b) can be accounted for by making the additional assumption that on their differential interpretations, these adjectives include the ZERO function as part of their meanings, so that the logical representations of (67a)–(67b) are (70a) and (70b), respectively.

(70)a. \( \text{ZERO}(\text{fast}_\delta(w_\mu)) \geq 15 \text{ minutes} \)

b. \( \text{ZERO}(\text{slow}_\delta(w_\gamma)) \geq 10 \text{ minutes} \)

Since the differential degrees onto which the adjectives map their arguments are bounded, closed intervals on the scale, they can be mapped by the ZERO function onto intervals in the set of degrees that includes those denoted by the measure phrases ‘15 minutes’ and ‘10 minutes’, and the ordering relations in (70) can be evaluated.

If this characterization of the meanings of these adjectives is correct, the analysis of the comparative in (66) is straightforward. This sentence can be assigned the interpretation in (71), which is just like a standard interpretation except for the addition of the ZERO function, which, by hypothesis, is built in to the adjectives’ meanings.

(71) \( \text{ZERO}(\text{fast}_\delta(w_\mu)) > \text{ZERO}(\text{slow}_\delta(w_\gamma)) \)
The result is a comparison between degrees of the same sort, which should be perfectly interpretable.

The third characteristic of antonymous adjectives in comparison of divergence constructions is that they fail to make substitution instances of (4) valid, as shown by the different uses of ‘fast’ and ‘slow’ in (72) ((72a) is false if both of our watches exceed the actual time but mine is farther ahead of it than yours is, and (consequently) neither of our watches is behind the actual time).

\[(72)\]
\[a. \text{My watch is faster than your watch if and only if your watch is slower than my watch. (INVALID)}\]
\[b. \text{My car is faster than your car if and only if your car is slower than my car. (VALID)}\]

This difference follows from the fact that positive and negative degrees in instances of ‘differential’ antonymy are structurally distinct from positive and negative degrees in the more standard cases of ‘complementary’ antonymy in an important way: unlike the latter, the former do not (obligatorily) correspond to complementary regions of a scale. As shown in Section 3.1, it is this property of positive and negative degrees in standard cases of antonymy that accounts for the validity of examples like (72b). If ‘fast’ and ‘slow’ in (72a) do not map their arguments onto complementary regions of the scale, however, then the deductions that explain the validity of (72b) do not apply.

3.5. Types of Polar Opposition

As observed in Section 2.2, even though differential antonyms like ‘fast’ and ‘slow’ do not show the same set of properties as complementary antonyms like ‘tall’ and ‘short’ (or ‘fast’ and ‘slow’ on complementary interpretations, as in (72b)) – in particular, they do not give rise to cross-polar anomaly – they are clearly antonymous in some sense. The question is in what sense, and how can the underlying similarities between these two classes of antonyms be captured in a way that also makes the type of semantic distinctions necessary to account for their different distributions. The discussion of comparison of divergence in the previous section suggests an answer to this question.

One property in particular that is shared by both complementary and differential antonyms is that the positive member of the pair maps its argument onto a degree that ranges from some lower bound upwards to some point, while the negative member maps its argument onto a degree that
ranges from some upper bound downwards to some point. This indicates that the crucial feature underlying both types of antonymy is one of directionality: in both complementary and differential uses, positive adjectives map their arguments onto degrees that have a natural ordering towards the upper end of the scale, and negative adjectives map their arguments onto degrees that have a natural ordering towards the lower end of the scale. The idea that directionality underlies antonymy goes back at least to Sapir (1944), and it appears in generative grammar in both traditional degree-as-point analyses in which positive and negative adjectives are treated as duals (see e.g., Rullmann (1995) and the discussion in Section 2.1), and in analyses that characterize the meaning of negative adjectives in terms of (some notion of) degree subtraction (see e.g., Bierwisch (1989), Faller (1998, to appear)).

The model of adjectival polarity that I have defended here differs from purely directional characterizations of polar opposition, however, by introducing an additional feature of complementarity. This feature provides a means of distinguishing between the two classes of antonyms under discussion here: complementary antonyms map their arguments onto complementary intervals of a scale; differential antonyms do not. As shown in the preceding sections, this difference in complementarity also plays a crucial role in the explanation of the distributions of antonymous adjectives in comparatives, thus making the crucial semantic distinction that the differing behavior of these two classes of antonyms demands.

4. Conclusion

Focusing on the distribution and interpretation of antonymous adjectives in comparative constructions, this paper has argued for a model in which degrees are formalized as intervals on a scale, and a structural distinction is made between two sorts of positive and negative degrees. Adjectival polarity in this model is characterized as a distinction in the ranges of polar adjectives: positive adjectives denote functions from objects to positive degrees, and negative adjectives denote functions from objects to negative degrees. Basing my claims on both constructed and naturally occurring

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25 The two classes of antonyms differ regarding the nature of the lower or upper bound, however. In the case of complementary antonyms, the lower and upper bounds for positive and negative degrees correspond to the lower and upper bounds of the scale itself. In the case of differential antonyms, however, the lower bound of the positive degree and the upper bound of the negative degree are literally the same point, which corresponds to some (conventionalized) value on the scale. This feature is related to the difference in complementarity discussed below.
data, I have shown that this type of model provides a comprehensive analysis of the complex range of facts in this domain, and moreover provides the basis for a more general theory of different types of adjectival polarity.

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