THE SEMANTICS OF DEGREE

This paper presents possible-worlds semantics for comparative constructions in English. In particular, a unified treatment is given of comparatives involving adjectives, mass nouns, and numerical comparisons. The paper is not “Montague grammar” in the strict sense of expounding or developing Montague’s own work, but it shares two important features: (a) It is based on possible-worlds semantics, and (b) it aims to make the underlying formal language as close as possible to the surface.\footnote{An example of an approach that satisfies (a) but does not pretend to satisfy (b) is to be found in Lewis (1972).} It in fact uses the procedure and terminology I adopted in Cresswell (1973), though I have taken pains to make the paper self-contained, and at certain points I reject analyses I once favored and present alternative ones.

In Section 1 I set out the formal languages I want to put forward as underlying natural languages, and summarize their semantic framework. Section 2 is the heart of the paper and presents a detailed semantic analysis of comparative adjectives. Section 3 discusses mass nouns and plurals; Section 4 looks at the metaphysics that underlie comparisons; Section 5 takes up the relation between the underlying formal language of comparatives and their surface form; and Section 6 lists a few problem cases, with tentative suggestions for the solution of some of them.

1. λ-CATEGORIAL LANGUAGES

This section is a summary of material from Part II of Cresswell (1973). For those familiar with that work the only difference here is that I treat propositions simply as sets of possible worlds rather than as sets of “heavens,” and I avoid any explicit reference to contexts of use.\footnote{Propositions need to be something more elaborate than sets of possible worlds for two reasons: first, because of context dependence (discussed later), and second, because of functors that represent propositional attitudes to logically equivalent propositions. This area is very obscure to me (see Cresswell, in prep.) and, I suspect, to others also e.g., Partee (1973).}

We first need the notion of a SYNTACTIC CATEGORY. For the purposes of this paper, there are two basic categories—0 of SENTENCE and 1 of NAME. Given categories $\tau$, $\sigma_1$, $\ldots$, $\sigma_n$, then $\langle \tau, \sigma_1, \ldots, \sigma_n \rangle$ is the category of a FUNCTOR that makes an expression of category $\tau$ out of expressions of category $\sigma_1$, $\ldots$, $\sigma_n$, respectively.
functions \( v \) such that where \( x \in \mathcal{X}_\sigma, v(x) \in \mathcal{D}_\sigma \). Where \( a \in \mathcal{D}_\sigma, (v, a/x)(x) \) is the function like \( v \) for all arguments except that \((v, a/x)(x) = a\).

We may show how \( V \) induces an assignment to all expressions of \( \mathcal{L}' \). Let \( V'(\alpha) \) mean "the value induced by \( V \) to the expression \( \alpha \), with respect to the assignment \( v \) to the variables"; \( V \) is defined as follows:

\[
\begin{align*}
(1.5) & \quad \text{If } \alpha \in \mathcal{E}_\sigma, \text{ then } V'(\alpha) = V(\alpha). \\
(1.6) & \quad \text{If } x \in \mathcal{X}_\sigma, \text{ then } V'(x) = v(x). \\
(1.7) & \quad \text{If } \alpha = \langle \delta, x_1, \ldots, x_n \rangle, \text{ then where } \omega = V'(\delta), \\
& \quad V'(\alpha) = \omega_\delta(V'(x_1)), \ldots, V'(x_n)). \\
(1.8) & \quad \text{If } \alpha = \langle \lambda, x, \beta \rangle, \beta \in \mathcal{E}_\sigma, \text{ then } V'(\omega) \text{ is the function } \omega \in \mathcal{D}_\sigma, \text{ such that for any } a \in \mathcal{D}_\sigma, (\omega(x) = V'_{\omega}(\omega)(\beta)).
\end{align*}
\]

We assume the usual distinction between free and bound variables. In the expression \( \langle \lambda, x, \alpha \rangle \) any \( x \) that occurs in \( \alpha \) will be bound. A variable that is not bound will be free. (The occurrence of \( x \) after the \( \lambda \) of course counts as neither bound nor free.) An expression without free variables is called closed. The value assigned to closed expressions does not depend upon the assignment to the variables. If \( x \) is closed and \( v \) and \( \mu \) are any assignments to the variables, then \( V'(\alpha) = V'_{\lambda}(\alpha) \). We frequently write simply \( V'_{\lambda}(\alpha) \) in such a case. (The \( + \) is to show that it is an induced assignment to the complex expression \( \alpha \), uniquely determined by the assignment \( V \) to the finite number of symbols of \( \mathcal{L}' \).)

Where \( \mathcal{W} \) is the set of all possible worlds, \( \mathcal{D}_\sigma \subseteq \mathcal{P}\mathcal{W} \). That is, the domain of propositions is a subset (possibly but not necessarily a proper subset) of the set of all possible worlds—which is just another way of saying that propositions are sets of possible worlds. If \( a \in \mathcal{D}_\sigma \) (i.e., if \( a \subseteq \mathcal{W} \)) and \( w \in \mathcal{W} \), then to say that \( a \) is true in \( w \) is to say that \( w \in a \). Where \( \alpha \) is a sentence of \( \mathcal{L}' \) (i.e., \( \alpha \in \mathcal{E}_\sigma \) and contains no free variables), \( \alpha \) is true in \( w \) under \( V \) iff \( w \in V'_{\lambda}(\alpha) \).

Because our domains include partial functions, there are many expressions that will not be defined. This kind of semantic anomaly must of course be clearly distinguished from grammatical ill-formedness. Only well-formed expressions are candidates for semantic oddness in \( \lambda \)-categorial languages.

In Section 5 I say something about the relation between \( \lambda \)-categorial and natural languages, and I have said a good bit about it elsewhere (Cresswell, 1973; in press). What we do is represent the symbols of \( \mathcal{L}' \) by English words printed in boldface type and then try to get expressions, sometimes called \( \lambda \)-deep structures, that look as though they could become English sentences with a bit of tinkering. In this particular work I am concerned more with the underlying semantic structure than with the

---

3 Variables are symbols that go in quantifiers or abstracts (see, e.g., Cresswell, 1973, p. 81). I mention this point because "variables in syntax" seems to mean something quite different.

4 Some motivation for treating propositions as sets of possible worlds will be found in Cresswell (1973, p. 24f.), see also Montague (1969) and Stalnaker (1972). We are, as noted, ignoring context dependence. Some remarks on the metaphysics of \( \mathcal{D}_\sigma \) and \( \mathcal{D}_1 \) are given in Cresswell (1973, Chapters 3, 7). Possible worlds are put to philosophical use in Montague (1969).
tinkering. There have been a number of recent syntactic studies of the comparative in English, and I have made use of some of these (Doherty and Schwartz, 1967; Hae, 1970; Lees, 1961; Smith, 1961). One in particular has guided much of my analysis, and that is Bresnan (1973). In Section 5 of this study, I discuss the relation between my $\lambda$-deep structures and her underlying structures.

Beginning with names (category 1) and sentences (category 0), we can produce symbols in certain other categories that look a bit like various parts of speech. For instance, intransitive verb expressions (whether simple or complex) look as though they should be treated as one-place predicates. This means that they make sentences out of names, which means that they are in category $\langle 0, 1 \rangle$. (Transitive verb expressions are in category $\langle 0, 1, 1 \rangle$, and so on.) Another class of expression in category $\langle 0, 1 \rangle$ is the class of common count nouns, whether simple or complex. Because they must be distinguished from intransitive verb expressions, Montague (1973, p. 223) has assigned them a different category. We might indicate this category by $\langle 0, 1 \rangle$. Expressions in category $\langle 0, 1 \rangle$ would still make sentences out of names, but they would be syntactically distinct from expressions of category $\langle 0, 1 \rangle$. However, in this study we shall not make such a distinction. In Cresswell (1973) the category 2 was introduced, with $D_2 = D_0 \cup \{\text{man}\}$, but I am now inclined to think that this was a mistake. I now believe that it is the function of the principles that relate the $\lambda$-deep structures and the sentences of ordinary language to distinguish nouns and verbs.

The meaning of common count nouns can be illustrated by man:

\[ (1.9) \quad I(\text{man}) = \{ \omega \in D_0 \mid \text{for any } a \in D_1, \text{ \text{a} is in the domain of } \omega \iff \text{if } a \text{ is a physical object in some } w \in W, \text{ and for any } w \in W, w \in \rho(a) \iff a \text{ is a man in } w. \] 

5 At the time this work was written I had not seen Seuren (1973) and had had to rely on Borowski (1973). What Seuren's article shows is, in effect, that if the basic primitive for an adjective like tall is taken as 'x is tall to degree at least y' then my er than can be expressed in terms of the concepts of ordinary quantification theory. For we can then say things like 'there is a degree y such that Bill is tall to degree at least u but it is not the case that Ophelia is long to degree at least u'. If one's aim is to make the object language look as much like quantification theory as possible, rather than as close to the surface as possible, then Seuren's analysis must be preferred to mine. Otherwise the issue must turn on how strong Seuren's (1973) syntactic arguments on pages 532–537 that there is an underlying negative in comparisons are felt to be. (I find myself agreeing with about half of Seuren's data.)

6 These semantic "rules" frequently provoke a charge of circularity. My reply is always that I am not attempting to teach anyone the meaning of English words. I assume that he understands English because English is the metalanguage of the paper. $I(\text{man})$ is a (set-theoretical) function of a certain kind, and if the analysis is correct, it is that function, however our English metalanguage serves to pick the function out. Those who think that rules like $I(\text{man})$ are circular seem to be making the impossible demand that we write articles on semantics without using a language.

I am taking the analysis of the semantics of common count nouns and intransitive verbs to be relatively unproblematic. I would hope at least that if I am wrong, the error will not affect what I say in this work. There is one obvious respect in which the semantics is not adequate as it stands, and this is that the meaning of nouns frequently depends on such things as the temporal context. In the case of man, there is perhaps the temptation to say once a man always a man (but is even this so certain?). However, in the case of a word like widow, we have to know not only what world is involved but also what point of time. Logicians have spoken of things like moments of time as "contextual indices." Another index is an utterer index, so that:

1. I am hungry.

could be said to be true for $u$ at time $t$ in world $w$ if $u$ is hungry at time $t$ in world $w$. Most of the semantics we go on to present ought to make reference to contextual indices, but it is in every case clear how it should be done, and we shall simply enjoy the relative simplicity afforded by forgetting about contexts of use. (It is in the analysis of features such as tenses and pronouns that context dependence becomes vital; vide Cresswell, 1973, Chapters 11, 12.) In what follows, a world $w$ can always be thought of as a world at a particular moment of time. This will take care of a great deal of the more important kinds of context.)

Another simplification is to treat names as expressions of category 1. In Cresswell (1973 p. 130f) it was argued that an important syntactic category is that of nominal. Nominals are in category $\langle 0, 1 \rangle$ and contain, as well as names, expressions that underlie phrases like:

2. at least one person present
3. no philosopher
4. the saggy baggy elephant

The need for nominals was argued in Cresswell (1973), but the questions we are currently interested in can be discussed without the irrelevant complexity they introduce.

2. COMPARATIVE ADJECTIVES

The approach adopted in Cresswell (1973) by me and in Montague (1970, p. 394) by Montague, and by other logicians besides, takes the attributive use of adjectives to be prior to their predicative use. In this we differ not only from almost all linguists (at least as far as my reading
goes) but also, I suspect, from most philosophers. Indeed I am coming to think that a great many philosophical puzzles have been generated by the taking of expressions like:

\[ (5) \quad \text{is red} \]

as paradigmatic instances of predicates.\(^7\) Be that as it may, adjectives are here treated as noun modifiers. In Cresswell (1973) they are treated as being of category \( \langle(0, 1), \langle 0, 1 \rangle \rangle \). One of the main tasks of this study is to refine the analysis given there in such a way as to give a semantically coherent account of comparatives. In so doing we shall also discover some important semantic features of mass nouns and the collectivizing function of plurals.

When we make comparisons we have in mind points on a scale. The scale can be represented by a relation, and the points on the scale by the field of that relation. (A relation in set theory is a set of ordered pairs; the field of a relation is the set of all things that are related in one direction or another to something else.) Where \( > \) is a relation we denote its field by \( F(>) \):

\[ (2.1) \quad \text{A DEGREE (of comparison) is a pair \( \langle u, > \rangle \), where \( > \) is a relation and } u \in F(>) \]

The \( > \) is heuristic, in order to suggest the direction of the comparison. It is tempting to think of \( > \) as at least a partial ordering (i.e., a transitive and antisymmetric relation); whether it should be strict or not or total or not seems unimportant, and perhaps we should even be liberal enough not to insist on transitivity and antisymmetry (Wheeler, 1972, p. 319).

We suppose that in phrases like:

\( (6) \quad \text{a tall man} \)

\( (7) \quad \text{a very tall man} \)

\( (8) \quad \text{a much too tall man} \)

\( (9) \quad \text{a taller man} \)

the underlying semantic concept is:

\[ (10) \quad x\text{-much tall man} \]

where \( x \) is a degree of tallness. In fact I am going to regard the word \textit{much} as a surface marker that signals the underlying degree variable (Bresnan, 1973, p. 277f). On this analysis \( \langle \text{tall, man} \rangle \) becomes in effect a two-place predicate with (roughly) the meaning of:

\[ (11)\quad x \text{ is a man who is tall to degree } y \]

We want to give a semantics for \textit{tall} that applies to any count noun to which it is applied. On this view \textit{tall} is of category \( \langle(0, 1), \langle 0, 1 \rangle \rangle \) and has the following semantics:

\[ (2.2) \quad I(\text{tall}) = \text{a function } \zeta \in D_{\langle(0, 1), \langle 0, 1 \rangle \rangle} \text{ such that } \omega \in D_{\langle 0, 1 \rangle} \text{ if } \omega \text{ is in the domain of } \zeta \text{ iff } \omega \text{ is a property whose domain contains only physical objects. For any } a, b \in D, \text{ in the domain of } \zeta(\omega), \text{ and any } w \in W, \ w \in \zeta(\omega)(a, b) \text{ iff } w \in \omega(a) \text{ and } b = \langle u, > \rangle, \text{ where } > \text{ is the relation whose field is the set of all } v \text{ such that } v \text{ is a spatial distance, and } \langle v_1, v_2 \rangle \in > \text{ iff } v_1 \text{ is a greater distance than } v_2, \text{ and } u \text{ is the distance between } a \text{ and } b \text{ in the case of most } c \text{'s such that } w \in \omega(c) \text{ this distance will typically be vertical.} \]

It must be constantly borne in mind that semantic “rules” like (2.2) are for purposes of illustration only. This study is not trying to tie down the English meaning of \textit{tall} but only to show the kind of thing that should represent it in a \( \lambda \)-categorical base for English. I would not go so far as Wheeler (1972 p. 319) and describe the study of the meanings of actual words as “physics” rather than “semantics,” but I would agree with him that an investigation into the logical structure underlying a natural language is not concerned primarily with getting the details of the meanings of particular words absolutely correct. Nevertheless the closer one can come, the more illuminating the examples will be.

I think a few words should be said about (2.2). The early part of the clause says something about the kinds of things that could be in the domain of the function. Where an expression is made up out of a functor whose arguments have as their meanings something that is not in the domain of the meaning of the functor, then the whole expression lacks a value. Thus in this case the phrase:

\[ (12) \quad \text{*a tall idea}^8 \]

would not have a value. This means that semantic anomalies do not require that any “feature theory” be present in the syntax. Example (12) will be perfectly well-formed but semantically anomalous. In the illustrations throughout the rest of this work, the domains of the functors will have to

\[ ^8 \text{Since reading Householder (1973) on asterisks (see also Bar Hillel, 1971 and Weydt, 1973, p. 578), I venture to use them with some trepidation. An asterisk means that there is something odd about the sentence. In each case I shall try to explain what sort of oddness I think it is, and why I think it a sort of oddness. An asterisk does not mean that the sentence is ungrammatical.} \]
be suitably restricted. It is, however, tedious to mention this most of the
time, and we shall always assume that when we say things like “for any
\(a \in D_i\)”, or “or any \(\omega \in D_{(0, 1)}\)”, what we really mean is “for any appropriate
\(a\) (or \(\omega\)).”

The difference between tall and long is that tall usually applies to distances
measured vertically and from the ground up, though of course one can find
out how tall a man is by measuring him lying down. I do not think (2.2)
has by any means captured all these nuances (e.g., that the distance between
the extremities, in the case of something like a man, will have to be measured
when he is stretched out and not curled up, and so on), but it will certainly
do for our purposes. Long can also be used to measure temporal distances.
We shall examine this use in (4.3).

We can now introduce the symbol for the comparative. We shall write
it as er than. For the present we shall regard it as a single symbol in category
\(\langle 0, \langle 0, 1 \rangle, \langle 0, 1 \rangle \rangle\), but there is no reason why a transformation cannot
regard it as two symbols and separate the two parts on the way to the surface.

The problem facing most analyses of er than is that the comparison
scales used vary from adjective to adjective. That is, if we define er than
by means of distances, then it may be all right for:

(13) Bill is taller than Arabella.

but it will not do for:

(14) Bill is uglier than Arabella.

The advantage of our approach is that the terms of the er than supply the
comparison scale: \(\lambda, \text{ er than}, \beta\) essentially means that the degree of \(\alpha\)
is higher than the degree of \(\beta\) on the scale that is common to the meaning
of \(\alpha\) and \(\beta\).

(2.3) \(V(\text{er than})\) is the function \(\zeta \in D_{(0, 0, 1 \rangle, \langle 0, 1 \rangle}\) such that where
\(\omega\) and \(\omega' \in D_{(0, 1)}, \omega\) and \(\omega'\) are in the domain of \(\zeta\) only if they are
functions whose domains consist entirely of degrees. For
any \(w \in W, w \in \zeta(\omega, \omega')\) iff there is an \(a\) and a \(b\) such that \(w \in \omega(a)\)
and \(w \in \omega(b)\), and for any such \(a\) and \(b\) there is some relation
\(>\) and some \(u_1\) and \(u_2\) in \(\mathcal{F}(>)\), and \(a = \langle u_1, >\rangle\) and
\(b = \langle u_2, >\rangle\) and \(u_1 > u_2\).

The generality of the analysis proposed here may be seen from a detailed
discussion of:

(15) Bill is a taller man than Ophidia is a long snake.

I have chosen this example because most logicians who have tried to ana-
lyze comparatives (Wheeler, 1972, p. 318; Wallace, 1972; Rennie, 1974,
pp. 146–151) have stayed with such examples as:

(16) Bill is a taller man than Tom.

If we can analyze (15), then we can regard (16) as a shortened version of:

(17) Bill is a taller man than Tom is a tall man.

Snake is a count noun, and its semantics ought to be obvious; and some-
ting was said about the semantics for long in the discussion of (2.2). The
important thing is that tall and long both involve the same scale.

A plausible \(\lambda\)-deep structure for (15) would seem to be:

(18) \(\langle \langle \lambda, x, \langle \text{Bill, (tall, man)}, x \rangle \rangle, \text{er than} \langle \lambda, x, \langle \text{Ophidia, (long, snake)}, x \rangle \rangle \rangle\)

(In this sentence \(\langle \text{tall, man} \rangle\) and \(\langle \text{long, snake} \rangle\) are in category \(\langle 0, 1, 1 \rangle\)
and are placed between their arguments; the first arguments are Bill
and Ophidia, respectively, and the second argument is the variable \(x\).)

Let us now see how (18) should be evaluated semantically. We first obtain,
for an assignment \(v\) to the variables:

(19) \(V_v(\langle \text{Bill, (tall, man)}, x \rangle)\)

Now \(V_v(\text{Bill}) = V(\text{Bill}) = \text{Bill}\), and \(V_v(x) = v(x)\). Suppose that \(v(x) = b\):
then (19) becomes:

(20) \(V_v(\langle \text{tall, man} \rangle)(V_v(\text{Bill}), V_v(x)) = V_v(\langle \text{tall, man} \rangle)(\text{Bill}, b)\)

Further,

\(V_v(\langle \text{tall, man} \rangle) = V_v(\text{tall})(V_v(\text{man})) = V(\text{tall})(V(\text{man}))\)

This means that (19) = \(V(\text{tall})(V(\text{man}))(\text{Bill}, b)\). Where \(V(\text{man})\) is \(\omega\) and
\(V(\text{tall})\) is \(\zeta\), then (19) = \(\zeta(\omega)(\text{Bill}, b)\). By (2.2), \(w \in \zeta(\omega)(\text{Bill}, b)\) iff Bill is
a man in \(w\) and \(b\) is a pair \(\langle u, > \rangle\) and \(>\) is the “greater than” relation with
respect to length, and \(u\) is the distance between Bill’s extremities in \(w\), and
this distance is typically vertical.

This means that:

(21) \(V^*(\langle \lambda, x, \langle \text{Bill, (tall, man)}, x \rangle \rangle)\)

is the \(\sigma\) such that \(u \in \sigma(\omega(a))\) if \(a = \langle u, > \rangle\), where \(u\) is Bill’s height in \(w\)
(i.e., the typically vertical distance between his extremities) and Bill is a
man in \(w\).
By analogous reasoning we discover that:

\[ V^*(\langle \lambda, x, \langle \text{Ophidia, long, snake}, x \rangle \rangle) \]

is the function \( \omega' \) such that \( w \in \omega'(b) \) iff \( b = \langle v, > \rangle \), where \( v \) is Ophidia's length in \( w \) and Ophidia is a snake in \( w \).

It is important that the > involved in both tall and long is, despite their differences in meaning, the same relation.

Now \( V^*(\langle 18 \rangle) = V(\text{er than})(\omega, \omega') \)

(where \( \omega \) and \( \omega' \) are as described earlier), and by (2.3), \( V(\text{er than}) \) is the function \( \zeta \) such that \( w \in \zeta(\omega, \omega') \) iff there are some \( a \) and \( b \) such that \( w \in \omega(a) \) and \( w \in \omega'(b) \) and for any such \( a \) and \( b \), \( a = \langle u, > \rangle \) and \( b = \langle v, > \rangle \), \( u > v \). In the particular case we have in mind, this will be true iff the distance that Tom's height is in \( w \) exceeds the distance that Ophidia's length is. That is, (18) will be true in \( w \) iff Tom is taller than Ophidia is long.

This sentence will also entail that Tom is a man and Ophidia a snake, because (2.3) requires that for (18) to be true in \( w \), there must be some \( a \) such that \( w \in \omega(a) \), and also that there is some \( b \) such that \( w \in \omega'(b) \); and as we saw in our discussion of (21) and (22), the former entails that Tom is a man in \( w \) and the latter that Ophidia is a snake.\(^{10}\)

The semantics also provides a natural explanation for why, although:

(i) Bill is the father of Arabella or Clarissa.

means (on its most natural interpretation):

(ii) Either Bill is the father of Arabella or Bill is the father of Clarissa.

yet:

(iii) Bill is taller than Arabella or Clarissa.

does not mean (at least not usually):

(iv) Bill is taller than Arabella or Bill is taller than Clarissa.

but means, rather:

(v) Bill is taller than Arabella and Bill is taller than Clarissa.

These points have been noticed by Jennings [13]. The underlying structure is:

\[ \langle \langle \lambda, x, \langle \text{Bill, tall, man}, x \rangle \rangle, \text{er than}, \langle \lambda, x, \langle \text{Arabella, tall, woman}, x \rangle \rangle, \text{or,} \langle \text{Clarissa, tall, woman}, x \rangle \rangle \]

And this will be true if for any \( u \) that is Bill's height and \( v \) that is Arabella's height or Clarissa's height, \( u > v \).

This example is possible because we have allowed the functions in the domain of \( V(\text{er than}) \) to be true of more than one degree. However, the members of their domains are still all degrees and, further, always degrees involving the same comparison scale. Lee (1961, p. 175) notes the semantic deviance of sentences like:

(vii) *I know him better than she doesn't.*

The reason for this deviance, on our account, would be that an expression like:

(viii) \( \langle \lambda, x, \langle \text{not, Bill, tall, man}, x \rangle \rangle \)

would denote a function \( \omega \) whose domain will include things that are not degrees. \( \omega \) would thus not be in the domain of \( 1(\text{er than}) \). And even the degrees in its domain can involve many different comparison scales (see Green, 1970).

More complicated cases pose no semantic problems. Thus examples (23) and (24) of Smith (1961 p. 352):

John has a bigger car than Bill has,

and

George climbed a higher mountain than Everest.

become (ix) and (x):

(ix) \[ \langle \langle \lambda, x, \langle \lambda, y, \langle \text{John, has, y}, x \rangle \rangle, \text{a,} \langle \lambda, z, \langle \text{big, car}, x \rangle \rangle, \text{er than,} \langle \lambda, x, \langle \lambda, y, \langle \text{Bill, has, y}, x \rangle \rangle, \text{a,} \langle \lambda, z, \langle \text{big, car}, y \rangle \rangle \rangle \]

(We assume that has is in category \( \langle 0, 1, 1 \rangle \) and a is in category \( \langle 0, 0, 1 \rangle \) with the semantics attributed to it in Cresswell (1973, p. 136). It occurs here between the arguments:

(x) \[ \langle \langle \lambda, x, \langle \lambda, y, \langle \text{George, climbed, y}, x \rangle \rangle, \text{a,} \langle \lambda, y, \langle \text{high, mountain}, x \rangle \rangle, \text{er than,} \langle \lambda, x, \langle \lambda, y, \langle \text{Everest, (high, mountain), x} \rangle \rangle \rangle \]

(We take climbed as a complete unit because tense is not our present concern.) In these representations er than is the main operator, and if we regard it as a conjunction, the analysis we propose is one of those Smith rejects (p. 353f.). However, her objection is in terms of the simplicity of the deletion and order changes, and it is not clear to me whether it still applies when the basic structure is in a 3-categorical language. Some other examples of this kind are given in Doherty and Schwartz (1967, p. 908).
among degrees of cleverness, and the semantics for ever than provides for no way of comparing the two.\footnote{See Vennemann (1973, p. 8).} Our underlying semantic concept for tall was in fact “x-much tall.” But of course we have sentences like:

(24) **Bill is a tall man.**
What seems to be meant in a case like this is something like:

(25) **Bill is taller than the average man.**
I propose to treat the ‘tall’ in (24) as composed of two symbols, pos and tall. tall satisfies (2.2), while pos conveys the “ever than the average—.”

pos (for positive, as opposed to comparative) is in category \(\langle\langle 0, 1\rangle, \langle 0, 1\rangle\rangle\), \(\langle\langle 0, 1\rangle, \langle 0, 1\rangle\rangle\). This looks very complicated, but all it means is that pos makes an ordinary noun modifier, say, \(\langle\text{pos}, \text{tall}\rangle\), out of a modifier like tall.\footnote{The comparison scales mentioned in the text have all been independent of the noun being modified. The following examples show that this need not always be so:}

\[
\begin{align*}
\langle\langle \text{pos}, \text{tall}\rangle, \text{man}\rangle \\
\langle\langle \text{pos}, \text{tall}\rangle, \text{man}\rangle \\
\end{align*}
\]

(26) In this case, if we let \(V(\text{tall}) = \zeta\), \(V(\text{man}) = \omega\), and \(V(\text{pos}) = \eta\), then \(V^+(26) = \eta(\zeta(\omega)(a))\), (2.2), (2.3), and (2.5) give us that \(w \in \eta(\zeta(\omega)(a))\) iff \(a\) is a man in \(w\) and there is exactly one \(b\) such that \(b = \langle u, \rangle\) and > is the greater than relation over spatial distances, and \(u\) is the distance between \(a\)’s extremities in \(w\), and \(u\) is towards the top of the scale determined by > when restricted to those \(v\) such that for some \(c, w \in \zeta(\omega)(c, \langle v, \rangle\rangle)\) and \(w \in \omega(c)\).

(27) If we apply this to tallest spy, we have:

\[
\begin{align*}
V^+(\langle\langle \text{tall}, \text{spy}\rangle, \text{est}\rangle) \\
V^+(\langle\langle \text{tall}, \text{spy}\rangle, \text{est}\rangle) \\
\end{align*}
\]

The phrase tall end alludes simply to the metaphor by which we speak of \(u\) being “higher than” \(u\) iff \(u\) is the tallest among the group. There is a certain amount of vagueness about this metalinguistic statement, but that is all to the good because the definiendum is likewise vague. What the rules amount to in the case of (26) is that to be a tall man is, in effect, to be taller than the average man. This explains why (16) does not entail that tall man (Vennemann, 1973, p. 8f).

The superlative seems best treated by a symbol est in category \(\langle\langle 0, 1\rangle, \langle 0, 1\rangle\rangle\):

(26) \(V(\text{est})\) is the function \(\zeta\) such that for \(a \in \mathbb{D}_{\langle 0, 1, 1 \rangle}\) and \(a \in \mathbb{D}_{\langle 0, 1, 1 \rangle}\), \(w \in \zeta(\omega)(a)\) iff there is a unique \(b\) such that \(w \in \omega(a)\) and \(b = \langle u, \rangle\) and \(u \in \mathcal{F}(\rangle\rangle)\) and \(u\) is the top of the scale determined by > when restricted to those \(v\) such that for some \(c, w \in \zeta(\omega)(c, \langle v, \rangle\rangle)\) and \(w \in \omega(c)\).

(27) If we apply this to tallest spy, we have:

\[
\begin{align*}
V^+(\langle\langle \text{tall}, \text{spy}\rangle, \text{est}\rangle) \\
V^+(\langle\langle \text{tall}, \text{spy}\rangle, \text{est}\rangle) \\
\end{align*}
\]

(28) The tallest spy in Australasia is not in New Zealand.

The est would seem to modify the whole:

(29) **tall spy in Australasia**

Bresnan (1973, p. 277) notes certain other locations that seem to correspond to ever than. We can have:

(30) **as tall as**

(31) **so tall (that)**

(32) **too tall (to)**

(33) **that tall**

We shall give a semantics for (30) by taking as as a single symbol of category \(\langle 0, \langle 0, 1\rangle\rangle\). (It splits as it gets toward the surface.) Examples (31)–(33) are a little more complicated, involving, as they seem to, references to human purposes and other matters beyond the scope of this paper.

\[
\begin{align*}
V(\text{as}) = \text{the function } \zeta \text{ such that for } a \in \mathbb{D}_{\langle 0, 1\rangle}, \text{ and } w \in \mathbb{F}: w \in \zeta(\omega, a) \text{ iff there is an } a \\
\end{align*}
\]
and \( w \in \omega'(b) \); and for any such \( a \) and \( b \), \( a = \langle u_1, > \rangle \) and \( b = \langle u_2, > \rangle \) and either \( u_1 > u_2 \) or \( u_1 = u_2 \).

The point is that Bill is as tall as Tom iff he is either taller than Tom or the same height as Tom. The modifier exactly would require that \( u_1 = u_2 \) and would not allow \( u_1 > u_2 \).

Another distinction we must capture is that between:

(34) \( \text{Bill is taller than Arabella.} \)

and:

(35) \( \text{Bill is taller than six feet.} \)

We shall for the present treat six feet as a single symbol in category 1 and let \( I(\text{six feet}) \) be a particular degree, viz. \( \langle u, > \rangle \), where \( > \) is the relation ordering lengths and \( u \) is the length of six feet. Thus (35) is:

(36) \( \langle \lambda, x, \langle \text{Bill}, \langle \text{tall, man} \rangle, x \rangle \rangle, \text{er than, six feet} \rangle \)

and of course:

(37) \( \text{Bill is six feet tall.} \)

is even simpler, viz. (with the functor following its arguments):

(38) \( \langle \text{Bill, six feet, \langle tall, man \rangle} \rangle \)

If six feet is the name of \( \langle u, > \rangle \), then it cannot also be the name of \( \langle u, < \rangle \). This may provide an explanation for:

(39) * \( \text{Bill is six feet short.} \)

3. MASS NOUNS AND PLURALS

Our approach to mass nouns assumes that the underlying semantic primitive of a word like \text{water} is a two-place predicate with some such meaning as:

(40) \( x \text{ is an amount of water of } y \)

\(^{13}\) This is not quite right, since many mass nouns seem able to be measured in several different ways, e.g., by both weight and volume. The meaning would have to supply several scales, and either the linguistic or the pragmatic context would select one of them. Mass nouns can of course be modified by adjectives. The \( j \)-categorial expression underlying Parson's \text{blue styrofoam} (1970, p. 364) would be:

(i) \( \langle \lambda, x, \langle \text{styrofoam, } x \rangle \rangle, x, \epsilon \rangle \)

which would mean, roughly:

(ii) \( x \text{ is } \epsilon \text{-much styrofoam of } j \text{ degree of blueness} \)

Because there are two degrees involved, we can explain the ambiguity in:

(iii) \( \text{more blue styrofoam} \)

\text{water} is in category \( \langle 0, 1, 1 \rangle \), and its semantics is as follows:

(3.1) \( V(\text{water}) \) is the function \( \omega \) such that for \( a, b \in D_1 \) and \( w \in W \):

\( w \in \omega(a, b) \) iff \( b = \langle u, > \rangle \), where \( > \) is the greater than relation with respect to volumes, and \( u \in \mathcal{F}(>) \) and \( u \) is the volume of \( a \) in \( w \) and \( a \) is water in \( w \).

We want to analyze a sentence like:

(41) \( \text{More water ebbs than mud flows.} \)

(This example, like so many that are candidates for formalization, is slightly unnatural because we want the irrelevant parts, in this case the verb phrases, to be as short as possible. The use of \text{ebbs} and \text{flows} rather than longer, albeit more natural, expressions corresponds to the use of “cop-out triangles” in phrase markers.) We take (41) to be something like:

(42) \( \text{The degree of the totality of ebbing water is greater than the degree of the totality of flowing mud.} \)

We introduce a symbol \text{tot} to mean “the degree of the totality of.” \text{tot} is in category \( \langle \langle 0, 1 \rangle, \langle 0, 1 \rangle, \langle 0, 1 \rangle \rangle \), and:

(3.2) \( V(\text{tot}) \) is the function \( \zeta \) such that for \( \omega \in D_{(0, 1, 1)} \) and \( \omega' \in D_{(0, 1)} \) and \( w \in W \): \( w \in (\omega, \omega')(a) \) iff there is a \( b \) such that \( w \in \omega'(b) \) and \( w \in \omega(b, a) \), and \( a = \langle u, > \rangle \) (for some \( u \in \mathcal{F}(>) \), for some \( > \)), and for any \( c \) and \( d \) such that \( w \in \omega'(d) \) and \( w \in \omega(d, c) \), \( c = \langle v, > \rangle \) for some \( v \) and either \( u = v \) or \( u > v \).

What \( V(\text{tot}) \) means is that \( a \) is the degree of the greatest part of whatever satisfies both \( \omega \) and \( \omega' \) in \( w \). So if \( V(\text{water}) = \omega \) and \( V(\text{ebbs}) = \omega' \), then:

(43) \( V^*(\langle \text{tot, water, ebbs} \rangle) \)

is the \( a \) such that \( a = \langle u, > \rangle \), where \( > \) is the greater than relation with respect to volume, and any amount of ebbing water not equal to \( u \) is less in volume than \( u \) (which means that \( u \) is the volume of the totality of ebbing water).

We can now formalize (41) as:

(44) \( \langle \langle \text{tot, water, ebbs} \rangle, \text{er than, } \langle \text{tot, mud, flows} \rangle \rangle \)

We assume here that the \( > \) that will form part of the meaning of \text{mud} is the same \( > \) as the one that forms part of the meaning of \text{water}, viz., “greater than” ranging over volumes. This means that the \text{er than} of (44) is the very same symbol as the one introduced for adjectives, whose semantics is captured by (2.3).
We shall later examine in more detail just how the surface sentence in (41) might be obtained from (44). At present we observe that much will be brought in as a signal of a degree and er much will be realized as more. tot water and tot mud will be realized as water and mud, respectively. The common surface realization of the two distinct expressions, water and tot water, perhaps explains why Quine (1960, pp. 97–100) was led to treat mass terms as ambiguous, a course followed also by Cresswell (1973, p. 139f). 14

In the sentence:

(45) Much water ebbs.

much looks as though it is a determiner like the or a and, indeed, can be so treated if we are prepared to give it a semantics that divorces it from its other uses (cf. Wheeler, 1972, pp. 326–329). What seems more plausible is that we have here a case analogous to the positive in adjectives. pos has a null surface realization but means something like “more than average.” Here we use a symbol (again with null realization) with a somewhat analogous meaning, something like “a high degree of.” We shall use the symbol deg in category \( \langle 0, 0, 1 \rangle \).

(3.3) \( V(\text{deg}) \) is the function \( \zeta \) such that for \( \omega \in D_{0, 1} \) and \( \omega' \in D_{0, 1} \) and \( w \in W \), \( w \in \zeta(\omega, \omega') \) as follows: Suppose that there is a unique \( a \) such that; \( w \in \omega'(a) \) and there is some \( b \) such that \( w \in \omega(a, b) \), and \( b = \langle u, > \rangle \), and for any \( c \) and \( d \) such that \( w \in \omega(c) \) and \( w \in \omega(c, d) \), where \( d = \langle v, > \rangle, u > v. \) In such a case \( w \in \zeta(\omega, \omega') \) iff \( u \) is considerably toward the top of \( v(d) \) when restricted to \( \{ u : \zeta(\omega, \omega')(d) \} \).

14 In addition, mass nouns have been discussed by Moravcsik (1973), Parsons (1970), Pelleier (1974), and by other authors. The only fact about them that I want to insist on is that they are subject to “more” and “less” and that their underlying semantics involves the notion of a degree. The following few points will, I hope, make clear my attitudes toward some things these and other authors have said.

1. The same surface word can have both a mass and a count sense. It seems to me that what we have here are distinct symbols, though with a very close semantics.
2. The mass/count distinction has no connection with the abstract/concrete distinction. Thus we can have more water, more virtue, a man, a mistake.
3. I do not take a logigrammatical analysis of the syntax and semantics of mass terms to require an analysis of, e.g., what it is to be a volume of water. (Obviously I do not think such an analysis philosophically unimportant, only unnecessary for my present task.) In particular, I have made no statement on whether there can be a volume of water that is not an individual something or collection of individual somethings.

What all this means is that the sentence:

(46) \( \langle \text{deg}, \text{water, ebbs} \rangle \)

will be true iff the total volume of ebbing water is considerably toward the top end of the greater than scale when this scale is restricted to volumes of water in \( w. \) Although deg has a null realization, the surface degree marker much frequently masquerades, as in (45), as the realization of deg. [Much does not appear in front of adjectives, vide (Bresnan, 1973, p. 278)]. We can, however, say simply:

(47) Water ebbs.

There is in English a species of construction that forms a (complex) count noun out of a mass noun. This construction can be illustrated by the phrase:

(48) drop of water 15

Drop will be a count noun (and so is in category \( \langle 0, 1 \rangle \)):

(3.4) \( V(\text{drop}) \) is the function \( \omega \) such that for any \( a \in D_1 \) and \( w \in W \), \( w \in \omega(a) \) iff \( a \) is in the form of a drop in \( w. \)

Prepositions are difficult words for a \( \lambda \)-categorial language. Perhaps they should not be in the \( \lambda \)-deep structure at all. If they are, we seem to need many different ones with an identical surface realization. This particular of will be in category \( \langle 0, 1 \rangle, \langle 0, 1 \rangle, \langle 0, 1, 1 \rangle \):

(3.5) \( V(\text{of}) \) is the function \( \zeta \) such that where \( \omega \in D_{0, 1} \) and \( \omega' \in D_{0, 1} \) and \( a \in D_1 \) and \( w \in W \), \( w \in \zeta(\omega, \omega')(a) \) iff there is some \( b \) such that \( w \in \omega'(a, b) \) and \( w \in \omega(a). \)

This means that \( \langle \text{drop of, water} \rangle \) has the following semantics: \( w \in V^+(\langle \text{drop of, water} \rangle)(a) \) iff \( w \in \zeta(\omega, \omega')(a), \) where \( V(\text{of}) = \zeta, V(\text{drop}) = \omega \) and \( V(\text{water}) = \omega'. \) By (3.1), (3.4), and (3.5), this will be so iff \( a \) is in the form of a drop in \( w \) and there is some degree \( b \) such that \( b = \langle u, > \rangle \) and \( u \) is a volume and \( a \) is water in \( w \) with volume \( u. \)

We now want to show that pluralization has a semantic effect that makes plurals behave like mass nouns. In particular, the semantic effect of pluralization is to turn a common count noun like man into a two-place predicate that means something like:

(49) \( x \) is a \( \gamma \)-membered set of men

15 This point was brought to my attention by G. D. Kennedy, who tells me that John Robert Ross called something like these “measure phrases” in a lecture at UCLA.
This predicate involves a comparison scale that consists of the positive integers under their standard ordering. The collectivizing use of the plural enables us to unify a large number of apparently semantically disparate phenomena in a way that accords neatly with their syntactically similar behavior. Consider:

(50) Arabella is more beautiful than Tom is clever.
(51) More water ebbs than mud flows.
(52) More men walk than birds fly.

It is my belief that in all these sentences more has a single meaning, and that the clue to understanding how it works in (52) lies in the following semantics for pluralization of count nouns:

pl in category \( \langle 0, 1, 1 \rangle, \langle 0, 1 \rangle \) and has the following semantics:

(3.6) \( V(\text{pl}) \) is the function \( \zeta \) such that, for \( \omega \in D_{\langle 0, 1 \rangle}, a, b \in D_{1}, \) and \( w \in W: \mu \in \zeta(\omega)(a, b) \) iff \( a = \{ c : w \in \omega(c) \} \) (i.e., \( a \) is the set of all \( c \) such that \( w \in \omega(c) \)), and \( b = \langle u \rangle \), where \( \langle u \rangle \) is the standard greater than ordering of the natural numbers and \( u \) is the number of members of \( a \).

This means that \( \langle \text{pl}, \text{man} \rangle \) indeed means something like (49).17

16 Much of what follows replaces the analysis offered by Cresswell (1973, pp. 161–165). It obviously has links with the observations in J. McCawley (1968, pp. 140–155, 161–169), though I have nothing to say here on the “respectively” phenomenon. There are also many similarities with the analysis of plurals offered in Bartsch (1973).

17 Notice that the analysis of a drop of water given earlier will carry over to a group of men. Since we have to account anyway for sentences like:

(i) The team walked to Cleveland.

we must suppose that verbs can predicate walking of sets. The plural ending in verbs seems to be sensitive only to pl; thus mass nouns and singular count nouns appear on the surface to be more similar than mass nouns and plurals. Probably the verb is sensitive to pl because it signals a set whose members can be counted. Where the subject term denotes a set but is not made up using the pl morpheme, English seems to be unable to make up its mind, and there is a wide variety of dialectal variation. Thus we can get both:

(ii) the team walk

and:

(iii) the team walks

[Some discussion of this occurs in (Perlmutter 1972).]

Certain mass nouns such as furniture also seem to denote sets. Furniture takes the singular because pl is not involved in its underlying representation. But its semantic equivalent pieces of furniture takes the plural because pl is involved in pieces.

The plural can appear on the surface without a determiner. It is not that, as suggested in Cresswell, (1973, p. 191), pl is itself a determiner; it is, rather, that the surface sentence Men run has as its \( \lambda \)-deep structure:

(53) \( \langle \text{deg}, \langle \text{pl}, \text{man} \rangle, \text{run} \rangle \)

Example (53) will (by the semantics for deg) be true in \( w \) iff the total number of running men in \( w \) is considerably toward the top end of the “numerically greater than” scale when restricted to the class of men in \( w \). Where much is allowed to appear, it will be realized, in the presence of pl, as many.

Numerical comparisons work in the following way. Example (52) is:

(54) \( \langle \langle \text{tot}, \langle \text{pl}, \text{man} \rangle, \text{walk} \rangle, \text{er than}, \langle \text{tot}, \langle \text{pl}, \text{bird} \rangle, \text{fly} \rangle \rangle \)

Literally:

(55) The degree of the totality of walking men is greater than the degree of the totality of flying birds.

The collectivizing function of the plural can explain the difference between:

(56) All men walk.

and:

(57) Every man walks.

and between:

(58) Some men walk.

and:

(59) A man walks.

It has usually been supposed that the plural is without semantic effect in sentences like this. In fact I have taken this view myself (Cresswell, 1973, p. 191). Some of the syntactic differences have been noticed by J. McCawley (1968) and others, but they have not been given a possible-worlds analysis.

The structure that underlies (56) would be something like:

(60) \( \langle \text{all}, \langle \text{pl}, \text{man} \rangle, \text{walk} \rangle \)

All would be in category \( \langle 0, 0, 1, 1 \rangle, \langle 0, 1 \rangle \rangle \) and would have the following semantics:

(3.7) \( V(\text{all}) \) is the function \( \zeta \) such that if \( \omega \in D_{\langle 0, 1 \rangle}, \) and \( \omega' \in D_{\langle 0, 1 \rangle}, \) then for any \( w \in W: w \in \zeta(\omega, \omega') \) iff there is an \( a \) and a \( b \) such that \( w \in \omega(a, b) \), and for any such \( a, w \in \omega'(a) \).
This semantics makes no reference to pluralization, and all, as so defined, will work equally well for mass nouns; thus we can form:

(61) \langle all, water, ebbs \rangle

This is why all is different from every. Every in Cresswell (1973, p. 136) is in category \langle 0, \langle 0, 1 \rangle, \langle 0, 1 \rangle \rangle with the following semantics:

(3.8) \( V(\text{every}) \) is the function \( \zeta \) such that for \( \omega \) and \( \omega' \in D_{(0,1)} \):

\[ w \in \zeta(\omega, \omega') \text{ iff for every } a \text{ such that } w \in \omega(a), w \in \omega'(a). \]

This shows why (56) and (57) are synonymous, though their meanings are obtained differently. A similar distinction obtains between (58) and (59).\(^{18}\)

4. THE METAPHYSICAL REALITY OF DEGREES OF COMPARISON

The key semantic notion of our logicolinguistic analysis is the concept of a degree of comparison, in terms of a pair \( \langle u, > \rangle \), where > is a relation and \( u \in F(> ) \). The kinds of degrees that have cropped up in our examples have been things like spatial distances, volumes, etc. In other words, we have chosen relatively unproblematic ones. But what if we say:

(62) Arabella is more beautiful than Clarissa.

(63) Bill is more manly than Tom.

\(^{18}\) This explains the deviance of:

(i) *Some man runs.

except in logician’s English. In Cresswell (1973, p. 191) it was assumed that some with the plural meant “at least two.” That now seems to me to be wrong. As far as I can see, some men run simply means “at least one man run.” The beauty of the semantics presented in this study is that it allows one to see how this can be so. This semantics ought also to be able to deal naturally with sentences like:

(ii) The apostles are twelve.

and the notorious example (Russell’s, I believe) of an invalid inference:

(iii) Men are numerous.

Socrates is a man.

Socrates is numerus.

It may, however, be that (3.7) as it stands needs amending to take care of the “collective” flavor of all.

Must we postulate the kalon as a degree of beauty or the andron as a degree of manliness? Degrees of beauty may be all right for the purposes of illustration but may seem objectionable in hard-core metaphysics. The first task of this section is to show that the possible-worlds-cum-set-theoretical framework on which our whole semantics is based is rich enough to support all the entities we need for degrees of comparison.

We take as basic data that we can and do make comparisons, i.e., that comparative sentences can be true or false; further, that we have the ability to make counterfactual comparisons. That is to say, our competence in this area is not limited to judgments involving how things actually are but can encompass judgments about how things might be. This means that we can make other-world and even trans-world comparisons.

It is not, in my opinion, the business of logic or linguistics (at least syntax) to explain how it is that we make the comparisons we do make or what the principles are by which we make them. But it is the business of linguistics, and of logic when used in the service of linguistics, to tell us how we put the comparisons we make into the linguistic forms into which we put them.

Consider adjectival comparisons. These can be reduced to sentences of the form:

(64) \( x \text{ is more } F \text{ than } y \text{ is } G \)

This schema generates a relation \( \phi \) between two things. In possible-worlds semantics the things that are compared are in particular possible worlds. E.g., \( x \) may be more beautiful than \( y \) in \( w \) but not in \( w' \); or \( x \) in \( w \) may be more beautiful than \( y \) in \( w' \). So, strictly, the terms of the relation should be pairs consisting of a thing and a world. Let \( a, b, c, \text{ etc.} \) denote such pairs, \( \langle a, e \rangle \) and let \( \phi \) denote the relation that corresponds to some particular comparison. \( \phi \) determines the following equivalence relation:

(4.1) \( a \approx b \text{ iff for all } c, \phi(a, c) \text{ iff } \phi(b, c) \text{ and } \phi(c, a) \text{ iff } \phi(c, b). \)

Here \( \approx \) is obviously an equivalence relation. We can now define the relation \( > \) between degrees based on \( \phi, F(>) \) is the set of all the \( \phi \)-equivalence classes. I.e., \( u \in F(>) \text{ iff, for some } a \in F(\phi), u = \{ b : b \approx a \}. \) In this case we can refer to \( u \) as \( \bar{a}. \)

(4.2) \( \bar{a} > \bar{b} \text{ iff } \phi(a, b) \)

Definition (4.2) is a consistent definition. For suppose that \( a \approx c \) and \( b \approx c \) and \( b \approx d \). Since \( \bar{a} = \bar{c} \) and \( \bar{b} = \bar{d} \), we must show that \( \phi(a, b) \text{ iff } \phi(c, d) \): Suppose that \( \phi(a, b) \). Then since \( a \approx c, \phi(c, b), \) and since \( b \approx d, \phi(c, d) \). Thus if \( \phi(a, b) \), then \( \phi(c, d) \). The converse is proved analogously.
What all this shows is that to the extent to which we do make comparisons, i.e., to the extent to which there is a body of data to be accounted for, then to that extent the formal theory can be accounted for in the manner of this study.

We have already noted how the method will also account correctly for the semantic deviance of many grammatically well-formed comparative sentences, e.g.:

(65)  *Bill is taller than Arabella is beautiful.

On the account given in the present paper this sentence is anomalous because Bill's tallness will be represented on a different scale from Arabella's beauty. We may, if we choose, classify comparison scales using feature theory, as Hale (1970) does, but we must be clear that this is not, strictly, involved in the syntax of comparative sentences.

We do occasionally make comparisons of a prima facie anomalous sort. We sometimes say things like:

(66)  I am older than you are wise.

What I would like to suggest here is that in the particular context of this sentence the speaker is using these words as if their meanings produced a scale common to age and wisdom. Exactly how this should be represented I do not know, but I am sure that its difficulties arise because the words are not being used literally, and that the difficulties are not solely ones that are tied to the idea of comparison. In some cases what is involved seems to be a sort of order-preserving mapping from one scale to another. This mapping represents the "as if."

A different case arises when the same word alludes to different scales when it modifies different nouns (Hale, 1970, p. 38). Thus although one can say:

(67)  The meeting was long.

and:

(68)  The road is long.

One cannot say:

(69)  *The meeting was longer than the road.

And the reason is that we have in the one case a time scale and in the other a spatial scale. The rule for long would be:

This explains the semantic anomaly of (69).

Something now on measurability. It seems clear that we can make comparisons without having units of measurement. I hope it is equally clear that nothing in the analysis so far presented has implied that degrees of comparison necessitate units of measurement. A careful look at the metaphysical analysis given at the beginning of this section should be enough to show this. Nevertheless our examples have been taken mainly from cases (the spatial and temporal distances) in which there are units of measurement. Or, rather, they have been taken from relations on which units of measurement can be imposed, whether or not the language has chosen to do so. Where a relation > has a natural metric,19 say d, we can define a new relation > d ⊆ F(>) × F(>) such that ⟨u1, u2⟩ > d ⟨v1, v2⟩ iff d(u1, u2) is greater than d(v1, v2) by the standard ordering of the real numbers. It is not at all clear how to specify precisely when a relation has a natural metric and just what the metric is, but in many cases we can do so. We can say, e.g., that x is twice as tall as y or twice as long, though when it comes to "twice as beautiful" things are less clear.

The analysis of comparatives and quantities presented in this study has been set in a framework based on possible worlds. This framework has been adopted by Montague, Lewis, Stalnaker, Thomason, myself, and others for the purposes of formal semantics, for what we believe to be adequate reasons. Many problems in formal semantics, however, can be solved independently of the need for possible worlds, and this applies to most, perhaps all, of the analysis so far in this work. Analyses of comparatives have been given by Wallace (1972) and Wheeler (1972). Much of what they have written has affinities with what has been said here, although they have not attempted to tackle the range of linguistic phenomena I have discussed.

One of the strongest reasons for using a possible-worlds approach is so that combinations that involve modality can be accommodated. A typical

19 A metric on a set X (Mansfield, 1973, p. 82) is a function from X × X into the real numbers satisfying certain conditions that ensure that d(x, y) for x and y ∈ X can be interpreted as the distance between x and y. We need also to ensure that d "respects" <, e.g., where u1 < u2, and u2 < u3, d(u1, u2) is not greater than d(u1, u3).
example is a sentence like:

\[(70) \quad \text{If Bill had been a smoker he would be shorter than he is.}\]

For simplicity I shall adopt the Stalnaker analysis of this conditional\(^{20}\) and assume that \(x \square \rightarrow \beta\) is to be true in a world \(w\) iff in the nearest world to \(w\) in which \(x\) is true \(\beta\) is true also. Accepting this, we can represent (70) as:

\[(71) \quad \langle \langle \lambda, x, \langle \langle \text{Bill, smokes}, \square \rightarrow, \langle \text{Bill, \langle \text{short, man}, x\rangle}, \text{er than}, \langle \lambda, x, \langle \text{Bill, \langle \text{short, man}, x\rangle}, x\rangle \rangle \rangle \text{21}\]

By principles that should by now be clear:

\[(72) \quad V^+(\langle \lambda, x, \langle \text{Bill, \langle \text{short, man}, x\rangle}, x\rangle)\]

is the function \(\omega \in D_{(0,1)}\) such that for \(a \in D\), and \(w \in W\), \(w \in \omega(a)\) iff Bill is a man in \(w\) and \(a = \langle u, <\rangle\), where \(<\) is the "less than" relation for spatial distances and \(u\) is Bill's height in \(w\).

The left term is a little more complex and brings out how other worlds come into the picture:

\[(73) \quad V^+(\langle \langle \lambda, x, \langle \langle \text{Bill, smokes}, \square \rightarrow, \langle \text{Bill, \langle \text{short, man}, x\rangle}, x\rangle \rangle \rangle)

is the function \(\omega' \in D_{(0,1)}\) such that for any \(b \in D\), and \(w \in W\): \(w \in \omega'(b)\) if in the nearest world \(w'\) to \(w\) such that \(w' \in V^+(\langle \langle \text{Bill, smokes}\rangle))\) (and this of course means the nearest world to \(w\) in which Bill smokes), \(w' \in \omega(a)\) (where \(\omega\) is as in (72)), and this means that in \(w'\) Bill is a man and \(b = \langle v, <\rangle\) and \(v\) is Bill's height in \(w'\).

When we combine these by (2,3), we find that \(w \in V^+(71)\) iff, where \(a = \langle u, <\rangle\) and \(b = \langle v, <\rangle\), \(v < u\).

But \(u\) is Bill's height in \(w\), while \(v\) is Bill's height in the nearest world to \(w\) in which he smokes. So (71) will be true in \(w\) iff in the nearest world to \(w\) in

\[\text{which Bill smokes his height is less than his height in } w\]—which is precisely what (70) says (assuming that \(\square \rightarrow\) does capture the conditional properly).

5. The Syntax of Comparatives

So far the analysis has been entirely semantic, and indeed semantics is the burden of my song; but a little needs to be said about how the \(\lambda\)-deep structures are related to the ordinary surface sentences. There are two ways one can go here, and I shall say a little about each. One is to attach the \(\lambda\)-deep structures to the bottom end of a syntactic study, and as an example of this I shall look at how the formulas I have presented might hook onto the underlying representations in Bresnan (1973). The other approach is to proceed, as did Cresswell (1973), directly to the surface with a formula that becomes a sentence of English when certain parts are knocked out. The first approach has the advantage of showing how possible-worlds semantics can fit in with transformational grammar; the second has the advantage of indicating the likelihood that some principles can be found that generate the acceptable structures, without any commitment to the form these principles might take.

The kind of underlying structure Bresnan proposes can be exhibited by the following tree (it is actually made up by using (245) and (251) of Bresnan (1973, p. 318f.)):
My \( \lambda \)-deep structure when represented as a tree would have to be something like:

\[\text{(I trust that the connection between (5.2) and (1.1)–(1.3) is reasonably clear.)}\]

I shall not attempt a detailed formulation of a set of transformations that get us from (5.2) to (5.1) but shall simply say enough to make their possibility plausible. We need principles of two kinds. One set of principles associates the labels of nodes in (5.2) with those in (5.1). Some are obvious, e.g., \( E_0 \) corresponds to S. The other principles move elements and insert elements. If we look at the “shapes” of (5.1) and (5.2), and forget about the labels, we see that certain things have to happen.

1. \textbf{much} should be inserted immediately after each \( x \) with something like the rule:

\[X_1 \quad \Rightarrow \quad QP\]

(Suitable restrictions must of course be placed on the environment within which this rule applies.)

2. The subtrees:

\[\lambda \ x \ y \ x \]

must be deleted and the tree “pruned” as a result.

3. In the subtrees in (5.2) ending in [my father tall man \( x \)] and [my mother tall woman \( x \)], the phrases [my father] and [my mother] must be raised to become the subjects of their respective clauses.\(^{22}\)

4. In some manner the predicates [tall man \( x \) much] and [tall woman \( x \) much] must be turned into [is \( x \) much tall a man] and [is \( x \) much tall a woman]. I shall make any suggestions about this, since it raises the whole question of the logical priority of the attributive or predicative use of adjectives.\(^{23}\)

5. \textbf{er} must move to replace the \( x \) in the leftmost \( x \) much.

6. The phrase my father must be raised so that it becomes the subject of the whole sentence.

As far as I am aware, all these operations are legitimate kinds of transformations, though I have no detailed syntactic arguments for them. If one desired to maintain Bresnan’s underlying structures as the deep syntactical level, one would have to postulate a global derivational constraint (see Lakoff, 1971, p. 234) to say that all derivations of comparatives must pass through that level.

The other way of arguing for the validity of a semantic analysis is this: If we can give apparently plausible possible-world semantics for symbols of a \( \lambda \)-categorial language and can then show that a large range of English sentences can be represented as what remains when \( \lambda \)’s and variables are deleted and when certain combinations (e.g., \textbf{er much}) are realized as single words (e.g., more),\(^{24}\) and if this can be done in such a way that the truth conditions of the \( \lambda \)-categorial sentence correspond with the truth conditions of the original surface sentence, then this is evidence for the correctness of our analysis. It is hard to compare an argument of this kind with the arguments one usually finds in linguistics, because even linguists who advocate a semantic base in formal logic seem to have little to offer on the nature of the entities they are postulating. Notice that a base language that uses possible-worlds semantics can be quite close to the surface because the

---

\(^{22}\) In (5.2) my father is treated as a name, in category 1. It should probably be analyzed as a nominal. And in any case indefinite phrases like someone or many lovers of wisdom will have to be nominals. In such a case “quantifier lowering” rules of the kind discussed by Lakoff (1971, p. 238f) may have to be brought into play.

\(^{23}\) Since I adopt an analysis of adjectives that takes their attributive use as prior to their predicative use, the way I would derive the former phrases from the latter would be a little different from the way they would be derived by someone who thinks the predicative use is prior (as I suspect Bresnan does). It might be that the structure that underlies (5.1) is not (5.2) but something more like (74) or (75). Such a structure would be equivalent to (5.2) by \( \lambda \)-conversion, given appropriate rules for is and a (Cresswell, 1973, pp. 136, 182).

\(^{24}\) In Cresswell (1973, p. 210) this is done by what is called an “amalgamation function.”
detailed semantic analysis can be expressed in a metalinguistic statement of the meaning of the symbol.

What I shall do is turn (16), (41), and (52) into formulas that become English sentences when certain changes are made. This section is intended primarily for readers of Cresswell (1973), and the examples all make use of \( \lambda \)-conversion. As noted briefly there (p. 224), principles of \( \lambda \)-conversion look very like principles that underlie many transformations.\(^{25}\)

In these sentences we shall split \textit{er} from \textit{than} up into its two parts and suppose that \textit{than} is the operator that carries the semantic weight. \textit{Than} is to be in category \( \langle 0, \langle 0, 1 \rangle, \langle 0, 1 \rangle \rangle \), whose semantics satisfies (2.3), while \textit{er} is a symbol whose function is simply to mark one of the terms of the comparative. We shall also have \textit{much} sometimes occurring in the same role.

\[
\langle\langle \lambda, x_1, \langle \text{Bill, is, } x_1 \rangle \rangle, \langle\lambda, x_{\langle 0, 1 \rangle}, \langle\langle \lambda, y_1, \langle \lambda, z_1, \langle a, \langle \lambda, y_1, \\
\langle\langle \langle \text{tall, er}, \text{man} \rangle, y_1, z_1 \rangle, r_{\langle 0, 1 \rangle} \rangle \rangle, \langle\lambda, x_{\langle 0, 1 \rangle}, r_{\langle 0, 1 \rangle}, \langle\langle x_{\langle 0, 1 \rangle}, r_{\langle 0, 1 \rangle}, \text{Tom} \rangle \rangle \rangle \rangle
\]

In this example \textit{er} is of category \( \langle 0, 1, 1 \rangle, \langle 0, 1, 1 \rangle \rangle \). It is in fact an identity operator in that category, in that its value is a function \( \xi \) such that \( \xi(\omega) = \omega \). (We have to say "an" identity operator, since it will be a partial function and there could be different ones simply because they had different domains.)

Principles of \( \lambda \)-conversion make this equivalent to:

\[
\langle\langle \lambda, y_1, \langle a, \langle \lambda, x_1, \langle\langle \text{tall, er}, \text{man} \rangle, x_1, y_1 \rangle \rangle, \langle\lambda, x_1, \langle\text{Bill, is, } x_1 \rangle \rangle \rangle, \text{than} \langle\langle \lambda, y_1, \langle\langle \text{tall, er}, \text{man} \rangle, \text{Tom}, y_1 \rangle \rangle \rangle
\]

Our second example involves mass nouns (\textit{er} is, as before, in category \( \langle 0, 1, 1 \rangle, \langle 0, 1, 1 \rangle \rangle \), and so is \textit{much}; \textit{much} is also an identity function):

\[
\langle\langle \text{tot}, \langle\text{much, er, water} \rangle, \text{ebbs}, \text{than}, \langle\text{tot, mud, flows} \rangle \rangle
\]

\textit{Tot} has a null realization, and \textit{much er} is realized as 'more'.

Our third and final example concerns pluralization:

\[
\langle\langle \text{tot}, \langle\text{much, er, pl, man} \rangle, \text{walk}, \text{than}, \langle\text{tot, pl, bird} \rangle \rangle, \text{fly} \rangle \rangle
\]

Since \textit{tot} has a null realization, with \textit{much er} as \textit{more} and \textit{pl man} as \textit{men} and \textit{pl bird} as 'birds' we get the right result.

The task of obtaining these forms for other cases will be tedious but possible. The examples will, I hope, illustrate both those claims.

\(^{25}\) The principles of \( \lambda \)-conversion allow us to identify (i.e., delete repetitions of) symbols and permute (i.e., change the order of) symbols. Indeed they look very like the operations licensed by the elementary transformations. (See Kimball, 1973, pp. 41–43.)

6. ODDS AND ENDS

This section makes a few relatively unsystematic remarks on some of the topics that we have not adequately dealt with.

The first topic is adverbs. Since I regard the attributive use of adjectives as primary, it ought to be clear that most of what has been said will carry over to adverbs. The reason I have not tackled them explicitly is that I am uncertain about their precise semantic analysis. This uncertainty is reflected in the analysis of adverbs that I have presented in Cresswell (1974) (see also Thomason and Stalnaker, 1968), but it is independent of the analysis of degree and quantity adopted in this paper. One use of adverbs is, however, important, and that is adverbs used to modify adjectives, e.g., those called intensifiers.\(^{26}\)

Take considerably. The semantics of considerably should entail that a phrase like \( \langle\langle \text{considerably, tall}, \text{man} \rangle \rangle \) means a man whose degree of tallness is considerably toward the top end of the degrees-of-tallness scale when this scale is restricted to tallnesses of men. This semantics can be formalized without too great difficulty. The slight problem arises when we intensify comparatives.

For we can say:

\[
\langle\langle \text{Bill is considerably taller than Arabella} \rangle \rangle
\]

On the semantics given for \textit{er than}, it is not possible to have degrees of "taller-than-ness." In this particular case, given the relation > of \textit{taller than}, there is a very natural metric \( d \) that goes with it. Given this metric, we can define a new relation > among degrees of "taller-than-ness" in the way described in Section 4. But it is not clear to me what to do when there is no natural metric. I do not know whether to be worried by this because I am not sure whether we can compare comparatives in the absence of a natural metric.\(^{27}\)

Where we can compare comparatives we can frequently also put definite measurements on, as with:

\[
\langle\langle \text{Bill is a foot taller than Arabella} \rangle \rangle
\]

The second topic concerns cases where verb phrases look as though they

\(^{26}\) Wheeler (1972, pp. 325–330) has some interesting things to say about modifiers of adjectives. For example, he takes very tall man to mean "tall by comparison with the set of tall men" (tall man of course means "tall by comparison with the set of men", so that "very" is an instruction to iterate the attributive which follows it [p. 325]."

\(^{27}\) There is certainly no syntactic objection to the iteration of modifiers of this kind. Bresnan (1973, pp. 289–299) has quite a long section on it.
are subject to comparative constructions, as in:

(80)  Bill runs more than Arabella swims.

My inclination with these cases is to say that more here really means something like “more often” and is therefore a case of adverbial comparison. However, cases like weighs more (Moravcsik, 1973, p. 264) are not so easy, and I recognize that I have not treated these cases in detail.

Finally, a slight reservation about the view that much is a word without semantic effect at the λ-deep level whose sole function is to be a surface indication of the presence of a degree of comparison. It may be that this is overly simple. Consider, e.g., the difference between:

(81)  Drink this water.

and:

(82)  Drink this much water.

One possibility is to say that mass nouns are, after all, of category ⟨0, 1⟩ and do not have a degree of comparison built into them. Thus water would mean simply:

(83)  x is an amount of water

The function of the word much would then be to produce the complex expression:

(84)  ⟨much, water⟩

and it would be this expression that would mean:

(85)  x is an amount of water of degree y

This would explain why much is required before mass nouns but not before adjectives. For adjectives, on my account, already have a degree of comparison as part of their meaning. It might also explain why we can say things like:

(86)  Bill is this much of a man.\(^{28}\)


My main reason for hesitation at this point is that I find it difficult to give a clear semantics for much. What much has to do is to discover the appropriate degree of comparison for a predicate, given merely that we know

\(^{28}\) I think the of in much of a man is a different of from the of in a drop of water, but I am not sure what kind of semantics should be given for it.
CAPTURING THE RUSSIAN ADJECTIVE

In Russian there are two forms of qualitative adjective—the long form and the short form. The short form is used in predicate position only, while the long form may be used in predicate position or prenominally as an attributive adjective. The morphological process that relates the two forms is so transparently productive that nearly every qualitative adjective may be said to have both forms, although one or the other may rarely or never be used owing to semantic considerations. To form a short adjective from a long one, one drops the regular long adjectival endings—ýj (masculine), åja (feminine), öe (neuter), and ye (plural)—and adds, instead, no ending for the masculine (but sometimes an epenthetic stem vowel), å for the feminine, ö for the neuter, and ye for the plural:

1. **LONG FORM (LF)**                                **SHORT FORM (SF)**
   
   `Novyj`    `Nov`   `Nova`   `New`
   `Umnýj`    `Umen`   `Uma`   `Intelligent`
   `Trudoljubivyj` `Trudoljubiv` `Trudoljubiva` `Industrious`

Passive participles, which have the same morphological shape as long-form adjectives, also have short forms, which must always be used when the participle is in predicate position:

2. a. *Dver' byla zakrytaja (LF).*
    b. *Dver' byla zakryta (SF).*
      'The door was closed.'

Like the short-form adjectives, short-form passive participles may never be used attributively:

3. a. *Nei'zja perexodit' cherez zakrytju (LF) dver'*
    b. *Nei'zja perexodit' cherez zakryta (SF) dver'*
      'One must not go through a closed door.'

Regular adjectives behave differently from the participles in that long-form adjectives may appear in predicate position. In this study I shall explore